

Notes on the General Equilibrium

Reflections on a Especial Edgeworth Box

FRD

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Introduction

These notes aim to clarify whether an increase in one consumer's endowment within a standard two-good, two-consumer Edgeworth box can make that consumer *worse off* in the resulting Walrasian equilibrium. The goal of these notes is to articulate the standard classroom intuition and then provide a more formal explanation of why, under the usual hypotheses, such a paradoxical outcome cannot arise.

Question 1. In an Edgeworth box exchange economy with two consumers and two goods, suppose consumer 1 receives a higher endowment of both goods, while consumer 2's endowment remains unchanged. Preferences of both consumers are strictly increasing and strictly convex. Is it possible that, in the new Walrasian equilibrium of the perturbed economy, consumer 1 becomes *worse off* while consumer 2 becomes *better off*? You should answer using words and graphs only.

A first approximation

Intuition. This is an example of what in the macro literature is sometimes called 'Dutch disease'.

Think of good 1 as a numeraire ("money") and good 2 as the "real" commodity. Suppose that in the initial equilibrium consumer 1 is very well endowed with good 2 and, relative to consumer 2, does not value it as much. In equilibrium, consumer 1 is therefore a *net seller* of good 2. Consumer 2, in contrast, has little of good 2 and values it highly, so that she is a *net buyer* of good 2.

Now consider an increase in the endowment of consumer 1 in both goods, but especially in good 2. At the aggregate level, this looks like an outward shift in the supply of good 2. Holding preferences fixed, the market for good 2 clears at a lower relative price: the price of the real commodity falls in terms of the numeraire.

This price movement has opposite welfare effects for the two consumers. For consumer 2, who is a net buyer of good 2, the fall in its price is beneficial: she can purchase more of the com-

modity she likes at a cheaper relative price, and her equilibrium consumption bundle moves to a higher indifference curve. For consumer 1, there are two opposing forces. On the one hand, the additional endowment is a direct gain: she now owns more units of both goods. On the other hand, as a large net seller of good 2, she suffers from the deterioration in her terms of trade: all the (many) units of good 2 she was initially endowed with and intended to sell now command a lower relative price.

If consumer 1 is sufficiently large in the market for good 2, and if the equilibrium price of good 2 is sufficiently sensitive to changes in aggregate supply, the terms-of-trade loss can dominate the direct endowment gain. In that case, the new Walrasian equilibrium leaves consumer 1 on a *lower* indifference curve than before, while consumer 2, who benefits from the lower price of her most desired good, moves to a *higher* indifference curve.

The quasilinear preferences example

Proposition (Existence of the paradoxical shift in endowment). *There exists a 2–good, 2–consumer Edgeworth box exchange economy with strictly increasing and strictly convex preferences, and two endowment vectors $\omega_1, \omega'_1 \in \mathbb{R}_{++}^2$ for consumer 1 with $\omega'_1 \gg \omega_1$ (componentwise larger), such that consumer 2's endowment is unchanged and, denoting by (p^0, x^0) and (p^1, x^1) the Walrasian equilibria corresponding to (ω_1, ω_2) and (ω'_1, ω_2) respectively, we have*

$$u_1(x_1^1) < u_1(x_1^0) \quad \text{and} \quad u_2(x_2^1) > u_2(x_2^0).$$

We construct the example with quasilinear preferences in good 1.

Step 1: Preferences and individual demand. Let goods be indexed by 1 and 2, and let consumer $i \in \{1, 2\}$ have utility

$$u_i(x_1, x_2) = x_1 + v_i(x_2),$$

where $v_i : \mathbb{R}_+ \rightarrow \mathbb{R}$ is C^2 , strictly increasing and strictly concave. These utilities are strictly increasing in both goods and strictly convex in the usual sense.

