SLIP ESTIMATION USING THE UNSCENTED KALMAN FILTER FOR THE TRACKING CONTROL OF MOBILE ROBOTS

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Abstract. *Research on autonomous navigation for tracked mobile robots operating in unstructured environments has received renewed attention in the last years due to its increasing use in tasks as forestry, mining, agriculture, military applications and space exploration. For these tasks, the slip phenomena is an important factor that must be taken into account during the control design. If the slip is not considered, the control objective may not be completed and a stable system may even become unstable. Accurate estimation of the slip is essential for the implementation of efficient control strategies. This paper shows that the longitudinal slip of the tracks can be estimated from the robot pose and velocity using the unscented Kalman filter (UKF). A control strategy that uses the estimation of the slip is proposed to achieve the trajectory tracking objective. The control strategy is based on the kinematic and dynamic models which include the longitudinal slip of the left and right tracks as two unknown parameters. Numerical results show the performance of the proposed control strategy.*

Keywords: *tracked mobile robot, nonlinear control, UKF estimation, dynamic model, longitudinal slip.*

1. INTRODUCTION

Tracked mobile robots have been extensively investigated due to their capacity of operating in unstructured and even hazardous environments, where a high degree of autonomy is required. Locomotion based on tracks has a large ground contact patch that provides satisfactory stability and traction on various terrain conditions, making the tracked mobile robots useful in applications such as agriculture, forestry, mining, military, search and rescue, and space exploration (Nourbakhsh and Siegwart, 2004). However, all these tasks require an efficient solution to the robot navigation problem.

One of the main navigation problem, the trajectory tracking problem, consists in designing control inputs that stabilize the mobile robot in a trajectory generated by a reference model. In general, this is a difficult problem, since tracked mobile robots are typical examples of systems that has nonholonomic constraints (Kolmanovsky and McClamroch, 1995). An overview of the most recent tracking control methods for nonholonomic mobile robots can be seen in Morin and Samson (2006). It is in general difficult to control tracked robots during applications in unstructured environment due to slip phenomena, which is an important factor that must be taken into account during the control design (Fan *et al.*, 1995).

Many researches have addressed the slip phenomena in the navigation of mobile robots. Wang and Low (2008) give a general presentation on modeling of wheeled mobile robots in the presence of wheel skid and slip from the perspective of control design. Sidek and Sarkar (2008) provide a theoretical and systematic framework to include the slip into the overall system dynamics of wheeled mobile robots. Ward and Iagnemma (2008) propose a model-based approach to estimating longitudinal wheel slip and detecting immobilized conditions of autonomous mobile robots operating on outdoor terrain. Song *et al.* (2008) present a nonlinear sliding mode observer for the estimation of tracked-vehicle slip parameters based on the vehicle kinematic equations and sensor measurements. Gonzales *et al.* (2009) present the synthesis of a control law for a wheeled mobile robot under slip condition using an LMI-based approach. Michalek *et al.* (2009) proposes a nonlinear feed-forward loop that compensates for the skid-slip effects of a mobile robot. The paper Iossaqui *et al.* (2010b) presents an adaptive tracking control strategy, based on the robot kinematic model, that is able to compensate for the longitudinal slip, assuming that the left and right slip are the same. This result is extended in Iossaqui *et al.* (2010a) to account for the robot dynamics. Assuming that the left and the right slip can be different, Iossaqui *et al.* (2011b) provide an adaptive control law, based on the kinematic model, that achieves trajectory tracking. The paper Iossaqui *et al.* (2011a) shows that the kinematic controller, from Iossaqui *et al.* (2010b), and the dynamic controller, from Iossaqui *et al.* (2010a), are still able to provide satisfactory performance even if the states are estimated using the unscented Kalman filter and the extended Kalman filter. Others works that also consider the slip problem are found in Iagnemma and Ward (2009); Tarakameh *et al.* (2010).

This paper proposes a control strategy for the tracking control of a tracked mobile robot under slip condition that considers the dynamics of the robot. A slip parameter dependent control law is derived, however, instead of using an adaptive update rule as done in Iossaqui *et al.* (2010a), a nonlinear filter is used to estimate the slip parameters from the measured states (pose and velocity) of the robot. Similar idea is developed in Le *et al.* (1997), where an extended Kalman filter (EKF) is used to estimate the slip parameters. However, the estimated parameters are not used in the feedback control law. The paper Zhou *et al.* (2007) also uses the unscented Kalman filter (UKF) to estimate the slip, but the dynamics of the robot are not consider.

The paper is organized as follows. Section 2 presents the dynamic model of a tracked mobile robot. Section 3 introduces a control strategy for the tracked robot under slip condition. Section 4 presents the numerical results using the proposed control strategy. Section 5 presents the conclusions and the Appendix briefly presents the nonlinear UKF algorithm.

