Relu

```
In [ ]: import tensorflow as tf
        from tensorflow.keras.models import Sequential
        from tensorflow.keras.layers import Dense
        import numpy as np
        import matplotlib.pyplot as plt
        # Generate training and testing data
        x_{train} = np.random.uniform(-10, 10, 5000)
        y_train = np.cos(x_train)
        x_{test} = np.random.uniform(-10, 10, 1000)
        y_{\text{test}} = np.cos(x_{\text{test}})
        # Build the model
        model = Sequential([
            Dense(64, input_shape=(1,), activation='relu'), # 1st hidden layer with relu
            Dense (64),
            Dense(64),
            Dense(1)
        1)
        # Compile the model
        model.compile(optimizer='adam', loss='mean squared error')
        # Fit the model
        model.fit(x_train, y_train, epochs=150, batch_size=64, validation_data=(x_test, y_t
        # Test the model
        val loss = model.evaluate(x test, y test, verbose=0)
        print(f"Validation loss: {val_loss}")
        import matplotlib.pyplot as plt
        num_test_samples = 1000
        X_test = np.linspace(-10, 10, num=num_test_samples).reshape(-1, 1)
        y_true = np.cos(X_test)
        y_pred = model.predict(X_test)
        plt.figure(figsize=(10, 6))
        plt.plot(X_test, y_true, label='True Cosine Values', color='b', linewidth=2)
        plt.plot(X_test, y_pred, label='Model Predictions', color='r', linestyle='--', line
        plt.xlabel('Input Value')
        plt.ylabel('Cosine Value')
        plt.title('Cosine Function and Model Predictions')
        plt.legend()
        plt.grid()
        plt.show()
```

```
Epoch 1/150
80
Epoch 2/150
79/79 [=========== ] - 0s 928us/step - loss: 0.5009 - val_loss: 0.
5013
Epoch 3/150
79/79 [========== ] - 0s 928us/step - loss: 0.5046 - val_loss: 0.
5296
Epoch 4/150
79/79 [========== ] - 0s 919us/step - loss: 0.5025 - val_loss: 0.
Epoch 5/150
79/79 [=========== ] - 0s 941us/step - loss: 0.5005 - val loss: 0.
5050
Epoch 6/150
79/79 [=========== ] - 0s 926us/step - loss: 0.4985 - val_loss: 0.
4948
Epoch 7/150
79/79 [=========== ] - 0s 909us/step - loss: 0.4875 - val_loss: 0.
4820
Epoch 8/150
79/79 [========== ] - 0s 916us/step - loss: 0.4892 - val_loss: 0.
4815
Epoch 9/150
79/79 [========== ] - 0s 910us/step - loss: 0.4941 - val_loss: 0.
4911
Epoch 10/150
79/79 [=========== ] - 0s 901us/step - loss: 0.4767 - val_loss: 0.
4831
Epoch 11/150
4579
Epoch 12/150
79/79 [=========== ] - 0s 890us/step - loss: 0.4533 - val_loss: 0.
4470
Epoch 13/150
4326
Epoch 14/150
79/79 [========== ] - 0s 899us/step - loss: 0.4468 - val_loss: 0.
4380
Epoch 15/150
79/79 [=========== ] - 0s 890us/step - loss: 0.4433 - val loss: 0.
4472
Epoch 16/150
79/79 [============ ] - 0s 908us/step - loss: 0.4456 - val loss: 0.
Epoch 17/150
79/79 [=========== ] - 0s 903us/step - loss: 0.4433 - val loss: 0.
4261
Epoch 18/150
79/79 [=========== ] - 0s 890us/step - loss: 0.4466 - val loss: 0.
4493
Epoch 19/150
79/79 [=========== ] - 0s 896us/step - loss: 0.4354 - val loss: 0.
```

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4246
Epoch 20/150
79/79 [=========== ] - 0s 909us/step - loss: 0.4341 - val loss: 0.
4157
Epoch 21/150
79/79 [=========== ] - 0s 890us/step - loss: 0.4309 - val loss: 0.
Epoch 22/150
79/79 [=========== ] - 0s 890us/step - loss: 0.4299 - val loss: 0.
4764
Epoch 23/150
79/79 [============ ] - 0s 881us/step - loss: 0.4256 - val loss: 0.
4190
Epoch 24/150
79/79 [============ ] - 0s 886us/step - loss: 0.4262 - val loss: 0.
5059
Epoch 25/150
79/79 [=========== ] - 0s 884us/step - loss: 0.4313 - val_loss: 0.
4771
Epoch 26/150
79/79 [=========== ] - 0s 890us/step - loss: 0.4319 - val_loss: 0.
4134
Epoch 27/150
79/79 [========== ] - 0s 897us/step - loss: 0.4163 - val_loss: 0.
Epoch 28/150
79/79 [=========== ] - 0s 886us/step - loss: 0.4266 - val_loss: 0.
4151
Epoch 29/150
79/79 [===========] - 0s 897us/step - loss: 0.4260 - val_loss: 0.
4091
Epoch 30/150
79/79 [========== ] - 0s 903us/step - loss: 0.4172 - val_loss: 0.
4108
Epoch 31/150
79/79 [=========== ] - 0s 897us/step - loss: 0.4221 - val_loss: 0.
4137
Epoch 32/150
79/79 [=========== ] - 0s 897us/step - loss: 0.4245 - val_loss: 0.
4496
Epoch 33/150
79/79 [========== ] - 0s 893us/step - loss: 0.4206 - val_loss: 0.
4105
Epoch 34/150
79/79 [============ ] - 0s 886us/step - loss: 0.4213 - val loss: 0.
4078
Epoch 35/150
79/79 [========== ] - 0s 897us/step - loss: 0.4186 - val_loss: 0.
4045
Epoch 36/150
4152
Epoch 37/150
4268
Epoch 38/150
```

6/6/23, 9:54 AM

```
79/79 [=========== ] - 0s 883us/step - loss: 0.4219 - val_loss: 0.
4098
Epoch 39/150
79/79 [=========== ] - 0s 874us/step - loss: 0.4153 - val loss: 0.
4144
Epoch 40/150
79/79 [=========== ] - 0s 881us/step - loss: 0.4245 - val_loss: 0.
4678
Epoch 41/150
79/79 [=========== ] - 0s 884us/step - loss: 0.4179 - val loss: 0.
4308
Epoch 42/150
79/79 [=========== ] - 0s 877us/step - loss: 0.4215 - val_loss: 0.
4081
Epoch 43/150
79/79 [=========== ] - 0s 885us/step - loss: 0.4310 - val loss: 0.
Epoch 44/150
79/79 [=========== ] - 0s 880us/step - loss: 0.4219 - val loss: 0.
4099
Epoch 45/150
79/79 [=========== ] - 0s 891us/step - loss: 0.4217 - val loss: 0.
4110
Epoch 46/150
79/79 [=========== ] - 0s 880us/step - loss: 0.4195 - val loss: 0.
4176
Epoch 47/150
79/79 [=========== ] - 0s 890us/step - loss: 0.4198 - val loss: 0.
4138
Epoch 48/150
79/79 [=========== ] - 0s 903us/step - loss: 0.4196 - val loss: 0.
4023
Epoch 49/150
79/79 [=========== ] - 0s 897us/step - loss: 0.4093 - val loss: 0.
4358
Epoch 50/150
79/79 [=========== ] - 0s 890us/step - loss: 0.4112 - val loss: 0.
4227
Epoch 51/150
79/79 [=========== ] - 0s 896us/step - loss: 0.4118 - val_loss: 0.
4103
Epoch 52/150
79/79 [========== ] - 0s 890us/step - loss: 0.3911 - val_loss: 0.
3851
Epoch 53/150
79/79 [=========== ] - 0s 890us/step - loss: 0.3643 - val_loss: 0.
3274
Epoch 54/150
79/79 [========== ] - 0s 909us/step - loss: 0.3044 - val_loss: 0.
2543
Epoch 55/150
79/79 [===========] - 0s 897us/step - loss: 0.2361 - val_loss: 0.
3679
Epoch 56/150
79/79 [========== ] - 0s 903us/step - loss: 0.1848 - val_loss: 0.
1525
```