Normalize the price of good 1 to 1 and denote by $p > 0$ the price of good 2. Given wealth w_i , consumer i solves

$$\max_{x_1, x_2 \geq 0} x_1 + v_i(x_2) \quad \text{s.t.} \quad x_1 + px_2 = w_i.$$

Substituting $x_1 = w_i - px_2$, this is equivalent to

$$\max_{x_2 \geq 0} w_i - px_2 + v_i(x_2).$$

The first-order condition (for interior solutions) is

$$v'_i(x_2) = p.$$

Since v'_i is strictly decreasing and onto its range, this defines a demand for good 2 that depends

only on p :

$$x_2^i(p) = d_i(p) := (v'_i)^{-1}(p), \quad d'_i(p) < 0.$$

Demand for the numeraire is then

$$x_1^i(p, w_i) = w_i - p d_i(p).$$

Step 2: Walrasian equilibrium and the price function. Let consumer i 's endowment be $\omega_i = (\omega_{1i}, \omega_{2i})$ and define total endowment (supply) of good 2 by

$$S_2 := \omega_{21} + \omega_{22}.$$

At price p , consumer i 's wealth is

$$w_i(p) = \omega_{1i} + p \omega_{2i}.$$

Market clearing in good 2 requires

$$d_1(p) + d_2(p) = S_2.$$

Because each d_i is continuous and strictly decreasing, there is a unique solution $p(S_2) > 0$ to this equation, which we regard as an equilibrium price function of total endowment S_2 .

Step 3: Indirect utilities and “surplus” terms. Up to now we have described, for each consumer i ,

- their *direct* utility function

$$u_i(x_1, x_2) = x_1 + v_i(x_2),$$

- their *Marshallian* demand for good 2 at price p ,

$$x_2^i(p) = d_i(p) := (v'_i)^{-1}(p),$$

which does not depend on income because of quasilinearity¹, and

- their demand for the numeraire given wealth w_i ,

$$x_1^i(p, w_i) = w_i - p d_i(p).$$

For the comparative statics we are interested in, it is more convenient to summarize each consumer's behavior by an *indirect utility function*, that is, by their maximal utility as a function of prices and wealth. The reason is twofold:

¹This is the key feature of quasilinearity: at given prices, the optimal consumption of the non-numeraire good (or the bundle of goods entering $v(\cdot)$) is determined solely by marginal utilities and prices, independently of income. Only after choosing these goods do we allocate the remaining wealth to the numeraire.

- (i) At equilibrium, consumers optimally choose their consumption bundles given prices and their budget set. When we change endowments, what moves *first* are wealth levels and equilibrium prices; consumption responds only indirectly through the demand functions. Working with indirect utility allows us to keep track of welfare changes *through* these primitive objects (prices and wealth) rather than having to recompute the full allocation each time.
- (ii) In the quasilinear case, the indirect utility function has a particularly simple structure: it decomposes into a purely “monetary” term (wealth) plus a term that captures the net gain from access to the non-numeraire good at the prevailing price. This second term will be our “surplus” term, and it is precisely what reacts to changes in the relative price caused by endowment shifts.

Formally, fix a price $p > 0$ and a wealth level w_i . The indirect utility of consumer i is

$$v_i^{\text{ind}}(p, w_i) := \max_{x_1, x_2 \geq 0} \{x_1 + v_i(x_2) : x_1 + px_2 = w_i\}.$$

Using the optimal demands derived in Step 1, we know that the maximizing bundle is $(x_1^i, x_2^i) = (w_i - p d_i(p), d_i(p))$, so

$$\begin{aligned} v_i^{\text{ind}}(p, w_i) &= x_1^i(p, w_i) + v_i(x_2^i(p)) \\ &= w_i - p d_i(p) + v_i(d_i(p)). \end{aligned}$$

This can be written as

$$v_i^{\text{ind}}(p, w_i) = w_i + s_i(p),$$

where we define the *surplus* term

$$s_i(p) := v_i(d_i(p)) - p d_i(p).$$

The interpretation is straightforward:

- the component w_i is the value of consumer i 's wealth measured in units of the numeraire; in other words, if the consumer spent all income on good 1, she would be able to buy w_i units of that good.
- the function $s_i(p)$ measures how much additional utility consumer i gets from being able to transform the numeraire into good 2 at price p , beyond simply holding all wealth in the numeraire. It is the extra utility that comes from having access to buy units of x_2 (which yield $v_i(x_2)$ units of utility) in exchange for px_2 units of the numeraire (a direct sacrifice of numeraire that could have been consumed instead).