2. KINEMATIC AND DYNAMIC MODEL OF THE TRACKED MOBILE ROBOT

This section presents a simplified dynamic model of a tracked mobile robot with slip. The longitudinal slip of the left and right tracks are described by two unknown parameters. These parameters are incorporated on the kinematic equations. It is assumed that the robot will operate at low speed, since the lateral slip is zero during straight line motion and it can be neglected when the robot turns on the spot (Gonzales *et al.*, 2009). Moreover, the kinematic model of a tracked robot can be approximated by the one of a wheeled robot if slow motions are assumed (Martínez *et al.*, 2005).

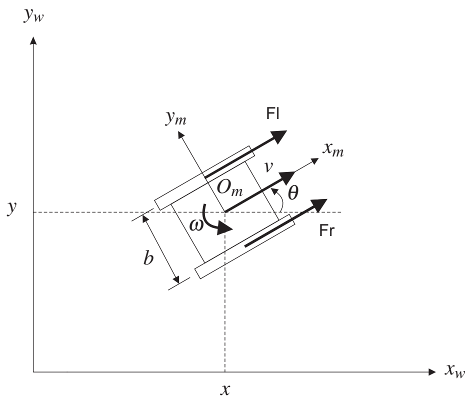


Figure 1. Geometric model of a tracked mobile robot.

Figure 1 shows the geometric model of a tracked mobile robot. It is assumed that the tracked robot is formed by a rigid body with two independent tracks. The motion of the robot is described by its position (x, y) and its orientation θ in an inertial coordinate frame $F_1(x_w, y_w)$. The robot position is given by the coordinate of its geometric center O_m, which is also the origin of the local coordinate frame $F_2(x_m, y_m)$. The robot orientation is given by the rotation of the frame F_2 in relation to the frame F_1. The distance between the two tracks is b and the equivalent radius of each track is r. Furthermore, the motion of the robot is composed of the translation velocity v and the rotational velocity $\omega = d\theta/dt$.

As presented in Zhou *et al.* (2007), the kinematic equation of the tracked robot with longitudinal slip is given by

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \frac{1}{2b} \begin{bmatrix} br(1-i_L)\cos\theta & br(1-i_R)\cos\theta \\ br(1-i_L)\sin\theta & br(1-i_R)\sin\theta \\ -2r(1-i_L) & 2r(1-i_R) \end{bmatrix} \begin{bmatrix} \omega_L \\ \omega_R \end{bmatrix} \quad \Leftrightarrow \quad \dot{q} = S(q)\xi \tag{1}$$

where $q = (x, y, \theta)^T$ is the robot configuration, $\xi = (\omega_L, \omega_R)^T$ is the vector composed of the angular velocities of the left and right tracks, i_L and i_R denote the longitudinal slip ratio of the left and right wheels, respectively, which are assumed unknown. As shown in Wong (2001), the unknown slip parameters can be defined as

$$i_L = \frac{(r\omega_L - v_L)}{r\omega_L} \quad \text{and} \quad i_R = \frac{(r\omega_R - v_R)}{r\omega_R}, \quad 0 \le i_L, i_R < 1$$

where v_L and v_R are respectively the linear velocities of the left and right wheels with relation to the terrain.

As presented in (Iossaqui *et al.*, 2010b), the dynamic model of the tracked robot is given by

$$\dot{q} = S(q)\xi \tag{2}$$
$$\overline{M}\dot{\xi} = \overline{B}(q)\tau \tag{3}$$

where q, $S(q)$ and ξ are defined in (1), $\tau = (\tau_L, \tau_R)^T$ is the input vector that represents the generalized forces on the left and right tracks, $\overline{M} = S^T(q)MS(q)$ and $\overline{B}(q) = S^T(q)B(q)$, with the matrices M and $B(q)$ given by

$$M = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I \end{bmatrix}, \quad B(q) = \begin{bmatrix} \cos\theta & \cos\theta \\ \sin\theta & \sin\theta \\ -b/2 & b/2 \end{bmatrix}$$

where m is the total mass of the robot and I is the moment of inertia about a vertical axis normal to the x_m-y_m plane through O_m.

3. CONTROL DESIGN USING THE UNSCENTED KALMAN FILTER

Although the kinematic model (Zhou *et al.*, 2007) may be suitable for certain control objectives, models that include dynamic effects are required for other purposes. An UKF-based controller for the dynamic model is presented in this section. The proposed method consists in converting the desired velocity input into a force control that take into account the dynamics of the system.

To apply the proposed control design, the dynamic model is represented in an integrator backstepping form (Fierro and Lewis, 1997). For this purpose, let u be an auxiliary control input, then by applying the following input force

$$\tau = \overline{B}(q)^{-1}\overline{M}u \tag{4}$$

we obtain the backstepping form

$$\dot{q} = S(q)\xi \tag{5}$$

$$\dot{\xi} = u \tag{6}$$

To design the control law u using the backstepping technique, it is necessary to specify a velocity input ξ. This input ξ will be generated as the solution of a kinematic tracking problem. Thereafter, the solution of the kinematic tracking problem will be denote as the desired velocity input $\xi_d = (\omega_{Ld}, \omega_{Rd})^T$. Before presenting the methodology to obtain the desired velocity ξ_d, we present the design of the auxiliary input u.

