Appendix to Submission 455 "Game-Theoretic Goal Recognition in Time-Sensitive Applications"

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APPENDIX A

Proposition 4 *The following results hold: The following results hold:*

(i) *Assume that* $q(x) = x/4$. *Then* (σ_D, σ_A) *is a Nash equilibrium if and only if*

$$\sigma_D = (2/3)\sigma_D^{C_o} + p_1\sigma_D^{C_1} + p_2\delta^{(C_2,T_2)} \qquad p_1 + p_2 = 1/3$$
$$\sigma_A = (2/3)\delta^{Q_1} + (1/3)\delta^{Q_4}$$

(ii) *Assume that* $q(x) = \min\{x/3, 1\}$. *Then,* (σ_D, σ_A) *is a Nash equilibrium if and only if*

$$\sigma_D = p_0^1\delta^{(C_0,T_0^1)} + p_0^2\delta^{(C_0,T_0^2)} \qquad p_0^1 + p_0^2 = 1$$
$$\sigma_A = q_1\delta^{Q_1} + q_4\delta^{Q_4} \qquad 0 < q_1 < 1/2 \quad q_1 + q_4 = 1$$

Proof From the descriptions (7) and (8) in the paper, we can represent mixed strategies for the attacker and the defender as, respectively,

$$\sigma_A = q_1\delta^{Q_1} + q_2\delta^{Q_2} + q_3\delta^{Q_3} + q_4\delta^{Q_4}$$
$$\sigma_D = p_0^1\delta^{(C_0,T_0^1)} + p_0^2\delta^{(C_0,T_0^2)} + p_1^1\delta^{(C_1,T_1^1)} + p_1^2\delta^{(C_1,T_1^2)} + p_2\delta^{(C_2,T_2)}$$

where all coefficients are non-negative and sum to 1 in each expression. We gather all the q_i's in a vector q and the p_i^j's in a vector p. We define $p_0 = p_0^1 + p_0^2$ and $p_1 = p_1^1 + p_1^2$. The value function can be characterized as follows:

$$\begin{array}{rcl} \Phi(\sigma_A, \sigma_D) & = & \frac{1}{2}q(3)p_0q_1 + \frac{1}{2}q(4)p_0q_4 \\ & + & \frac{1}{2}\left[\left(q(4)p_0^1 + q(3)p_0^2\right)q_2 + \frac{1}{2}\left(q(3)p_0^1 + q(4)p_0^2\right)\right]q_3 \\ & + & q(2)p_1q_1 + \frac{1}{2}q(2)p_1q_4 \\ & + & q(2)\left(p_1^1 + \frac{1}{2}p_1^2\right)q_2 + q(2)\left(\frac{1}{2}p_1^1 + p_1^2\right)q_3 \\ & + & q(2)p_2q_1 + q(1)p_2q_4 \\ & + & \frac{1}{2}\left(q(1) + q(2)\right)p_2(q_2 + q_3) \end{array} \tag{1}$$

Preliminary, we notice that the sum of the second and fourth expressions in eq. (1) can be written as $Aq_2 + Bq_3$ where

$$A = \frac{1}{2}\left(q(4)p_0^1 + q(3)p_0^2\right) + q(2)\left(p_1^1 + \frac{1}{2}p_1^2\right)$$
$$B = \frac{1}{2}\left(q(3)p_0^1 + q(4)p_0^2\right) + q(2)\left(\frac{1}{2}p_1^1 + p_1^2\right)$$

Notice that A and B only depend on the quaduple $p_0^1, p_0^2, p_1^1, p_1^2$ and that $A + B$ only depends on p_0 and p_1. In particular, $A + B$ does not change if we modify the quaduple $p_0^1, p_0^2, p_1^1, p_1^2$ keeping invariant p_0 and p_1. Notice finally that we can only achieve $A = B$ choosing $p_0^1 = p_0^2 = p_0/2, p_1^1 = p_1^2 = p_1/2$. As the remaining expressions in (24) only depend on p and q through the indices p_0, p_1, p_2 and $q_1, q_2 + q_3, q_4$, in any Nash equilibrium, the quantity $Aq_2 + Bq_3$ must be a maximum for the defender under the quadruples $p_0^1, p_0^2, p_1^1, p_1^2$ that keep invariant p_0 and p_1 and a minimum for the attacker under the possible choices of q_2 and q_3 for which $q_2 + q_3$ is given. This yields $A = B$. Assuming this, we can replace $Aq_2 + Bq_3$ in (24) with $(A + B)(q_2 + q_3)/2$ and obtain

$$\begin{array}{rcl} \Phi(\sigma_A, \sigma_D) & = & \frac{1}{2}q(3)p_0q_1 + \frac{1}{2}q(4)p_0q_4 \\ & + & \frac{1}{2}\left(q(3) + q(4)\right)p_0(q_2 + q_3) \\ & + & q(2)p_1q_1 + \frac{1}{2}q(2)p_1q_4 \\ & + & \frac{3}{2}q(2)p_1(q_2 + q_3) \\ & + & q(2)p_2q_1 + q(1)p_2q_4 \\ & + & \frac{1}{2}\left(q(1) + q(2)\right)p_2(q_2 + q_3) \end{array} \tag{2}$$

Moreover, since by construction $\delta_{Q_1} + \delta_{Q_4} = \delta_{Q_2} + \delta_{Q_3}$ and the expression above depends on q_2 and q_3 only through their sum $q_2 + q_3$, with no loss of generality we assume that $q_2 = q_3 = 0$ and we further rewrite the value function as

$$\begin{array}{rcl} \Phi(\sigma_A, \sigma_D) & = & \frac{1}{2}(q(3)q_1 + q(4)q_4)p_0 \\ & & +q(2)\left(q_1 + \frac{1}{2}q_4\right)p_1 + (q(2)q_1 + q(1)q_4)p_2 \\ & = & \left(\frac{1}{2}q(3)p_0 + q(2)p_1 + q(2)p_2\right)q_1 \\ & & +\left(\frac{1}{2}q(4)p_0 + \frac{1}{2}q(2)p_1 + q(1)p_2\right)q_4 \end{array} \tag{3}$$

We now analyze the two cases.

(i) The case when $q(x) = x/4$. Since, in this case, $\Phi(\sigma_A, \sigma_D)$ depends on p_1 and p_2 exclusively through their sum $p_1 + p_2$, we consider first the case when $p_1 = 0$. A direct check shows that, in a Nash equilibrium, we cannot have $q_1, q_4 \in \{0, 1\}$ or $p_0, p_2 \in \{0, 1\}$. Imposing that (σ_A, σ_D) is a Nash equilibrium, the two expressions in (3) then imply

$$\begin{cases} \frac{1}{2}q(3)p_0 + q(2)p_2 = \frac{1}{2}q(4)p_0 + q(1)p_2 \\ \frac{1}{2}(q(3)q_1 + q(4)q_4 = (q(2)q_1 + q(1)q_4) \end{cases}$$

whose unique solution is given by $q_1 = 2/3$, $q_2 = 1/3$, $p_0 = 2/3$, $p_2 = 1/3$. Reintegrating p_1, the last constraint becomes $p_1 + p_2 = 1/3$. If we now consider that for such function q, the equality $A = B$ above only holds when $p_0^1 = p_0^2$ and $p_1^1 = p_1^2$, we have that the set of Nash equilibria is the one described in (i).

(ii) The case when $q(x) = \max\{x/3, 1\}$. As in the previous case, we can assume, with no loss of generality, that $p_1 = 0$. We prove that in a Nash equilibrium, necessarily, $p_2 = 0$. If not, from the second expression in (3), using that $q(3) = q(4)$, we would deduce that $q_1 = 0$ and $q_4 = 1$. Substituting in the

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first expression in (3), we would then obtain

$$\Phi(\rho, \mu) = \frac{1}{2} q(4) p_0 + q(1) p_2$$

and since $q(4)/2 > q(1)$, we have that the best response for the defender is $p_2 = 0$. Looking again at the first expression in (3), we deduce that, for $p_2 = 0$ to be a (strict) best response for the defender, it must be that

$$\frac{1}{2}(q(3)q_1 + q(4)q_4) > q(2)q_1 + q(1)q_4$$

which can be shown to be satisfied if and only if $0 < q_1 < 1/2$. Since for such function q, the equality $A = B$ above holds when $p_1^1 = p_1^2$ and any possible p_0^1 and p_0^2, we have that the set of Nash equilibria is the one described in (ii).