Next we express this indirect utility in terms of endowments and the aggregate supply of

good 2. Recall that consumer i 's wealth at price p is

$$w_i(p) = \omega_{1i} + p\omega_{2i},$$

and that the equilibrium price depends on total endowment of good 2, $S_2 = \omega_{21} + \omega_{22}$, via the market-clearing condition in Step 2. Writing $p(S_2)$ for the unique equilibrium price associated with S_2 , the equilibrium indirect utility of consumer i can be written as

$$u_i(\omega_i; S_2) = v_i^{\text{ind}}(p(S_2), w_i(p(S_2))) = \omega_{1i} + p(S_2)\omega_{2i} + s_i(p(S_2)).$$

Thus, when we vary endowments, the effect on u_i comes from:

- (a) the direct change in the “value” of the bundle $\omega_i = (\omega_{1i}, \omega_{2i})$, given by $\omega_{1i} + p(S_2)\omega_{2i}$, and
- (b) the indirect change in the surplus term $s_i(p)$ induced by the change in the equilibrium price $p(S_2)$.

Decomposition allow us, in the next steps, to separate the pure endowment effect from the terms-of-trade effect generated by changes in S_2 .

Step 4: Changing the distribution of endowment of good 2.

We now use the indirect utility representation from Step 3 to understand what happens when we change how much of good 2 is initially held by consumer 1. Recall that, for any total endowment of good 2,

$$S_2 := \omega_{21} + \omega_{22},$$

the equilibrium price $p(S_2)$ is determined by the market-clearing condition

$$d_1(p(S_2)) + d_2(p(S_2)) = S_2,$$

and that the equilibrium indirect utility of consumer i can be written as

$$u_i(\omega_i; S_2) = \omega_{1i} + p(S_2)\omega_{2i} + s_i(p(S_2)),$$

with $s'_i(p) = -d_i(p)$ (using the envelope theorem to derive this).

Now we fix ω_{11} , ω_{12} and ω_{22} , and let ω_{21} vary. Economically, this means that we inject additional units of good 2 into the economy through consumer 1's endowment, while consumer 2's endowment of good 2 remains fixed. Both the *distribution* of good 2 and its *total supply*

$$S_2(\omega_{21}) = \omega_{21} + \omega_{22}$$

change with ω_{21} . Since S_2 changes, the equilibrium price $p(S_2)$ will also adjust.

Price response to total endowment. The equilibrium price p is determined by the market-clearing

condition

$$d_1(p) + d_2(p) = S_2.$$

Differentiating with respect to S_2 and using the chain rule,

$$(d'_1(p) + d'_2(p)) \frac{dp}{dS_2} = 1,$$

so

$$\frac{dp}{dS_2} = \frac{1}{d'_1(p) + d'_2(p)} < 0, \quad (0.1)$$

because $d'_i(p) < 0$ for each i . Thus, when the total endowment of good 2 increases, its equilibrium price falls. In our experiment, $S_2(\omega_{21}) = \omega_{21} + \omega_{22}$ with ω_{22} fixed, so $dS_2/d\omega_{21} = 1$ and therefore $dp/d\omega_{21} = dp/dS_2 < 0$.

Effect on consumer 1. Write consumer 1's equilibrium utility as a function of ω_{21} :

$$u_1(\omega_{21}) = \omega_{11} + p(S_2(\omega_{21})) \omega_{21} + s_1(p(S_2(\omega_{21}))).$$

Using the chain rule,

$$\begin{aligned} \frac{du_1}{d\omega_{21}} &= \frac{\partial u_1}{\partial \omega_{21}} + \frac{\partial u_1}{\partial S_2} \frac{dS_2}{d\omega_{21}} \\ &= p(S_2) + [\omega_{21} + s'_1(p(S_2))] \frac{dp}{dS_2} \cdot 1. \end{aligned}$$

Using $s'_1(p) = -d_1(p)$, this simplifies to

$$\frac{du_1}{d\omega_{21}} = p(S_2) + \frac{dp}{dS_2} (\omega_{21} - d_1(p)). \quad (0.2)$$

Equation (0.2) has a natural economic interpretation:

- The first term, $p(S_2) > 0$, is a *direct wealth effect*: giving consumer 1 one more unit of good 2 at the prevailing price increases the value of her endowment by $p(S_2)$ units of the numéraire.
- The second term, $\frac{dp}{dS_2} (\omega_{21} - d_1(p))$, is a *terms-of-trade effect*. The factor $\omega_{21} - d_1(p)$ is consumer 1's *net trade* of good 2: her endowment of good 2 minus her equilibrium consumption of that good. If $\omega_{21} > d_1(p)$, she is a net seller of good 2; when S_2 increases, the price p falls (by (0.1)), and she loses on all the infra-marginal units of good 2 that she was planning to sell. The negative product captures this loss $\frac{dp}{dS_2} (\omega_{21} - d_1(p))$.

Thus, if consumer 1 is a sufficiently large net seller of good 2 ($\omega_{21} - d_1(p)$ large and positive), the negative terms-of-trade effect can dominate the positive direct effect $p(S_2)$, making $\frac{du_1}{d\omega_{21}} < 0$.

Effect on consumer 2. A similar computation for consumer 2 yields

$$u_2(\omega_{21}) = \omega_{12} + p(S_2(\omega_{21}))\omega_{22} + s_2(p(S_2(\omega_{21}))),$$

and

$$\frac{du_2}{d\omega_{21}} = \frac{dp}{dS_2}(\omega_{22} - d_2(p)). \quad (0.3)$$

Here again, $\omega_{22} - d_2(p)$ is consumer 2's net trade of good 2 (endowment minus demand). If $d_2(p) > \omega_{22}$, consumer 2 is a net *buyer* of good 2, so $\omega_{22} - d_2(p) < 0$, and using (0.1) we obtain

$$\frac{du_2}{d\omega_{21}} > 0.$$

Intuitively, as more of good 2 is injected into the economy through consumer 1's endowment, the price of good 2 falls; a net buyer of good 2 strictly gains from this improvement in her terms of trade.

Step 5: Choosing an initial distribution.

We now choose an initial allocation of good 2 between the two consumers such that:

- (a) consumer 1 is a *large* net seller of good 2, and
- (b) consumer 2 is a net buyer of good 2.

In this way, we can exploit the signs in (0.2) and (0.3).

Fix the primitives (v_1, v_2) and consider the total endowment $S_2 > 0$ of good 2, together with the resulting equilibrium price $p = p(S_2)$ solving

$$d_1(p) + d_2(p) = S_2.$$

Given S_2 and $p(S_2)$, we can distribute S_2 between the two consumers by choosing $(\omega_{21}, \omega_{22})$ such that

$$\omega_{21} + \omega_{22} = S_2.$$

Because $d_1(p)$ and $d_2(p)$ are finite for any $p > 0$, we can select $(\omega_{21}, \omega_{22})$ satisfying

$$\omega_{21} > d_1(p) \quad \text{and} \quad \omega_{22} < d_2(p),$$

that is, consumer 1 holds more of good 2 than she demands at (p, ω_1) (she is a net seller), while consumer 2 holds less than she demands (she is a net buyer). Then $\omega_{21} - d_1(p) > 0$ and $\omega_{22} - d_2(p) < 0$.

Substituting into (0.2) and (0.3), we obtain

$$\frac{du_1}{d\omega_{21}} = p(S_2) + \frac{dp}{dS_2}(\omega_{21} - d_1(p)), \quad \frac{du_2}{d\omega_{21}} = \frac{dp}{dS_2}(\omega_{22} - d_2(p)).$$

Since $dp/dS_2 < 0$ by (0.1) and $\omega_{22} - d_2(p) < 0$, we immediately have

$$\frac{du_2}{d\omega_{21}} > 0.$$

That is, as we increase ω_{21} (injecting more of good 2 via consumer 1), consumer 2's utility increases because she is a net buyer of good 2 and benefits from the lower equilibrium price.

For consumer 1, the sign of $\frac{du_1}{d\omega_{21}}$ is a priori ambiguous, since it combines a positive direct wealth effect $p(S_2) > 0$ and a negative terms-of-trade effect $\frac{dp}{dS_2}(\omega_{21} - d_1(p))$. However, the latter term is strictly negative when $\omega_{21} > d_1(p)$, and it becomes more negative as $\omega_{21} - d_1(p)$ grows. By choosing an allocation with consumer 1 holding a sufficiently large amount of good 2 relative to her demand, we can ensure that the terms-of-trade loss dominates the direct wealth gain and thus

$$\frac{du_1}{d\omega_{21}} < 0.$$

In summary, we can select an initial distribution of good 2 (together with arbitrary positive endowments ω_{11}, ω_{12} of the numeraire) such that, at the corresponding Walrasian equilibrium,

$$\frac{du_1}{d\omega_{21}} < 0 \quad \text{and} \quad \frac{du_2}{d\omega_{21}} > 0.$$

At this initial allocation, a marginal increase in ω_{21} (holding $\omega_{11}, \omega_{12}, \omega_{22}$ fixed) makes consumer 1 worse off and consumer 2 better off.

Step 6: Increasing consumer 1's endowment in both goods.

We now extend the previous marginal comparison to an endowment change that is strictly positive in *both* goods for consumer 1.

Consider a small change in consumer 1's endowment

$$\Delta\omega_1 = (\Delta\omega_{11}, \Delta\omega_{21}), \quad \Delta\omega_{11} > 0, \quad \Delta\omega_{21} > 0,$$

leaving consumer 2's endowment ω_2 unchanged. Let $\omega'_1 := \omega_1 + \Delta\omega_1$ denote the new endowment of consumer 1. By construction, $\omega'_1 \gg \omega_1$ componentwise.

Using a first-order (local) approximation around the initial equilibrium, the corresponding change in utilities is

$$\Delta u_1 \approx \Delta\omega_{11} \frac{\partial u_1}{\partial\omega_{11}} + \Delta\omega_{21} \frac{du_1}{d\omega_{21}}, \quad \Delta u_2 \approx \Delta\omega_{21} \frac{du_2}{d\omega_{21}}.$$

Because good 1 is the numeraire and enters utility linearly, we have

$$\frac{\partial u_1}{\partial\omega_{11}} = 1.$$

From Step 5, at our chosen initial distribution we know that

$$\frac{du_1}{d\omega_{21}} < 0 \quad \text{and} \quad \frac{du_2}{d\omega_{21}} > 0.$$

Hence, for consumer 2 we immediately obtain

$$\Delta u_2 \approx \Delta\omega_{21} \frac{du_2}{d\omega_{21}} > 0$$

for any $\Delta\omega_{21} > 0$ sufficiently small.

For consumer 1, the first-order change is

$$\Delta u_1 \approx \Delta\omega_{11} + \Delta\omega_{21} \frac{du_1}{d\omega_{21}}.$$

Since $\frac{du_1}{d\omega_{21}} < 0$, we can choose $\Delta\omega_{11} > 0$ small enough (relative to a given $\Delta\omega_{21} > 0$) so that

$$\Delta\omega_{11} + \Delta\omega_{21} \frac{du_1}{d\omega_{21}} < 0.$$

In words: we increase consumer 1's endowment of good 2 by some fixed small amount, and we increase her endowment of the numeraire by a smaller amount, so that the negative terms-of-trade effect dominates the direct benefit from extra numeraire.

By continuity of the indirect utility functions with respect to endowments, these first-order inequalities translate into strict inequalities for the actual changes in utilities provided that $\Delta\omega_1$ is small enough. Thus we can choose a vector $\Delta\omega_1 \gg 0$ such that, at the Walrasian equilibrium corresponding to (ω'_1, ω_2) , we have

$$u_1(\omega'_1, \omega_2) < u_1(\omega_1, \omega_2) \quad \text{and} \quad u_2(\omega'_1, \omega_2) > u_2(\omega_1, \omega_2).$$

This completes the construction of an exchange economy with quasilinear, strictly increasing and strictly convex preferences in which increasing consumer 1's endowment in both goods (leaving consumer 2's endowment fixed) makes consumer 1 strictly worse off and consumer 2 strictly better off in Walrasian equilibrium.

A brief digression: why Cobb–Douglas preferences cannot generate the paradox

The construction above crucially used quasilinear preferences to make consumer 1 a very large net seller of good 2 whose consumption of that good does not rise with income. This allows an increase in her endowment of good 2 to translate almost one-for-one into extra supply on the market, pushing the equilibrium price down and generating a large negative terms-of-trade effect.

With Cobb–Douglas preferences, this mechanism breaks down. To see why, it is useful to contrast the two structures.