To obtain the input u that guarantee that the velocity ξ, applied to the system, follows the desired velocity ξ_d, we need to define the velocity error

$$e_d = \begin{bmatrix} e_4 \\ e_5 \end{bmatrix} = \xi - \xi_d$$

It is possible to shown (see Iossaqui *et al.* (2010a)) that the auxiliary input u, that guarantees that e_d converges to zero, is given by

$$u = \dot{\xi}_d - \begin{bmatrix} k_4 & 0 \\ 0 & k_5 \end{bmatrix} (\xi - \xi_d) \tag{7}$$

with k_4 and k_5 positive constants.

Now, to obtain the desired velocity ξ_d, consider an auxiliary velocity $\eta = (v, \omega)^T$, for the kinematic model, that is related to the desired velocity ξ_d according to the equation $\eta = T\xi_d$ given by

$$\begin{bmatrix} v \\ \omega \end{bmatrix} = \frac{r}{2b} \begin{bmatrix} b\omega_{Ld}(1-i_L) + b\omega_{Rd}(1-i_R) \\ -2\omega_{Ld}(1-i_L) + 2\omega_{Rd}(1-i_R) \end{bmatrix} = T \begin{bmatrix} \omega_{Ld} \\ \omega_{Rd} \end{bmatrix}$$

with

$$T = \frac{r}{2b} \begin{bmatrix} b(1-i_L) & b(1-i_R) \\ -2(1-i_L) & 2(1-i_R) \end{bmatrix}$$

The inverse relation $\xi_d = T^{-1}\eta$ is given by

$$\begin{bmatrix} \omega_{Ld} \\ \omega_{Rd} \end{bmatrix} = \frac{1}{2r} \begin{bmatrix} 2(1-i_L)^{-1} & -b(1-i_L)^{-1} \\ 2(1-i_R)^{-1} & b(1-i_R)^{-1} \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \tag{8}$$

Thus, the desired velocity $\xi_d = (\omega_{Ld}, \omega_{Rd})^T$ can always be obtained if an auxiliary velocity $\eta = (v, \omega)^T$ is provided. The auxiliary velocity η will be computed to solve a tracking problem for the kinematic model

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \iff \dot{q} = S_a(q)\eta \tag{9}$$

where $q = (x, y, \theta)^T$ is the robot configuration. The tracking problem is stated as follows: find a velocity $\eta = (v, \omega)^T$ such that

$$\lim_{t\to\infty}(q_r - q) = 0$$

where $q_r = (x_r, y_r, \theta_r)^T$ is a reference trajectory generated using the kinematic equation

$$\dot{q}_r = S_a(q_r)\eta_r$$

that is

$$\begin{bmatrix} \dot{x}_r \\ \dot{y}_r \\ \dot{\theta}_r \end{bmatrix} = \begin{bmatrix} \cos\theta_r & 0 \\ \sin\theta_r & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_r \\ \omega_r \end{bmatrix} \tag{10}$$

where $\eta_r = (v_r, \omega_r)^T$ is a given constant reference input, described by a linear velocity $v_r > 0$ and an angular velocity ω_r.

To address the kinematic tracking problem, we define the tracking error $e = (e_1, e_2, e_3)^T$ as

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x \\ y_r - y \\ \theta_r - \theta \end{bmatrix} \tag{11}$$

The dynamics of the error e, derived using (9), (10) and (11), is given by

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{bmatrix} = \begin{bmatrix} \omega e_2 + v_r \cos e_3 - v \\ -\omega e_1 + v_r \sin e_3 \\ \omega_r - \omega \end{bmatrix}$$

Neglecting the slip, it is shown in Kim and Oh (1998) that the velocity input

$$\begin{aligned} v &= v_r \cos e_3 - k_3 e_3 \omega + k_1 e_1 \\ \omega &= \omega_r + \frac{v_r}{2}\left[k_3\left(e_2 + k_3 e_3\right) + \frac{1}{k_2}\sin e_3\right], \quad k_i > 0 \end{aligned} \tag{12}$$

drives the error signal e to zero.

Now, if the parameters i_L and i_R that appear in (8) are unknown, we cannot directly calculate the desired velocity ξ_d and thus an estimation procedure is necessary. Instead of using the adaptive update rule presented in Iossaqui *et al.* (2011b), we use a nonlinear filtering algorithm to estimate the slip parameters i_L and i_R. Equation (8), considering now the estimate i_{Le} and i_{Re}, is given by

$$\begin{bmatrix} \omega_{Ld} \\ \omega_{Rd} \end{bmatrix} = \frac{1}{2r} \begin{bmatrix} 2(1-i_{Le})^{-1} & -b(1-i_{Le})^{-1} \\ 2(1-i_{Re})^{-1} & b(1-i_{Re})^{-1} \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \tag{13}$$

Note that the input force given by equation (4) also depends on the slip parameters i_L and i_R and thus needs to be reformulated in therms of the estimated values i_{Le} and i_{Re}.