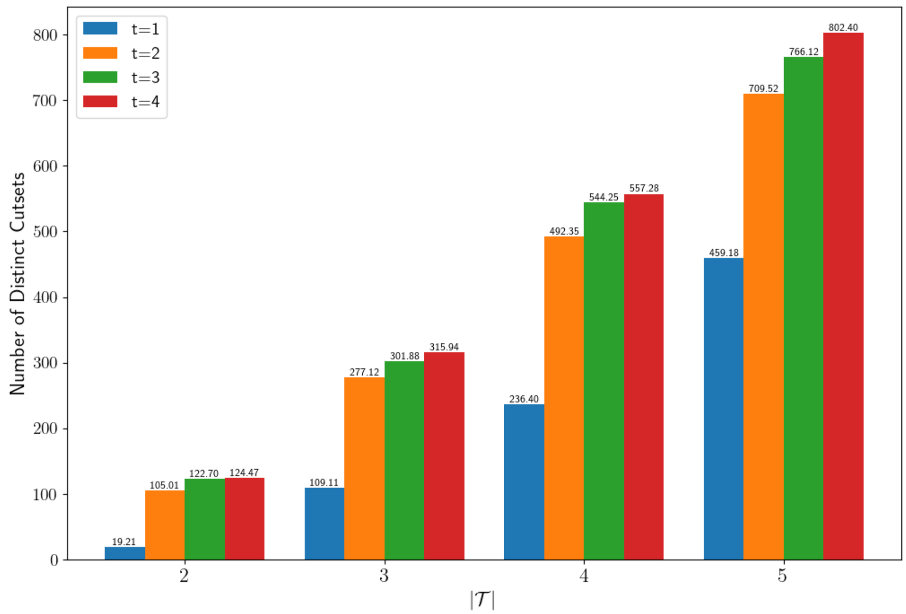
APPENDIX B

Figures 1a) and 1b) show the average number of distinct prefix cuts generated during the fictitious play iterations. A wider variety of prefix cuts indicates that the defender must employ complex strategies to achieve a Nash equilibrium and a guaranteed performance against the attacker. Adding more targets makes the problem harder and increases the complexity of the strategies. Increasing the saturation parameter t favors using more prefix cuts due to the lower penalty for delayed predictions.

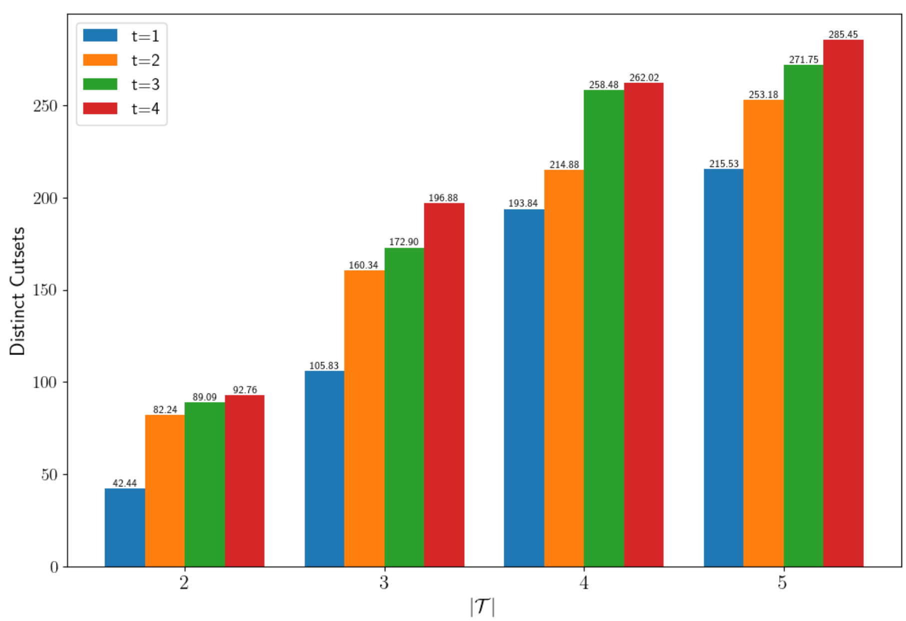
Table 1 shows Φ, the number of prefix cuts, and runtime (average and standard deviation) for the FP algorithm on the 30x30 grid. Origin and targets are picked randomly and uniformly. We run ten thousand FP iterations per problem, 200 problems per category, and up to a thousand paths per target.