```
Epoch 57/150
79/79 [========== ] - 0s 897us/step - loss: 0.1157 - val_loss: 0.
1171
Epoch 58/150
79/79 [=========== ] - 0s 897us/step - loss: 0.0679 - val_loss: 0.
9626
Epoch 59/150
79/79 [=========== ] - 0s 888us/step - loss: 0.0459 - val_loss: 0.
1001
Epoch 60/150
79/79 [========== ] - 0s 891us/step - loss: 0.0401 - val_loss: 0.
Epoch 61/150
79/79 [=========== ] - 0s 884us/step - loss: 0.0341 - val loss: 0.
0467
Epoch 62/150
79/79 [=========== ] - 0s 903us/step - loss: 0.0207 - val_loss: 0.
0176
Epoch 63/150
79/79 [========== ] - 0s 903us/step - loss: 0.0176 - val_loss: 0.
0188
Epoch 64/150
79/79 [========== ] - 0s 891us/step - loss: 0.0167 - val_loss: 0.
0170
Epoch 65/150
79/79 [========== ] - 0s 895us/step - loss: 0.0197 - val_loss: 0.
0152
Epoch 66/150
79/79 [=========== ] - 0s 890us/step - loss: 0.0225 - val loss: 0.
0557
Epoch 67/150
0115
Epoch 68/150
79/79 [========== ] - 0s 896us/step - loss: 0.0213 - val_loss: 0.
0342
Epoch 69/150
0128
Epoch 70/150
79/79 [========== ] - 0s 914us/step - loss: 0.0182 - val_loss: 0.
0229
Epoch 71/150
79/79 [=========== ] - 0s 901us/step - loss: 0.0172 - val loss: 0.
Epoch 72/150
79/79 [=========== ] - 0s 889us/step - loss: 0.0239 - val loss: 0.
Epoch 73/150
79/79 [=========== ] - 0s 899us/step - loss: 0.0192 - val loss: 0.
0129
Epoch 74/150
79/79 [========== ] - 0s 903us/step - loss: 0.0151 - val_loss: 0.
0226
Epoch 75/150
79/79 [=========== ] - 0s 904us/step - loss: 0.0178 - val loss: 0.
```

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0322
Epoch 76/150
79/79 [=========== ] - 0s 890us/step - loss: 0.0174 - val loss: 0.
0118
Epoch 77/150
79/79 [=========== ] - 0s 905us/step - loss: 0.0188 - val loss: 0.
Epoch 78/150
79/79 [=========== ] - 0s 891us/step - loss: 0.0154 - val loss: 0.
0200
Epoch 79/150
79/79 [=========== ] - 0s 889us/step - loss: 0.0155 - val loss: 0.
0103
Epoch 80/150
79/79 [=========== ] - 0s 893us/step - loss: 0.0181 - val loss: 0.
0237
Epoch 81/150
79/79 [========== ] - 0s 897us/step - loss: 0.0155 - val_loss: 0.
0118
Epoch 82/150
79/79 [========== ] - 0s 895us/step - loss: 0.0163 - val_loss: 0.
0244
Epoch 83/150
79/79 [=========== ] - 0s 900us/step - loss: 0.0170 - val_loss: 0.
Epoch 84/150
79/79 [=========== ] - 0s 891us/step - loss: 0.0190 - val_loss: 0.
0181
Epoch 85/150
79/79 [========== ] - 0s 897us/step - loss: 0.0147 - val_loss: 0.
0131
Epoch 86/150
79/79 [========== ] - 0s 890us/step - loss: 0.0149 - val_loss: 0.
0105
Epoch 87/150
79/79 [=========== ] - 0s 947us/step - loss: 0.0171 - val_loss: 0.
0137
Epoch 88/150
79/79 [========== ] - 0s 897us/step - loss: 0.0143 - val_loss: 0.
Epoch 89/150
79/79 [========== ] - 0s 890us/step - loss: 0.0153 - val_loss: 0.
0322
Epoch 90/150
43
Epoch 91/150
79/79 [========== ] - 0s 890us/step - loss: 0.0187 - val_loss: 0.
0230
Epoch 92/150
79/79 [============= ] - 0s 903us/step - loss: 0.0176 - val_loss: 0.
0148
Epoch 93/150
Epoch 94/150
```

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79/79 [========== ] - 0s 890us/step - loss: 0.0184 - val_loss: 0.
0124
Epoch 95/150
79/79 [=========== ] - 0s 881us/step - loss: 0.0186 - val loss: 0.
0195
Epoch 96/150
79/79 [========== ] - 0s 916us/step - loss: 0.0184 - val_loss: 0.
0144
Epoch 97/150
79/79 [=========== ] - 0s 886us/step - loss: 0.0140 - val loss: 0.
0122
Epoch 98/150
79/79 [=========== ] - 0s 890us/step - loss: 0.0152 - val loss: 0.
Epoch 99/150
79/79 [=========== ] - 0s 896us/step - loss: 0.0149 - val loss: 0.
0170
Epoch 100/150
79/79 [=========== ] - 0s 895us/step - loss: 0.0133 - val loss: 0.
Epoch 101/150
79/79 [=========== ] - 0s 890us/step - loss: 0.0160 - val loss: 0.
0150
Epoch 102/150
79/79 [=========== ] - 0s 890us/step - loss: 0.0151 - val loss: 0.
0126
Epoch 103/150
79/79 [=========== ] - 0s 897us/step - loss: 0.0175 - val loss: 0.
0379
Epoch 104/150
79/79 [=========== ] - 0s 903us/step - loss: 0.0199 - val loss: 0.
0266
Epoch 105/150
79/79 [=========== ] - 0s 884us/step - loss: 0.0157 - val loss: 0.
0103
Epoch 106/150
79/79 [=========== ] - 0s 890us/step - loss: 0.0162 - val loss: 0.
0156
Epoch 107/150
79/79 [========== ] - 0s 878us/step - loss: 0.0151 - val_loss: 0.
0166
Epoch 108/150
79/79 [========== ] - 0s 888us/step - loss: 0.0156 - val_loss: 0.
0119
Epoch 109/150
79/79 [========== ] - 0s 899us/step - loss: 0.0172 - val_loss: 0.
0328
Epoch 110/150
79/79 [========== ] - 0s 895us/step - loss: 0.0207 - val_loss: 0.
Epoch 111/150
79/79 [=========== ] - 0s 875us/step - loss: 0.0154 - val_loss: 0.
0147
Epoch 112/150
79/79 [========== ] - 0s 891us/step - loss: 0.0158 - val_loss: 0.
0242
```

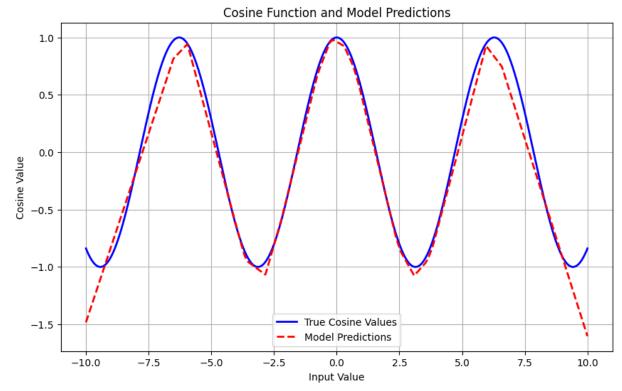
```
Epoch 113/150
79/79 [========== ] - 0s 883us/step - loss: 0.0161 - val_loss: 0.
0218
Epoch 114/150
79/79 [=========== ] - 0s 884us/step - loss: 0.0167 - val_loss: 0.
0120
Epoch 115/150
79/79 [========== ] - 0s 889us/step - loss: 0.0137 - val_loss: 0.
Epoch 116/150
79/79 [========== ] - 0s 888us/step - loss: 0.0149 - val_loss: 0.
Epoch 117/150
79/79 [=========== ] - 0s 891us/step - loss: 0.0205 - val loss: 0.
0114
Epoch 118/150
79/79 [=========== ] - 0s 873us/step - loss: 0.0171 - val_loss: 0.
0269
Epoch 119/150
79/79 [========== ] - 0s 871us/step - loss: 0.0168 - val_loss: 0.
0155
Epoch 120/150
79/79 [========== ] - 0s 909us/step - loss: 0.0175 - val_loss: 0.
0133
Epoch 121/150
79/79 [========== ] - 0s 897us/step - loss: 0.0157 - val_loss: 0.
0438
Epoch 122/150
79/79 [=========== ] - 0s 879us/step - loss: 0.0174 - val loss: 0.
0136
Epoch 123/150
0270
Epoch 124/150
79/79 [=========== ] - 0s 886us/step - loss: 0.0154 - val_loss: 0.
0112
Epoch 125/150
0177
Epoch 126/150
79/79 [=========== ] - 0s 884us/step - loss: 0.0145 - val_loss: 0.
0132
Epoch 127/150
79/79 [=========== ] - 0s 884us/step - loss: 0.0140 - val loss: 0.
0127
Epoch 128/150
79/79 [=========== ] - 0s 897us/step - loss: 0.0187 - val loss: 0.
Epoch 129/150
79/79 [=========== ] - 0s 897us/step - loss: 0.0132 - val loss: 0.
0386
Epoch 130/150
79/79 [=========== ] - 0s 903us/step - loss: 0.0168 - val loss: 0.
0231
Epoch 131/150
79/79 [=========== ] - 0s 903us/step - loss: 0.0171 - val loss: 0.
```

```
0166
Epoch 132/150
79/79 [=========== ] - 0s 909us/step - loss: 0.0145 - val loss: 0.
0110
Epoch 133/150
79/79 [=========== ] - 0s 890us/step - loss: 0.0135 - val loss: 0.
Epoch 134/150
79/79 [============ ] - 0s 892us/step - loss: 0.0155 - val loss: 0.
0148
Epoch 135/150
79/79 [============ ] - 0s 884us/step - loss: 0.0166 - val loss: 0.
0316
Epoch 136/150
79/79 [=========== ] - 0s 884us/step - loss: 0.0167 - val loss: 0.
0129
Epoch 137/150
79/79 [========== ] - 0s 890us/step - loss: 0.0168 - val_loss: 0.
0148
Epoch 138/150
79/79 [=========== ] - 0s 890us/step - loss: 0.0147 - val_loss: 0.
0336
Epoch 139/150
79/79 [========== ] - 0s 887us/step - loss: 0.0170 - val_loss: 0.
Epoch 140/150
79/79 [=========== ] - 0s 897us/step - loss: 0.0156 - val_loss: 0.
0154
Epoch 141/150
79/79 [========== ] - 0s 878us/step - loss: 0.0167 - val_loss: 0.
0142
Epoch 142/150
79/79 [========== ] - 0s 883us/step - loss: 0.0150 - val_loss: 0.
0126
Epoch 143/150
79/79 [=========== ] - 0s 891us/step - loss: 0.0176 - val_loss: 0.
Epoch 144/150
79/79 [========== ] - 0s 887us/step - loss: 0.0172 - val_loss: 0.
Epoch 145/150
79/79 [=========== ] - 0s 884us/step - loss: 0.0143 - val_loss: 0.
0171
Epoch 146/150
79/79 [=========== ] - 0s 922us/step - loss: 0.0131 - val loss: 0.
0146
Epoch 147/150
79/79 [===========] - 0s 878us/step - loss: 0.0138 - val_loss: 0.
0199
Epoch 148/150
0146
Epoch 149/150
79/79 [==========] - 0s 890us/step - loss: 0.0199 - val_loss: 0.
Epoch 150/150
```