The unscented Kalman filter (UKF) is now needed to jointly estimate the states and the slip parameters. Figure 2 shows the schematic representation of the proposed strategy. The numbering inside the blocks in Fig. 2 indicate the corresponding equation number. The UKF algorithm are given in the Appendix. The UKF acts like an observer by which the slip parameters i_L and i_R are recovered from the states (the pose and the velocity) of the robot $(x, y, \theta, \omega_L, \omega_R)^T$. To apply this filter, it is necessary to form an augmented state $(x, y, \theta, \omega_L, \omega_R, i_L, i_R)^T$. The estimate $(x_e, y_e, \omega_{Le}, \omega_{Re}, \theta_e, i_{Le}, i_{Re})^T$ is obtained from the measurements $(x_m, y_m, \theta_m, \omega_{Lm}, \omega_{Rm})^T$ that are corrupted by additive noises respectively denoted by δ_x, δ_y, δ_θ, δ_{ω_L} and δ_{ω_R}.

4. NUMERICAL RESULTS

This section presents the numerical results for the proposed tracking control algorithm. The numerical data for the robot, taken from Wang *et al.* (2007), are $b = 0.50$ m, $r = 0.25$ m, $m = 27.0$ kg and $I = 1.125$ kg.m^2. The control gains are heuristically selected as $k_1 = k_3 = 1$, $k_2 = 20$ and $k_4 = k_5 = 10$. In order to simplify the analysis, two basic reference trajectories are used. First, a linear trajectory generated by the reference velocity $v_r = 0.3$ m/s and $\omega_r = 0$ rad/s. Second, a circular trajectory generated by the reference velocity $v_r = 0.3$ m/s and $\omega_r = 0.1$ rad/s. The initial conditions are taken as $q_r(0) = (0,0,0)^T$. The total time for the computer simulation is $t = 200$ s. The slip parameter i_L and i_R are taken as

$$\begin{aligned} 0\text{s} \le t < 40\text{s}: &\quad i_L = 0.0 \quad \text{and} \quad i_R = 0.3 \\ 40\text{s} \le t < 50\text{s}: &\quad i_L = 0.0 \quad \text{and} \quad i_R = 0.0 \\ 50\text{s} \le t < 100\text{s}: &\quad i_L = 0.2 \quad \text{and} \quad i_R = 0.0 \\ 100\text{s} \le t < 120\text{s}: &\quad i_L = 0.0 \quad \text{and} \quad i_R = 0.0 \\ 120\text{s} \le t < 150\text{s}: &\quad i_L = 0.0 \quad \text{and} \quad i_R = 0.15 \\ 150\text{s} \le t < 200\text{s}: &\quad i_L = 0.0 \quad \text{and} \quad i_R = 0.0 \end{aligned}$$

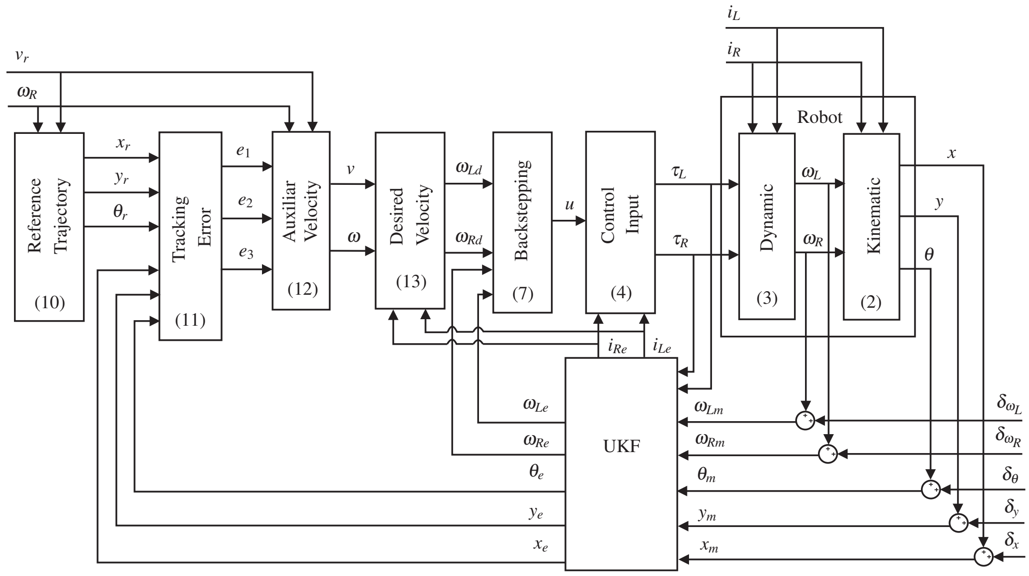


Figure 2. Schematic representation of the proposed control strategy.