(i) *Prices depend only on total endowments.* In both the quasilinear and the Cobb–Douglas case, the equilibrium price of good 2 is pinned down by a market-clearing condition of the form

$$\text{aggregate demand for good 2 at } p = \text{total endowment of good 2}.$$

In a two-good, two-consumer exchange economy, the price p is therefore a function of the *total endowment* $S_2 = \omega_{21} + \omega_{22}$ of good 2. When we “inject” more units of good 2 through consumer 1, we increase S_2 , and the equilibrium price $p(S_2)$ falls: good 2 becomes cheaper in terms of the numéraire. So far, this is common to both preference structures: more aggregate supply of good 2 pushes down its price.

(ii) *What quasilinearity buys us.* With quasilinear utility

$$u_i(x_1, x_2) = x_1 + v_i(x_2),$$

the key property is that the demand for good 2 is independent of income:

$$x_{2i}(p) = d_i(p),$$

solving $v'_i(x_2) = p$. Giving consumer 1 more endowment of good 2 does *not* change how much of good 2 she wants to consume at a given price; it only changes how much she can sell. In other words:

- consumption $x_{21}(p)$ is pinned down by p ;
- endowment ω_{21} can be made arbitrarily large;
- net supply of good 2 by consumer 1, $\omega_{21} - x_{21}(p)$, can thus be made arbitrarily large.

This is exactly what allows us to engineer a situation where consumer 1 is a huge net seller of good 2: when the price of good 2 falls because S_2 increases, she loses heavily on all those infra-marginal units she is trying to sell. The negative terms-of-trade effect can be made so strong that it dominates the direct gain from receiving extra endowment.

(iii) *Why Cobb–Douglas kills this effect.* Now consider Cobb–Douglas preferences

$$u_i(x_{1i}, x_{2i}) = x_{1i}^{\alpha_i} x_{2i}^{1-\alpha_i}, \quad 0 < \alpha_i < 1.$$

At prices $(1, p)$ and income w_i , the Marshallian demands are

$$x_{1i}(p, w_i) = \alpha_i w_i, \quad x_{2i}(p, w_i) = \frac{(1 - \alpha_i)w_i}{p}.$$

Two features are crucial:

- (a) *Demands are homothetic.* Both x_{1i} and x_{2i} are proportional to income w_i . If we increase consumer 1's endowment (of either good), her income w_1 rises, and she wants to consume more of *both* goods in fixed proportions. In particular, when we give her more of good 2, she does not simply "dump" the extra units on the market: her desired consumption $x_{21}(p, w_1)$ of good 2 increases with w_1 . The gap

$$\omega_{21} - x_{21}(p, w_1)$$

does not grow arbitrarily: part of the extra endowment is absorbed by higher own consumption.

- (b) *Indirect utility is monotone in own wealth.* In a two-good Cobb–Douglas economy, indirect utility takes the form

$$v_i^{\text{ind}}(p, w_i) = C_i w_i p^{\alpha_i - 1},$$

where $C_i > 0$ is a constant depending on preferences. When we express p as a function of total endowments and w_i as the value of consumer i 's endowment at those prices, one can show that v_i^{ind} is strictly increasing in each component of consumer i 's own endowment. Intuitively, Cobb–Douglas preferences treat extra wealth as an increase in "real income" measured against a price index; the associated change in prices never offsets the direct gain from higher wealth.

Put differently, the Cobb–Douglas consumer always adjusts her consumption bundle so that she spends a fixed share of her (now higher) income on each good. When you give her more of good 2, two things happen at once:

- her income goes up, so she *demands* more of good 2 herself;
- the price of good 2 falls because aggregate supply increased, but this price change affects all consumers symmetrically and with a strength that is tied to the expenditure shares.

These forces ensure that extra endowment always raises her equilibrium utility: there is no way to make her such a large net seller of good 2 that a fall in its price can outweigh the direct wealth gain.

In contrast, with quasilinear preferences, the demand for the non–numéraire good does not scale with income. This allows us to hold consumption of good 2 fixed while increasing endowment, turning consumer 1 into an arbitrarily large net seller of that good. When the equilibrium price adjusts to the larger total endowment, the terms-of-trade loss on her large net supply can dominate the direct benefit of the additional endowment. That asymmetry between "how much I want to consume" and "how much I am endowed with" is precisely what Cobb–Douglas preferences do *not* permit.