Advanced Trade

Week 2: Eaton and Kortum (2002)

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Consideraciones iniciales

- El modelo es "normativo" en el sentido de que no se basa en datos empíricos, sino en lo que "debería ser", partiendo de supuestos similares a los de DFS (1977) y ampliandoló a muchos paises e incorporando la geografía.
- Entrega 4 hechos estilizados:
  1. All countries benefit from freer world trade, with small countries gaining more than big ones.
  2. A decline in geographic barriers from their current level tends to work against the largest countries and favor the smallest.
  3. Improvement in technology raises welfare. The gains abroad approach those at home only in countries enjoying proximity to the source and the flexibility to downsize manufacturing.
  4. Big countries are likely to suffer if they drop tariffs unilaterally.

¿Why the EK model?

- The EK model is a general equilibrium model of international trade that incorporates the role of geography and technology in shaping trade patterns.
- It provides a framework for understanding how countries interact in the global economy, taking into account factors such as distance, size, and technological differences.
- The model is based on the idea that countries with similar characteristics are more likely to trade with each other, leading to a more efficient allocation of resources.
- In this sense, they want to show that the model can explain that (i) the size of countries matters, and (ii) the distance between countries matters.
- The EK model has been widely used in empirical research to analyze trade patterns and the impact of trade policies on economic outcomes.

El set up del modelo EK

- **Continuo de bienes:** cada país produce potencialmente un conjunto [0,1] de bienes.
- **Preferencias CES:** la demanda final (y de insumos) se modela con una función CES sobre ese continuo.
- **Iceberg trade costs:** mover 1 unidad de un bien desde el país i hasta el país n requiere enviar d_{ni} unidades.
- **Tecnología Ricardiana:** la productividad para cada bien en cada país se extrae de una distribución Fréchet con parámetros $\{T_i, \theta\}$. ▸ Why Fréchet?
- **Competencia perfecta:** cada bien se compra al productor (país) que lo ofrece al menor costo puesto en destino.

Cost and prices

- Cada país tiene diferente acceso a tecnología, por lo que la eficiencia varía entre commodities y entre paises.
- Given this, the cost of producing a unit of good j in country i is $c_i/z_i(j)$ where c_i is the input cost of country i.
- Delivering a unit of good j produced in country i to country n costs

$$p_{ni}(j) = \left(\frac{c_i}{z_i(j)}\right) d_{ni}$$

  which is the unit cost multiplied by the geographic barrier.
- Given perfect competition, the actual price paid for good j in country n is:

$$p_n(j) = \min\left\{p_{ni}(j); i = 1, \ldots, N\right\}$$

Consumers: Utility Maximization

- En cada país, un consumidor representativo elige $\{Q(j)\}_{j\in[0,1]}$ para maximizar:

$$U = \left[\int_0^1 Q(j)^{(\sigma-1)/\sigma} dj\right]^{\sigma/(\sigma-1)}$$

donde $\sigma > 0$ es la elasticidad de sustitución.

- La restricción presupuestaria (con gasto total X_n) es:

$$\int_0^1 p_n(j)\, Q(j)\, dj \;\; \leq \;\; X_n,$$

donde $p_n(j)$ es el precio de cada bien j en el país n.

- Dadas las condiciones de primer orden, la demanda individual $Q(j)$ depende de los precios relativos y del gasto total X_n.

Tecnología: distribución Fréchet

- Cada país i obtiene un "sorteo" de productividad $z_i(j)$ para cada bien $j \in [0,1]$.
- Se asume que $z_i(j)$ proviene de una **distribución Fréchet**:

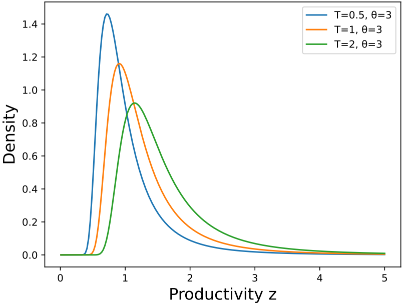
$$F_i(z) = \Pr\Big(Z_i \leq z\Big) = e^{-T_i\, z^{-\theta}}, \quad T_i > 0,\ \theta > 1.$$

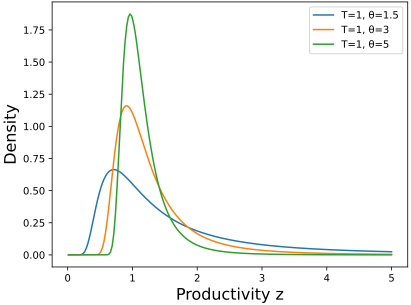
  by the law of large numbers, $F_i(z)$ is also the fraction of goods for which country i's efficiency is below z.