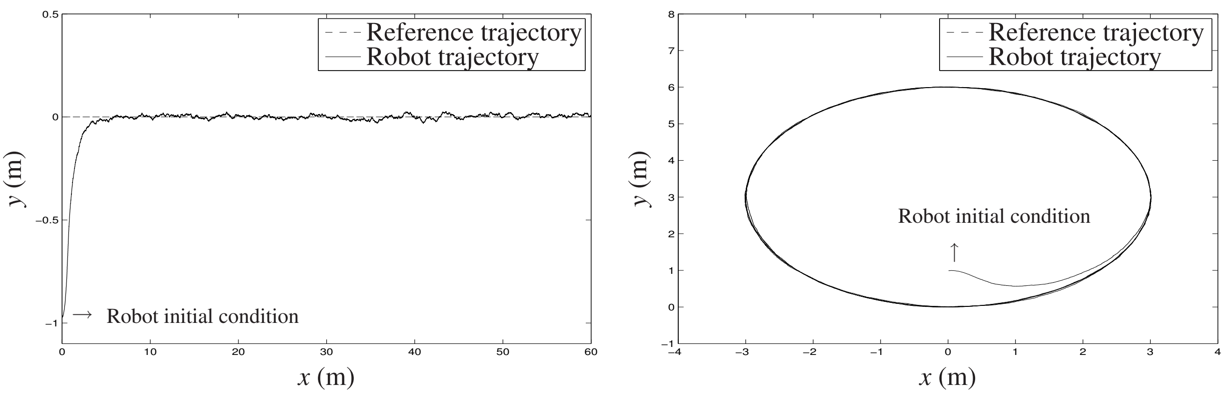
Note that the kinematics and the dynamics of the robot are described by the continuous-time equations (2) and (3). On the other hand, the UKF is a discrete-time algorithm. Thus, to perform the computer simulation, the continuous-time equations (2) and (3) are discretized using Euler's forward-difference scheme with a sampling period of $T_s = 0.01$ s.

The input for the UKF algorithm are the forces τ_L, τ_R and the measurements $(x_m, y_m, \theta_m, \omega_{Lm}, \omega_{Rm})^T$, which are corrupted by an additive noise $(\delta_x, \delta_y, \delta_\theta, \delta_{\omega_{Lm}}, \delta_{\omega_{Rm}})^T$ with covariance matrix $R_m = \mathrm{diag}(0.01, 0.01, 0.01, 0.001, 0.001)$, where $\mathrm{diag}(x)$ is the diagonal matrix with the vector x down the diagonal. The output of the UKF algorithm are the estimated $(x_e, y_e, \theta_e, \omega_{Le}, \omega_{Re})^T$ of the robot states and the estimated i_{Le} and i_{Re} of the slip parameters. To apply the filter, it is necessary to define an augmented state given by $(x_e, y_e, \theta_e, \omega_{Le}, \omega_{Re}, i_{Le}, i_{Re})^T$ that includes the dynamics of the robot and the dynamics of the unknown slip parameters, which is taken as

$$\begin{bmatrix} i_{Le} \\ i_{Re} \end{bmatrix}_{k+1} = \begin{bmatrix} i_{Le} \\ i_{Re} \end{bmatrix}_{k} + \begin{bmatrix} w_{i_{Le}} \\ w_{i_{Re}} \end{bmatrix}_{k}$$

with $w_{i_{Le}}$ and $w_{i_{Re}}$ the additive process noises.

To achieve satisfactory result, the filter parameters were heuristically chosen as $\alpha = 1$, $\beta = 2$ and $\gamma = 0$. The initial state, for the filter, is chosen as $x_0 = (1, 1, \pi/4, 0, 0, 0, 0)^T + 0.01(1, 1, 1, 1, 1, 1, 1)^T \varepsilon$, where ε is a random value drawn from a normal distribution with mean zero and standard deviation one. The initial state covariance is the identity matrix $P_0 = I_{7\times 7}$. The covariance of the process and the measurement noise, needed by the UKF algorithm, are selected respectively as $Q_k = 10^{-4} I_{7\times 7}$ and $R_k = \mathrm{diag}(0.02, 0.015, 0.01, 0.0015, 0.0005)$. We have assumed that the signal τ_L and τ_R are measured without noise.



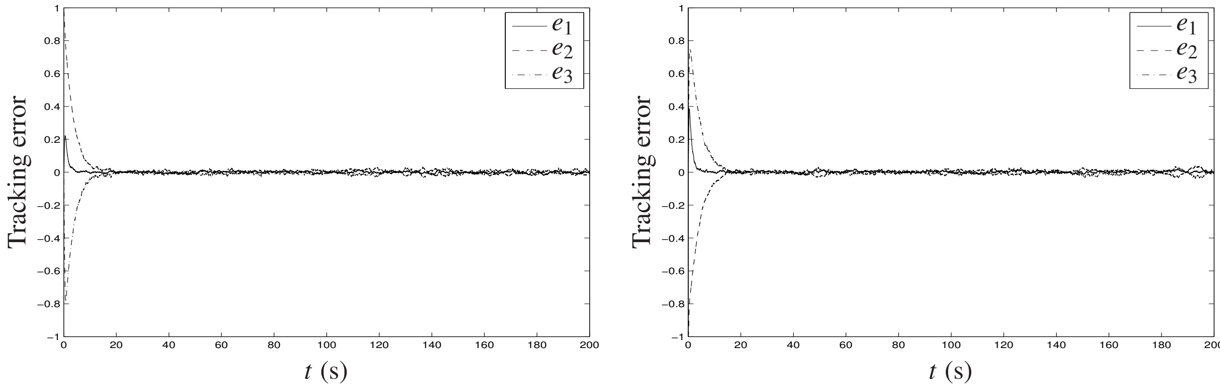
(a) Linear reference trajectory.