```
79/79 [============] - 0s 897us/step - loss: 0.0148 - val_loss: 0.0214

Validation loss: 0.021354960277676582

32/32 [==========] - 0s 480us/step
```



Epochs: 150

Loss: 0.021

Tanh

```
In [ ]: import tensorflow as tf
         from tensorflow.keras.models import Sequential
         from tensorflow.keras.layers import Dense
         import numpy as np
         import matplotlib.pyplot as plt
         # Generate training and testing data
         x_{train} = np.random.uniform(-10, 10, 5000)
         y_train = np.cos(x_train)
         x_{test} = np.random.uniform(-10, 10, 1000)
         y_{\text{test}} = np.cos(x_{\text{test}})
         # Build the model
         model = Sequential([
             Dense(64, input_shape=(1,), activation='tanh'), #1st hiddn Layer with tanh
             Dense(64),
             Dense(64),
             Dense(1)
```

```
])
# Compile the model
model.compile(optimizer='adam', loss='mean_squared_error')
# Fit the model
model.fit(x_train, y_train, epochs=150, batch_size=64, validation_data=(x_test, y_t
# Test the model
val_loss = model.evaluate(x_test, y_test, verbose=0)
print(f"Validation loss: {val_loss}")
import matplotlib.pyplot as plt
num test samples = 1000
X_test = np.linspace(-10, 10, num=num_test_samples).reshape(-1, 1)
y_true = np.cos(X_test)
y_pred = model.predict(X_test)
plt.figure(figsize=(10, 6))
plt.plot(X_test, y_true, label='True Cosine Values', color='b', linewidth=2)
plt.plot(X_test, y_pred, label='Model Predictions', color='r', linestyle='--', line
plt.xlabel('Input Value')
plt.ylabel('Cosine Value')
plt.title('Cosine Function and Model Predictions')
plt.legend()
plt.grid()
plt.show()
```

```
Epoch 1/150
14
Epoch 2/150
79/79 [========== ] - 0s 904us/step - loss: 0.5205 - val_loss: 0.
5495
Epoch 3/150
79/79 [========== ] - 0s 934us/step - loss: 0.5159 - val_loss: 0.
5612
Epoch 4/150
79/79 [========== ] - 0s 970us/step - loss: 0.5201 - val_loss: 0.
Epoch 5/150
79/79 [=========== ] - 0s 915us/step - loss: 0.5069 - val loss: 0.
5516
Epoch 6/150
79/79 [=========== ] - 0s 905us/step - loss: 0.5130 - val_loss: 0.
6422
Epoch 7/150
79/79 [========== ] - 0s 916us/step - loss: 0.5159 - val_loss: 0.
5156
Epoch 8/150
79/79 [=========== ] - 0s 928us/step - loss: 0.4989 - val_loss: 0.
5376
Epoch 9/150
79/79 [========== ] - 0s 916us/step - loss: 0.4928 - val_loss: 0.
4884
Epoch 10/150
79/79 [=========== ] - 0s 917us/step - loss: 0.4787 - val_loss: 0.
Epoch 11/150
4291
Epoch 12/150
79/79 [=========== ] - 0s 894us/step - loss: 0.4272 - val_loss: 0.
4068
Epoch 13/150
3722
Epoch 14/150
79/79 [========== ] - 0s 894us/step - loss: 0.3706 - val_loss: 0.
3888
Epoch 15/150
79/79 [=========== ] - 0s 898us/step - loss: 0.3621 - val loss: 0.
3975
Epoch 16/150
79/79 [========== ] - 0s 903us/step - loss: 0.3118 - val_loss: 0.
Epoch 17/150
79/79 [=========== ] - 0s 919us/step - loss: 0.2897 - val loss: 0.
3365
Epoch 18/150
79/79 [============ ] - 0s 908us/step - loss: 0.2457 - val loss: 0.
2622
Epoch 19/150
79/79 [============ ] - 0s 891us/step - loss: 0.1956 - val loss: 0.
```

```
1725
Epoch 20/150
79/79 [=========== ] - 0s 886us/step - loss: 0.1633 - val loss: 0.
Epoch 21/150
79/79 [=========== ] - 0s 890us/step - loss: 0.1466 - val loss: 0.
Epoch 22/150
79/79 [=========== ] - 0s 904us/step - loss: 0.1231 - val loss: 0.
1112
Epoch 23/150
79/79 [============ ] - 0s 890us/step - loss: 0.1057 - val loss: 0.
0840
Epoch 24/150
79/79 [=========== ] - 0s 892us/step - loss: 0.0874 - val loss: 0.
1142
Epoch 25/150
79/79 [========== ] - 0s 885us/step - loss: 0.0769 - val_loss: 0.
1004
Epoch 26/150
79/79 [=========== ] - 0s 885us/step - loss: 0.0743 - val_loss: 0.
0551
Epoch 27/150
79/79 [========== ] - 0s 912us/step - loss: 0.0615 - val_loss: 0.
Epoch 28/150
79/79 [=========== ] - 0s 888us/step - loss: 0.0712 - val_loss: 0.
0623
Epoch 29/150
79/79 [========== ] - 0s 888us/step - loss: 0.0629 - val_loss: 0.
0450
Epoch 30/150
79/79 [========== ] - 0s 890us/step - loss: 0.0618 - val_loss: 0.
0878
Epoch 31/150
79/79 [=========== ] - 0s 893us/step - loss: 0.0501 - val_loss: 0.
0781
Epoch 32/150
79/79 [=========== ] - 0s 888us/step - loss: 0.0617 - val_loss: 0.
0522
Epoch 33/150
79/79 [========== ] - 0s 893us/step - loss: 0.0541 - val_loss: 0.
0692
Epoch 34/150
79/79 [=========== ] - 0s 893us/step - loss: 0.0599 - val loss: 0.
0402
Epoch 35/150
79/79 [===========] - 0s 885us/step - loss: 0.0468 - val_loss: 0.
0362
Epoch 36/150
79/79 [=========== ] - 0s 887us/step - loss: 0.0447 - val_loss: 0.
0336
Epoch 37/150
79/79 [===========] - 0s 898us/step - loss: 0.0449 - val_loss: 0.
0748
Epoch 38/150
```

```
79/79 [=========== ] - 0s 884us/step - loss: 0.0432 - val_loss: 0.
0352
Epoch 39/150
79/79 [=========== ] - 0s 882us/step - loss: 0.0419 - val loss: 0.
0333
Epoch 40/150
79/79 [=========== ] - 0s 890us/step - loss: 0.0450 - val loss: 0.
0495
Epoch 41/150
79/79 [=========== ] - 0s 912us/step - loss: 0.0441 - val loss: 0.
0329
Epoch 42/150
79/79 [=========== ] - 0s 879us/step - loss: 0.0493 - val loss: 0.
0443
Epoch 43/150
79/79 [=========== ] - 0s 887us/step - loss: 0.0417 - val loss: 0.
0428
Epoch 44/150
79/79 [=========== ] - 0s 883us/step - loss: 0.0459 - val loss: 0.
0804
Epoch 45/150
79/79 [=========== ] - 0s 909us/step - loss: 0.0490 - val loss: 0.
0943
Epoch 46/150
79/79 [=========== ] - 0s 878us/step - loss: 0.0568 - val loss: 0.
0470
Epoch 47/150
79/79 [=========== ] - 0s 903us/step - loss: 0.0373 - val loss: 0.
0404
Epoch 48/150
79/79 [=========== ] - 0s 888us/step - loss: 0.0465 - val loss: 0.
0703
Epoch 49/150
79/79 [============ ] - 0s 886us/step - loss: 0.0446 - val loss: 0.
0911
Epoch 50/150
79/79 [=========== ] - 0s 893us/step - loss: 0.0570 - val loss: 0.
0404
Epoch 51/150
79/79 [========== ] - 0s 878us/step - loss: 0.0386 - val_loss: 0.
0974
Epoch 52/150
79/79 [=========== ] - 0s 890us/step - loss: 0.0403 - val loss: 0.
1054
Epoch 53/150
79/79 [========== ] - 0s 901us/step - loss: 0.0400 - val_loss: 0.
0308
Epoch 54/150
79/79 [========== ] - 0s 907us/step - loss: 0.0386 - val_loss: 0.
Epoch 55/150
79/79 [=========== ] - 0s 909us/step - loss: 0.0425 - val_loss: 0.
0389
Epoch 56/150
79/79 [=========== ] - 0s 884us/step - loss: 0.0381 - val_loss: 0.
0484
```