Table 2 shows Φ, the number of prefix cuts, and runtime (average and standard deviation) for the FP algorithm on the Shanghai map. Origin and targets are picked randomly and uniformly. We run one thousand FP iterations per problem, 200 problems per category, and up to a thousand paths per target.

We provide the code and a README file with exact instructions on how to run each of the experiments reported in the paper at the following link: AnonymizedCodeRepository.



(a) 30x30 Grid



(b) Shanghai Grid

Figure 1: Number of prefix cuts generated by the FP algorithm.

Table 1: Φ, prefix cuts and runtime (avg. and SD) for the FP algorithm on the 30x30 grid.

t	$\lvert\mathcal{T}\rvert$	$\Phi \pm$ SD	Number of Prefix Cuts ± SD	Runtime ± SD
1	2	0.56 ± 0.10	19.21 ± 34.31	7,088.95 ± 6,146.47
1	3	0.42 ± 0.10	109.11 ± 209.42	12,066.39 ± 8,486.23
1	4	0.34 ± 0.07	236.40 ± 291.71	17,600.05 ± 10,621.89
1	5	0.29 ± 0.06	459.18 ± 474.88	24,461.05 ± 14,217.74
2	2	0.70 ± 0.20	105.01 ± 220.87	7,194.42 ± 6,662.03
2	3	0.58 ± 0.15	277.12 ± 364.95	12,436.83 ± 9,091.61
2	4	0.51 ± 0.12	492.35 ± 476.97	17,694.62 ± 10,997.01
2	5	0.45 ± 0.10	709.52 ± 601.06	24,372.72 ± 14,035.35
3	2	0.76 ± 0.21	122.70 ± 255.57	7,341.78 ± 7,117.27
3	3	0.65 ± 0.17	301.88 ± 393.26	12,553.84 ± 9,280.97
3	4	0.58 ± 0.13	544.25 ± 512.47	17,951.53 ± 11,211.37
3	5	0.53 ± 0.11	766.12 ± 629.32	24,785.88 ± 14,605.67
4	2	0.78 ± 0.21	124.47 ± 254.45	7,236.88 ± 7,024.59
4	3	0.69 ± 0.17	315.94 ± 425.73	12,166.80 ± 9,139.02
4	4	0.63 ± 0.14	557.28 ± 542.59	17,249.77 ± 10,998.86
4	5	0.57 ± 0.11	802.40 ± 653.44	23,522.68 ± 13,420.93

Table 2: Φ, prefix cuts and runtime (avg. and SD) for the FP algorithm on the Shanghai map (256x256).

t	$\lvert\mathcal{T}\rvert$	$\Phi \pm$ SD	Number of Prefix Cuts ± SD	Runtime ± SD
1	2	0.60 ± 0.19	42.44 ± 63.83	15,873.13 ± 11,426.86
1	3	0.47 ± 0.13	105.83 ± 99.41	22,089.08 ± 14,628.47
1	4	0.37 ± 0.10	193.84 ± 118.20	33,433.94 ± 15,410.67
1	5	0.38 ± 0.09	215.53 ± 124.21	34,849.86 ± 14,915.25
2	2	0.72 ± 0.19	82.24 ± 101.21	15,606.27 ± 8,332.11
2	3	0.63 ± 0.15	160.34 ± 116.73	24,286.54 ± 13,093.64
2	4	0.57 ± 0.13	214.88 ± 129.85	31,624.08 ± 16,609.73
2	5	0.58 ± 0.13	253.18 ± 138.40	33,351.72 ± 15,584.53
3	2	0.79 ± 0.18	89.09 ± 112.08	16,755.87 ± 8,786.13
3	3	0.7 ± 0.14	172.9 ± 127.86	24,979.44 ± 11,513.62
3	4	0.65 ± 0.13	258.48 ± 134.90	32,666.24 ± 14,668.99
3	5	0.66 ± 0.12	271.75 ± 143.62	32,966.29 ± 14,714.87
4	2	0.81 ± 0.18	92.76 ± 120.02	16,268.2 ± 8,593.37
4	3	0.73 ± 0.13	196.88 ± 136.42	24,697.12 ± 12,301.72
4	4	0.69 ± 0.12	262.02 ± 142.14	33,903.87 ± 16,019.83
4	5	0.7 ± 0.12	285.45 ± 154.62	33,105.98 ± 12,672.85