(b) Circular reference trajectory.

Figure 3. Robot trajectories in the inertial frame.

Figure 3 shows the robot trajectory in the fixed frame F_1 obtained using our proposed controller. The results for the linear and circular reference trajectories are respectively shown in Figs. 3(a) and 3(b). The dashed line stands for the reference trajectory, while the solid line stands for the robot trajectory. The robot initial condition for the linear and circular reference trajectories are respectively given by $q(0) = (0, -1, 0)^T$ and $q(0) = (0, 1, 0)^T$. This figure shows that the robot trajectory is able to follow to reference trajectory.

Figure 4 shows the tracking errors e_1, e_2 and e_3. The results for the linear and circular reference trajectories are respectively shown in Figs. 4(a) and 4(b). The high initial error is due to the fact that the reference trajectory starts at a point that is far from the initial position of the robot. As we can see in Fig. 4(a) and Fig. 4(b), the mobile robot tracking error already approach zero around $t = 20$s.

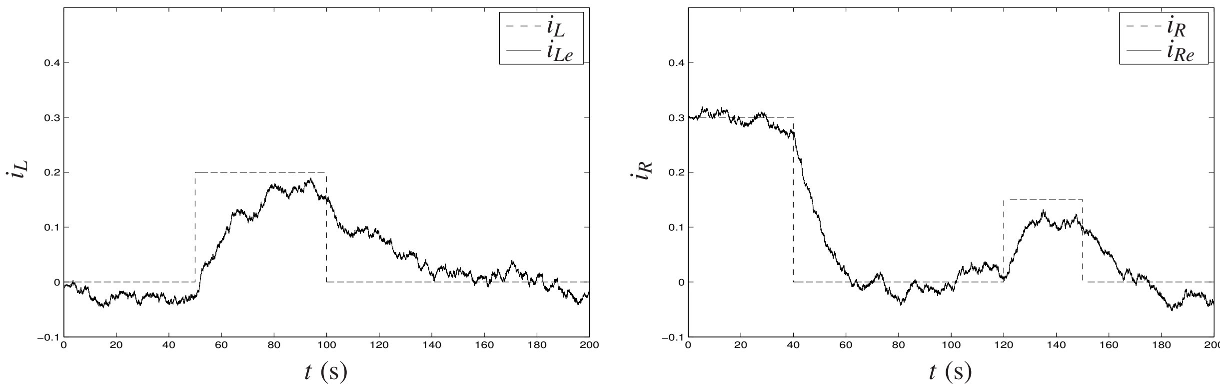


(a) Linear reference trajectory.

(b) Circular reference trajectory.

Figure 4. Time evolution of the tracking error.

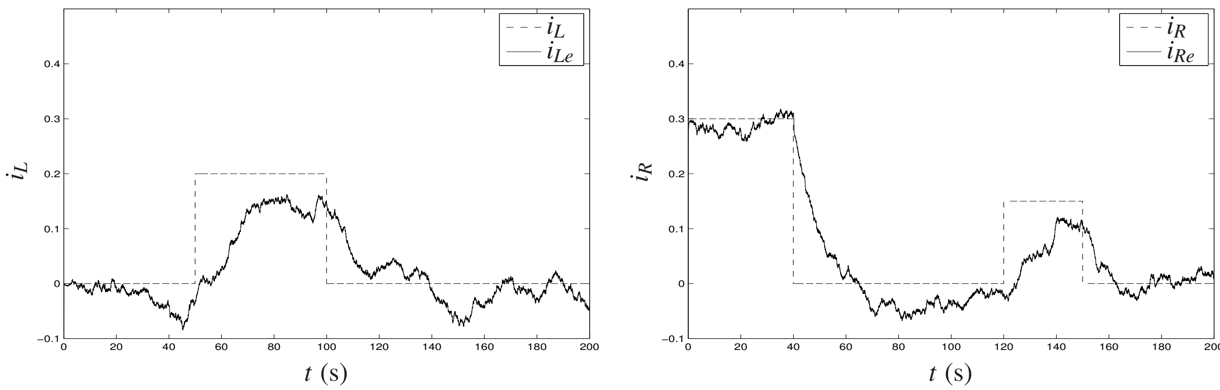
Figures 5 and 6 show the estimated slip parameters i_{Le} and i_{Re} for the linear and for the circular reference trajectories, respectively. The dashed line denotes the true value of the unknown slip parameters i_L and i_R and the solid line denotes the estimated value i_{Le} and i_{Re}. Note that the parameters converge to the true values at a slow rate (slower than the tracking error).



(a) Estimated slip parameter i_{Le}.

(b) Estimated slip parameter i_{Re}.