```
Epoch 57/150
79/79 [========== ] - 0s 897us/step - loss: 0.0356 - val_loss: 0.
0421
Epoch 58/150
79/79 [=========== ] - 0s 893us/step - loss: 0.0349 - val_loss: 0.
0291
Epoch 59/150
79/79 [=========== ] - 0s 880us/step - loss: 0.0436 - val_loss: 0.
0379
Epoch 60/150
79/79 [========== ] - 0s 891us/step - loss: 0.0435 - val_loss: 0.
Epoch 61/150
79/79 [============ ] - 0s 871us/step - loss: 0.0377 - val loss: 0.
0373
Epoch 62/150
79/79 [=========== ] - 0s 891us/step - loss: 0.0431 - val_loss: 0.
0310
Epoch 63/150
79/79 [=========== ] - 0s 880us/step - loss: 0.0346 - val_loss: 0.
0552
Epoch 64/150
79/79 [========== ] - 0s 898us/step - loss: 0.0503 - val_loss: 0.
1261
Epoch 65/150
79/79 [========== ] - 0s 887us/step - loss: 0.0481 - val_loss: 0.
0416
Epoch 66/150
79/79 [=========== ] - 0s 884us/step - loss: 0.0449 - val loss: 0.
Epoch 67/150
0358
Epoch 68/150
79/79 [========== ] - 0s 886us/step - loss: 0.0328 - val_loss: 0.
0313
Epoch 69/150
0837
Epoch 70/150
79/79 [========== ] - 0s 909us/step - loss: 0.0384 - val_loss: 0.
0361
Epoch 71/150
79/79 [=========== ] - 0s 896us/step - loss: 0.0462 - val loss: 0.
Epoch 72/150
79/79 [=========== ] - 0s 904us/step - loss: 0.0408 - val loss: 0.
0896
Epoch 73/150
79/79 [=========== ] - 0s 891us/step - loss: 0.0360 - val loss: 0.
0295
Epoch 74/150
79/79 [========== ] - 0s 986us/step - loss: 0.0338 - val_loss: 0.
0597
Epoch 75/150
79/79 [============ ] - 0s 884us/step - loss: 0.0453 - val loss: 0.
```

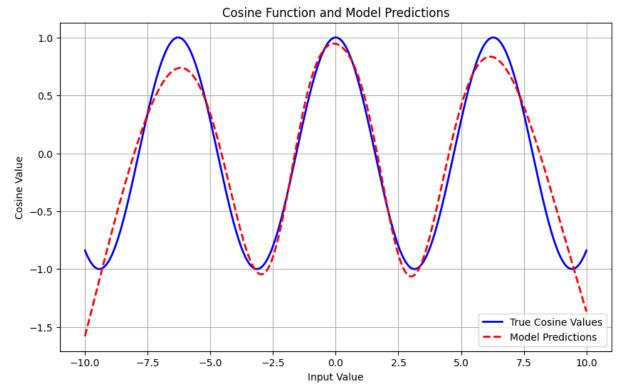
```
0338
Epoch 76/150
79/79 [=========== ] - 0s 897us/step - loss: 0.0344 - val loss: 0.
0296
Epoch 77/150
79/79 [=========== ] - 0s 909us/step - loss: 0.0328 - val loss: 0.
Epoch 78/150
79/79 [=========== ] - 0s 879us/step - loss: 0.0357 - val loss: 0.
0274
Epoch 79/150
79/79 [============ ] - 0s 890us/step - loss: 0.0447 - val loss: 0.
0569
Epoch 80/150
79/79 [=========== ] - 0s 878us/step - loss: 0.0377 - val loss: 0.
0991
Epoch 81/150
79/79 [=========== ] - 0s 872us/step - loss: 0.0470 - val_loss: 0.
0449
Epoch 82/150
79/79 [========== ] - 0s 887us/step - loss: 0.0384 - val_loss: 0.
0337
Epoch 83/150
79/79 [========== ] - 0s 892us/step - loss: 0.0354 - val_loss: 0.
Epoch 84/150
79/79 [=========== ] - 0s 878us/step - loss: 0.0393 - val_loss: 0.
0307
Epoch 85/150
79/79 [========== ] - 0s 891us/step - loss: 0.0361 - val_loss: 0.
0856
Epoch 86/150
79/79 [========== ] - 0s 878us/step - loss: 0.0387 - val_loss: 0.
0521
Epoch 87/150
79/79 [=========== ] - 0s 897us/step - loss: 0.0359 - val_loss: 0.
Epoch 88/150
79/79 [========== ] - 0s 912us/step - loss: 0.0402 - val_loss: 0.
0299
Epoch 89/150
79/79 [=========== ] - 0s 900us/step - loss: 0.0347 - val_loss: 0.
0358
Epoch 90/150
79/79 [=========== ] - 0s 891us/step - loss: 0.0330 - val loss: 0.
0401
Epoch 91/150
79/79 [===========] - 0s 907us/step - loss: 0.0428 - val_loss: 0.
0286
Epoch 92/150
79/79 [============= ] - 0s 881us/step - loss: 0.0308 - val_loss: 0.
0709
Epoch 93/150
79/79 [===========] - 0s 887us/step - loss: 0.0408 - val_loss: 0.
Epoch 94/150
```

```
79/79 [========== ] - 0s 876us/step - loss: 0.0354 - val_loss: 0.
0326
Epoch 95/150
79/79 [=========== ] - 0s 894us/step - loss: 0.0344 - val loss: 0.
0825
Epoch 96/150
79/79 [=========== ] - 0s 904us/step - loss: 0.0438 - val loss: 0.
0305
Epoch 97/150
79/79 [=========== ] - 0s 908us/step - loss: 0.0345 - val loss: 0.
0971
Epoch 98/150
79/79 [=========== ] - 0s 890us/step - loss: 0.0351 - val loss: 0.
1453
Epoch 99/150
79/79 [=========== ] - 0s 903us/step - loss: 0.0448 - val loss: 0.
Epoch 100/150
79/79 [=========== ] - 0s 886us/step - loss: 0.0334 - val loss: 0.
0279
Epoch 101/150
79/79 [=========== ] - 0s 890us/step - loss: 0.0427 - val loss: 0.
0405
Epoch 102/150
79/79 [=========== ] - 0s 909us/step - loss: 0.0347 - val loss: 0.
0497
Epoch 103/150
79/79 [=========== ] - 0s 913us/step - loss: 0.0317 - val loss: 0.
0273
Epoch 104/150
79/79 [=========== ] - 0s 893us/step - loss: 0.0330 - val loss: 0.
0498
Epoch 105/150
79/79 [=========== ] - 0s 928us/step - loss: 0.0391 - val loss: 0.
0526
Epoch 106/150
79/79 [=========== ] - 0s 916us/step - loss: 0.0350 - val loss: 0.
0282
Epoch 107/150
79/79 [========== ] - 0s 890us/step - loss: 0.0363 - val_loss: 0.
0270
Epoch 108/150
79/79 [========== ] - 0s 886us/step - loss: 0.0398 - val_loss: 0.
0295
Epoch 109/150
79/79 [========== ] - 0s 920us/step - loss: 0.0319 - val_loss: 0.
0275
Epoch 110/150
79/79 [========== ] - 0s 935us/step - loss: 0.0326 - val_loss: 0.
Epoch 111/150
79/79 [=========== ] - 0s 897us/step - loss: 0.0431 - val_loss: 0.
0387
Epoch 112/150
79/79 [========== ] - 0s 884us/step - loss: 0.0328 - val_loss: 0.
0466
```

```
Epoch 113/150
79/79 [========== ] - 0s 903us/step - loss: 0.0390 - val_loss: 0.
0331
Epoch 114/150
79/79 [=========== ] - 0s 903us/step - loss: 0.0322 - val_loss: 0.
0386
Epoch 115/150
79/79 [========== ] - 0s 909us/step - loss: 0.0352 - val_loss: 0.
Epoch 116/150
79/79 [========== ] - 0s 902us/step - loss: 0.0384 - val_loss: 0.
Epoch 117/150
79/79 [============ ] - 0s 892us/step - loss: 0.0392 - val loss: 0.
0355
Epoch 118/150
79/79 [=========== ] - 0s 883us/step - loss: 0.0397 - val_loss: 0.
1090
Epoch 119/150
79/79 [========== ] - 0s 890us/step - loss: 0.0414 - val_loss: 0.
0270
Epoch 120/150
79/79 [========== ] - 0s 902us/step - loss: 0.0334 - val_loss: 0.
0539
Epoch 121/150
79/79 [=========== ] - 0s 884us/step - loss: 0.0383 - val loss: 0.
0513
Epoch 122/150
79/79 [=========== ] - 0s 889us/step - loss: 0.0334 - val loss: 0.
Epoch 123/150
0348
Epoch 124/150
79/79 [========== ] - 0s 897us/step - loss: 0.0324 - val_loss: 0.
0714
Epoch 125/150
0508
Epoch 126/150
79/79 [========== ] - 0s 895us/step - loss: 0.0387 - val_loss: 0.
0292
Epoch 127/150
79/79 [=========== ] - 0s 893us/step - loss: 0.0354 - val loss: 0.
Epoch 128/150
79/79 [=========== ] - 0s 884us/step - loss: 0.0424 - val loss: 0.
0276
Epoch 129/150
79/79 [=========== ] - 0s 903us/step - loss: 0.0316 - val loss: 0.
0275
Epoch 130/150
79/79 [=========== ] - 0s 887us/step - loss: 0.0308 - val loss: 0.
0323
Epoch 131/150
79/79 [============ ] - 0s 884us/step - loss: 0.0368 - val loss: 0.
```