- T_i mide el estado de la tecnología (ventaja absoluta): cuanto mayor T_i, mayor probabilidad de obtener productividades altas.
- θ mide la dispersión de las productividades (ventaja comparativa): valores menores de θ implican mayor heterogeneidad.

Visualizando la distribución Fréchet

- **Izquierda:** Al incrementar T_i, se incrementa la probabilidad de obtener productividades altas. Esto representa un *mayor estado de tecnología*, o *ventaja absoluta*, el país puede producir (en promedio) con mayor eficiencia.
- **Derecha:** Un θ mayor reduce la varianza. Con *menor dispersión* se debilita la fuerza de la ventaja comparativa, ya que es menos probable que un país "gane" por outliers.





¿Quién produce cada bien?

- Ya dijimos que para cada bien j, el productor efectivo es el que logre el menor costo puesto en el destino n.

$$p_n(j) = \min_i \left\{ \left(\frac{c_i}{z_i(j)} \right) d_{ni} \right\}.$$

- La probabilidad de que el país i venda en n equivale a la **fracción de bienes para los cuales i es la fuente más barata en n**:

$$\pi_{ni} \;=\; \Pr\left[\left(\frac{c_i}{z_i} \right) d_{ni} \;\leq\; \min_{\forall k \neq i} \left\{ \left(\frac{c_k}{z_k} \right) d_{nk} \right\} \right] = \frac{T_i (c_i d_{ni})^{-\theta}}{\sum_{s=1}^{N} T_s (c_s d_{ns})^{-\theta}}.$$

- Dado que z_i se distribuye con Fréchet, la fracción de bienes que n compra a i resulta en una forma muy sencilla.

Distribución de precios

- Country i *presents* country n with a distribution of prices:

$$G_{ni}(p) \;=\; \Pr\Big(P_{ni} \leq p\Big) \;=\; 1 \;-\; F_i\Big(\tfrac{c_i d_{ni}}{p}\Big),$$

donde $P_{ni} = \left(\frac{c_i}{z_i}\right) d_{ni}$.

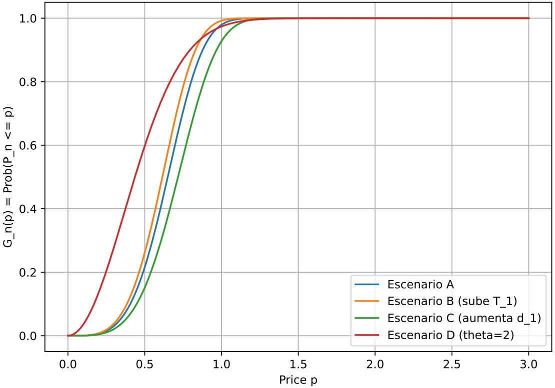
- El precio *efectivo* que paga n por cada bien es $P_n = \min_i \{P_{ni}\}$.
- Hence the distribution of actually prices paid for country n is

$$G_n(p) \;=\; \Pr(P_n \leq p) \;=\; 1 - \prod_{i=1}^{N} \Big[1 - G_{ni}(p)\Big].$$

Distribución de precios

- Substituyendo $G_{ni}(p)$ en la forma Fréchet de F_i, se demuestra que

$$G_n(p) \;=\; 1-\exp\left(-\Phi_n\, p^{\theta}\right), \quad \text{donde} \quad \Phi_n \;=\; \sum_{i=1}^{N} T_i\, (c_i d_{ni})^{-\theta} \;=\; \sum_{i=1}^{N} \frac{T_i}{(c_i d_{ni})^{\theta}}.$$



Distribución de precios

- En particular, cada factor en Φ_n:
  1. Cambios en $\mathbf{T_i}$: $\frac{\partial \Phi_n}{\partial T_i} > 0$ y $\frac{\partial G_n(p)}{\partial \Phi_n} > 0 \Rightarrow \frac{\partial G_n(p)}{\partial T_i} > 0$.

     $G_n(p)$ se "desplaza hacia arriba", lo cual significa que el país n enfrenta *mayor probabilidad* de encontrar precios bajos, y en promedio paga menos.
  2. Cambios en c_i y d_{ni} afectan en la misma dirección. El país i se vuelve menos competitivo $\Rightarrow n$ enfrenta precios más altos.