Figure 5. Estimation of the slip parameters for the linear reference trajectory.

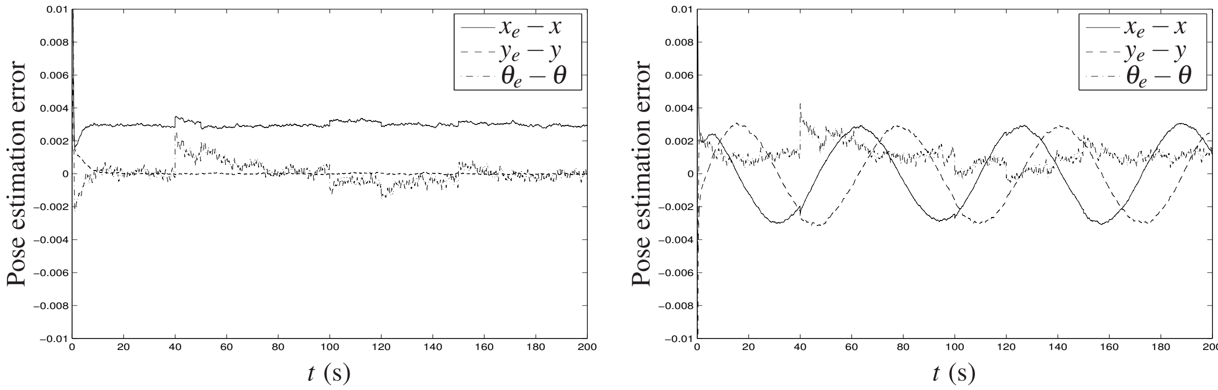


(a) Estimated slip parameter i_{Le}.

(b) Estimated slip parameter i_{Re}.

Figure 6. Estimation of the slip parameters for the circular reference trajectory.

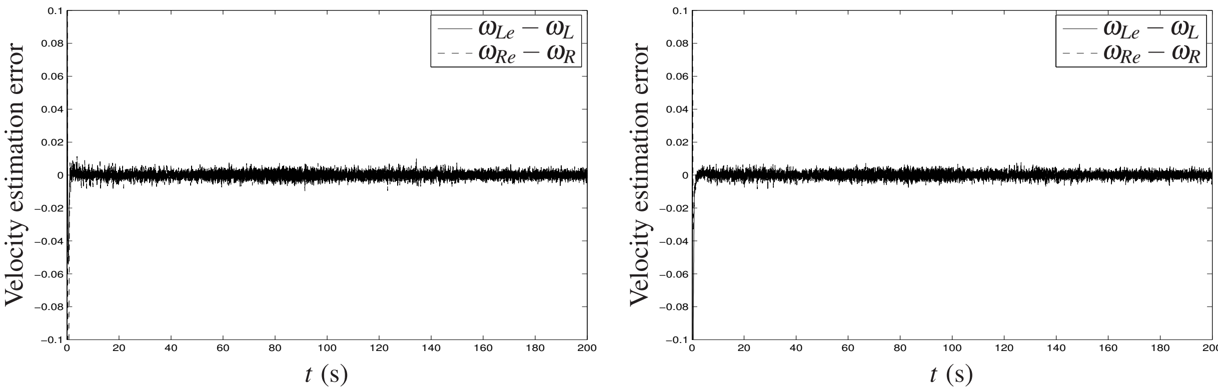
Figure 7 shows the estimation error for the pose $x_e - x$, $y_e - y$ and $\theta_e - \theta$, obtained using the UKF algorithm. The results for the linear and circular reference trajectories are respectively shown in Figs. 7(a) and 7(b). Although the estimation error does not converge to zero, the robot trajectory is still able to follow the reference trajectory. The estimation error is sensitive to the increment-time used in the first-order Euler approximation.



(a) Linear reference trajectory. (b) Circular reference trajectory.

Figure 7. The estimation error for the pose.

Figure 8 shows the estimation error for the angular velocities $\omega_{Le} - \omega_L$ and $\omega_{Re} - \omega_R$, obtained using the UKF algorithm. The results for the linear and circular reference trajectories are respectively shown in Figs. 8(a) and 8(b).



(a) Linear reference trajectory. (b) Circular reference trajectory.

Figure 8. The estimation error for the angular velocity.

5. CONCLUSIONS

This paper provides a tracking control strategy for a tracked mobile robot under longitudinal slip condition. The proposed control strategy is based on the dynamic model of the tracked robot, in which the longitudinal slip of the left and right tracks are described by two unknown parameters. A nonlinear feedback control law is proposed to achieve the trajectory-tracking objective, using estimation of the slip parameters. The unscented Kalman filter (UKF) is introduced to joint estimate the states and the slip parameters. Although the estimation of the slip parameters are slower than the robot pose, the proposed tracking control strategy is able to ensure that the robot can appropriately follow both linear and circular reference trajectories.