```
0418
Epoch 132/150
79/79 [=========== ] - 0s 886us/step - loss: 0.0395 - val loss: 0.
0433
Epoch 133/150
79/79 [=========== ] - 0s 888us/step - loss: 0.0296 - val loss: 0.
Epoch 134/150
79/79 [=========== ] - 0s 878us/step - loss: 0.0347 - val loss: 0.
0258
Epoch 135/150
79/79 [=========== ] - 0s 879us/step - loss: 0.0332 - val loss: 0.
0509
Epoch 136/150
79/79 [=========== ] - 0s 880us/step - loss: 0.0366 - val loss: 0.
0306
Epoch 137/150
79/79 [========== ] - 0s 909us/step - loss: 0.0346 - val_loss: 0.
0302
Epoch 138/150
79/79 [========== ] - 0s 911us/step - loss: 0.0335 - val_loss: 0.
0275
Epoch 139/150
79/79 [========== ] - 0s 890us/step - loss: 0.0346 - val_loss: 0.
Epoch 140/150
79/79 [=========== ] - 0s 885us/step - loss: 0.0336 - val_loss: 0.
0311
Epoch 141/150
79/79 [========== ] - 0s 897us/step - loss: 0.0368 - val_loss: 0.
0900
Epoch 142/150
79/79 [========== ] - 0s 935us/step - loss: 0.0378 - val_loss: 0.
0296
Epoch 143/150
79/79 [=========== ] - 0s 912us/step - loss: 0.0414 - val_loss: 0.
Epoch 144/150
79/79 [========== ] - 0s 894us/step - loss: 0.0306 - val_loss: 0.
Epoch 145/150
79/79 [========== ] - 0s 898us/step - loss: 0.0345 - val_loss: 0.
0352
Epoch 146/150
79/79 [=========== ] - 0s 893us/step - loss: 0.0389 - val loss: 0.
0445
Epoch 147/150
79/79 [===========] - 0s 903us/step - loss: 0.0331 - val_loss: 0.
0463
Epoch 148/150
0289
Epoch 149/150
79/79 [===========] - 0s 890us/step - loss: 0.0377 - val_loss: 0.
Epoch 150/150
```

```
79/79 [============] - 0s 878us/step - loss: 0.0318 - val_loss: 0.0262  
Validation loss: 0.026232393458485603  
32/32 [==========] - 0s 503us/step
```



Epochs: 150

Loss: 0.026

Sigmoid

```
In [ ]: import tensorflow as tf
         from tensorflow.keras.models import Sequential
         from tensorflow.keras.layers import Dense
         import numpy as np
         import matplotlib.pyplot as plt
         # Generate training and testing data
         x_{train} = np.random.uniform(-10, 10, 5000)
         y_train = np.cos(x_train)
         x_{test} = np.random.uniform(-10, 10, 1000)
         y_{\text{test}} = np.cos(x_{\text{test}})
         # Build the model
         model = Sequential([
             Dense(64, input_shape=(1,), activation='sigmoid'), #1st hiddn Layer with tanh
             Dense(64),
             Dense(64),
             Dense(1)
```

```
])
# Compile the model
model.compile(optimizer='adam', loss='mean_squared_error')
# Fit the model
model.fit(x_train, y_train, epochs=150, batch_size=64, validation_data=(x_test, y_t
# Test the model
val_loss = model.evaluate(x_test, y_test, verbose=0)
print(f"Validation loss: {val_loss}")
import matplotlib.pyplot as plt
num test samples = 1000
X_test = np.linspace(-10, 10, num=num_test_samples).reshape(-1, 1)
y_true = np.cos(X_test)
y_pred = model.predict(X_test)
plt.figure(figsize=(10, 6))
plt.plot(X_test, y_true, label='True Cosine Values', color='b', linewidth=2)
plt.plot(X_test, y_pred, label='Model Predictions', color='r', linestyle='--', line
plt.xlabel('Input Value')
plt.ylabel('Cosine Value')
plt.title('Cosine Function and Model Predictions')
plt.legend()
plt.grid()
plt.show()
```

```
Epoch 1/150
93
Epoch 2/150
79/79 [===========] - 0s 936us/step - loss: 0.5485 - val_loss: 0.
5744
Epoch 3/150
79/79 [========== ] - 0s 923us/step - loss: 0.5338 - val_loss: 0.
5265
Epoch 4/150
79/79 [========== ] - 0s 907us/step - loss: 0.5313 - val_loss: 0.
Epoch 5/150
79/79 [=========== ] - 0s 909us/step - loss: 0.5280 - val loss: 0.
5467
Epoch 6/150
79/79 [=========== ] - 0s 909us/step - loss: 0.5280 - val_loss: 0.
5253
Epoch 7/150
79/79 [========== ] - 0s 918us/step - loss: 0.5286 - val_loss: 0.
5292
Epoch 8/150
79/79 [========== ] - 0s 906us/step - loss: 0.5326 - val_loss: 0.
5231
Epoch 9/150
79/79 [========== ] - 0s 920us/step - loss: 0.5264 - val_loss: 0.
5287
Epoch 10/150
79/79 [========== ] - 0s 903us/step - loss: 0.5235 - val_loss: 0.
5227
Epoch 11/150
5274
Epoch 12/150
79/79 [========== ] - 0s 890us/step - loss: 0.5262 - val_loss: 0.
5296
Epoch 13/150
5215
Epoch 14/150
79/79 [========== ] - 0s 909us/step - loss: 0.5221 - val_loss: 0.
5236
Epoch 15/150
79/79 [=========== ] - 0s 941us/step - loss: 0.5232 - val loss: 0.
5177
Epoch 16/150
79/79 [========== ] - 0s 941us/step - loss: 0.5221 - val_loss: 0.
Epoch 17/150
79/79 [=========== ] - 0s 902us/step - loss: 0.5207 - val loss: 0.
5276
Epoch 18/150
79/79 [=========== ] - 0s 884us/step - loss: 0.5183 - val loss: 0.
5153
Epoch 19/150
79/79 [=========== ] - 0s 902us/step - loss: 0.5210 - val loss: 0.
```

```
5208
Epoch 20/150
79/79 [=========== ] - 0s 909us/step - loss: 0.5221 - val loss: 0.
Epoch 21/150
79/79 [=========== ] - 0s 909us/step - loss: 0.5204 - val loss: 0.
Epoch 22/150
79/79 [=========== ] - 0s 923us/step - loss: 0.5146 - val loss: 0.
5132
Epoch 23/150
79/79 [=========== ] - 0s 947us/step - loss: 0.5143 - val loss: 0.
5130
Epoch 24/150
79/79 [=========== ] - 0s 992us/step - loss: 0.5129 - val loss: 0.
5263
Epoch 25/150
79/79 [========== ] - 0s 890us/step - loss: 0.5158 - val_loss: 0.
5253
Epoch 26/150
79/79 [========== ] - 0s 897us/step - loss: 0.5198 - val_loss: 0.
5144
Epoch 27/150
79/79 [========== ] - 0s 910us/step - loss: 0.5144 - val_loss: 0.
Epoch 28/150
79/79 [=========== ] - 0s 898us/step - loss: 0.5107 - val_loss: 0.
5156
Epoch 29/150
79/79 [=========== ] - 0s 901us/step - loss: 0.5158 - val_loss: 0.
5165
Epoch 30/150
79/79 [========== ] - 0s 897us/step - loss: 0.5139 - val_loss: 0.
5165
Epoch 31/150
79/79 [===========] - 0s 897us/step - loss: 0.5133 - val_loss: 0.
5162
Epoch 32/150
79/79 [========== ] - 0s 897us/step - loss: 0.5136 - val_loss: 0.
Epoch 33/150
79/79 [=========== ] - 0s 894us/step - loss: 0.5115 - val_loss: 0.
5155
Epoch 34/150
79/79 [=========== ] - 0s 900us/step - loss: 0.5120 - val loss: 0.
5160
Epoch 35/150
79/79 [===========] - 0s 897us/step - loss: 0.5121 - val_loss: 0.
5153
Epoch 36/150
79/79 [============= ] - 0s 896us/step - loss: 0.5104 - val_loss: 0.
5133
Epoch 37/150
79/79 [===========] - 0s 905us/step - loss: 0.5103 - val_loss: 0.
5283
Epoch 38/150
```