     In autarky, $d_{ni} \to \inf, \forall i \neq n \Rightarrow \Phi_n = T_n c_n^{-\theta}$ (prices only depends on local conditions).
  3. Aumentos en θ disminuyen la varianza de la distribución $rightarrow$ se debilita la ventaja comparativa (menos outliers) $\Rightarrow n$ enfrenta precios más altos.
- Un Φ_n *grande* significa más "competencia" de proveedores eficientes/cercanos, reduciendo los precios que paga n.

Tres propiedades clave de la distribución de precios

(a) Fracción de bienes que país n compra a i: ‣ Math

$$\pi_{ni} = \frac{T_i \left(c_i d_{ni}\right)^{-\theta}}{\Phi_n} \quad \Longrightarrow \quad \text{“probabilidad de que } i \text{ sea la fuente más barata”.}$$

(b) Distribución del precio comprado: ‣ Math

El precio de cualquier bien que n *efectivamente* compra en i sigue la misma $G_n(p)$.

(c) Índice de precios CES: ‣ Math

$$p_n = \gamma \, \Phi_n^{-\frac{1}{\theta}}, \quad \text{con} \;\; \gamma = \left[\Gamma\!\left(\frac{\theta+1-\sigma}{\theta}\right)\right]^{1/(1-\sigma)}.$$

Al depender Φ_n de $\{T_i, c_i, d_{ni}\}$, surgen *desviaciones de paridad de poder de compra* (PPP).

Corolario de la propiedad (b): *Conditional = Unconditional*

- Esto es bien bien importante:
  - Countries that have higher $c's$, $d's$, or lower $T's$, sell a smaller range of goods, but charge the same average prices. **Adjustment is at the extensive margin.**
  - Aunque cada proveedor i sea distinto en costos/tecnología, la dispersión de los precios comprados es la misma para todo i.

- Esto implica que el gasto promedio **por bien** del país n no varía según el origen del bien.

- La fracción de bienes que el país n compra al país i también representa la fracción del gasto total del país n en bienes provenientes del país i:

$$\pi_{ni} = \frac{X_{ni}}{X_n} = \frac{T_i(c_i d_{ni})^{-\theta}}{\Phi_n} = \frac{T_i(c_i d_{ni})^{-\theta}}{\sum_{k=1}^{N} T_k(c_k d_{nk})^{-\theta}} \qquad X's \text{ are expenditures.}$$

  Aquí, el numerador $T_i(c_i d_{ni})^{-\theta}$ captura la competitividad de i, y el denominador agrega esta medida para todos los exportadores. En consecuencia, si, por ejemplo, n compra el 30% de sus bienes a i, también gasta el 30% de su presupuesto en bienes de i.

Aggregate Trade Flows

- At this point, we can also solve for aggregate trade flows.
- **Total Production of Country i:**

$$Q_i = \sum_{m=1}^{N} X_{mi} = T_i(c_i)^{-\theta} \sum_{m=1}^{N} \frac{d_{mi}^{-\theta} X_m}{\Phi_m}$$

- Solving for $T_i(c_i)^{-\theta}$ and plugging this into our expression from the last slide for X_{ni}/X_n and also with $\Phi_n = (p_n)^{-\theta}\gamma^{\theta}$, we get:

$$X_{ni} = \frac{\left(\frac{d_{ni}}{p_n}\right)^{-\theta} X_n}{\sum_{m=1}^{N} \left(\frac{d_{mi}}{p_m}\right)^{-\theta} X_m} Q_i$$

- Lo cual exhibe una forma “gravity-like” en la que X_{ni} depende del tamaño de mercado X_n, de la competitividad Q_i, y de las barreras d_{ni} relativas al nivel de precios p_n.

Gravity in Trade Flows

If we take logs of the equation on the previous side, we get something that looks a lot like a gravity regression:

$$\ln X_{ni} = \underbrace{-\ln\left(\sum_{m=1}^{N}\left(\frac{d_{mi}}{p_m}\right)^{-\theta} X_m\right)}_{\text{Country } i \text{ f.e.}} + \underbrace{\theta \ln p_n}_{\text{Country } n \text{ f.e.}} - \underbrace{\theta \ln d_{ni}}_{\text{distance}} + \underbrace{\ln X_n + \ln Q_i}_{\text{Country } i \text{ and } n\text{'s sizes.}}$$

- Gross output, X_n and Q_i, enters linearly (unit elasticities).
- Distance is regulated by $-\theta$—more dispersion in productivity (lower θ), more likely goods will travel farther distances.

Closing the Model

- So far, the model has taken input costs c_i as given (constant). But in a GE setting, these costs are endogenous: they depend on wages and prices of intermediate inputs, which themselves adjust to global changes.
- This matters especially in counterfactuals: when trade barriers or technology change, input costs must respond, otherwise we'd miss key general equilibrium effects.
- Therefore, to close the model, they explicitly model how c_i depends on wages and intermediate prices, and how those in turn are determined by trade and production structure.

Production Structure

- EK assume production is Cobb-Douglas in labor and a basket of intermediate goods.
- Intermediate goods form a CES bundle of the same goods consumed.

  $\Rightarrow$ Input costs are now affected by trade.
- This implies that the cost of an input bundle in country i will be equal to:

$$c_i = w_i^{\beta} \, p_i^{1-\beta}$$

where w_i is wages and p_i is the same CES price index we derived earlier.

Real Wages

Next, we can write down an expression for real wages. Start with the price index, p_i:

$$p_n = \gamma \left(\Phi_n\right)^{-\frac{1}{\theta}} \implies \Phi_i = (p_i)^{-\theta}\gamma^{\theta}$$

$$\frac{X_{ni}}{X_n} = \frac{T_i(c_i d_{ni})^{-\theta}}{\Phi_n} \implies \pi_{ii} = \frac{T_i(c_i)^{-\theta}}{\Phi_i}$$

Plugging in for c_i and Φ_i, we have:

$$\pi_{ii} = \frac{T_i(w_i^{\beta} p_i^{1-\beta})^{-\theta}}{(p_i)^{-\theta}\gamma^{\theta}} \implies \frac{w_i}{p_i} = \gamma^{-\frac{1}{\beta}} \left(\frac{T_i}{\pi_{ii}}\right)^{\frac{1}{\beta\theta}}$$

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Trade Gains Insight

In autarky, $\pi_{ii} = 1$. $\Rightarrow$ We can infer **gains from trade** from the share of imports in total purchases $(1 - \pi_{ii})$. Importantly, for a given import share, **gains are larger when:**

- θ is **small** (i.e., greater heterogeneity in efficiency);
- β is **large** (i.e., higher share of intermediates in production).

Equilibrium Prices

Next, we can plug in input costs to get an expression for the price level:

$$p_n = \gamma \left(\Phi_n\right)^{-\frac{1}{\theta}} = \gamma \left(\sum_{i=1}^{N} T_i (c_i d_{ni})^{-\theta} \right)^{-\frac{1}{\theta}}$$

$$= \gamma \left(\sum_{i=1}^{N} T_i (w_i^{\beta} p_i^{1-\beta} d_{ni})^{-\theta} \right)^{-\frac{1}{\theta}}$$

Given w_i, this generally needs to be solved numerically. We can also plug in input costs here:

$$\frac{X_{ni}}{X_n} = \pi_{ni} = T_i \left(\frac{\gamma d_{ni} w_i^{\beta} p_i^{1-\beta}}{p_n} \right)^{-\theta}$$

Labor Market Equilibrium

Labor income equals labor's share of output value:

$$w_i L_i = \beta Q_i = \beta \sum_{i=1}^{N} X_{ni} = \beta \sum_{n=1}^{N} \pi_{ni} X_n$$

Total expenditures in country n are:

$$X_n = \frac{1-\beta}{\beta} w_n L_n + w_n L_n = \frac{1}{\beta} w_n L_n$$

Wages thus satisfy:

$$w_i L_i = \sum_{n=1}^{N} \pi_{ni} w_n L_n$$

Labor Market Equilibrium

Labor Income from Manufacturing

$$Q_i = \sum_{n=1}^{N} X_{ni} \quad \Rightarrow w_i L_i = \beta Q_i = \beta \sum_{n=1}^{N} X_{ni} = \beta \sum_{n=1}^{N} \pi_{ni} X_n$$

The model closes by focusing on **manufacturing**:

- Value added in manufacturing: $Y_n^M = w_n L_n$
- Final total expenditure: $Y_n = \alpha Y_n^M + (1-\alpha) Y_n^O \to Y_n^M = \alpha Y_n$

By Cobb-Douglas production, intermediate input demand is (using $\beta Q_n = w_n L_n$):

$$(1-\beta) Q_n = (1-\beta) \frac{w_n L_n}{\beta} = \frac{1-\beta}{\beta} w_n L_n$$

Then total manufacturing expenditure is:

$$X_n = \frac{1-\beta}{\beta} w_n L_n + \alpha Y_n = \frac{1-\beta}{\beta} w_n L_n + \alpha w_n L_n$$

Special Case

We can assume $\alpha = 1$ (all expenditure on manufactures).
This is a special case of the model, but it's not the only one. In this case, total expenditure on manufactures is:

$$X_n = \frac{1}{\beta} w_n L_n$$

And then

$$w_i L_i = \sum_{n=1}^{N} \pi_{ni} w_n L_n$$

Model Closure: Two Labor Market Assumptions

E&K consider two polar cases regarding how labor interacts with the manufacturing sector:

Case 1: Mobile Labor (wage is exogenous)

- Workers freely reallocate between manufacturing and nonmanufacturing.
- Nonmanufacturing productivity pins down the wage w_n.
- Manufacturing employment L_i adjusts to satisfy

$$w_i L_i = \sum_{n=1}^{N} \pi_{ni} \left[(1-\beta) w_n L_n + \alpha \beta Y_n \right] \quad \text{where } Y_n \text{ is exogenous.}$$

Case 2: Immobile Labor (employment is exogenous)

- Each country has a fixed manufacturing labor force L_n.
- Wage w_i adjusts to clear the market:

$$w_i L_i = \sum_{n=1}^{N} \pi_{ni} \left[(1-\beta+\alpha\beta) w_n L_n + \alpha \beta Y_n^O \right] \quad \text{where } Y_n^O \text{ is exogenous.}$$

More assumptions

$\alpha = 1$ and mobile labor

If $\alpha = 1$ then there is no nonmanufacturing sector. This simplification makes the equilibrium to be determined by three sets of equations:

Wages:

$$w_i L_i = \sum_{n=1}^{N} \pi_{ni} w_n L_n, \quad i = 1, 2, ..., N$$

Trade Shares:

$$\pi_{ni} = T_i \left(\frac{\gamma d_{ni} w_i^{\beta} p_i^{1-\beta}}{p_n} \right)^{-\theta}, \quad i, n = 1, 2..., N$$

Prices:

$$p_n = \gamma \left[\sum_{i=1}^{N} T_i \left(d_{ni} w_i^{\beta} p_i^{1-\beta} \right)^{-\theta} \right]^{-\frac{1}{\theta}}, \quad n = 1, 2, ..., N$$

This particular case makes the model easier to interpret and solve.

Special Case: Free Trade

EK show that in **free trade**, $d_{ni} = 1$, real GDP per worker (Welfare) is:

$$W_i = \frac{Y_i/L_i}{p_i} = \frac{w_i}{p_i} = \gamma^{-1/\beta} T_i^{1/(1+\theta\beta)} \left[\sum_{k=1}^{N} T_k^{1/(1+\theta\beta)} \left(\frac{L_k}{L_i} \right)^{\theta\beta/(1+\theta\beta)} \right]^{1/(\theta\beta)}$$

Take L_i outside the bracket:

$$W_i = \gamma^{-1/\beta} T_i^{1/(1+\theta\beta)} L_i^{-1/(1+\theta\beta)} \left[\sum_{k=1}^{N} T_k^{1/(1+\theta\beta)} L_k^{\theta\beta/(1+\theta\beta)} \right]^{1/(\theta\beta)}$$

- Welfare is increasing in T_k everywhere.
- Increase in home T also benefits foreign wages.
- Gains depend on relative labor force size. Small countries benefit more from technological improvements abroad.

Special Case: Autarky

Since $\pi_{ii} = 1$, Eaton-Kortum show that:

$$\frac{w_i}{p_i} = \gamma^{-\frac{1}{\beta}} \left(\frac{T_i}{\pi_{ii}}\right)^{\frac{1}{\beta\theta}} \implies \frac{w_i}{p_i} = \gamma^{-\frac{1}{\beta}} T_i^{\frac{1}{\theta\beta}}$$

Welfare simply increases in own technology, T_i. Using the free trade solution (with $d_{ni} = 1$), we can express welfare as:

$$= \underbrace{\gamma^{-1/\beta} T_n^{1/(\theta\beta)}}_{\text{Autarky}} \underbrace{\left[\sum_{i=1}^{N} \left(\frac{T_i}{T_n}\right)^{1/(1+\theta\beta)} \left(\frac{L_i}{L_n}\right)^{\theta\beta/(1+\theta\beta)}\right]^{1/(\theta\beta)}}_{>1}$$

See derivation

Why > 1?

If all countries were identical to n, then:

$$\frac{T_i}{T_n} = 1 \quad \text{and} \quad \frac{L_i}{L_n} = 1$$

$\Rightarrow$ The term in brackets would equal 1.
If at least one country has higher T_i or L_i, n can import cheaper goods.
With a positive exponent, these differences raise the bracketed term. **Hence, welfare is higher under trade.**

Empirical Analisys

- In order to perform counterfactuals, we need to estimate several paramenters.
- The model is estimated using a two-step procedure:
  1. Estimate θ. Several ways depending on the data available:
     - Eaton and Kortum (2002) use a panel of trade flows in manufactured goods from 19 OECD countries in 1990 and a method of moments.
     - Simonovska and Waugh (2011) said that the EK method of moments is not consistent, and developed a simulated method of moments and then applied it to disaggregate price and trade-flow data for the year 2004.
     - Others who have used similar approaches are Constinot, Donaldson and Komunjer (2012) (estimated θ using sectoral productivities and a dif-in-dif strategy) and Caliendo and Parro (2014) (used another dif-in-dif strategy).
     - In summary, the estimates of θ are between 2 and 12, but the most common value is ~ 8.
  2. Given θ, estimate T_i and d_{ni}.

EK Counterfactuals

- About the gains from trade, EK show that smaller countries tend to be hurt more by trade barriers (i.e. when we move to autarky). Link to the appendix table
- Moving to zero gravity $d_{ni} = 1$ would imply bigger gains for all countries. Link to the appendix table

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4. **Comparative advantage works at the extensive margin**: bad locations sell fewer goods, not the same goods at higher prices.
5. **Empirical fit**: with three parameters EK matches bilateral trade, price dispersion and wage gaps across 19 OECDs.

Main Lessons from Eaton–Kortum

1. **Gravity emerges from technology + geography**—no Armington or monopolistic competition needed.
2. **Two sufficient statistics** (θ, π_{ii}) pin down welfare gains from trade.
3. **Small countries gain more** from both trade liberalization and foreign technological progress.
4. **Comparative advantage works at the extensive margin**: bad locations sell fewer goods, not the same goods at higher prices.
5. **Empirical fit**: with three parameters EK matches bilateral trade, price dispersion and wage gaps across 19 OECDs.

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Apéndice A (I): Definición y paso a la integral

$$\pi_{ni} = \Pr\left[P_{ni}(j) \leq \min_{k \neq i}\{P_{nk}(j)\}\right] = \int_0^\infty \underbrace{\prod_{s \neq i} \Pr\left[P_{ns} > p\right]}_{\substack{\text{Ningún otro país ofrece} \\ \text{precio menor a } p}} \; \underbrace{d\, \Pr[P_{ni} \leq p]}_{\substack{\text{País } i \text{ sí} \\ \text{ofrece } \leq p}}.$$

- Es la probabilidad de que, para un bien j, el país i ofrezca el precio más bajo entre todos los países.
- **Idea:** We take the set of all prices for which any country is offering a price lower than p and integrate over that set for the products in which country i is offering a price lower than p.

Apéndice A (II): Uso de la distribución Fréchet

Pero recuerden que por la distribución Fréchet para z_i:

$$G_{ni}(p) \;=\; \Pr\big(P_{ni} \leq p\big) \;=\; 1 - \exp\Big[-T_i\big(\tfrac{c_i\, d_{ni}}{p}\big)^{-\theta}\Big].$$

$$\begin{aligned}
\Rightarrow \pi_{ni} &= \int_0^\infty \prod_{s\neq i} [1 - G_{ns}(p)]\, dG_{ni}(p) \\
&= \int_0^\infty \prod_{s\neq i} \Big[e^{-\,T_s\,(c_s\,d_{ns})^{-\theta}\,p^\theta}\Big]\;\Big[\big(T_i\,(c_i\,d_{ni})^{-\theta}\big)\,\theta\,p^{\theta-1} e^{-\,T_i\,(c_i d_{ni})^{-\theta}\,p^\theta}\Big]\;dp \\
&= \Big[T_i\,(c_i\,d_{ni})^{-\theta}\Big] \int_0^\infty \prod_{s\neq i} \Big[e^{-\,T_s\,(c_s\,d_{ns})^{-\theta}\,p^\theta}\Big]\,\theta\,p^{\theta-1}\,dp \\
&= \Big[T_i\,(c_i\,d_{ni})^{-\theta}\Big] \int_0^\infty e^{-\,\Phi_n\,p^\theta}\;\theta\,p^{\theta-1}\,dp
\end{aligned}$$

donde $\Phi_n = \sum_{s=1}^{N} T_s\,(c_s\,d_{ns})^{-\theta}$

Apéndice A (III): Resultado

Finally, we have:

$$
\begin{aligned}
\pi_{ni} &= \left[T_i\,(c_i\,d_{ni})^{-\theta}\right]\,\int_0^\infty \theta\,p^{\theta-1}\,e^{-\,\Phi_n\,p^\theta}\,dp \\
&= \left[T_i\,(c_i\,d_{ni})^{-\theta}\right]\left[-\frac{1}{\Phi_n}e^{-\,\Phi_n\,p^\theta}\right]_{p=0}^{\infty} = \left[T_i\,(c_i\,d_{ni})^{-\theta}\right]\,\frac{1}{\Phi_n} \\
&= \frac{T_i\,(c_i\,d_{ni})^{-\theta}}{\Phi_n} = \frac{T_i\,(c_i\,d_{ni})^{-\theta}}{\sum_m T_m\,(c_m\,d_{nm})^{-\theta}}
\end{aligned}
$$

which is the fracción de bienes que n compra a i.

- Al aumentar T_i, $c_i\,d_{ni}$ se vuelve "más competitivo" en promedio, subiendo π_{ni}.
- θ modula la sensibilidad a esas diferencias (menos dispersión $\rightarrow$ menor "ventaja comparativa").

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Apéndice B: Derivación de $G_n(p)$

- Rememeber that

$$P_n(j) = \min_i \{P_{ni}(j)\}.$$

- La CDF de P_n en p es

$$G_n(p) = Pr[P_n \leq p] = \Pr\Big[\min_i \{P_{ni}\} \leq p\Big].$$

- Reescribir la prob. de la "mínima" como

$$G_n(p) = 1 - \Pr\Big[\min_i \{P_{ni}\} > p\Big] = 1 - \underbrace{\prod_{i=1}^{N} \Pr[P_{ni} > p]}_{\text{todos los precios} > p}.$$

- Using $\Pr[P_{ni} > p] = 1 - G_{ni}(p)$, $P_{ni} = \frac{c_i}{z_i} d_{ni}$ y $z_i \sim$ Fréchet,

$$G_{ni}(p) = 1 - \exp\Big[-T_i\,(c_i\,d_{ni})^{-\theta}\,p^{\theta}\Big].$$

- Entonces

$$\Pr[P_{ni} > p] = \exp\Big[-T_i\,(c_i\,d_{ni})^{-\theta}\,p^{\theta}\Big].$$

- Sustitución y definición de Φ_n:

$$G_n(p) = 1 - \prod_{i=1}^{N} \exp\Big[-T_i\,(c_i\,d_{ni})^{-\theta}\,p^{\theta}\Big] = 1 - \exp\Big[-p^{\theta}\underbrace{\sum_{i=1}^{N} T_i\,(c_i\,d_{ni})^{-\theta}}_{\Phi_n}\Big].$$

- Finalmente, el índice de precios es

$$G_n(p) = 1 - \exp[-\Phi_n\,p^{\theta}], \quad \Phi_n = \sum_i T_i\,(c_i d_{ni})^{-\theta}.$$

- Now think that Si país n compra un bien a i a precio q, *i.e.* i es el proveedor más barato, queremos ver la distribución de ese q.
- Probabilidad de "i es el más barato dado que vende a q"

$$\prod_{s \neq i} \Pr\left[P_{ns} \geq q\right] = \prod_{s \neq i}\left[1 - G_{ns}(q)\right] = e^{-\Phi_n^{-i} q^\theta}$$

- Integrar sobre todos los $q \leq p$

$$\int_0^p e^{-\Phi_n^{-i} q^\theta} dG_{ni}(q) = \pi_{ni} \, G_n(p)$$

- Es decir que la distribución de q condicionada a que i es el más barato coincide con $G_n(p)$.
- El país i ajusta el rango de bienes que vende de modo que "la parte del precio" en la que sí compite exitosamente *hereda* la distribución G_n.

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Apéndice C: Derivación del índice de precios p_n

- We need to show that

$$p_n = \left[\int_0^1 \Big(p_n(j)\Big)^{1-\sigma} dj\right]^{\frac{1}{1-\sigma}} = \gamma\, \Phi_n^{-\frac{1}{\theta}}.$$

$$\begin{aligned}
p_n^{1-\sigma} &= \int_0^1 p_n(j)^{\,1-\sigma} dj = \int_0^\infty p^{\,1-\sigma}\, dG_n(p) && \text{(cambiando variable } j \mapsto p) \\
&= \int_0^\infty p^{\,1-\sigma} \left[\Phi_n\, \theta\, p^{\,\theta-1}\, e^{-\Phi_n\, p^\theta}\right] dp && \text{(usando } G_n'(p) \text{ y } G_n(p) = 1 - e^{-\Phi_n p^\theta}) \\
&= \Phi_n\, \theta \int_0^\infty p^{\theta-\sigma} e^{-\Phi_n p^\theta}\, dp \\
&= \Phi_n^{\frac{1-\sigma}{\theta}} \int_0^\infty x^{\frac{1-\sigma}{\theta}}\, e^{-x} \frac{dx}{\theta\, x^{\frac{\theta-1}{\theta}}} && \text{(cambio de variable } x = \Phi_n p^\theta) \\
&= \Phi_n^{\frac{1-\sigma}{\theta}} \int_0^\infty x^{\frac{1-\sigma}{\theta}+\frac{\theta-1}{\theta}}\, e^{-x}\, dx = \Phi_n^{-\frac{1-\sigma}{\theta}}\, \Gamma\!\left(\tfrac{\theta+1-\sigma}{\theta}\right)
\end{aligned}$$

- Entonces, el índice de precios es

$$p_n^{1-\sigma} = \Phi_n^{-\frac{1-\sigma}{\theta}} \Gamma\left(\frac{\theta+1-\sigma}{\theta}\right).$$

- Finalmente, el índice de precios es

$$p_n = \underbrace{\left[\Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)\right]^{\frac{1}{1-\sigma}}}_{\gamma} \Phi_n^{-\frac{1}{\theta}}.$$

$$p_n = \gamma\, \Phi_n^{-\frac{1}{\theta}}.$$

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Model Closure: Additional Notes

- **Mobile labor case:**
  - Wage is pinned down by nonmanufacturing productivity (since wages are equalized across sectors).
  - L_n adjusts endogenously in manufacturing.
  - T_i reflects then not only absolute advantage within manufactures, but *comparative advantage in manufacturing relative to nonmanufacturing*.
- **Immobile labor case:**
  - L_n is exogenous and fixed, then:

$$X_n = \frac{1-\beta}{\beta} w_n L_n + \alpha Y_n = \frac{1-\beta}{\beta} w_n L_n + \alpha (w_n L_n + Y_n^O)$$

  - L_n fixed $\Rightarrow$ specialization is exogenous.
  - T_i affects w_i through its impact on trade shares π_{ni}: higher technology T_i makes country i more competitive, increasing π_{ni} and thus raising labor income $w_i L_i$; with L_i fixed, this implies higher w_i.
- In both cases, θ governs the strength of comparative advantage *within* manufacturing.

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Appendix: Deriving Welfare

Using the free trade solution (with $d_{ni} = 1$), we can express welfare as:

$$
\begin{aligned}
\frac{w_n}{p_n} &= \gamma^{-1/\beta} T_n^{1/(1+\theta\beta)} \left[\sum_{i=1}^{N} T_i^{1/(1+\theta\beta)} \left(\frac{L_i}{L_n} \right)^{\theta\beta/(1+\theta\beta)} \right]^{1/(\theta\beta)} \\
&= \gamma^{-1/\beta} \left[T_n^{(\theta\beta)/(1+\theta\beta)} \sum_{i=1}^{N} T_i^{1/(1+\theta\beta)} \left(\frac{L_i}{L_n} \right)^{\theta\beta/(1+\theta\beta)} \right]^{1/(\theta\beta)} \\
&= \gamma^{-1/\beta} \left[T_n^{(\theta\beta)/(1+\theta\beta)} T_n^{1/(1+\theta\beta)} \sum_{i=1}^{N} \left(\frac{T_i}{T_n} \right)^{1/(1+\theta\beta)} \left(\frac{L_i}{L_n} \right)^{\theta\beta/(1+\theta\beta)} \right]^{1/(\theta\beta)}
\end{aligned}
$$

Rewriting it:

$$
= \underbrace{\gamma^{-1/\beta} T_n^{1/(\theta\beta)}}_{\text{Autarky}} \underbrace{\left[\sum_{i=1}^{N} \left(\frac{T_i}{T_n} \right)^{1/(1+\theta\beta)} \left(\frac{L_i}{L_n} \right)^{\theta\beta/(1+\theta\beta)} \right]^{1/(\theta\beta)}}_{>1}
$$

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