6. ACKNOWLEDGMENTS

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8. Responsibility notice

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APPENDIX

The Unscented Kalman Filter

The UKF uses a single Gaussian distribution to match the first and second-order moments of the required probability density function to different accuracy levels (Cui *et al.*, 2005). Furthermore, the UKF does not approximate the nonlinear process and measurement models. Instead, it uses the true nonlinear models and approximates the distribution of the state random variable. By preserving the higher order information of the system, UKF improves both accuracy and convergence properties of the solution.

The basic structure of the unscented Kalman filter (UKF) involves the estimation of the state of a discrete-time nonlinear dynamic system of the form

$$\begin{aligned} x_{k+1} &= f(x_k, u_k) + w_k \\ y_k &= h(x_k) + \delta_k \end{aligned}$$

where x_k is the state of the system, u_k is the control input and y_k is the measured output. The process and measurement noises are respectively given by w_k and δ_k. It is assumed that w_k and δ_k are independent zero-mean Gaussian random variables with covariance matrices respectively given by $Q_k \geq 0$ and $R_k > 0$.

The UKF algorithm (Haykin, 2001) can be summarized as follows. Let the n-dimensional state vector x_{k-1}, with mean $\hat{x}_{k-1}$ and covariance P_{k-1}, be approximated by $2n+1$ weighted samples or sigma points. Initialize with:

$$\begin{aligned} &\hat{x}_0 = \mathbb{E}[x_0] \\ &P_0 = \mathbb{E}[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T] \end{aligned}$$

where $\mathbb{E}[\cdot]$ denotes the expectation. For $k \in \{1, ..., \infty\}$, calculate the sigma points:

$$\begin{aligned} X^i_{k-1} &= \hat{x}_{k-1}, \qquad i = 0 \\ X^i_{k-1} &= \hat{x}_{k-1} + \sqrt{(n+\lambda)P_{k-1}}, \quad i = 1, ..., n \\ X^i_{k-1} &= \hat{x}_{k-1} - \sqrt{(n+\lambda)P_{k-1}}, \quad i = n+1, ..., 2n \end{aligned}$$

where $\lambda = \alpha^2(n+\kappa) - n$ for given $\alpha > 0$ and $\kappa \geq 0$. Propagate the sigma points and obtain the mean and covariance of the state using the following time-update equations:

$$\begin{aligned} X^i_{k|k-1} &= f(X^i_{k-1}, u_{k-1}), \quad i = 0, ..., 2n \\ \hat{x}^-_k &= \sum_{i=0}^{2n} W_i^{(m)} X^i_{k|k-1} \\ P^-_k &= \sum_{i=0}^{2n} W_i^{(c)} \left[X^i_{k|k-1} - \hat{x}^-_k\right] \left[X^i_{k|k-1} - \hat{x}^-_k\right]^T + Q_k \end{aligned}$$

with the weights given by

$$\begin{aligned} W_i^{(m)} &= \lambda/(n+\lambda), \quad i = 0 \\ W_i^{(c)} &= \lambda/(n+\lambda) + (1 - \alpha^2 + \beta), \quad i = 0 \\ W_i^{(m)} &= W_i^{(c)} = 1/2(n+\lambda), \quad i = 1, 2, ..., n \end{aligned}$$

where β is a non-negative weighting term. The size of the sigma point distribution is regularized by the non-negative weighting terms α and β, which can be used to compensate for the information of the higher order moments of the distribution. Finally, calculate the measurement sigma points $Y^i_{k|k-1}$ using $h(\cdot)$ and update the mean and covariance using the following measurement-update equation:

$$
\begin{aligned}
Y^{i}_{k|k-1} &= h(X^{i}_{k|k-1}), \quad i = 0, ..., 2n \\
\hat{y}^{-}_{k} &= \sum_{i=0}^{2n} W_i^{(m)} Y^{i}_{k|k-1} \\
P_{\tilde{y}_k \tilde{y}_k} &= \sum_{i=0}^{2n} W_i^{(c)} \left(Y^{i}_{k|k-1} - \hat{y}^{-}_{k} \right) \left(Y^{i}_{k|k-1} - \hat{y}^{-}_{k} \right)^T + R_k \\
P_{x_k y_k} &= \sum_{i=0}^{2n} W_i^{(c)} \left(X^{i}_{k|k-1} - \hat{x}^{-}_{k} \right) \left(Y^{i}_{k|k-1} - \hat{y}^{-}_{k} \right)^T \\
K_k &= P_{x_k y_k} P^{-1}_{\tilde{y}_k \tilde{y}_k} \\
\hat{x}_k &= \hat{x}^{-}_{k} + K_k \left(y_k - \hat{y}^{-}_{k} \right) \\
P_k &= P^{-}_{k} - K_k P_{\tilde{y}_k \tilde{y}_k} K^{T}_{k}
\end{aligned}
$$

It is worth to emphasize that the EKF algorithm could have also been used for parameter estimation. However, it may suffer from large estimate errors when the system has strong nonlinearities. On the other hand, the UKF uses the true nonlinear models and can achieve more accurate estimations without linearization. As stated by Haykin (2001), the UKF algorithm was first proposed by Julier *et al.* (1995) and further developed by Wan and van der Menve (2000).