```
79/79 [========== ] - 0s 912us/step - loss: 0.5156 - val_loss: 0.
5115
Epoch 39/150
79/79 [=========== ] - 0s 924us/step - loss: 0.5112 - val loss: 0.
5091
Epoch 40/150
79/79 [========== ] - 0s 947us/step - loss: 0.5108 - val_loss: 0.
5112
Epoch 41/150
79/79 [=========== ] - 0s 906us/step - loss: 0.5095 - val loss: 0.
5141
Epoch 42/150
79/79 [=========== ] - 0s 903us/step - loss: 0.5131 - val loss: 0.
5120
Epoch 43/150
79/79 [=========== ] - 0s 913us/step - loss: 0.5089 - val loss: 0.
Epoch 44/150
79/79 [=========== ] - 0s 892us/step - loss: 0.5156 - val loss: 0.
Epoch 45/150
79/79 [=========== ] - 0s 900us/step - loss: 0.5080 - val loss: 0.
5184
Epoch 46/150
79/79 [=========== ] - 0s 912us/step - loss: 0.5102 - val loss: 0.
5134
Epoch 47/150
79/79 [============ ] - 0s 906us/step - loss: 0.5087 - val loss: 0.
5139
Epoch 48/150
79/79 [=========== ] - 0s 915us/step - loss: 0.5091 - val loss: 0.
5104
Epoch 49/150
79/79 [=========== ] - 0s 909us/step - loss: 0.5073 - val loss: 0.
5110
Epoch 50/150
79/79 [=========== ] - 0s 897us/step - loss: 0.5074 - val loss: 0.
5077
Epoch 51/150
79/79 [=========== ] - 0s 892us/step - loss: 0.5110 - val_loss: 0.
5114
Epoch 52/150
79/79 [========== ] - 0s 897us/step - loss: 0.5080 - val_loss: 0.
5238
Epoch 53/150
79/79 [========== ] - 0s 935us/step - loss: 0.5086 - val_loss: 0.
5111
Epoch 54/150
79/79 [========== ] - 0s 922us/step - loss: 0.5058 - val_loss: 0.
5133
Epoch 55/150
79/79 [=========== ] - 0s 919us/step - loss: 0.5089 - val_loss: 0.
5072
Epoch 56/150
79/79 [========== ] - 0s 909us/step - loss: 0.5049 - val_loss: 0.
5341
```

```
Epoch 57/150
86
Epoch 58/150
79/79 [=========== ] - 0s 922us/step - loss: 0.5096 - val_loss: 0.
5109
Epoch 59/150
79/79 [========== ] - 0s 916us/step - loss: 0.5088 - val_loss: 0.
5087
Epoch 60/150
79/79 [=========== ] - 0s 910us/step - loss: 0.5075 - val_loss: 0.
Epoch 61/150
79/79 [============ ] - 0s 922us/step - loss: 0.5088 - val loss: 0.
5068
Epoch 62/150
79/79 [=========== ] - 0s 928us/step - loss: 0.5084 - val_loss: 0.
5089
Epoch 63/150
79/79 [========== ] - 0s 922us/step - loss: 0.5080 - val_loss: 0.
5215
Epoch 64/150
79/79 [========== ] - 0s 941us/step - loss: 0.5085 - val_loss: 0.
5105
Epoch 65/150
79/79 [=========== ] - 0s 909us/step - loss: 0.5091 - val loss: 0.
5097
Epoch 66/150
79/79 [=========== ] - 0s 941us/step - loss: 0.5064 - val loss: 0.
5075
Epoch 67/150
5053
Epoch 68/150
79/79 [========== ] - 0s 930us/step - loss: 0.5060 - val_loss: 0.
5081
Epoch 69/150
5079
Epoch 70/150
79/79 [========== ] - 0s 928us/step - loss: 0.5084 - val_loss: 0.
5074
Epoch 71/150
79/79 [=========== ] - 0s 928us/step - loss: 0.5060 - val loss: 0.
5066
Epoch 72/150
79/79 [========== ] - 0s 935us/step - loss: 0.5069 - val_loss: 0.
Epoch 73/150
79/79 [=========== ] - 0s 947us/step - loss: 0.5063 - val loss: 0.
5130
Epoch 74/150
79/79 [=========== ] - 0s 941us/step - loss: 0.5084 - val loss: 0.
5053
Epoch 75/150
79/79 [============ ] - 0s 935us/step - loss: 0.5048 - val loss: 0.
```

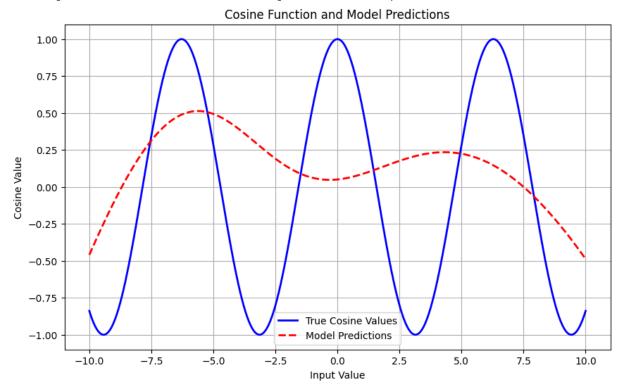
```
5080
Epoch 76/150
79/79 [=========== ] - 0s 928us/step - loss: 0.5086 - val loss: 0.
5200
Epoch 77/150
79/79 [=========== ] - 0s 954us/step - loss: 0.5053 - val loss: 0.
Epoch 78/150
79/79 [=========== ] - 0s 973us/step - loss: 0.5060 - val loss: 0.
5147
Epoch 79/150
79/79 [=========== ] - 0s 973us/step - loss: 0.5068 - val loss: 0.
5056
Epoch 80/150
79/79 [=========== ] - 0s 913us/step - loss: 0.5071 - val loss: 0.
5155
Epoch 81/150
79/79 [========== ] - 0s 902us/step - loss: 0.5059 - val_loss: 0.
5115
Epoch 82/150
79/79 [========== ] - 0s 892us/step - loss: 0.5070 - val_loss: 0.
5078
Epoch 83/150
79/79 [========== ] - 0s 897us/step - loss: 0.5091 - val_loss: 0.
Epoch 84/150
79/79 [=========== ] - 0s 914us/step - loss: 0.5065 - val_loss: 0.
5106
Epoch 85/150
79/79 [========== ] - 0s 894us/step - loss: 0.5047 - val_loss: 0.
5043
Epoch 86/150
79/79 [========== ] - 0s 901us/step - loss: 0.5057 - val_loss: 0.
5045
Epoch 87/150
79/79 [===========] - 0s 884us/step - loss: 0.5038 - val_loss: 0.
5255
Epoch 88/150
79/79 [========== ] - 0s 903us/step - loss: 0.5057 - val_loss: 0.
Epoch 89/150
79/79 [========== ] - 0s 909us/step - loss: 0.5040 - val_loss: 0.
5058
Epoch 90/150
79/79 [=========== ] - 0s 909us/step - loss: 0.5083 - val loss: 0.
5097
Epoch 91/150
79/79 [===========] - 0s 897us/step - loss: 0.5086 - val_loss: 0.
5058
Epoch 92/150
79/79 [============= ] - 0s 892us/step - loss: 0.5026 - val_loss: 0.
5046
Epoch 93/150
79/79 [===========] - 0s 905us/step - loss: 0.5030 - val_loss: 0.
Epoch 94/150
```

```
79/79 [========== ] - 0s 974us/step - loss: 0.5053 - val_loss: 0.
5036
Epoch 95/150
79/79 [=========== ] - 0s 897us/step - loss: 0.5075 - val loss: 0.
5097
Epoch 96/150
79/79 [========== ] - 0s 947us/step - loss: 0.5031 - val_loss: 0.
5200
Epoch 97/150
79/79 [=========== ] - 0s 890us/step - loss: 0.5078 - val loss: 0.
5065
Epoch 98/150
79/79 [=========== ] - 0s 885us/step - loss: 0.5033 - val loss: 0.
5028
Epoch 99/150
79/79 [=========== ] - 0s 895us/step - loss: 0.5020 - val loss: 0.
Epoch 100/150
79/79 [=========== ] - 0s 892us/step - loss: 0.5052 - val loss: 0.
Epoch 101/150
79/79 [=========== ] - 0s 889us/step - loss: 0.5036 - val loss: 0.
5021
Epoch 102/150
79/79 [=========== ] - 0s 895us/step - loss: 0.5061 - val loss: 0.
5037
Epoch 103/150
79/79 [=========== ] - 0s 920us/step - loss: 0.5020 - val loss: 0.
5057
Epoch 104/150
79/79 [=========== ] - 0s 902us/step - loss: 0.5024 - val loss: 0.
5154
Epoch 105/150
79/79 [=========== ] - 0s 916us/step - loss: 0.5023 - val loss: 0.
5037
Epoch 106/150
79/79 [=========== ] - 0s 909us/step - loss: 0.5026 - val loss: 0.
5180
Epoch 107/150
79/79 [========== ] - 0s 904us/step - loss: 0.5098 - val_loss: 0.
5270
Epoch 108/150
79/79 [========== ] - 0s 897us/step - loss: 0.5049 - val_loss: 0.
5057
Epoch 109/150
79/79 [========== ] - 0s 909us/step - loss: 0.5012 - val_loss: 0.
5065
Epoch 110/150
79/79 [========== ] - 0s 884us/step - loss: 0.5035 - val_loss: 0.
5045
Epoch 111/150
79/79 [=========== ] - 0s 894us/step - loss: 0.5044 - val_loss: 0.
5183
Epoch 112/150
79/79 [========== ] - 0s 890us/step - loss: 0.5029 - val_loss: 0.
5309
```

```
Epoch 113/150
79/79 [========== ] - 0s 895us/step - loss: 0.5041 - val_loss: 0.
5046
Epoch 114/150
79/79 [=========== ] - 0s 896us/step - loss: 0.5008 - val_loss: 0.
5054
Epoch 115/150
79/79 [========== ] - 0s 899us/step - loss: 0.4997 - val_loss: 0.
Epoch 116/150
79/79 [=========== ] - 0s 901us/step - loss: 0.4981 - val_loss: 0.
Epoch 117/150
79/79 [=========== ] - 0s 893us/step - loss: 0.5009 - val loss: 0.
5006
Epoch 118/150
79/79 [=========== ] - 0s 915us/step - loss: 0.5001 - val_loss: 0.
4974
Epoch 119/150
79/79 [========== ] - 0s 903us/step - loss: 0.4999 - val_loss: 0.
4998
Epoch 120/150
79/79 [=========== ] - 0s 899us/step - loss: 0.4988 - val_loss: 0.
5132
Epoch 121/150
79/79 [========== ] - 0s 897us/step - loss: 0.4985 - val_loss: 0.
4974
Epoch 122/150
79/79 [=========== ] - 0s 897us/step - loss: 0.5005 - val loss: 0.
Epoch 123/150
5038
Epoch 124/150
79/79 [========== ] - 0s 897us/step - loss: 0.4989 - val_loss: 0.
5066
Epoch 125/150
5065
Epoch 126/150
79/79 [========== ] - 0s 887us/step - loss: 0.4976 - val_loss: 0.
4944
Epoch 127/150
79/79 [=========== ] - 0s 890us/step - loss: 0.4931 - val loss: 0.
4907
Epoch 128/150
79/79 [=========== ] - 0s 890us/step - loss: 0.4945 - val loss: 0.
Epoch 129/150
79/79 [=========== ] - 0s 886us/step - loss: 0.4941 - val loss: 0.
4933
Epoch 130/150
79/79 [=========== ] - 0s 892us/step - loss: 0.4945 - val loss: 0.
5179
Epoch 131/150
79/79 [============ ] - 0s 909us/step - loss: 0.4916 - val loss: 0.
```

```
5077
Epoch 132/150
79/79 [=========== ] - 0s 909us/step - loss: 0.4935 - val loss: 0.
4893
Epoch 133/150
79/79 [=========== ] - 0s 916us/step - loss: 0.4884 - val loss: 0.
Epoch 134/150
79/79 [============ ] - 0s 922us/step - loss: 0.4865 - val loss: 0.
4923
Epoch 135/150
79/79 [============ ] - 0s 899us/step - loss: 0.4933 - val loss: 0.
4772
Epoch 136/150
79/79 [=========== ] - 0s 897us/step - loss: 0.4849 - val loss: 0.
4760
Epoch 137/150
79/79 [=========== ] - 0s 889us/step - loss: 0.4816 - val_loss: 0.
4803
Epoch 138/150
79/79 [=========== ] - 0s 897us/step - loss: 0.4807 - val_loss: 0.
Epoch 139/150
79/79 [========== ] - 0s 910us/step - loss: 0.4792 - val_loss: 0.
4734
Epoch 140/150
79/79 [============ ] - 0s 895us/step - loss: 0.4772 - val_loss: 0.
4807
Epoch 141/150
79/79 [=========== ] - 0s 903us/step - loss: 0.4816 - val_loss: 0.
4706
Epoch 142/150
79/79 [=========== ] - 0s 922us/step - loss: 0.4684 - val_loss: 0.
4971
Epoch 143/150
79/79 [=========== ] - 0s 913us/step - loss: 0.4735 - val_loss: 0.
Epoch 144/150
79/79 [=========== ] - 0s 894us/step - loss: 0.4717 - val_loss: 0.
Epoch 145/150
79/79 [========== ] - 0s 916us/step - loss: 0.4768 - val_loss: 0.
4772
Epoch 146/150
79/79 [=========== ] - 0s 973us/step - loss: 0.4701 - val loss: 0.
4601
Epoch 147/150
79/79 [===========] - 0s 973us/step - loss: 0.4661 - val_loss: 0.
4539
Epoch 148/150
79/79 [==========] - 0s 913us/step - loss: 0.4679 - val_loss: 0.
5211
Epoch 149/150
79/79 [===========] - 0s 916us/step - loss: 0.4696 - val_loss: 0.
4695
Epoch 150/150
```

```
79/79 [===========] - 0s 895us/step - loss: 0.4571 - val_loss: 0.4766
Validation loss: 0.47656092047691345
32/32 [==========] - 0s 496us/step
```



Epochs: 150

Loss: 0.47

Our Latest Hermite

```
class H2Layer(Layer):
   def __init__(self, h1, **kwargs):
        super(H2Layer, self).__init__(**kwargs)
        self.h1 = h1
   def call(self, x):
        return (2*x*(self.h1(x)))-2
class H3Layer(Layer):
   def __init__(self, h1, h2, **kwargs):
       super(H3Layer, self).__init__(**kwargs)
        self.h1 = h1
        self.h2 = h2
   def call(self, x):
        return (2*x*(self.h2(x)))-(4*self.h1(x))
class H4Layer(Layer):
   def __init__(self, h2, h3, **kwargs):
        super(H4Layer, self).__init__(**kwargs)
        self.h2 = h2
        self.h3 = h3
   def call(self, x):
        return (2*x*(self.h3(x)))-(6*self.h2(x))
class H5Layer(Layer):
   def __init__(self, h3, h4, **kwargs):
        super(H5Layer,self).__init__(**kwargs)
        self.h3 = h3
        self.h4 = h4
   def call(self,x):
        return (2*x*(self.h4(x)))-(8*self.h3(x))
class H6Layer(Layer):
   def init (self, h4, h5, **kwargs):
        super(H6Layer,self).__init__(**kwargs)
        self.h4 = h4
        self.h5 = h5
   def call(self,x):
        return (2*x*(self.h5(x)))-(10*self.h4(x))
class TensorDecompositionLayer(Layer):
   def __init__(self, rank, **kwargs):
        self.rank = rank
        super(TensorDecompositionLayer, self).__init__(**kwargs)
   def build(self, input_shape):
        self.factors_a = self.add_weight(shape=(input_shape[-1], self.rank),
                                         initializer='random normal',
                                         trainable=True)
        self.factors_b = self.add_weight(shape=(self.rank, input_shape[-1]),
                                         initializer='random normal',
```

```
trainable=True)
        super(TensorDecompositionLayer, self).build(input_shape)
    def call(self, x):
        return tf.matmul(tf.matmul(x, self.factors_a), self.factors_b)
def build_model(input_shape, filters):
    rank = 3
    input layer = Input(shape=input shape)
    x = input_layer
    h1 = H1Layer()
    h2 = H2Layer(h1)
    h3 = H3Layer(h1,h2)
    h4 = H4Layer(h2,h3)
    h5 = H5Layer(h3,h4)
    h6 = H6Layer(h4, h5)
    x = Dense(filters)(x)
    x = h2(x)
    x = Dense(filters)(x)
    x = TensorDecompositionLayer(rank)(x)
    x = h3(x)
    x = Dense(filters)(x)
    x = TensorDecompositionLayer(rank)(x)
    x = h4(x)
    x = Dense(filters)(x)
    x = TensorDecompositionLayer(rank)(x)
    x = h5(x)
    x = Dense(filters)(x)
    x = TensorDecompositionLayer(rank)(x)
    x = h6(x)
    x = Dense(filters)(x)
    x = TensorDecompositionLayer(rank)(x)
    output_layer = Dense(1)(x)
    model = Model(inputs=input layer, outputs=output layer)
    return model
input\_shape = (1,)
filters = 16
model = build model(input shape, filters)
optimizer = Adam(learning_rate=0.0001) # Reduce Learning rate
model.compile(optimizer=optimizer, loss='mse')
import numpy as np
np.random.seed(42)
n_samples = 10000
#Lower_bound = -2 * np.pi
#upper bound = 2 * np.pi
lower bound = -10
upper_bound = 10
```

```
\# X = np.random.uniform(lower_bound, upper_bound, size=(n_samples, 1))
X = np.arange(lower_bound, upper_bound, 0.001)
y = np.cos(X)
from sklearn.model_selection import train_test_split
X_train, X_val, y_train, y_val = train_test_split(X, y, test_size=0.2, random_state
batch size = 64
epochs = 150
history = model.fit(X_train, y_train,
                    batch_size=batch_size,
                    epochs=epochs,
                    verbose=1,
                    validation_data=(X_val, y_val))
val_loss = model.evaluate(X_val, y_val, verbose=0)
print(f"Validation loss: {val_loss}")
import matplotlib.pyplot as plt
num_test_samples = 1000
X_test = np.linspace(lower_bound, upper_bound, num=num_test_samples).reshape(-1, 1)
y_true = np.cos(X_test)
y_pred = model.predict(X_test)
plt.figure(figsize=(10, 6))
plt.plot(X_test, y_true, label='True Cosine Values', color='b', linewidth=2)
plt.plot(X_test, y_pred, label='Model Predictions', color='r', linestyle='--', line
plt.xlabel('Input Value')
plt.ylabel('Cosine Value')
plt.title('Cosine Function and Model Predictions')
plt.legend()
plt.grid()
plt.show()
```

```
Epoch 1/150
5020
Epoch 2/150
250/250 [============= ] - 0s 1ms/step - loss: 0.4726 - val_loss: 0.
4570
Epoch 3/150
250/250 [============] - 0s 1ms/step - loss: 0.4099 - val_loss: 0.
3958
Epoch 4/150
250/250 [============] - 0s 1ms/step - loss: 0.3890 - val_loss: 0.
Epoch 5/150
3875
Epoch 6/150
250/250 [============] - 0s 1ms/step - loss: 0.3845 - val_loss: 0.
3869
Epoch 7/150
250/250 [============] - 0s 1ms/step - loss: 0.3834 - val_loss: 0.
3864
Epoch 8/150
250/250 [=============] - 0s 1ms/step - loss: 0.3820 - val_loss: 0.
3839
Epoch 9/150
250/250 [============= ] - 0s 1ms/step - loss: 0.3838 - val_loss: 0.
3888
Epoch 10/150
250/250 [============= ] - 0s 1ms/step - loss: 0.3811 - val_loss: 0.
Epoch 11/150
3826
Epoch 12/150
250/250 [============= ] - 0s 1ms/step - loss: 0.3797 - val_loss: 0.
3828
Epoch 13/150
3808
Epoch 14/150
250/250 [============= ] - 0s 1ms/step - loss: 0.3782 - val_loss: 0.
3848
Epoch 15/150
Epoch 16/150
250/250 [============= ] - 0s 1ms/step - loss: 0.3752 - val_loss: 0.
Epoch 17/150
3741
Epoch 18/150
250/250 [============= ] - 0s 1ms/step - loss: 0.3702 - val loss: 0.
3647
Epoch 19/150
```

```
2013
Epoch 20/150
250/250 [============= ] - 0s 1ms/step - loss: 0.2070 - val loss: 0.
1967
Epoch 21/150
250/250 [============= ] - Os 1ms/step - loss: 0.1984 - val loss: 0.
Epoch 22/150
250/250 [============= ] - 0s 1ms/step - loss: 0.1793 - val loss: 0.
1500
Epoch 23/150
250/250 [============ ] - 0s 1ms/step - loss: 0.1301 - val loss: 0.
1270
Epoch 24/150
0557
Epoch 25/150
250/250 [=============] - 0s 2ms/step - loss: 0.0334 - val_loss: 0.
Epoch 26/150
250/250 [============= ] - 0s 1ms/step - loss: 0.0142 - val_loss: 0.
0106
Epoch 27/150
250/250 [============= ] - 0s 1ms/step - loss: 0.0099 - val_loss: 0.
Epoch 28/150
0074
Epoch 29/150
250/250 [============= ] - 0s 1ms/step - loss: 0.0071 - val_loss: 0.
0064
Epoch 30/150
250/250 [============= ] - 0s 1ms/step - loss: 0.0063 - val_loss: 0.
0065
Epoch 31/150
250/250 [============] - 0s 1ms/step - loss: 0.0053 - val_loss: 0.
0045
Epoch 32/150
250/250 [============] - 0s 1ms/step - loss: 0.0044 - val_loss: 0.
Epoch 33/150
250/250 [============= ] - 0s 1ms/step - loss: 0.0032 - val_loss: 0.
0026
Epoch 34/150
250/250 [============= ] - 0s 1ms/step - loss: 0.0020 - val_loss: 0.
0017
Epoch 35/150
250/250 [============= ] - Os 1ms/step - loss: 0.0014 - val_loss: 8.
8657e-04
Epoch 36/150
7842e-04
Epoch 37/150
s: 7.6063e-04
Epoch 38/150
```

```
250/250 [============= ] - 0s 1ms/step - loss: 0.0013 - val_loss: 6.
0266e-04
Epoch 39/150
s: 6.4347e-04
Epoch 40/150
s: 4.6990e-04
Epoch 41/150
3777e-04
Epoch 42/150
s: 4.1122e-04
Epoch 43/150
s: 3.2674e-04
Epoch 44/150
s: 3.1717e-04
Epoch 45/150
s: 2.6097e-04
Epoch 46/150
s: 2.5650e-04
Epoch 47/150
s: 2.2475e-04
Epoch 48/150
s: 2.5106e-04
Epoch 49/150
s: 2.3455e-04
Epoch 50/150
s: 1.9452e-04
Epoch 51/150
s: 6.2152e-04
Epoch 52/150
s: 2.9750e-04
Epoch 53/150
s: 2.1964e-04
Epoch 54/150
s: 1.6598e-04
Epoch 55/150
s: 1.5522e-04
Epoch 56/150
s: 1.4666e-04
```

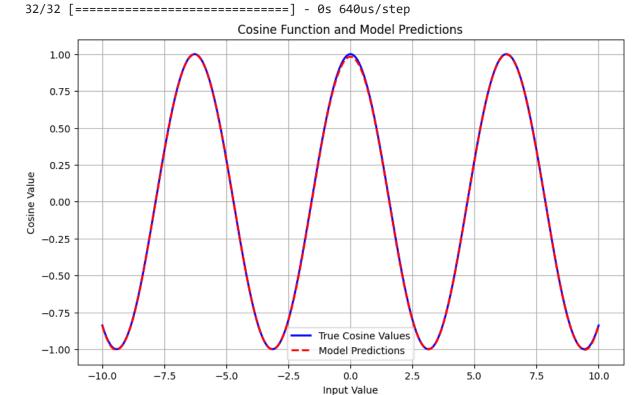
```
Epoch 57/150
s: 2.7621e-04
Epoch 58/150
s: 1.7632e-04
Epoch 59/150
s: 1.1176e-04
Epoch 60/150
s: 1.1285e-04
Epoch 61/150
s: 1.5897e-04
Epoch 62/150
s: 1.0183e-04
Epoch 63/150
s: 8.7182e-05
Epoch 64/150
s: 2.7299e-04
Epoch 65/150
s: 1.4050e-04
Epoch 66/150
s: 7.2869e-05
Epoch 67/150
s: 1.3102e-04
Epoch 68/150
s: 2.4055e-04
Epoch 69/150
s: 2.5392e-04
Epoch 70/150
s: 5.6846e-05
Epoch 71/150
s: 2.3769e-04
Epoch 72/150
s: 1.5903e-04
Epoch 73/150
s: 1.5717e-04
Epoch 74/150
s: 4.5748e-05
Epoch 75/150
```

```
s: 4.1174e-05
Epoch 76/150
s: 3.8007e-05
Epoch 77/150
s: 9.6931e-05
Epoch 78/150
s: 3.1775e-05
Epoch 79/150
s: 4.2438e-05
Epoch 80/150
s: 1.3028e-04
Epoch 81/150
s: 4.8657e-05
Epoch 82/150
s: 3.4311e-05
Epoch 83/150
s: 3.0396e-05
Epoch 84/150
s: 2.3950e-05
Epoch 85/150
s: 7.2661e-05
Epoch 86/150
s: 2.8421e-05
Epoch 87/150
s: 1.0546e-04
Epoch 88/150
s: 2.3402e-05
Epoch 89/150
s: 2.5783e-05
Epoch 90/150
s: 3.5875e-05
Epoch 91/150
s: 4.9289e-05
Epoch 92/150
s: 1.3369e-05
Epoch 93/150
s: 2.6491e-05
Epoch 94/150
```

```
s: 2.4816e-05
Epoch 95/150
s: 3.6988e-05
Epoch 96/150
s: 2.6605e-05
Epoch 97/150
s: 1.5069e-05
Epoch 98/150
s: 2.3351e-05
Epoch 99/150
s: 1.0409e-05
Epoch 100/150
s: 1.7738e-05
Epoch 101/150
s: 2.0019e-05
Epoch 102/150
s: 7.6052e-06
Epoch 103/150
s: 1.4429e-05
Epoch 104/150
s: 1.7476e-05
Epoch 105/150
s: 1.6682e-04
Epoch 106/150
s: 2.3678e-05
Epoch 107/150
s: 1.0784e-05
Epoch 108/150
s: 1.8008e-05
Epoch 109/150
s: 1.4265e-05
Epoch 110/150
s: 2.9633e-05
Epoch 111/150
s: 1.0939e-05
Epoch 112/150
s: 8.2533e-06
```

```
Epoch 113/150
s: 2.4087e-05
Epoch 114/150
s: 7.8610e-06
Epoch 115/150
s: 7.9300e-04
Epoch 116/150
s: 6.8962e-06
Epoch 117/150
s: 5.9142e-06
Epoch 118/150
s: 7.6758e-05
Epoch 119/150
s: 1.5949e-05
Epoch 120/150
s: 7.8777e-06
Epoch 121/150
s: 6.5308e-06
Epoch 122/150
s: 1.0196e-05
Epoch 123/150
s: 5.5124e-06
Epoch 124/150
s: 1.0095e-05
Epoch 125/150
s: 1.1249e-05
Epoch 126/150
s: 1.1293e-04
Epoch 127/150
s: 9.7145e-06
Epoch 128/150
s: 2.1120e-05
Epoch 129/150
s: 4.8920e-05
Epoch 130/150
s: 3.5531e-05
Epoch 131/150
```

```
s: 2.5309e-05
Epoch 132/150
s: 2.7792e-05
Epoch 133/150
s: 6.7250e-06
Epoch 134/150
s: 2.3094e-05
Epoch 135/150
s: 1.1117e-05
Epoch 136/150
s: 8.0138e-06
Epoch 137/150
s: 9.6207e-06
Epoch 138/150
s: 1.2769e-05
Epoch 139/150
s: 8.3861e-06
Epoch 140/150
s: 9.4143e-05
Epoch 141/150
s: 6.8808e-05
Epoch 142/150
s: 5.4198e-06
Epoch 143/150
s: 3.4500e-05
Epoch 144/150
s: 4.6880e-06
Epoch 145/150
s: 6.4382e-04
Epoch 146/150
s: 3.9778e-05
Epoch 147/150
s: 1.5246e-05
Epoch 148/150
s: 7.2196e-06
Epoch 149/150
s: 6.3672e-05
Epoch 150/150
```



Epochs: 150

Loss: 0.000031

In []: