(In)formal Growth: Wage Dynamics in Developing Economies*

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Abstract

Labor informality is pervasive in developing economies. In this paper, we investigate the interconnection between informal labor, human capital accumulation, and economic growth. Using panel data from Chile, we first explore the dynamics of the formal and informal sectors by documenting four new empirical facts. First, a substantial portion of the formal wage premium is attributable to workers' skill-based sorting. Second, formal experience is associated with higher wages, while informal experience is not. Third, wages for formal workers increase significantly more over the life cycle than wages for informal workers. Fourth, there is a dynamic formal wage premium: formal workers earn higher future wages in the short and medium run. To rationalize these patterns, we build an endogenous growth model where heterogeneous workers sort into formal and informal labor markets based on their potential earnings. Workers' human capital increases over their life cycle through interactions with other workers. In equilibrium, more knowledgeable workers sort into the formal sector, and the growth rate of the economy is determined by the rate at which all workers meet more knowledgeable formal workers. We structurally estimate the parameters of the model and use it to quantify the effect of formalization policies. We structurally estimate the parameters of the model and use it to quantify the effect of formalization policies. We find that policies that decrease the cost of hiring formal workers are more effective in reducing the size of the informal sector compared to policies that increase the cost of hiring informal workers. However, both types of policies have adverse effects on economic growth by lowering the quality of interactions among more skilled workers.

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1 Introduction

Informal labor markets are prevalent in developing economies. In Latin America, between 35% and 80% of workers do not have a formal labor contract, and the informal sector accounts for 40% of the national GDP.[1] Despite the coexistence of formal and informal firms within the same industries, producing similar products, wages for informal and formal workers vary widely.[2] Understanding the determinants of these wage differentials is crucial because they can be important factors shaping overall income inequality, and governments spend vast resources every year to decrease informality. Importantly, when wage differences reflect differences in human capital between workers, wage dynamics over the life cycle of formal and informal workers can have key implications for economic growth and long-term welfare. How do informal labor markets affect the formation of human capital? How does the stock of talent in an economy determine the size of the informal sector? How does this loop feed into economic growth and welfare?

This paper addresses these questions with three main contributions. First, utilizing worker panel data from Chile, we document four new empirical findings on wage dynamics for formal and informal workers. By exploiting the panel structure of our data to account for unobserved worker heterogeneity, we find that: (i) a significant portion of the formal wage premium is attributable to workers sorting based on skill, (ii) formal experience is associated with higher wages, while informal experience is not, (iii) formal workers experience higher wage growth throughout their life cycle, and (iv) formal workers earn higher future wages in the short and middle run. Second, to rationalize these findings, we develop a heterogeneous agent endogenous growth model with formal and informal labor markets. The model explains wage differentials between formal and informal sectors through workers' sorting based on skill and differential human capital accumulation over the life cycle. Third, we estimate the model and use it to quantify the aggregate effects of different types of formalization policies on informality, the growth rate of the economy, and welfare.

In the first part of the paper, we examine wage differentials between formal and informal workers using a longitudinal survey in Chile.[3] We classify workers as formal or informal by examining their affiliation with pension funds and whether or not they have a labor contract. Additionally, we have access to workers' full formal and informal job histories since 1980. Therefore, compared to previous studies, the structure of our data allows us to study wage changes for the same worker across the formal and informal sectors and analyze wage dynamics for individual workers over the life cycle. By exploiting these data characteristics, we document four new empirical facts.

First, consistent with previous evidence from other developing countries, we observe a significant wage premium for formal workers compared to their informal counterparts. Controlling for observable characteristics, formal workers in Chile earn wages 17% higher than their informal counterparts. These findings align with the results of Bargain & Kwenda (2014) and Ulyssea (2018). However, once we account for workers' unobserved time-invariant characteristics using worker fixed effects, the formal wage premium reduces to 7.4%. This reduction in the formal wage premium,

[1] See Perry et al. (2007).

[2] See Ulyssea (2020b) for a comprehensive review of informal firms and workers.

[3] We use the EPS, *Encuesta de Protección Social*. We also use the Colombian ELCO, *Encuesta Longitudinal de Colombia*, for robustness exercises.

when including worker fixed effects, suggests that a substantial portion of this wage gap can be attributed to high-skilled workers sorting into the formal sector.

Second, we examine the relationship between work experience and wages in the formal and informal sectors. We find that experience in the formal sector is positively correlated with higher wages, whereas experience in the informal sector does not yield similar benefits. Specifically, accounting for a large set of workers' observable characteristics and fixed effects, we find that a 1% increase in workers' formal experience is associated with a 0.4% increase in contemporary wages. However, an increase in workers' informal experience does not appear to be associated with wage changes and, if anything, seems to reduce wages.

Third, we document that wages for formal workers grow significantly more over their life cycle compared to wages for informal workers. We find that the wages of formal workers aged 55-65 are 60% higher than those of formal workers aged 15-25. In contrast, the wages of informal workers increase modestly over the years, with informal workers aged 55-65 earning wages that are only 3% higher than those of informal workers aged 15-25. We show that this pattern is robust, even when accounting for observable characteristics and time-invariant unobserved heterogeneity.

Fourth, we report the existence of a dynamic formal premium. Specifically, we use individual worker wage trajectories to relate future wage dynamics to formality status and various observable characteristics. We find that formal workers experience larger future wage increases compared to informal workers. Over one year, formal workers earn wages 2.7% higher than their informal counterparts, and this effect increases to 5.7% over a ten-year horizon. This dynamic formal premium persists even when controlling for firm size. We believe this provides suggestive evidence that formal workers learn relatively more than informal workers, and that a substantial portion of this learning is not explained by firm characteristics and is likely to occur outside the workplace.

The four facts altogether illustrate that more skilled workers tend to enter the formal sector, while less skilled workers remain in the informal sector, leading to inherent differences in learning opportunities. Skilled workers in the formal sector benefit from interactions with other knowledgeable individuals, whereas less skilled workers in the informal sector interact primarily with similarly less skilled individuals. This sorting creates superior learning opportunities for formal workers, resulting in a steeper wage growth trajectory over time compared to informal workers. This dynamic formal wage premium is the key factor driving the higher wage increases for formal workers throughout their careers.

In the second part of the paper, to rationalize our empirical findings, we propose an endogenous growth model with heterogeneous workers and informal labor markets. Workers differ in their levels of human capital (skills) and choose between operating in the formal and informal sectors. A representative firm offers both formal and informal jobs. To open a formal job, the firm incurs a registration cost and pays payroll taxes. This aspect of the model aims to encompass the variable costs of the formal sector, such as pension contributions, as well as fixed costs, like registering with government authorities. In contrast, when opening an informal job, the representative firm can avoid these overhead costs and payroll taxes but is subject to potential fines imposed by the government, which increase with the level of production of a specific firm-worker match. As production levels increase, the likelihood of government detection also rises.[4] The government rebates labor taxes as lump-sum transfers to all workers, regardless of their formal status.[5]

Wages increase over the life cycle as workers accumulate human capital by learning from more skilled peers. Workers interact with each other at a common exogenous rate. Building on the observations of Jarosch et al. (2021), we allow the probability of an informal worker interacting with a formal worker, conditional on a meeting, to differ from that of a formal worker interacting with another formal worker. Consequently, workers choose to operate in each sector based not only on the static returns, which depend on their current skill and labor market regulations, but also on the potential learning opportunities that shape future wage paths. The model is sufficiently tractable to imply a unique cutoff in the productivity distribution: workers with higher skills sort into the formal sector, while those with lower skills enter the informal sector. We characterize the model's outcomes in a Balanced Growth Path (BGP) and demonstrate that the economy's growth along this path is determined by the dispersion of productivity and a weighted average of meeting rates with the most knowledgeable agents.

The size of the informal labor market has implications for growth and welfare. First, a large informal sector results in high output losses for the economy, as more resources are used to conceal operations from the government. Additionally, if learning frictions are high enough, a larger labor informality diminishes human capital improvements for the least-skilled workers who sort into the informal sector. Second, a smaller informal sector has two effects on the quality of interactions in the formal sector. First, it increases the expected human capital improvements for workers who formalize, as they have better opportunities to learn from more skilled workers. Second, it decreases the quality of human capital in the formal sector, as the marginal agent that formalizes is less productive. With less skilled workers crowding out meetings in the formal sector, it becomes harder for all formal workers to meet more skilled counterparts. The relative strength of these two forces determines whether reductions in labor informality have a positive or negative effect on growth and welfare. Therefore, depending on parameter values, reductions in the informal sector might actually decrease growth and welfare in the long run, something that we find in our quantitative exercises.

In the final section of the paper, we estimate the model's parameters to perform counterfactual exercises. We employ a Simulated Method of Moments (SMM) approach to estimate most of the model's parameters. Leveraging the model structures, we infer the parameters that determine workers' dynamic returns from estimated transition matrices between the formal and informal sectors and the aggregate growth rate. These parameters include the worker's meeting rates, the meeting probabilities for formal and informal workers, and the tail of the overall initial productivity distribution. Moreover, we infer parameters that determine workers' static returns from the aggregate share of informality, and by informality rates by age bins. These parameters encompass the formal sector registering cost, payroll tax, and the informal sector hiring costs.The remaining parameters are estimated directly from the data using a Generalized Method of Moments (GMM).

[4] This can also be interpreted as expenses that firms must bear to avoid detection by authorities.

[5] This structure is consistent with social security programs in developing economies, in which high-income households subsidize access for low-income households.

We use our estimated model to assess the effects of two types of commonly used formalization policies. On the one hand, governments often implement policies that increase the prosecution of informal activity (sticks). In our model, we represent these policies as an increase in the expected costs firms face when hiring an informal worker, capturing increments in government auditing efforts or fines. On the other hand, governments also implement policies that decrease the cost of operating formally (carrots). We model this type of policies as a reduction in the fixed costs associated with hiring a formal worker. We highlight three main results that arise from comparing the effects of implementing each policy.

Firstly, a reduction in the cost of hiring formal workers is more effective at reducing labor informality increasing the cost of hiring an informal worker. Decreasing registering cost in the formal sector by 20% decreases informality from four percentage points, from 31% to 27%. In contrast, increasing auditing efforts by the same amount has almost negligible effects, decreasing the share of informal labor only by one percentage point.

Secondly, both types of formalization policies lower the growth rate of the economy. While initially surprising, the intuition behind this result can be understood through the equity-efficiency trade-off at the core of our framework. Both policies move less skilled workers into the formal sector, generating two countervailing effects on the growth rate. First, they improve the learning opportunities for the newly formalized workers, which boosts growth. However, as these newly formalized workers have lower skills, both policies also lower the stock of human capital in the formal sector. Consequently, both policies create a crowding-out effect on the learning opportunities of formal workers by decreasing the quality of interactions in this sector. This crowding-out effect dominates the potential gains from human capital improvements for the newly formalized workers, leading to a decrease in the growth rate. This logic suggests a tension between formalizing low-skilled workers to equalize learning opportunities and segmenting the market so that only the most skilled workers interact, thereby expanding the knowledge frontier and boosting economic growth.

Finally, we find that the policies have opposite effects on welfare: increasing the costs of hiring informal workers has a negative effect, while decreasing the registration costs for formal workers has a positive effect. In our setting, aggregate welfare along the balanced growth path (BGP) can be decomposed into two components. First, a level component captures the baseline consumption of the average worker. Second, a growth component captures how much the consumption for the average worker grows along that BGP, which is determined by the growth rate of the economy. Both types of policies lower the growth rate and therefore decrease welfare along the second dimension. However, increasing the cost of hiring informal workers creates deadweight losses for the informal workers who do not formalize after the policy, in the form of lower wages. These losses lower the level component in the new BGP after the policy is implemented. In contrast, reducing the costs of hiring formal workers increases wages for all formal workers, including the newly formalized workers, which boosts the level of consumption in the new BGP. The increase in the level of consumption under the 'carrot' policy offsets the lower growth rate and ultimately generates an increase in welfare. These results suggest that governments aiming to reduce labor informality should focus on lowering worker hiring costs rather than increasing government auditing or fines for informal establishments.

Related Literature. Our work contributes to three strands of the literature. The first strand involves the study of knowledge diffusion and economic growth. Our model aligns with the concept of a mean-field game, as introduced by Lasry & Lions (2007), with the productivity distribution serving as a state of the economy. Previous studies, such as those by Lucas Jr. (2009), Lucas & Moll (2014), and Perla & Tonetti (2014), have developed frameworks in which knowledge diffusion occurs through interactions or imitation. We extend this work by introducing multiple sectors. In our model, workers endogenously sort into formal and informal sectors based on their static returns and learning opportunities, following the approach of Akcigit et al. (2018). Moreover, we allow learning opportunities to vary between sectors, capturing the idea that more productive colleagues imply greater spillovers, as demonstrated by Jarosch et al. (2021). Consequently, our model establishes a productivity cutoff that effectively sorts workers into formal and informal sectors, akin to the framework presented by Perla et al. (2021).

Secondly, our work contributes to the literature that examines wage dynamics in developing economies. The study of wage profiles throughout the life cycle in developing economies is limited, and our research takes a step toward understanding frictional labor markets in these settings. Lagakos et al. (2018) is among the first to explore wages across the life cycle for both developing and developed economies. They find steeper wage profiles throughout the life cycle for developed economies and discuss potential mechanisms driving this pattern. In contrast, our work identifies one friction that contributes to this phenomenon. We demonstrate that wages grow faster in the formal sector, and the presence of a large informal sector suppresses average wage growth. Beneath the differing wage profiles for formal and informal workers, we develop a theory explaining variations in human capital accumulation. Our findings align with Engbom (2022), where higher contact rates lead to increased human capital and, consequently, higher wage growth. However, in our framework, the accumulation of human capital is the primary driver of wage growth throughout the life cycle, consistent with the insights from Ben-Porath (1967) and Heckman et al. (1998).

Finally, our work contributes to the literature on informal labor markets and their impact on economic performance. There is a growing body of literature examining labor informality from a macro perspective, with notable contributions from Ulyssea (2018), Dix-Carneiro et al. (2021), and Meghir et al. (2015). More closely related to our work, Bobba et al. (2022) investigates the relationship between informal markets and human capital investments. However, these studies primarily focus on the static effects of informality. In contrast, we forefront the dynamics and explore how informal markets not only affect aggregate production but also influence economic growth. Similarly, Lopez Garcia (2015) examines how informal markets shape incentives to invest in human capital, yet their work overlooks the feedback effect of human capital in determining the size of the informal sector. To address this gap, we build a general equilibrium model where the stock of human capital and the share of informality are endogenously determined. Lastly, a recent group of articles investigates the effects of informality on economic growth, including works by Akcigit et al. (2024) and Lopez-Martin (2019). However, these papers approach informality from the firm perspective and do not delve into the intricacies of human capital and economic performance, as emphasized by Manuelli & Seshadri (2014).[6]

[6] Ulyssea (2020a) highlights the need to study workers dynamic decision regarding formality status.

The rest of the paper follows this structure. Section 2 describes the institutional settings, presents our empirical findings, and discusses the data. Section 3 introduces our model and outlines the equilibrium. Section 4 details our estimation strategy and provides an overview of our sources of identification. In Section 5, we present the results of our counterfactual exercises. Finally, Section 6 offers concluding remarks.

2 Descriptive Analysis

This section first describes the labor informality definitions and the data used in our empirical exercises. Then, it presents four empirical facts on wages for formal and informal workers. Our main analysis focuses on Chile, where we can exploit the richness of the formal and informal workers' survey data. Appendix B.3 confirms that some of our main findings also hold in Colombia, another emerging economy with high informality rates.

2.1 Definitions and Data

We define an informal worker as any employee whose contract does not comply with local labor regulations. This definition is consistent with Chilean regulations and previous definitions in the literature.[7] Specifically, an informal worker is an employee who either does not have a formal labor contract or does not contribute to the Chilean pension system.[8]

Our main data source is the Chilean *Encuesta de Protección Social* (EPS). The EPS is a comprehensive longitudinal survey designed to gather data on various aspects of social protection. The survey spans five waves conducted in 2002, 2004, 2006, 2009, and 2015.[9] We utilize two blocks of the survey. First, we use the block containing information on each individual's demographics, such as age and education. Second, we use the block containing the individual's job history dating back to 1980.[10] The worker's job history includes information on job types, occupation, economic sector, region, weekly hours worked, starting and ending dates, type of contract (if any), pension fund contributions, job category (e.g., employed by a firm or self-employed), and the number of workers in the firm. Since 2004, individuals also provide information on monthly disposable income for each job. The first wave conducted in 2002 only considered workers' affiliation with a pension fund, making it representative only of formal workers. We restrict our sample starting from the 2004 wave, the year in which individuals without pension affiliation were included, making the survey representative of all workers.

The EPS offers four advantages for analyzing employment dynamics for formal and informal workers. First, the detailed information for each job allows us to classify workers as formal or informal based on their contract type and pension fund contributions. Second, the data on disposable

[7] For Brazil, Ulyssea (2018) defines an informal worker as an employee who does not hold a formal labor contract.

[8] *Ley 3500* (1980) states that at the start of any labor relation, any non-affiliated worker generates automatic affiliation to the pension system and the obligation to contribute 10% of her (his) salary to the fund.

[9] We exclude the 2012 and 2019 waves from our analysis. The 2012 wave is not representative at the national level, and the questions for 2019 changed because of the COVID-19 pandemic.

[10] When an individual enters the survey, they are asked for their job history since 1980. If, in a specific wave, the individual was already part of the survey, they are asked about their job history since the survey's last wave.

monthly income and the number of hours worked allows us to compute the hourly wage rate for jobs starting from 2002 onwards.[11] Third, despite the survey being conducted over scattered years, the job history block enables us to reconstruct the employment history for each worker since 1980. We aggregate different jobs for each individual to construct our baseline sample, which is an unbalanced year-level panel with 18,650 workers from 2002 to 2016.[12] The length and coverage of our sample allow us to overcome the shortcomings of previous studies by tracking formal and informal workers over long periods. For example, we can follow 50% of the workers in our sample for at least five years and 20% for at least ten years.[13] Finally, although we only use data for jobs active from 2002 onwards due to wage availability, we use workers' job histories since 1980 to measure experience, which we can further disaggregate into formal and informal experiences.[14]

We exclude certain types of workers from our analysis. First, in accordance with Chilean labor regulations, we include male workers aged 15 to 65 and female workers aged 15 to 60, aligning with the country's legal starting working age and retirement age for men and women, respectively. Second, in our baseline specifications, we focus exclusively on private-sector salaried workers, omitting self-employed individuals, entrepreneurs, workers without remuneration, as well as public-sector and armed forces employees. Notably, the exclusion of self-employed workers deviates from previous informality studies, such as Meghir et al. (2015). This exclusion is due to the tendency of self-employed workers to be informal, and their wage trajectories likely respond to different dynamics than those of workers employed by a firm. Hence, we exclude self-employed workers to make the formal versus informal wage comparison more transparent.

Table 1 displays summary statistics for our baseline sample. There are three key points worth highlighting. First, on average, formal workers are paid more than informal workers. Nevertheless, wages for informal workers are more dispersed than those for formal workers, consistent with previous findings. Second, formal workers tend to work more hours per week and have more years of experience. Third, transition probabilities between the formal and informal sectors are highly asymmetric. On average, 13% of informal workers migrate to the formal sector the following year, whereas only 2% of formal workers become informal the next year. Importantly, these transition probabilities are computed by restricting our sample to active workers, which does not count migrations into and from unemployment. Appendix B.1 provides additional descriptive statistics such as informality rates by age, education, firm size, occupation, and economic sector.

We complement the EPS with two additional data sources. First, we use Consumer Price Index (CPI) data from the Central Bank of Chile to construct real hourly wages. Second, we use the World Bank's Gross Domestic Product (GDP) time series data to calculate the average GDP growth rate for Chile during our sample period. We use the average GDP growth rate as a targeted moment later in our quantitative analysis.

[11] We construct wages for 2002 and 2003 using job history information.

[12] For workers with multiple jobs in a given year, we take the job with the highest total hours worked.

[13] In contrast, Meghir et al. (2015) use a rotating panel in Brazil that follows individuals for five consecutive months and then for another four months one year after their entry into the sample. Samaniego De La Parra & Fernández Bujanda (2020) use a rotating panel in Mexico that follows workers for only five quarters.

[14] We compute experience by dividing the cumulative hours worked up to a particular year by the number of hours corresponding to a full work schedule of 48 hours per week.

Table 1: Summary Statistics

	(1) Informal	(2) Formal	(3) All
Fraction of workers	0.18	0.78	1.00
Mean log wage	8.15	8.53	8.46
Std. deviation low wage	0.62	0.58	0.61
Mean weekly working hours	40.96	46.01	45.10
Fraction of male workers	0.49	0.61	0.59
Mean experience (years)	12.34	15.67	15.07
Transitions (by initial status)			
Frac. of informal workers next year	0.87	0.02	...
Frac. of formal workers next year	0.13	0.98	...
Number of observations	16,763	76,102	92,902
Number of workers	5,141	12,503	14,732

Notes: Table 1 displays summary statistics for the baseline sample 2002-2016. We include male workers aged 15 to 65 and female workers aged 14 to 60. We exclude self-employed workers, entrepreneurs, workers without remuneration, and public-sector and armed forces employees. Wage is the real hourly wage. The variable experience corresponds to the equivalent in years of a full-time work schedule of 48 hours per week.

2.2 Stylized Facts

This section presents four facts about wages for formal and informal workers. The first two facts provide new insights into the cross-sectional wage differences between formal and informal workers, while facts three and four analyze wage dynamics for formal and informal workers over the years.

Fact 1: There is a Formal-Informal Wage Gap, even after Controlling for Workers' Unobserved Characteristics

We begin by revisiting the well-established fact of the existence of a formal-informal wage gap. As highlighted by Perry et al. (2007), the literature has found evidence that, on average, formal workers earn higher wages than informal workers. This formal-informal wage gap persists even after controlling for several observable worker characteristics. By exploiting the panel structure of our data, we provide new evidence that this formal wage premium remains even after accounting for workers' time-invariant unobserved characteristics.

The formal-informal wage gap, which is the average difference in wages for formal and informal workers, is given by the parameter β in the following regression equation:

$$\log w_{i,t} = \beta \text{Formal}_{i,t} + \omega_{\text{age}} + \omega_{\text{educ}} + \omega_{\text{gender}} + \omega_{\text{occ}} + \omega_{\text{sec}} + \omega_{\text{reg}} + \omega_{\text{fsize}} + \omega_t + \omega_i + \varepsilon_{i,t}, \qquad (1)$$

where $\log w_{i,t}$ is worker's i log wage at time t, $\text{Formal}_{i,t}$ is a dummy variable equal to one if worker i is formal in year t and zero if they are informal, and ω_{age}, ω_{educ}, ω_{gender}, ω_{occ}, ω_{sec}, ω_{reg}, ω_{fsize}, ω_t and

Table 2: Formal-Informal Wage Gap

	Dependent variable: $\log w_{i,t}$		
	(1)	(2)	(3)
Formal	0.412***	0.155***	0.0709***
	(0.0188)	(0.0150)	(0.0203)
Observations	44,330	44,330	44,330
Adj R-squared	0.0675	0.512	0.863
Controls	No	Yes	Yes
Worker Fixed Effects	No	No	Yes

Notes: Table 2 reports estimates for β in equation (1). Column (1) displays the results without including any fixed effects. Column (2) shows the results when controlling for worker characteristics but without including worker-fixed fixed-effects. Column (3) illustrates results when controlling for worker characteristics and worker-fixed fixed-effects. Standard errors in parenthesis are clustered at the worker level. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

ω_i represent age-bin, educational attainment, gender, occupation, sector, region, firm size quartiles, time and individual fixed effects, respectively. Regarding the inclusion of firm size quantiles fixed effects, we perform this exercise with the caveat that we do not observe firm identifiers. The EPS has information on the size of the firm of each worker, but it is not possible to determine whether two different individuals work at the same firm. Nevertheless, by controlling for firm size, we use the variation between wages of formal and informal workers in similarly sized firms to better isolate the role of formality status in explaining wage differentials.[15]

Table 2 displays different estimates of the formal-informal wage gap. In Column 1, we estimate the gap without including any controls or worker fixed effects. In Column 2, we add observable worker characteristics, and in Column 3, we include both worker characteristics and worker fixed effects. As our focus is on the estimate in Column 3, we restrict our sample across all specifications to workers whom we observe in both the formal and informal sectors in different years. This restriction enhances the transparency of the comparison of the results across the columns. The inclusion of worker fixed effects implies that we estimate a *within-worker* formal-informal wage gap by exploiting the variation in formality status for a given worker.

We find a substantial formal-informal wage gap across all the specifications in Table 2. Column 1 shows that formal workers earn, on average, more than 50% more than informal workers. Once we control for worker characteristics in Column 2, the formal-informal wage gap reduces to 17%. The reduction in the wage gap from Column 1 to Column 2 is consistent with the fact that formal workers tend to be more educated and work in larger firms, which positively affects wages and confounds the estimate of β. Finally, the estimate in Column 3 illustrates that, even after controlling for worker characteristics and fixed effects, formal workers earn, on average, 7.4% higher

[15] Consistent with the large majority of firm dynamics models, one interpretation of this exercise is that firm size proxies for firm productivity. Hence, to some extent, our estimates of the formal-informal gap account for the fact that formal and informal workers can be employed by firms of different productivity levels.

wages than informal workers. All our estimates are statistically significant at the 1 percent level.

Our results align with some findings of previous literature while contrasting with others. First, our raw estimates of the formal-informal wage gap in Chile are consistent with the estimates of Perry et al. (2007), which finds wage gaps of 43%, 67%, and 46% between salaried formal and informal workers for Argentina, Bolivia, and the Dominican Republic, respectively. Moreover, our estimates of the wage gap, when controlling for observable worker characteristics, are similar to the 20% gap documented by Ulyssea (2010) for Brazil. At the same time, our estimate in Column 3 contrasts with the findings of Ulyssea (2018). In the context of Brazil, Ulyssea (2018) estimates a similar regression to equation (1) but controls for firm fixed effects instead of worker fixed effects. He finds that the formal-informal wage gap becomes statistically insignificant when including such fixed effects. Two factors can help to reconcile the difference between our estimates and his. First, by controlling for firm fixed effects, he estimates a *within-firm* wage gap, while we estimate a *within-worker* wage gap. Because both specifications use different variations to estimate the formal-informal wage gap, it is not straightforward to expect similar results. Second, his estimates use workers employed by firms with five or fewer employees, whereas our estimate includes workers employed by firms across the entire firm size distribution.

We interpret the formal-informal wage gap estimates as suggestive evidence of worker sorting. As previously mentioned, the comparison between the estimates of Column 1 and Column 2 in Table 2 is consistent with the fact that formal workers are more educated. Similarly, the fact that the formal-informal wage gap decreases between Columns 2 and 3 suggests that workers with higher unobserved skills, captured by the worker fixed effects, sort into the formal sector. Therefore, altogether, our results suggest that more skilled workers (measured by observed and unobserved characteristics) tend to be formal. Of course, we do not interpret our estimates as causal as their might be worker unobserved time-variant characteristics that could be potentially correlated with the formality status. Hence, our results only provide suggestive evidence of this sorting pattern.

Fact 2: Experience In The Formal Sector Is Associated With Higher Wages, While Experience In The Informal Sector Is Not

We now enrich equation (1) by including workers total experience:

$$\log w_{i,t} = \beta \text{Formal}_{i,t} + \gamma \operatorname{asinh} \exp_{i,t} + \omega_{\text{age}} + \omega_{\text{educ}} + \omega_{\text{gender}} + \omega_{\text{occ}} + \omega_{\text{sec}} + \omega_{\text{reg}} + \omega_{\text{fsize}} + \omega_t + \omega_i + \varepsilon_{i,t}, \quad (2)$$

where $\exp_{i,t}$ is the worker's i total experience (in years) at time t, and *asinh* is the inverse hyperbolic sine function. Because workers enter the workforce with zero experience, we use the inverse hyperbolic sine transformation instead of the logarithmic one.[16] Moreover, we further exploit the richness of our data and estimate a variant of equation (2) in which we control for worker's i formal and informal experiences separately. We denote by $\exp^F_{i,t}$ and $\exp^I_{i,t}$ worker's i total formal and informal experiences at time t, respectively.

Table 3 reports estimates of equation (2). Columns 1 and 2 examine the formal-informal wage gap and the effect of total experience on wages. Positive point estimates in both columns indicate that

[16]See Bellemare & Wichman (2020).

Table 3: Formal-Informal Wage Gap with Formal and Informal Experience

	Dependent variable: $\log w_{i,t}$			
	(1)	(2)	(3)	(4)
Formal	0.136***	0.0697***	0.0833***	0.0585***
	(0.0150)	(0.0203)	(0.0161)	(0.0207)
$\text{asinh}\,\text{exp}_{i,t}$	0.0535***	0.0243		
	(0.00955)	(0.0182)		
$\text{asinh}\,\text{exp}^F_{i,t}$			0.0442***	0.0357**
			(0.00678)	(0.0155)
$\text{asinh}\,\text{exp}^I_{i,t}$			-0.0125**	-0.0199
			(0.00508)	(0.0151)
Observations	44,330	44,330	44,330	44,330
Adj R-squared	0.515	0.863	0.516	0.863
Controls	Yes	Yes	Yes	Yes
Worker Fixed Effects	No	Yes	No	Yes

Notes: Table 3 reports estimates for β in equation (2). Column (1) displays the results without including any fixed effects. Column (2) shows the results when controlling for worker characteristics but without including worker-fixed fixed-effects. Column (3) illustrates results when controlling for worker characteristics and worker-fixed fixed-effects. Standard errors in parenthesis are clustered at the worker level. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

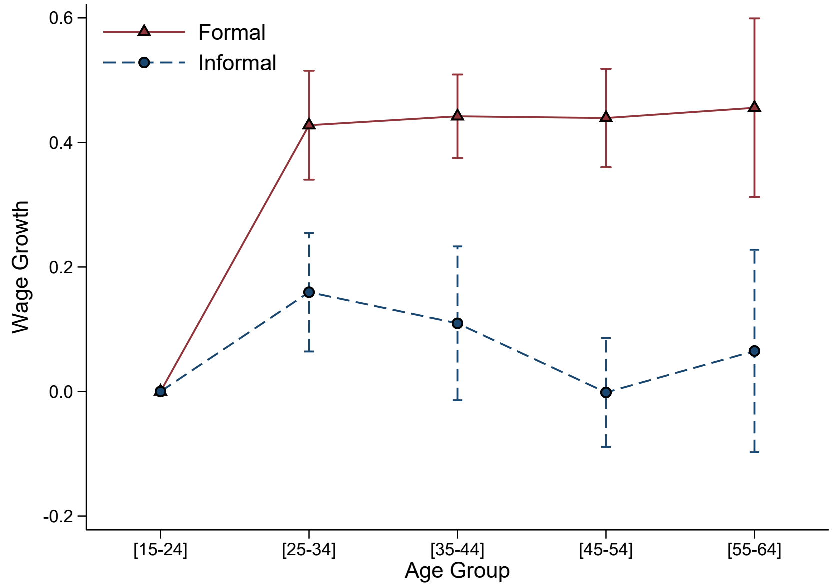
more experienced workers earn higher wages, conditional on the controls and worker fixed effects. However, this positive relationship ceases to be statistically significant when including worker fixed effects. Columns 3 and 4 consider a more flexible specification in which we allow formal and informal experience to correlate with wages differently. Surprisingly, in Column 3, we find that formal and informal experiences affect wages in opposite directions. As workers accumulate experience in the formal sector, they also earn higher wages. Nevertheless, accumulating experience in the informal sector is associated with lower wages. When we add the worker fixed effects in Column 4, the point estimates retain their sign, but informal experience is no longer statistically significant.[17] Our more complete specification (Column 4) suggests that experience in the formal sector is associated with higher wages, while experience in the informal sector is not. Across all specifications, the formal-informal wage gap remains positive and statistically significant.

Fact 3: Formal Workers' Wages Increase Significantly More Over the Life Cycle

Now we turn to study wage dynamics for formal and informal workers over the years. To do so, we start by presenting age-wage profiles for formal and informal workers. These profiles are used to analyze life cycle wage dynamics for both groups. We construct the age-wage profiles for formal and informal workers as follows. First, we classify workers as formal or informal according to the definition provided in the previous section. Second, we divide formal and informal workers into five 10-year age bins: 15-24, 25-34, 35-44, 45-54, and 55-64. Then, we compute the average wage

[17] The opposite effects of formal and informal experience could potentially explain why total experience becomes statistically insignificant in Column 2.

Figure 1: Wages Over the Life Cycle for Formal and Informal Workers



Notes: Figure 1 displays wage paths over the life cycle for formal and informal workers. We restrict our sample to salaried workers, excluding self-employed workers, entrepreneurs, workers without remuneration, and public-sector and armed forces employees. We compute the average wage for each 10-year age bin relative to the average wage for workers less than 24 years. Vertical dashed lines denote 90 percent confidence intervals.

for each age bin for both formal and informal workers. Finally, to highlight the differential wage growth patterns between formal and informal workers, we normalize the average wages for each bin by the average wage of the lowest age bin (15-24). This normalization eliminates any wage-level differences between formal and informal workers at entry into the workforce. As Facts 1 and 2 showed, formal workers earn higher wages.

Figure 1 displays raw wage paths for formal and informal workers over the life cycle. This figure conveys three important messages. First, there is a significant difference in wages for formal and informal workers throughout the life cycle. Even if formal and informal workers earned the same wage when entering the workforce, formal workers would earn wages 60% higher than their informal counterparts by the end of their careers. Second, a larger fraction of the wage divergence between formal and informal workers occurs early in the life cycle. While formal workers experience a wage increase of 40% between their 20s and 30s, informal workers only experience a wage increase of around 20%. Third, wages for informal workers barely increase throughout the life cycle. Although informal workers experience a wage increase at an early age, this increase dissipates by the end of their careers. Notably, informal workers aged 55-64 experience an almost null wage increase relative to informal workers in the 15-24 age bin.

Life cycle wage patterns for formal and informal workers are consistent with recent findings on wage growth across countries. Lagakos et al. (2018) find that wages in developed countries increase relatively more over the life cycle than wages in developing economies. Figure 1 highlights that the formal sector behaves like a "developed economy labor market", whereas the informal sector displays the patterns of a "developing economy labor market." In Appendix B.2 , we report wage trajectories over the life cycle for all formal and informal workers, including self-employed workers, entrepreneurs, workers without remuneration, and public-sector and armed forces employees. Interestingly, the wage-growth patterns Lagakos et al. (2018) document for Chile, which combine wage trajectories for formal and informal workers, lie between the formal and informal wage-growth patterns displayed in Figure B6. Hence, our results extend previous analyses on life cycle wage growth in developing countries by decomposing wage growth patterns for formal and informal workers.

Appendix B.2 also provides additional exercises analyzing life-cycle wage growth for formal and informal workers. First, we display age-wage profiles while controlling for a large vector of workers' observable characteristics: education, gender, occupation, sector, region, firm size, and year. Although the inclusion of these controls reduces the difference in wage growth rates for younger workers, formal workers still experience a wage increase 20% larger than informal workers.[18] Second, instead of presenting age-wage profiles for formal and informal workers, we present experience-wage profiles as a measure of life-cycle wage growth.[19] Experience-wage profiles display similar patterns to age-wage profiles: wages for formal workers increase significantly more for more experienced workers, whereas wages for informal workers seem not to increase as workers accumulate experience. These patterns hold with and without controlling for the workers' characteristics.

Potential Mechanisms. Three potential theories can rationalize the fact that formal workers experience a larger wage increase over the life cycle.[20] First, we will discuss the theories that, given our empirical findings, we believe fail to explain the differential wage trajectories for formal and informal workers. Then, we will discuss our preferred interpretation.

The first theory that could potentially explain the dynamics in Figure 1 is the theory of differential search frictions for formal and informal workers. In a frictional labor market, the steeper wage curve for formal workers could indicate that they face fewer frictions. Similar to the concept in Burdett (1978), formal workers may have higher wages at the end of their careers simply because they received more job offers over the years. The more job offers they received, the greater the probability of finding a high-wage job. If this explanation is correct, one would expect informal workers to have fewer job-to-job transitions. However, previous studies have documented the opposite. Meghir et al. (2015) show that informal-to-informal transition rates in the Brazilian context are between 50% and 300% higher than formal-to-formal transitions.[21] Thus, a narrative of differential labor market frictions is inconsistent with other aspects of our data.

The second theory that could potentially rationalize our findings is the theory of back-loaded contracts. In the presence of moral hazard or limited monitoring, firms could offer wage schedules that pay more in the future to incentivize workers as highlighted by Lazear (1979). These wage back-loading schemes could potentially explain the differential wage trajectories for formal and in-

[18]The inclusion of these controls naturally increases the standard errors. Hence, the estimates become slightly noisier, particularly for salaried workers.

[19]This contrasts with Lagakos et al. (2018), who, lacking an explicit measure of individual-level experience, compute potential experience as the number of years elapsed since a worker finished schooling or turned 18.

[20]These three potential environments have been widely discussed in previous literature. See, for instance, Lagakos et al. (2018) and Jarosch et al. (2021).

[21]The range comes from analyzing male and female workers in two different labor markets.

formal workers. However, there are two reasons this explanation is unappealing. First, if workers receive back-loaded wages due to moral hazard, the fact that formal workers experience higher wages at the end of their careers would suggest that moral hazard issues are more salient in the formal sector. This is implausible, as formal workers have labor contracts that improve enforcement, whereas informal workers do not. Second, these types of contractual relationships are likely linked to occupations and sectors where it is harder to monitor worker performance. Figure B7 in Appendix B.2 presents wage-age profiles controlling for worker occupation and firm economic sector, among other controls. The resulting wage-age profiles indicate that formal workers experience higher wage growth over the life cycle, even after controlling for these two variables.

The last and our preferred theory is the one on human capital accumulation. It has been popular in previous studies to interpret wage-age and wage-experience profiles as reflecting human capital accumulation. Under this interpretation, our findings imply that formal workers accumulate more human capital over the life cycle than informal workers. Different incentives to accumulate human capital can explain the differential learning over the life cycle. If there are frictions to switch from the informal to the formal sector, informal workers might have fewer incentives to learn because higher skills would be associated with higher earnings and, therefore, a higher probability of being detected by the government.[22] Alternatively, workers can accumulate human capital by interacting with others, as described by Lucas Jr. (2009); Lucas & Moll (2014); Jarosch et al. (2021). If informal workers have fewer interactions with other workers or if they meet less knowledgeable individuals, they will experience flatter wage-age profiles than their formal counterparts. Both explanations are consistent with our findings.

Fact 4: Formal Workers Earn Higher Future Wages

We use individual worker wage trajectories to further explore the life cycle wage dynamics documented by the previous fact. In the spirit of Jarosch et al. (2021), we relate workers' future wage dynamics to formality status and various observable characteristics. Specifically, we estimate the following relationship separately for different time horizons h:

$$\log w_{i,t+h} = \beta \text{Formal}_{i,t} + \gamma \log w_{i,t} + \theta \text{asinhexp}_{i,t} + \omega_{\text{age}} + \omega_{\text{educ}} + \omega_{\text{gender}} + \omega_{\text{occ}} + \omega_{\text{sec}} + \omega_{\text{reg}} + \omega_t + \varepsilon_{i,t}, \quad (3)$$

where $\log w_{i,t+h}$ is worker i's log wage in year $t+h$, $\log w_{i,t}$ is the log wage at time t, and the rest of the variables are as defined in (1) and (2). Relative to specification (2), we add the current-year worker's wage, and we purposely exclude firm size quartile fixed effects as we explore the role of firm size on learning below.

The parameter of interest in equation (3) is β. This parameter captures the average wage gap at time $t+h$ between formal and informal workers at time t, conditional on observable worker characteristics. We run this regression by pooling all observations across the 15 years in our sample (2002 - 2016). Moreover, we consider different time horizons by varying $h = 1, 2, \ldots, 10$. The different time horizons imply that we consider a subset of years for each value of h. For

[22] Manuelli & Seshadri (2014) propose a framework in which low total factor productivity depresses the returns to human capital accumulation, resulting in flat wage-experience profiles.

Table 4: Dynamic Formal-Informal Wage Gap

	Dependent var.: $\log w_{i,t+h}$					
	(1)	(2)	(3)	(4)	(5)	(6)
Time Horizon in Years	$h = 1$	$h = 2$	$h = 3$	$h = 4$	$h = 5$	$h = 10$
Formal	0.0269***	0.0442***	0.0454***	0.0473***	0.0516***	0.0536**
	(0.00612)	(0.0107)	(0.0146)	(0.0165)	(0.0176)	(0.0244)
Observations	36,319	29,884	24,884	21,480	18,705	9,437
Adj R-squared	0.831	0.686	0.577	0.532	0.509	0.422

Notes: Table 4 reports estimates for β in equation (3). Column titles indicate the time horizon h. The regressions include current log wage, current log total experience, and fixed effects for age group, educational attainment, gender, occupation, sector, region, and time. Standard errors in parenthesis are clustered at the individual level. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

instance, for $h = 1$, we use 14 years, while for $h = 5$, we only have 10 years available. Finally, we restrict our sample to salaried workers only and cluster standard errors at the worker level.

Table 4 reports the estimates of β in equation (3) for different time horizons. We report estimates for $h = 1, 2, \ldots, 5$ and $h = 10$.[23] We highlight three main findings from these exercises. First, formal workers experience a larger future wage increase relative to their informal counterparts. The average gap in future wages is quantitatively large. On average, formal workers earn a wage 2.7% higher next year than their informal counterparts. Second, the formal worker's future wage premium persists over different time horizons. Over a period of 10 years, formal workers earn, on average, a wage 5.7% higher than their informal counterparts. Third, the formal future wage premium gradually increases as the time horizon extends.[24]

Figure 2(a) presents estimates for the formal future wage premium across all time horizons. We convert the estimates of β from equation (3) into percentage differences between formal and informal workers' future wages.[25] As Table 4 foreshadowed, we find a substantial future wage premium for formal workers across all time horizons. This premium peaks around seven years forward ($h = 7$), with formal workers earning, on average, 6% higher wages than informal workers. Although standard errors increase as the time horizon extends, the formal workers' future wage premium remains statistically significant at the 95% confidence level for all time horizons.

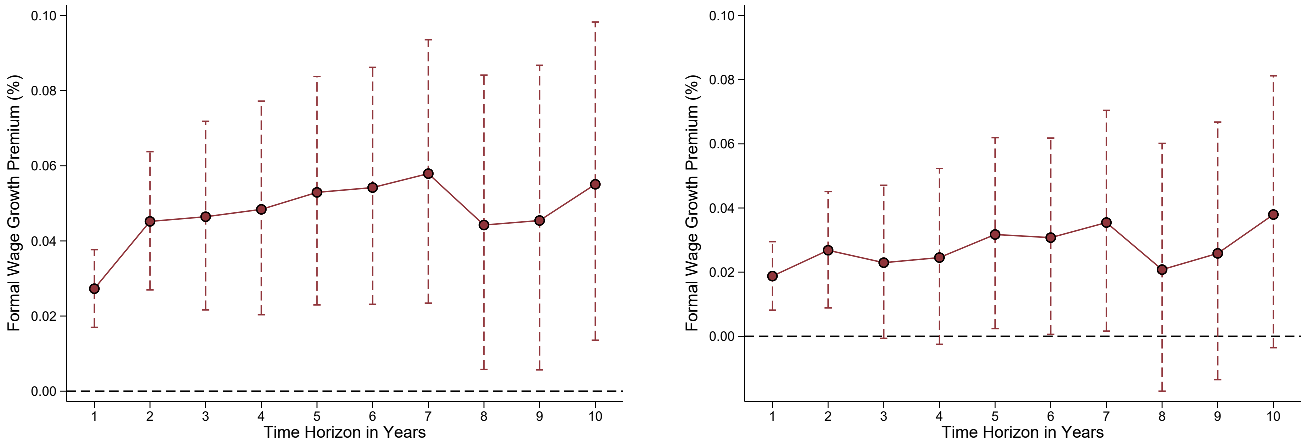
The results displayed in Table 4 and Figure 2(a) support the view that formal workers learn more than informal workers. In a labor market where wages are increasing in human capital (skills, knowledge), the steeper wage growth for formal workers is associated with higher learning. As formal workers learn more over the years, their wages increase more over the life cycle compared to those of informal workers, as shown in Figure 2.

[23] The estimates for the remaining years are very similar.

[24] Jarosch et al. (2021) find a similar increasing pattern over time horizons when studying the effect of having high-paid coworkers on future wages.

[25] As equation (3) is a log-dummy variable model, the estimated formal future wage premium is $\exp(\hat{\beta}) - 1$.

Figure 2: Dynamic Formal Wage Premium



(a) Without Firm Size

(b) With Firm Size

Notes: Figure 2(a) displays formal future wage premium based on the estimates of β from equation (3). Figure 2(b) reports the same estimates when controlling for quantiles of firm size distribution in equation (3). Each dot represents the estimated wage premium at the time horizon $t+h$: $\exp(\hat{\beta}_h) - 1$, for $h = 1, \cdots, 10$. Vertical dotted lines denote 90 percent confidence intervals.

One important aspect of this view remains to be explored: *where does the learning take place?* As shown by Jarosch et al. (2021) and Herkenhoff et al. (2024), workers learn at the workplace from their coworkers. To the extent that formal workers have more knowledgeable peers, learning from coworkers could plausibly explain the different learning patterns we observe in the data. To explore this mechanism, we estimate a variant of equation (3) in which we control for firm size quartile fixed effects.

Figure 2(b) displays the dynamic formal wage premium for different time horizons when controlling for firm size. When comparing Figures 2(a) and 2(b), we derive two key messages. First, point estimates of β for different time horizons drop by around half when controlling for firm size fixed effects. We interpret the reduction in these coefficients as evidence that the workplace plays an important role in the learning opportunities for formal and informal workers, consistent with Jarosch et al. (2021) and Herkenhoff et al. (2024). The idea that formal workers sort into better firms, where they interact with better coworkers, can potentially explain our findings. Nevertheless, the second message is that, even after controlling for firm size, formal workers experience higher wage increases over the years than informal workers. All point estimates in 2(b) are positive, and the ones for time horizons 1, 2, 5, 6, and 7 are statistically significant. We believe these patterns indicate that, although firm size plays a role in workers' learning, there is still a significant part of the learning process that might be taking place outside the firm. We propose some plausible candidates for these outside-the-firm learning opportunities while taking in the next subsection.

2.3 Taking Stock

In the previous subsection, we documented four facts:

1. There is a formal wage premium, and worker's sorting explains a portion of this premium.

2. Formal experience is associated with higher wages, while informal experience is not.
3. Wages for formal workers increase relatively more over the life cycle.
4. There is a dynamic wage premium: formal workers earn higher future wages.

We believe that these four facts support the narrative that more skilled workers sort into the formal sector and that there are endogenous differentials in learning across formal and informal sectors. More knowledgeable workers sort into the formal sector, where they interact with other more knowledgeable workers. In contrast, less skilled workers sort into the informal sector, where they interact with other less skilled workers. These sorting patterns create better learning opportunities for formal workers, which then translate into formal workers experiencing a steeper wage increase over the years relative to informal workers. This dynamic formal wage premium is the central mechanism that leads to formal workers experiencing a higher wage increase over the life cycle.

The interpretation that workers learn from others outside the workplace is central to our narrative. Our fourth fact shows that formal workers learn more than informal workers, even when employed in similar-sized firms. This result implies that there are learning frictions between formal and informal workers occurring outside the firm. Face-to-face interactions outside the workplace, like those studied in Atkin et al. (2022), Catalini et al. (2020), and Andrews (2023), are a natural explanation. To the extent that high-skilled formal workers tend to live or work close by, attend the same schools, or frequent the same bars and restaurants, the quality of interactions for formal workers would be higher than that for informal workers.

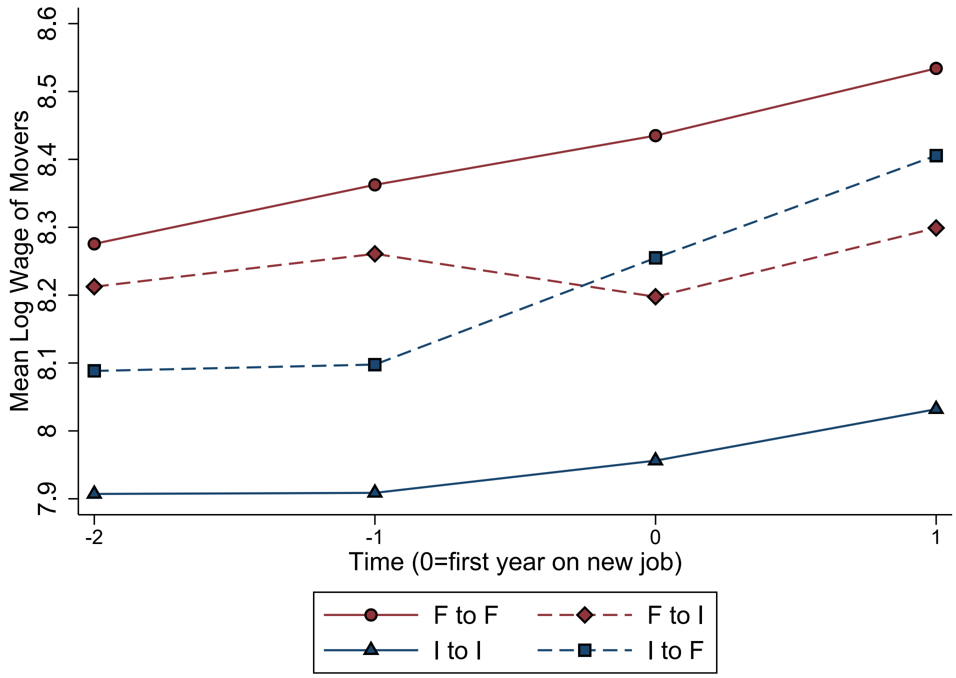
Figure 3 summarizes the economic mechanisms underlying our narrative. Similar to Card et al. (2013), we present wage dynamics for workers who switch sectors.[26] We focus on workers who change jobs and for whom we observe wages in both the old and new job sectors for two periods each. Specifically, we consider four types of workers: those who switched from a formal job to another formal job, those who switched from a formal job to an informal job, those who switched from an informal job to another informal job, and those who switched from an informal job to a formal job. For each type of worker, we require wage observations for two periods in the old job sector and two periods in the new job sector.[27] Finally, for each group, we compute mean log wages for the two periods before the job change and the two periods after.

We highlight three key messages from this figure. First, different mobility groups differ in wage levels before and after the move. Formal-to-formal workers have higher average wages both before and after the move. In contrast, informal-to-informal workers consistently have the lowest average wages. Interestingly, formal-to-informal workers begin with higher average wages than informal-to-formal workers, but after changing sectors, the former group ends up with lower average wages than the latter. This ordering of average wages before and after the move supports our view that

[26]Unlike Card et al. (2013), we do not claim causality between changing sectors and wage changes. Our goal in presenting this figure is to provide suggestive evidence of the economic mechanisms included in our quantitative model, motivated by our facts.

[27]Given the structure of the EPS, we cannot link jobs across waves for the same individual. Hence, we cannot distinguish whether workers within each group change jobs within the same sectors before or after the specific job change we consider.

Figure 3: Mean Wages of Sector Movers



Notes: Figure 3 shows mean wages for salaried workers who change sectors in the respective interval and held a job in the preceding sector for two years or more and stayed in the new sector for two or more years. F denotes "Formal Sector" and I "Informal Sector".

more skilled workers sort into the formal sector. Second, we do not observe any wage changes for informal workers before switching jobs, suggesting that learning opportunities in the informal sector are limited. Third, workers who move from the informal to the formal sector experience an increase in the slope of their wage profiles, whereas formal workers who switch to the informal sector do not. This suggests that informal-to-formal workers enhance their learning opportunities after the switch, while formal-to-informal workers do not.

We close this section by highlighting that some of the elements of the narrative have been proposed in previous studies, but never all together. Using dynamic life cycle models and the same data we use, Lopez Garcia (2015) and Joubert (2015) find that high-skilled individuals experience higher returns in the formal sector. On human capital accumulation, Bobba et al. (2021) find that formal workers in Mexico accumulate human capital more quickly than informal workers. Based on our findings, we propose that formal workers accumulate more human capital because they interact with better peers, aligning with theories of learning by interactions. While we find that learning from coworkers, as in Jarosch et al. (2021) and Herkenhoff et al. (2024), explains part of the differential learning, learning outside the workplace is equally important.[28] Hence, the framework we propose in Section 3 aligns with models where all individuals in a sector or region interact, similar to Lucas Jr. (2009), Lucas & Moll (2014), and Perla & Tonetti (2014).

[28]We leave for future research to explore the interaction between formality status and firm composition on workers' learning.

3 Endogenous Growth Model with Informal Labor

In this section, we develop a theory to rationalize the findings described in Section 2. We construct a heterogeneous-agent endogenous growth model that features an informal labor market. The model includes a representative firm that hires both formal and informal workers. The firm can observe the productivity of the workers and offers them two potential wages: a formal wage and an informal wage. The difference between these wages arises from the varying costs associated with formal and informal workers. We assume that these costs are fully passed through wages. Workers then observe both latent wages and choose to sort into formal or informal jobs. Over time, workers can increase their productivity by learning from other workers, which subsequently increases their future wages. However, the set of people a worker interacts with differs depending on their formality status. Therefore, when a worker decides on their formality status, they consider both their current wage and their learning opportunities. The fact that workers do not internalize their impact on others when they sort makes our model a mean-field game, as described by Lasry & Lions (2007). Our theory extends the work of Lucas & Moll (2014) by introducing multiple sectors and the sorting of workers.

3.1 Production and Wages

There is one final good in the economy produced by a representative firm, which we assume to be the numeraire of our economy. The firm has a total factor productivity (TFP) of A, and the only factor of production is labor. We assume that the labor market is perfectly competitive. Each period, the firm chooses a bundle of skills $n = \{n_f(z), n_i(z)\}_z$ to produce the final good using the production technology

$$Y(n) = A \int_0^{\infty} z \left(n_f(z) + n_i(z)\right) \mathrm{d}z. \tag{4}$$

Equation (4) has several notable properties. First, formal and informal workers are perfect substitutes in production. We abstract from any potential complementarity between formal and informal workers, loading all differences into the associated costs. Moreover, perfect substitutability provides a tractable way to derive the latent wages without adding an additional fixed point to the equilibrium. Second, skills are also perfect substitutes in production. While this is not a crucial assumption, it allows us to obtain a closed-form solution for the latent wages. Finally, the production function exhibits complementarities between TFP and skills.

On the other hand, hiring informal workers involves different costs compared to hiring formal workers. Formal workers receive a wage $w_f(z,t)$, which depends on their productivity. For each formal worker hired, the firm must pay a tax τ proportional to the worker's wage, along with a fixed cost $F(t)$ per formal worker. This fixed cost captures the administrative burden of reporting formal workers to the authorities. However, this cost does not depend on the worker's wage or productivity. Therefore, the total cost of hiring a formal worker with productivity z is

$$c_f(z,t) = (1+\tau) w_f(z,t) + F(t),$$

and the total cost for the formal payroll is

$$C_f(n,t) = \int_0^\infty n_f(z)c_f(z,t)\mathrm{d}z. \tag{5}$$

In contrast, informal workers avoid all costs associated with formality. However, the firm incurs a cost $\varphi(z,t)$ to conceal these workers from tax authorities. We assume that $\varphi(\cdot,t)$ satisfies the following properties:

$$\varphi(z,t) \geq 0, \qquad \varphi_z(z,t) \geq 0, \qquad \varphi_{zz}(z,t) > 0, \qquad \varphi(0,t) = 0,$$

where $\varphi_z(z,t)$ and $\varphi_{zz}(z,t)$ denote the first and second partial derivatives with respect to z, respectively. Intuitively, the firm must exert more effort to hide more productive workers from the authorities since output is directly proportional to worker productivity. Alternatively, $\varphi(z,t)$ can be interpreted as the expected fine for operating informally, as in Ulyssea (2018). We consider this cost a deadweight loss and thus a source of inefficiency in our model. The cost of an informal worker with productivity z is

$$c_i(z,t) = w_i(z,t) + \varphi(z,t),$$

where $w_i(z)$ denotes the wage paid to an informal worker with productivity z. The total cost for informal workers is

$$C_i(n,t) = \int_0^\infty n_i(z)c_i(z,t)\mathrm{d}z. \tag{6}$$

The firm's full problem is

$$\max_{n_f(z),n_i(z)_z} A\int_0^\infty z\left(n_f(z)+n_i(z)\right)\mathrm{d}z - \int_0^\infty n_i(z)c_i(z,t)\mathrm{d}z - \int_0^\infty n_f(z)c_f(z,t)\mathrm{d}z.$$

The first-order conditions and perfect competition imply that wages for formal workers are

$$w_f(z,t) = \frac{Az}{1+\tau} - \frac{F(t)}{1+\tau}, \tag{7}$$

while wages for informal workers are

$$w_i(z,t) = Az - \varphi(z,t). \tag{8}$$

Several properties are worth highlighting. First, both wages are smaller than the marginal product of labor for any productivity level z due to the complete pass-through of labor costs to wages. Second, the convexity of $\varphi(z,t)$ implies that the wages for informal workers are a concave function of productivity. A highly productive worker who sorts into an informal job produces a lot and requires significant effort to hide from tax authorities. The complete pass-through result implies that the worker bears this cost through a reduction in wages. Conversely, wages in the formal sector are linear functions of individual productivity. Thus, a highly productive worker would prefer to sort into the formal sector since the latent wage exhibits increasing returns to scale with productivity. However, a worker with low productivity might end up with a negative wage in the formal sector due to the fixed cost. This yields the first proposition:

Proposition 1. *There exists a unique* $\bar{z}_s(t) \in [0, \infty)$ *such that*

1. $w_f(z,t) \geq w_i(z,t)$ *for every* $z \geq \bar{z}_s(t)$
2. $w_f(z,t) \leq w_i(z,t)$ *for every* $z \leq \bar{z}_s(t)$

Proof. See Appendix XX. □

Proposition 1 implies there is a unique cut-off that perfectly sorts workers into the formal and informal sectors in a static framework. This will become relevant as the full dynamic model will inherit this property.

3.2 Workers Preferences and Life Cycle

Time is continuous and there is a constant unit mass of agents who discount the future at a rate ρ. Each agent is characterized by her productivity z. The cross-sectional distribution of productivity is a continuous distribution with a cumulative distribution function (CDF)

$$G(z,t) = \Pr(y \leq z, \text{ at time } t).$$

We assume that the support of $G(z,t)$ is $\Omega = [0, \infty)$ and is fixed over time. Each agent has one unit of labor and can choose to work in either the formal or informal sectors. We assume that workers are hand-to-mouth agents with a linear utility function for their consumption. Each period, workers receive a wage, depending on their formality status, and a lump-sum transfer $T(t)$ from the government. Hence, the disposable income for formal workers is

$$Y_f(z,t) = w_f(z,t) + T(t) = \frac{Az}{1+\tau} - \frac{F(t)}{1+\tau} + T(t), \tag{9}$$

and for informal workers, it is

$$Y_i(z,t) = w_i(z,t) + T(t) = Az - \varphi(z,t) + T(t). \tag{10}$$

For now, we assume that the transfer $T(t)$ is the same for all workers. However, it would be straightforward to extend the model so that the transfers received by formal and informal workers differ. Intuitively, $T(t)$ is meant to capture universal coverage of basic services such as healthcare access. Additionally, workers exit the economy at a Poisson rate δ. Upon exit, a new worker mechanically replaces the old worker. The new worker's productivity draw z comes from a continuous distribution with CDF $B(z,t)$ and p.d.f. $b(z,t)$. Later, we will impose some assumptions on $B(z,t)$, but for now, the only assumption needed is that for every $t \geq 0$, $B(z,t)$ is first-order stochastically dominated by $G(z,t)$. Intuitively, this assumption implies that young workers are, on average, less productive and become more productive over the life cycle. We do not characterize the cohort-specific productivity distribution but instead focus on the cross-sectional age distribution.[29] Let $H(a,t)$ be the cross-sectional CDF of age with p.d.f. $h(a,t)$. The Kolmogorov Forward Equation describing the evolution of h is given by

$$\frac{\partial h(a,t)}{\partial a} + \frac{\partial h(a,t)}{\partial t} = -\delta h(a,t). \tag{11}$$

We provide a formal derivation of equation (11) in Appendix XX. We focus our analysis on the stationary distribution of age $H(a)$.

Proposition 2. *There exists a unique stationary distribution of age described by the CDF*

$$H(a) = 1 - e^{-\delta(a-a_0)}. \tag{12}$$

where a_0 is the age at which workers enter the economy.

The resulting CDF in equation (12) provides a closed-form solution that allows us to estimate δ directly from the data. We provide a more detailed discussion later in Section 4.

3.3 Learning

Workers can increase their productivity throughout their life cycle by interacting and learning from others. Meetings occur at a Poisson rate α. When a worker with productivity $z(t)$ meets another worker with productivity $\tilde{z}(t)$ during a time interval Δt, they learn according to the following technology:[30]

$$z(t+\Delta t) = \max\left\{z(t), \tilde{z}(t)\right\}. \tag{13}$$

As in Akcigit et al. (2018), the meetings in our economy are asymmetric: $z(t)$ can learn from $\tilde{z}(t)$, but $\tilde{z}(t)$ cannot learn from $z(t)$. In fact, $\tilde{z}(t)$ may not be seeking meetings. Moreover, learning is probabilistic, meaning that a meeting does not guarantee that either worker will learn. We assume that a worker's productivity determines not only their wage but also their ability to learn from others. Specifically, when a worker with productivity $z(t)$ meets another worker with productivity $\tilde{z}(t)$, learning occurs with probability

$$k\left(\frac{\tilde{z}}{z}\right) = \begin{cases} 1 & \text{if } \tilde{z} \leq z \\ \sigma + (1-\sigma)\left(\frac{\tilde{z}}{z}\right)^{-\kappa} & \text{otherwise} \end{cases}. \tag{14}$$

Equation (14) introduces two new parameters and has a very intuitive interpretation. First, if worker $z(t)$ is more productive than worker $\tilde{z}(t)$, learning will happen with certainty. However, the productivity of worker $z(t)$ will remain unchanged due to our assumed learning technology in equation (14). Conversely, if worker $\tilde{z}(t)$ is more productive, the probability that worker $z(t)$ learns decreases as the relative productivity gap widens. The parameter σ captures the baseline probability of a worker learning from any other worker in the economy, with $\sigma \in (0,1]$. When $\sigma = 1$, learning always occurs regardless of productivity levels. The parameter $\kappa > 0$ captures the limits to intellectual range, with higher values implying that learning from more productive

[29] For a complete discussion on deriving the cohort-specific distribution, see Caicedo (2019).

[30] This learning technology is the one used in the baseline exercises in Lucas & Moll (2014).

workers occurs less frequently. Intuitively, workers are more likely to learn from others with similar productivity levels.

The probability of meeting formal or informal workers depends on the current sector of the worker. Conditional on having a meeting, a worker in the formal sector will meet another formal worker with probability

$$\mathbb{P}^f_f(t) = \frac{\pi_f \mu_f(t)}{\pi_f \mu_f(t) + (1-\pi_f)\mu_i(t)}, \tag{15}$$

where $\mu_i(t)$ and $\mu_f(t)$ denote the economy's share of informal and formal workers, and $\pi_f \in [0,1]$ is a parameter capturing the degree of segmentation between formal workers. Let $\Omega_f(t)$ be the set of formal workers and $\Omega_i(t)$ the set of informal workers at time t. Note that $\Omega_f(t), \Omega_i(t)$ is a partition of $[0,\infty)$ at every moment in time. The shares are defined as

$$\mu_i(t) = \int_{\Omega_i(t)} g(z,t)\mathrm{d}z, \qquad \mu_f(t) = \int_{\Omega_f(t)} g(z,t)\mathrm{d}z,$$

with $\mu_i(t) + \mu_f(t) = 1$ at every moment in time t. For notational convenience, we denote $\mathbb{P}^i_f(t) = 1 - \mathbb{P}^f_f(t)$. Similarly, conditional on a meeting, an informal worker will meet another informal worker with probability:

$$\mathbb{P}^i_i(t) = \frac{\pi_i \mu_i(t)}{(1-\pi_i)\mu_f(t) + \pi_i \mu_i(t)}, \tag{16}$$

where π_i is a parameter governing the segmentation of informal workers from formal workers in terms of meetings. In the spirit of Jarosch et al. (2021), we allow the probability of meeting someone within your sector to differ between sectors (i.e., π_i can differ from π_f). In the extreme case where $\pi_f = 1$, formal workers only meet other formal workers. Similarly, when $\pi_i = 1$, informal workers only meet other informal workers. Both equations (15) and (16) account for the scarcity of workers in each sector. If the share of formal workers, $\mu_f(t)$, increases, the probability of meeting a formal worker increases regardless of the worker's sector.

3.4 Sorting

Next, we consider the sorting of workers into each sector. We assume that workers can switch sectors at any time at no cost. Thus, if $V_f(z,t)$ and $V_i(z,t)$ are the value of being in the formal and informal sector, respectively, the value of a worker with productivity z is

$$V(z,t) = \max\Big\{V_i(z,t), V_f(z,t)\Big\}. \tag{17}$$

Furthermore, we can write the value of being a formal worker in a recursive form. More explicitly, letting $T(t)$ being the government's lump-sum transfer, the Hamilton-Jacobi-Bellman (HJB) equation for a formal worker of productivity z at time t is

$$\begin{aligned}(\rho+\delta)V_f(z,t) &= Y_f(z,t)+\dot{V}_f(z,t)\\ &\quad + \alpha\mathbb{P}_f^f(t)\int_{\Omega_f(t)} \max\Big\{V(\tilde{z},t)-V_f(z,t),0\Big\}k\left(\frac{\tilde{z}}{z}\right)\frac{g(\tilde{z},t)}{\mu_f(t)}\mathrm{d}\tilde{z}\\ &\quad + \alpha\mathbb{P}_f^i(t)\int_{\Omega_i(t)} \max\Big\{V(\tilde{z},t)-V_f(z,t),0\Big\}k\left(\frac{\tilde{z}}{z}\right)\frac{g(\tilde{z},t)}{\mu_i(t)}\mathrm{d}\tilde{z}.\end{aligned} \tag{18}$$

The left-hand side (LHS) of the first line in (18) captures the net discounted value of being a formal worker, whereas the right-hand side (RHS) reflects the formal static returns. The second line in (18) captures the formal worker's human capital improvements from interacting with other formal workers. With a probability of $\alpha\mathbb{P}_f^f(t)$, a formal worker meets another formal worker. Conditional on this meeting, a formal worker with productivity z learns only from formal workers with higher human capital. Meetings with workers with skills lower than z are discarded. Finally, the last term in this equation captures the formal worker's human capital improvements from interacting with informal workers. With a probability of $\alpha\mathbb{P}_f^i(t)$, a formal worker meets an informal worker. For a formal worker with productivity z, only meetings with informal workers with higher skills increase her human capital. [31] Similarly, the HJB equation for an informal worker is

$$\begin{aligned}(\rho+\delta)V_i(z,t) &= Y_i(z,t)+\dot{V}_i(z,t)\\ &\quad + \alpha\mathbb{P}_i^f(t)\int_{\Omega_f(t)} \max\Big\{V(\tilde{z},t)-V_i(z,t),0\Big\}k\left(\frac{\tilde{z}}{z}\right)\frac{g(\tilde{z},t)}{\mu_f(t)}\mathrm{d}\tilde{z}\\ &\quad + \alpha\mathbb{P}_i^i(t)\int_{\Omega_i(t)} \max\Big\{V(\tilde{z},t)-V_i(z,t),0\Big\}k\left(\frac{\tilde{z}}{z}\right)\frac{g(\tilde{z},t)}{\mu_i(t)}\mathrm{d}\tilde{z}.\end{aligned} \tag{19}$$

The structure of the informal worker's value function, (19), is similar to the formal worker's value function, (18). The first line's LHS captures the net discounted value of being an informal worker, while the RHS reflects the informal static returns. The second line illustrates human capital improvements by interacting with formal workers, while the last line displays human capital accumulation by interacting with other informal workers. When meeting with formal and other informal workers, an informal worker with skill z only learns from workers with higher skills.

We fully derive the HJB equations in Appendix A. One key feature of our model is that formal workers have increasing returns to scale on their productivity after tax and fixed costs. On the contrary, the convexity of the cost of informal implies decreasing returns to scale. This would generate a unique cutoff in a static setting that creates perfect sorting as shown in Proposition 5. Workers with a productivity draw above the static cutoff will sort into the formal sector, and the remaining will sort into the informal sector. This result holds also in our dynamic setting, as shown in Proposition 3.

Proposition 3. *There is exist a unique* $\bar{z}(t) \in [0,\infty)$ *such that*

[31] As shown later in (20), worker's sorting implies that the last term in (18) is zero. That is, in equilibrium, formal workers do not learn from informal workers.

1. $\Omega_i(t) = [0, \bar{z}(t)]$

2. $\Omega_f(t) = [\bar{z}(t), \infty)$

Moreover, $\mu_f(t) = 1 - G(\bar{z}(t), t)$ *and* $\mu_i(t) = G(\bar{z}(t), t)$.

Proof. See Appendix A. □

Observe that Proposition 3 holds if and only if

$$\forall z > \bar{z}(t), \quad V_i(z,t) < V_f(z,t),$$

and vice-versa. Moreover, at $\bar{z}(t)$, workers are indifferent between operating in the formal and informal sectors. An immediate consequence of sorting based on skills is that formal workers will only learn from more productive formal workers. On the other hand, informal workers will learn from more productive informal workers and any formal worker they meet. This implies that in equilibrium, the HJB equation (18) takes the form

$$\begin{aligned}(\rho+\delta)V_f(z,t) &= Y_f(z,t) + \dot{V}_f(z,t) \\ &+ \frac{\alpha \mathbb{P}_f^f(t)}{1 - G(\bar{z}(t),t)} \int_z^\infty \Big(V(\tilde{z},t) - V_f(z,t)\Big) k\left(\frac{\tilde{z}}{z}\right) g(\tilde{z},t) d\tilde{z}.\end{aligned} \tag{20}$$

Analogously, the HBJ equation for informal workers, in equilibrium, simplifies to:

$$\begin{aligned}(\rho+\delta)V_i(z,t) &= Y_i(z,t) + \dot{V}_i(z,t) \\ &+ \frac{\alpha \mathbb{P}_i^f(t)}{1 - G(\bar{z}(t),t)} \int_{\bar{z}(t)}^\infty \Big(V_f(\tilde{z},t) - V_i(z,t)\Big) k\left(\frac{\tilde{z}}{z}\right) g(\tilde{z},t) d\tilde{z} \\ &+ \frac{\alpha \mathbb{P}_i^i(t)}{G(\bar{z}(t),t)} \int_z^{\bar{z}(t)} \Big(V_i(\tilde{z},t) - V_i(z,t)\Big) k\left(\frac{\tilde{z}}{z}\right) g(\tilde{z},t) \mathrm{d}\tilde{z}.\end{aligned} \tag{21}$$

Note that in both equations, we exploited the monotonicity of the value functions and the sorting cut-off $\bar{z}(t)$ to further simplify the expressions for the learning dynamics. Furthermore, the only decision the worker is making is which sector to work in, as shown by equation (17). This approach differs from the model proposed by Lucas & Moll (2014), where each worker decides how to allocate their time between production and learning. We abstract from the mechanism proposed by Ben-Porath (1967) to isolate the externality generated by sorting. To better understand the sources of inefficiency, consider a worker with productivity $\bar{z}(t)$. This worker is indifferent between working in the formal and informal sectors. Suppose this worker decides to sort into the formal sector. According to Proposition 3, he will be the least productive worker in the formal sector. As a result, no other formal worker can learn from him, and any meeting with him will result in no learning. In other words, by sorting into the formal sector, the worker is crowding out learning opportunities for other workers. Hence, the externalities in our model come from the fact that when workers choose their sector, they do not internalize the effect it will have on other workers' learning opportunities.

3.5 Human Capital Dynamics

We now proceed to describe the evolution of the productivity distribution over time. At every instant t, there is a set of workers learning; thus, the distribution of productivity changes over time. We note that Proposition 3 implies that the distribution dynamics differ above and below $\bar{z}(t)$. Hence, the Kolmogorov Forward Equation (KFE) will be piece-wise defined. For notation convenience, define $\lambda^j_{j'}(t)$ as

$$\lambda^j_{j'}(t) = \frac{\mathbb{P}^j_{j'}(t)}{\mu_j(t)}, \quad j, j' \in \{i, f\}$$

A natural interpretation for $\lambda^j_{j'}(t)$ is the quality-adjusted meeting rate of a worker in sector j' with a worker of sector j. Perfect sorting resulting from Proposition 3 creates a trade-off between the frequency of a meeting and the quality of it. Consider the case of an informal worker with productivity $z < \bar{z}(t)$. Note that

$$\frac{\partial \mathbb{P}^f_i(t)}{\partial \bar{z}(t)} < 0,$$

because there are fewer formal workers in the economy. However, as $\bar{z}(t)$ increases, the effect on λ^f_i will depend on the probability that an informal worker meets another worker. Proposition 4 summarizes this result

Proposition 4. *Given $\pi_i, \pi_f \in [0, 1]$, the quality adjusted meeting rates are monotonic on the share of informality and satisfy*

1. $\frac{\partial \lambda^i_i(t)}{\partial G(\bar{z}(t),t)} < 0$ *and* $\frac{\partial \lambda^f_i(t)}{\partial G(\bar{z}(t),t)} < 0$ *if and only if* $\pi_i > 1/2$.
2. $\frac{\partial \lambda^f_f(t)}{\partial G(\bar{z}(t),t)} > 0$ *if and only if* $\pi_f > 1/2$.

Proof. See Appendix A. □

The intuition behind Proposition 4 is straightforward. The sorting given by Proposition 3 implies that the share of informality is $\mu_i(t) = G(\bar{z}(t), t)$. If $\pi_i > 1/2$, it implies that informal workers are more prone to meet with other informal workers. As $G(\bar{z}(t), t)$ increases, there will be more informal workers; hence, formal workers' meetings will be more scarce. Interestingly, if $\pi_i < 1/2$, then informal workers interact more with formal workers. These interactions are guaranteed to be productive, and while there is a decrease in the extensive margin of formality, the average quality of interactions will be higher.

Now, we proceed to define the KFE. We present the full derivation of the KFE in Appendix A. For any $z < \bar{z}(t)$, the evolution of the cumulative distribution function satisfies the following equation:

$$\begin{aligned}\frac{\partial g(z,t)}{\partial t} &= \alpha\lambda_i^i g(z,t)\int_0^z k\left(\frac{z}{y}\right)g(y,t)\mathrm{d}y \\ &- \alpha\lambda_i^f g(z,t)\int_{\bar{z}}^{\infty} k\left(\frac{y}{z}\right)g(y,t)\mathrm{d}y \\ &- \alpha\lambda_i^i g(z,t)\int_z^{\bar{z}} k\left(\frac{y}{z}\right)g(y,t)\mathrm{d}y \\ &- \delta(g(z,t)-b(z,t))\end{aligned} \tag{22}$$

Interpreting Equation (22) is useful for understanding the dynamics of the model. Begin with the inflows: Due to Proposition 3, inflows can only come from informality. The RHS of the first row accounts for all the informal workers that meet and learn from another informal worker with productivity z. Specifically, only informal workers with productivity $y \leq z$ will learn from a worker. They meet $g(z,t)$ workers of productivity z at a rate $\alpha\lambda_i^i(t)$. The probability of successful learning is $k\left(z/y\right)$. Integrating over that set of workers, the amount of informal workers that learn from a worker of productivity z

$$\int_0^z k\left(\frac{z}{y}\right)g(y,t)\mathrm{d}y$$

The second line depicts the outflows towards formality: there are $g(z,t)$ with productivity z and they meet a formal worker at a rate of $\alpha\lambda_i^f$ with productivity y. The probability of meeting a worker with productivity y is $g(y,t)$ and with probability $k(y/z)$ the worker will learn successfully. Then we integrate over the support of formal workers following Proposition 3. The intuition for the third line is very similar to the one just described with the sole difference of the relevant support for integration. Specifically, we integrate over the support of informal workers from which a worker with productivity z can potentially learn. Finally, the last line accounts for the net exit of workers with productivity z from the labor market. Correspondingly, for any $z \geq \bar{z}(t)$, the KFE that describes the dynamics of distribution is

$$\begin{aligned}\frac{\partial g(z,t)}{\partial t} &= \alpha\lambda_i^f g(z,t)\int_0^{\bar{z}} k\left(\frac{z}{y}\right)g(y,t)\mathrm{d}y \\ &+ \alpha\lambda_f^f g(z,t)\int_{\bar{z}}^{z} k\left(\frac{z}{y}\right)g(y,t)\mathrm{d}y \\ &- \alpha\lambda_f^f g(z,t)\int_{\bar{z}}^{z} k\left(\frac{y}{z}\right)g(y,t)\mathrm{d}y \\ &- \delta(g(z,t)-b(z,t))\end{aligned} \tag{23}$$

Equation (23) has a similar interpretation as equation (22), but now inflows come from both formality and informality. Note that for a fixed worker, productivity is a non-decreasing function of time that discontinuously jumps. But as the distribution shifts to the right, the informality threshold $\bar{z}(t)$ also shifts to the right. Consequently, a worker who initially sorted in the formal sector and by chance never had a productive meeting will eventually be caught by the informality threshold $\bar{z}(t)$ and, therefore, transition into the informal sector. With these, we can define an equilibrium in our economy.

Definition 1. *Given an initial distribution $g(z,0)$, an equilibrium is a tuple of functions (V, V_i, V_f, g) from $\mathbb{R}_+^2$ to $\mathbb{R}$ and a function $\bar{z}$ from $\mathbb{R}$ to $\mathbb{R}$ such that $\forall t \geq 0$:*

1. *$V_f(z,t)$ satisfies equation (20), and $V_i(z,t)$ satisfies equation (21),*
2. *For every $z < \bar{z}(t)$, $g(z,t)$ satisfies equation (22),*
3. *For every $z \geq \bar{z}(t)$, $g(z,t)$ satisfies equation (23),*
4. *$V(z,t)$ satisfies equation (17),*
5. *The indifference condition $V_f(\bar{z}(t),t) = V_i(\bar{z}(t),t)$ is satisfied,*
6. *The government has a balanced budget, satisfying*

$$T(t) = \int_{\bar{z}(t)}^{\infty} \tau w_f(z,t) g(z,t) \mathrm{d}z \tag{24}$$

The first condition in the definition of the equilibrium states that the value functions for formal and informal workers, $V_f(z,t)$ and $V_i(z,t)$, are consistent with workers sorting. Conditions 2 and 3 estate that human capital dynamics are dictated by the KFE equations, also consistent with worker sorting. Moreover, conditions 4 and 5 indicate that workers freely sort into the sector that yields higher returns and that there exists a marginal worker with skill $\bar{z}(t)$ who is indifferent between both sectors. Finally, the last condition states that the government runs a balanced budget, and this implies that per-capita transfers are equal to labor taxes levied on formal workers.

3.6 Balanced Growth Path

In the remainder of the paper, we focus on a particular equilibrium. In particular, as is common in the economic growth literature, we focus on a Balanced Growth Path (BGP) equilibrium. Intuitively, a BGP is an equilibrium that satisfies the conditions of Definition 1 in which the growth rate of the economy and the relative human capital distribution are constant over time. Formally,

Definition 2. *A balanced growth path (BGP) is a vector $(\gamma, \bar{x})$ and a tuple of real functions $(v, v_i, v_f, \phi, \Phi, \psi, \Psi)$ defined over $\mathbb{R}_+$ such that $\bar{z}(t) = \bar{x}e^{\gamma t}$ and*

$$\begin{aligned} V(z,t) &= e^{\gamma t} v\left(ze^{-\gamma t}\right) & V_i(z,t) &= e^{\gamma t} v_i\left(ze^{-\gamma t}\right) & V_f(z,t) &= e^{\gamma t} v_f\left(ze^{-\gamma t}\right) \\ g(z,t) &= e^{-\gamma t} \phi\left(ze^{-\gamma t}\right) & b(z,t) &= e^{-\gamma t} \psi\left(ze^{-\gamma t}\right) \end{aligned}$$

for every pair (z,t) and (V, V_i, V_f, g) together with $\bar{z}(t)$ define an equilibrium with initial distribution $g(z,0) = \phi(z)$ and $G(z,0) = \Phi(z)$.

From this definition, we see that a BGP is a path for the skill distributions for formal and informal workers in which all quantiles grow at the same rate γ. To further characterize the values of being a formal or informal worker in the BGP, we specialize the functional forms for the formal fixed cost, $F(t)$, and the expected informality cost, $\varphi(z,t)$. Specifically, we assume that:

$$F(t) = Fe^{\gamma t}, \quad \text{and} \quad \varphi(z,t) = \tilde{\varphi}\left(ze^{-\gamma t}|\eta\right)e^{\gamma t}, \tag{25}$$

for some constants, $F > 0$ and $\tilde{\varphi}(\cdot|\eta)$ is a time-invariant function, positive, increasing, and convex smooth function satisfying $\tilde{\varphi}(0) = 0$. In addition we assume that

$$\frac{\partial \tilde{\varphi}(x|\eta)}{\partial \eta} > 0$$

implying that η is a parameter that captures in a reduce form approach the cost of informality. Equation (25) reveals that formal workers' fixed cost is a fraction of the total production in the economy. Therefore, as the economy grows, the sunk costs of operating formally also grow. Similarly, the cost of informality, $\varphi(z,t)$, has two components. The first component $\tilde{\varphi}(x|\eta)$ depends only on the level of productivity relative to the initial distribution. The second term implies that the cost of informality also grows over time.

Letting $x \equiv ze^{-\gamma t}$ be the relative human capital of a given worker, equation (24) implies that government transfers in the BGP, T, are equal to

$$T = \int_{\bar{x}}^{\infty} \tau w_f(x)\phi(x)\mathrm{d}x = \frac{\tau}{1+\tau}\Big(A\mathbb{E}\left[x \mid x \geq \bar{x}\right] - F(1-\Phi(\bar{x}))\Big) \tag{26}$$

Note that equation (26) shows that the higher levels of fixed cost reduces the transfer of the government by the amount of people sorting out of formality. With this notation, along the BGP, the HJB equation for formal workers takes the form:

$$\begin{aligned}(\rho+\delta-\gamma)v_f(x) + \gamma x v_f'(x) = {} & \frac{Ax - F}{1+\tau} + T \\ & + \alpha\lambda_f^f \int_x^{\infty} \Big(v(\tilde{x}) - v_f(x)\Big) k\left(\frac{\tilde{x}}{x}\right)\phi(\tilde{x})\mathrm{d}\tilde{x},\end{aligned} \tag{27}$$

where we replaced the value for $Y_f(\cdot,t)$, $F(t)$, and $T(t)$ by the respective detrended values along the BGP. Similarly, the HJB equation for an informal worker with relative human capital x is

$$\begin{aligned}(\rho+\delta-\gamma)v_i(x) + \gamma x v_i'(x) = {} & Ax - \tilde{\varphi}(x) + T \\ & + \alpha\lambda_i^f \int_x^{\infty} \Big(v(\tilde{x}) - v_i(x)\Big) k\left(\frac{\tilde{x}}{x}\right)\phi(\tilde{x})\mathrm{d}\tilde{x} \\ & + \alpha\lambda_i^i \int_x^{\bar{x}} \Big(v(\tilde{x}) - v_i(x)\Big) k\left(\frac{\tilde{x}}{x}\right)\phi(\tilde{x})\mathrm{d}\tilde{x}\end{aligned} \tag{28}$$

Finally, we need additional assumptions on our initial distribution of productivity so that a BGP can be sustained. Although our results do not hinge on this assumption, it gives us closed-form solutions for the economy's growth rate, γ.

Assumption 1. *The initial productivity distribution, $G(z, 0)$, has a Pareto tail. That is, there exist $k, \theta > 0$ such that*

$$\lim_{z \to \infty} \frac{1 - G(z, 0)}{z^{-1/\theta}} = k \tag{29}$$

Assumption 1 implies that the initial distribution tail approaches 0 at the same rate as a Pareto distribution but encompasses a higher class of possible distributions. In addition, note that the support of the initial distribution is unbounded, suggesting that at time 0, all the knowledge in the economy already exists, and it is just waiting to be discovered. Given the definition of a BGP, the distribution moves steadily to the right as time progresses. Hence, very unlikely knowledge becomes feasible as time goes by. In addition, we need an assumption on the initial distribution from which new born workers draw their productivity $\psi(x)$.

Assumption 2. *The initial productivity distribution at birth, $B(z, 0)$, has a common Pareto tail with the initial productivity distribution, but a different location parameter. That is, there exist $k_0 < k$ such that*

$$\lim_{z \to \infty} \frac{1 - B(z, 0)}{z^{-1/\theta}} = k_0 \tag{30}$$

Observe that by imposing $k_0 < k$ we guarantee implies that the birth distribution is first order stochastic dominated by the productivity distribution. Assumption 1 and 2 are sufficient conditions for the economy to sustain a growth rate $\gamma > 0$.

Proposition 5. *Suppose that Assumptions 1 and 2 hold and $\pi_i < 1$. Then there is a number γ that holds a BGP, and it is defined as*

$$\gamma = \alpha\theta\sigma \left[\Phi(\bar{x})\lambda_i^f + (1 - \Phi(\bar{x}))\lambda_f^f\right] - \delta\theta \left(1 - \frac{k_0}{k}\right) \tag{31}$$

Proof. See Appendix A □

Equation (31) provides a clear interpretation of the growth rate along the BGP. Firstly, growth depends positively on α, which governs the rate at which agents interact. Thus, an economy where agents meet frequently will experience a higher growth rate. Secondly, the growth rate increases with θ. Recall that $1/\theta$ captures the dispersion of productivity in the economy. In the extreme case where $\theta \to 0$, the distribution becomes degenerate, and there is no dispersion. Hence, the growth rate increases as productivity is distributed more evenly among agents. We assume that $\theta \in [0, 1]$ so that the distribution has a well-defined first moment. Additionally, the growth rate depends positively on σ, the baseline probability of successful learning. Higher levels of σ increase the share of meetings in which learning occurs, thereby accelerating the rate at which productivity evolves. The latter term from the positive term is an endogenous object that encapsulates the weighted average rate at which agents meet formal workers. Proposition 3 implies that more productive workers sort into the formal sector, and consequently interactions with formal workers yield higher learning, translating into higher economic growth. Finally, the negative term depends on both the rate at which agents exit the economy δ and the relative gap between the birth distribution and the stationary distribution k_0/k. A smaller relative gap k_0/k results in higher growth γ. The intuition behind this is straightforward: if the gap is smaller, on average, newborn workers will be less productive and will have more opportunities to learn over their life cycle. Conversely, if δ increases, the growth rate decreases because workers exit the labor market faster, reducing the time available for learning. Moreover, if $k_0 = k$ then the birth distribution and the invariant distribution will have the same location and δ has no effect on growth. This highlights how workers at birth are different than other workers with higher tenure.

The Random Matching case: A particular case of interest arises when $\pi_i = \pi_f = 1/2$. In this case, the probability of meeting a formal worker is $\mu_f(t)$, and the probability of meeting an informal worker is $\mu_i(t)$ for any worker. In other words, meetings are random and independent of the sector, as in Lucas & Moll (2014). Learning opportunities will differ by sector whenever $(1 - \pi_i) \neq \pi_f$. Workers then decide to sort into each sector based not only on their static returns but also on their learning opportunities. However, for this particular case,

$$\lambda_i^i = \lambda_i^f = \lambda_f^i = \lambda_f^f = 1,$$

as all agents are equally likely to meet. In this specific scenario, sorting has no implications for learning and there are several interesting results. First, $\bar{x}_s = \bar{x}$, meaning that only the static returns will determined the formality cut-off. Moreover, the growth rate of the economy simplifies to

$$\gamma = \alpha\theta\sigma - \delta\theta\left(1 - \frac{k_0}{k}\right)$$

This case highlights a critical property of our model. While sorting is static, it has dynamic implications as it changes learning opportunities. When we remove friction segmentation in terms of learning, the growth rate depends solely on the frequency of meeting rates and the dispersion of productivity across agents.

4 Model Estimation

This section describes the estimation of the model parameters. We summarize the parameters described in our theory and discuss the various strategies employed for their estimation. Next, we examine the variation in the data that helps identify these parameters. Finally, we present the results and explore some of the equilibrium objects in our baseline economy.

4.1 Estimation

The model developed in Section 3 has 13 parameters:

$$\Gamma = \left\{ \underbrace{\tau, F, \eta}_{\text{Regulatory}}, \underbrace{\alpha, \pi_i, \pi_f, \sigma, \kappa,}_{\text{Learning}} \underbrace{k, \theta, k_0}_{\text{Distributional}}, \underbrace{\rho, \delta}_{\text{Macro}} \right\}$$

The first three parameters capture the regulatory status of the economy: τ is the tax on formal production, F is the fixed cost of formality, and η is a parameter that governs the cost of informality. The next set of parameters relates to learning. First, α is the Poisson arrival rate for a meeting with another agent, governing how frequently meetings with other workers occur. Additionally, π_i and π_f measure segmentation in terms of learning between informal and formal workers. In the extreme case where $\pi_f = 1$ and $\pi_i = 1$, learning occurs only within each sector. σ is the baseline probability of successful learning during a meeting, setting a lower bound for the share of meetings that result in actual learning from a frequentist perspective. Lastly, κ captures the limits of intellectual range.

The next set of parameters helps us pin down the invariant distribution of productivity. Three parameters describe the initial distribution of productivity: θ, which speaks to the dispersion of both the invariant and the birth distributions, and k_0 and k, which anchor the units of both distributions. Finally, ρ and δ are macro-parameters that capture the discount rate of agents and the Poisson rate at which agents retire from the economy, respectively.

Of the 13 parameters in our model, we externally calibrate three. First, we set $\rho = 0.05$ following Akcigit et al. (2021). Additionally, we normalize k to 2. This parameter is not identified, as it anchors the units of the productivity distribution, which is an unobserved object in the data. Anchoring the units of the distribution for both countries allows us to make cross-country comparisons.[32] Next, we directly match δ from the data and τ from the Chilean tax code. Note that our implied age distribution $H(\cdot)$ yields an equation that allows us to estimate δ from the data. Explicitly, by manipulating equation (12) we get a log-linear relation between age and the CDF:

$$\log\left(1 - H(a)\right) = -\delta a.$$

Hence, we run a regression of age a on the empirical cumulative distribution function, $H^e(a)$, without a constant. We estimate the remaining seven parameters, $\Gamma = \{F, \eta, \alpha, \pi_i, \pi_f, \kappa, k_0, \theta\}$, using a Simulated Method of Moments (SMM) estimator. Formally, let $M(\Gamma)$ be the vector of size S of moments generated by the model with parameters Γ. Similarly, let M^e be the vector of empirical moments from the data. We estimate our model by minimizing the distance between the empirical and the simulated moments. Given a vector of empirical moments M^e, our score function is

$$\Psi\left(M^e\right) = \min_{\Gamma} \sqrt{\sum_{m=1}^{S} \left[\frac{\omega_m \left(M^e_m - M_m(\Gamma)\right)}{1/2|M^e_m| + 1/2|M_m(\Gamma')|}\right]^2}$$

where ω_m is the relative weight of moment m. To find a global minimum, we implement the TikTak algorithm described by Arnoud et al. (2019) to search over the parameter space. In every iteration of the algorithm, we solve the equilibrium and calculate the simulated moments to calculate the score function. Next, we discuss how our parameters are identified.

[32]While this is an innocuous normalization, we found that the algorithm is not very robust for low levels of k as too much mass is allocated to the left tail of the distribution.

4.2 Identification

Now, we turn our attention to describing how our selected moments identify each of our parameters. We select moments associated with informality, growth, and the transition probability between sectors, as well as wage growth. Although we employ an indirect inference estimation strategy and all our parameters are identified jointly, we provide a heuristic discussion on how the selected moments aid in identifying our parameters.

Parameters determining dynamic returns. There are six parameters governing the dynamic returns for workers: α governs the frequency of meetings, θ governs the dispersion of the productivity distribution, influencing the returns of learning. Additionally, π_i and π_f represent the probability of interacting with other workers from the same sector, conditional on a meeting. Finally, σ and κ describe the shape of the learning technology. To identify the dynamic parameters, we estimate the transition matrix between the formal and informal sectors. To achieve this, we exploit the panel structure of our data and construct the empirical transition matrix based on workers who switch sectors within each wave of the survey. However, since the frequency of our survey is not yearly, we first need to calculate the annual transition matrix to match the model. Let g be the number of years within each wave of the survey, and P^e be the empirical transition matrix. To calculate the 1-year transition, we diagonalize P^e and then raise the diagonal to the power of the inverse of the gap between waves, denoted as g. More formally, let D be a diagonal matrix such that

$$P^e = A^{-1}DA,$$

for some matrix A. Then, the 1-year transition matrix can be expressed as

$$\tilde{P}^e = A^{-1}D^{1/g}A.$$

By definition, $\tilde{P}^e$ is a 2×2 matrix which satisfies

$$\sum_{i,j\in\{f,i\}} \tilde{P}^e_{ij} = 1,$$

determining two different moments. We target $\tilde{P}^e_{i,f}$ and $\tilde{P}^e_{f,i}$. Note that our model yields a closed-form solution for both. The probability of an informal worker transitioning into a formal worker in a year is

$$\Pr(i \to f) = \frac{\alpha\lambda_i^f}{\Phi(\bar{x})}\int_0^{\bar{z}(t)}\int_{\bar{z}(t+1)}^{\infty} k\left(\frac{\tilde{z}}{z}\right) g(\tilde{z}, t+1)g(z,t)\mathrm{d}\tilde{z}\mathrm{d}z$$

Alternatively, the probability of a formal worker can be written as

$$\begin{aligned}\Pr(f \to i) = \frac{1}{\mu_f(t)}\int_{\bar{z}(t)}^{\bar{z}(t+1)} & \left[(1-\alpha) + \alpha\lambda_f^i G(\bar{z}(t), t)\right] \\ & + \alpha\lambda_f^f \int_{\bar{z}(t+1)}^{\infty}\left[1 - k\left(\frac{\tilde{z}}{z}\right)\right] g(\tilde{z}, t+1)\mathrm{d}\tilde{z} \\ & + \alpha\int_0^1 \lambda_f^i \int_{\bar{z}(t)}^{\bar{z}(t+s)} g(\tilde{z}, t+s)\mathrm{d}\tilde{z} + \lambda_f^i \int_{\bar{z}(t+s)}^{\bar{z}(t+1)} g(\tilde{z}, t+s)\mathrm{d}\tilde{z}\mathrm{d}s\mathrm{d}z\end{aligned}$$

We provide a more formal derivation in Appendix A.1. Equation (31) provides an equation that relates GDP growth to all of our dynamic parameters. Note that k_0 also shows up in equation (31). While k_0 is not identified as it anchors the units of the distribution at birth, we identified ${}^{k_0}/_{k}$. By assumption 2, ${}^{k_0}/_{k} \in (0, 1]$ which gives us the relative position of the birth distribution relative to the stationary distribution. Hence, we use the share of informality for the group of new workers to identify this parameter. Moreover, to identify the rest of the dynamics we use the share of informality across the life-cycle of workers. Specifically, we target a binned version of Figure B1.

Parameters determining static returns: There are three parameters that govern the static returns of workers: τ represents the proportional tax on productivity, F stands for the fixed cost for formal workers, and η governs the cost of informality. Equation (17) implies that sorting considers both static returns and learning opportunities. To identify the static parameters, we focus on two particular moments as we are fixing τ to the value stated in the regulation. Specifically, we target the share of informal workers in the economy. In our model, the share of informality is expressed as $\Phi(\bar{x})$ as one of the main drivers of sorting is the static return as shown by equations (27), (28), and (17). Moreover, we also target the average formal premium as this is pinned down by both the static returns and the distribution of productivity. Specifically, in our model, the formal premium is defined as

$$\int_0^\infty \frac{w_f(x)}{w_i(x)} \phi(x) \mathrm{d}x.$$

Note that for low values of x, the formal premium can be negative as the fixed cost F might be too much of a burden.

4.3 Estimation Results

Table 5 presents the estimation results, highlighting several noteworthy points. First, the estimated value of α implies that the probability of a worker interacting with another worker within a year is 32%. This finding aligns with Lagakos et al. (2018), who observed that developing economies generally experience a less pronounced wage increase over the life cycle. Second, the estimate of the tail of the initial productivity distribution, θ, is close to the value estimated by Lucas Jr. (2009) ($\theta = 0.5$), who used a similar model to match the variance of earnings in the US. Our estimate of $\theta = 0.38$ reflects the lower variance of wages for workers at the end of their careers in Chile relative to the US. Third, the baseline learning probability, σ, indicates considerable limits to learning in the Chilean economy. Specifically, our estimate suggests that Chilean workers learn from more skilled peers in about one-third of their interactions, regardless of the skill of their counterparts. Fourth, the estimated informality cost η is close to the value estimated by Ulyssea (2018), who used a similar functional form to (25) and data from Brazil.

The estimation results also illustrate insights into the learning frictions between formal and informal workers. Conditional on meeting another worker, formal workers meet other formal workers with a probability of 80%. Similarly, conditional on a meeting, informal workers interact with formal workers with a probability of 45%. This suggests that there are asymmetries in learning environments in the formal and informal sectors. While formal workers tend to always inter-

Table 5: Estimated Parameters

Parameter	Description	Value
ρ	Discount Rate	0.05
k	Pareto Location	2
δ	Death Hazard Rate	0.064
σ	Learning Prob.	0.367
α	Meeting Rate	0.323
π_i	Probability of Meeting within Sector (Informals)	0.566
π_f	Probability of Meeting within Sector (Formals)	0.804
κ	Intelectual Range	3.56
θ	Pareto Tail	0.389
k_0/k	Birth Distribution Location	0.09
F	Hiring Costs	0.558
η	Informality Costs	1.168

Notes: Table 5 displays Simulated Method of Moments (SMM) estimation results for the model parameters.

act within the formal sector, informal workers tend to equally interact with formal and informal workers. These results suggest that barriers for informal workers to interact with their informal counterparts are quantitatively small and are in line with the no-dual economy results described by Ulyssea (2020a). Based on Proposition 4, this implies that in our estimated economy, both λ_i^f and λ_f^f are monotonically increasing on the share of informality $\Phi(\bar{x})$.

Table 6 presents the goodness of fit of the estimated model. The model closely matches the aggregate growth rate, the informality rate, and the yearly transition rates between formal and informal sectors. As displayed in the third column, these are the moments with a larger weight in our SMM estimation procedure. In this sense, the estimated model closely replicates key economic aggregates of the Colombian economy. Nevertheless, as discussed below, we still need to improve the model's ability to replicate the earnings paths for individual workers.

Table 6 also illustrates that the estimated model falls short of matching the wage growth paths for both formal and informal workers. This result is most likely to be implied by the low value of our estimate for α. We believe that there are potentially two features of the model that are causing these low-wage growth trajectories. First, there is the assumption that new entrants to the labor market draw their productivity from the stationary distribution. In this sense, workers who enter the labor market later in time benefit from human capital improvements from previous generations. Fixing the entry distribution of skills different from the stationary one could allow the model to generate steeper wage growth paths as new entrants do not have this later mover advantage, accumulating more human capital over the life cycle to catch up with incumbent workers.

Alternative learning technologies could also provide wage growth paths closer to the ones observed in the data. Learning technology (13) does not have barriers to learning. This means that the least skilled worker can acquire the skill of the most skilled worker in the economy, conditional on a meeting. Hence, to rationalize the patterns in the data, the model underestimates the meeting

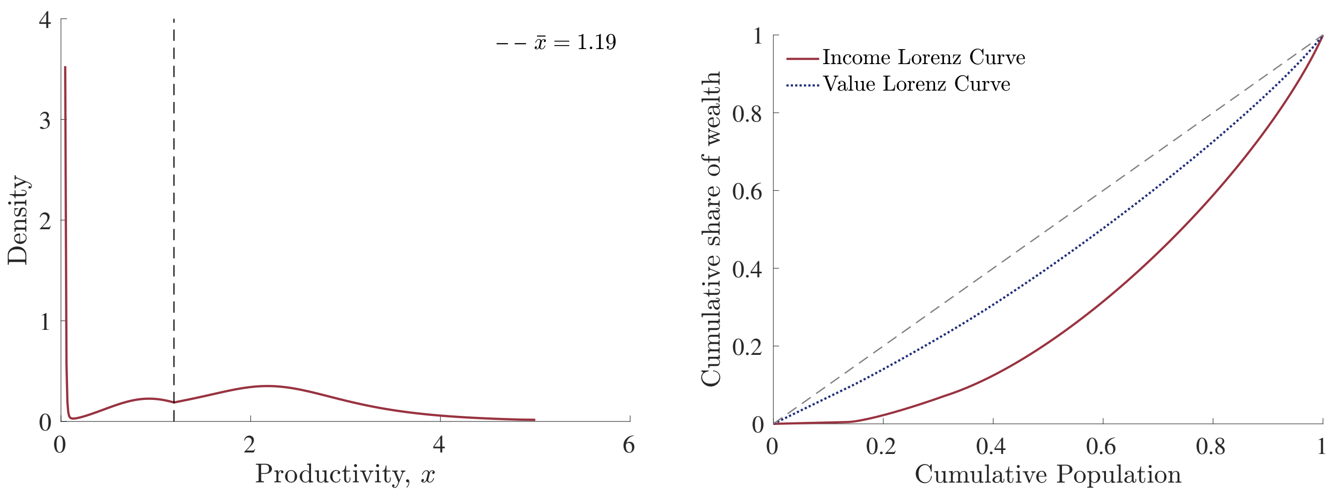
Table 6: Goodness of Fit

	Model	Data
Growth Rate (%)	3.175	4.104
Informality Rate (%)	30.172	18.584
Avg. Formal Premium	0.070	0.060
Informality Rate (%), 15-24	0.212	0.255
Informality Rate (%), 25-34	0.182	0.137
Informality Rate (%), 35-44	0.171	0.159
Informality Rate (%), 45-54	0.187	0.191
Informality Rate (%), 55-64	0.236	0.259
Transition Probability - I-F	0.082	0.128
Transition Probability - F-I	0.017	0.018

Notes: Table 6 displays the estimated model goodness of fit.

probability, α.[33] In reality, it is plausible that workers only acquire a fraction of the higher skills of the counterparts they meet. Similarly, workers could benefit relatively more by interacting with workers with similar skills. Thus, in the worker in progress, we are exploring alternative learning technologies such as those introduced by Lucas & Moll (2014), which incorporate features such as symmetric meetings, limits to learning, and exogenous learning shocks. To conclude this section,

Figure 4: Estimated Equilibrium



(a) Productivity Distribution **(b)** Lorenz Curves

Figure 4 displays different features of the BGP equilibrium with the estimated parameters. First, Figure 4(a) illustrates the stationary density of the relative productivity, x, and the estimated formality cutoff, $\bar{x} = 1.189$. The stationary human capital distribution, $\phi(x)$, inherits some of the properties of the initial Pareto skill distribution, $g(z, 0)$. In particular, the distribution exhibits a long tail that encodes a large dispersion for higher skill levels. Interestingly, in the BGP, the human capital stock of the most skilled formal workers (percentile 95th) is 2.14 times larger than

[33]Given the baseline learning technology, larger values of α are likely to deliver unrealistically steeper wage growth paths.

the level of human capital of the least skilled formal worker (marginal worker with $x = \bar{x}$). In contrast, human capital dispersion in the informal sector is significantly larger, with the human capital level of the most skilled informal worker being 3.6 times larger than the level of human capital of the least informal worker (percentile 5th). This result is driven by the larger share of informal workers in the economy as a whole.[34]

Figure 4(b) plots two Lorenz curves for the estimated BGP equilibrium. The solid (red) line shows the fraction of the total current income attributed to workers with productivity less than x. The cumulative share of wealth (measured as current income) is linear until the cumulative population reaches 63%, which is the formality cutoff. From this point onwards, the cumulative share becomes a concave function, reflecting the large skill dispersion of the invariant human capital distribution. The dashed (blue) line represents the fraction of the total present discounted income attributed to workers with productivity less than x. In contrast to the current income curve, the Lorenz value curve does not have any kinks, illustrating the smooth pasting property of the formal/informal value functions (27) - (28).

The income Lorenz curve exhibits less inequality than the income Lorenz curve. In line with the results in Jarosch et al. (2021), this property highlights the importance of examining present value rather than flow Lorenz curves in dynamic problems. Importantly, the value Lorenz curves account for the learning opportunities for both formal and informal workers. Interestingly, accounting for dynamic effects increases the reduction in inequality relatively more for low-skilled workers.

5 The Role of Formalization Policies

In this final section, we present two counterfactual exercises that quantify the aggregate effects of implementing two types of formalization policies. These policies aim to reduce the informal sector's size and increase government revenue. Moreover, they have become increasingly relevant as governments invest significant resources in their implementation. Such policies can be classified as either sticks, which are intended to increase the cost of operating in the informal sector, or carrots, which are designed to decrease the cost of operating in the formal sector.

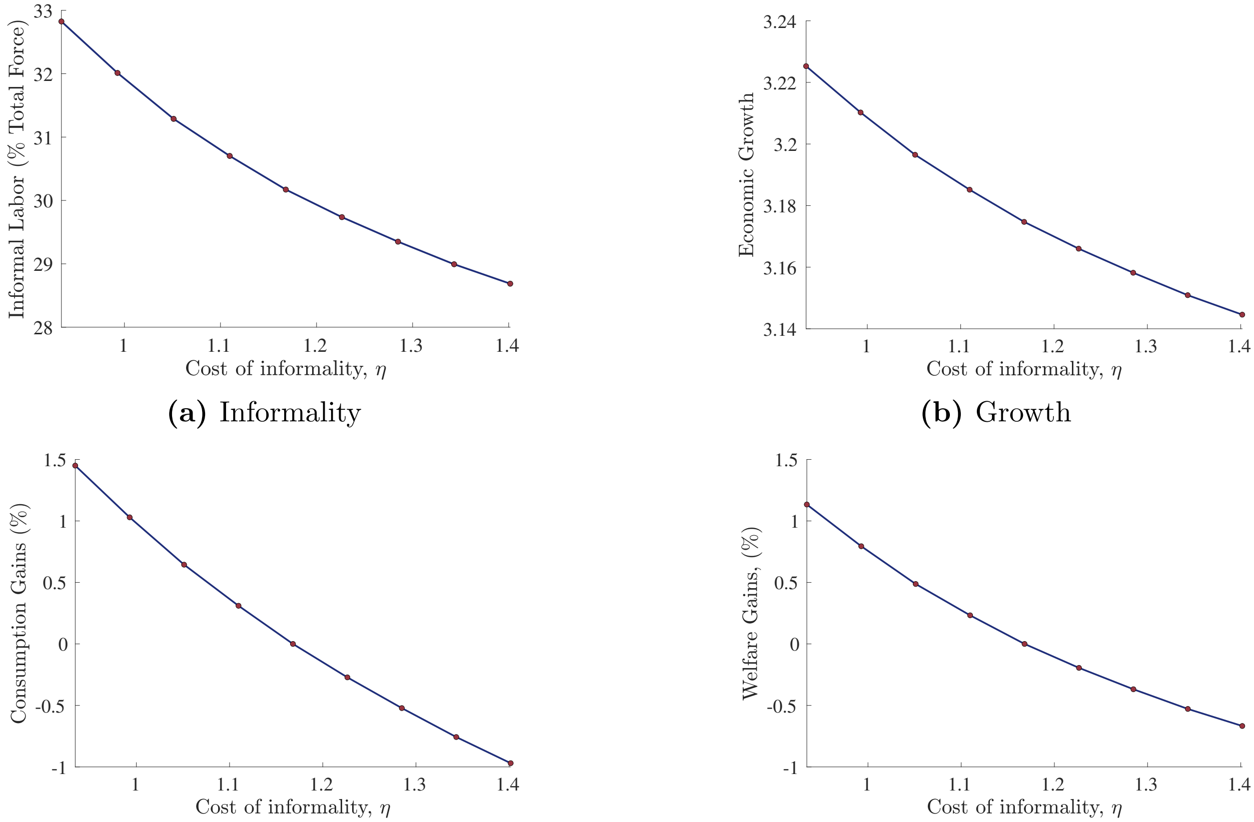
In our model, we investigate how the economy reacts to an increase in the cost of informality, η, versus a decrease in the fixed cost of formality, F. We explore the effects of these policies on growth, informality, government revenue, and welfare, which is defined as the mean-adjusted value across all workers with different levels of human capital:

$$W^* = \frac{\mathbb{E}_\Phi \left[v(x) \right]}{\rho + \delta - \gamma}. \tag{32}$$

We begin by computing the counterfactual economy upon a change in the cost of informality, and then we proceed to perform the same exercise by changing the formal sector's fixed cost.

[34]Note that the formality cutoff $\bar{x}$ is at the percentile 63th of the human capital invariant distribution.

Figure 5: Increase in the cost of informality, η



(a) Informality

(b) Growth

(c) Output

(d) Welfare

Notes: All figures depict the effect of changing informal workers' hiring costs, η, while keeping the rest of the parameters constant. We solve the model for every value of η in a grid with upper and lower bounds corresponding to a 20% increase and a 20% decrease around the estimated value of η in Table 5. Figure 5(a) shows the change in the percentage of informal workers. Figure 5(b) presents the changes in the growth rate of the economy. Figure 5(c) illustrates the instantaneous effect on consumption. Finally, Figure 5(d) depicts the welfare changes defined in (32) (consumption equivalent).

5.1 The Cost of Informality

We model an increase in the cost of informality by changing the parameters that govern the informality cost $\varphi(z, t)$. Specifically, given our assumed functional form in (25), we model a change in government prosecution of informality as a change to η. More explicitly, we take our estimated parameter from Table 5 and create a grid around that point. We set the lower bound of the grid at 20% of the estimated parameter and the upper bound at a 20% increase. Then, for each point on the grid, we solve the equilibrium and calculate the moments of interest. Figure 5 illustrates the results of this exercise.

In the first place, Figure 5(a) shows that the share of informality decreases with η. Higher values of η imply higher operating costs for informal firms. As equation (8) shows, informal firms pass these informality costs to workers in the form of lower salaries. Consequently, when faced with lower informal wages, marginal workers formalize, leading to a decrease in the aggregate number of informal workers. Furthermore, the exercise shows that the share of informality has a convex shape in η, indicating marginal diminishing returns from increasing the cost of informality. Therefore, the data shows that policies that increase the cost of informality are effective at reducing informal labor when the informal sector is large, but when the sector shrinks, they become less effective.

To better understand the effects of the policy on output, growth, and welfare, we define three groups of workers. First, *informal stayers* are initially informal workers who remained informal after the policy. Second, *informal movers* are workers who were informal before the policy but migrated to the formal sector afterward. Finally, we have the *formal stayers*, who are formal workers both before and after the policy.

Figure 5(b) shows the policy effect on the growth rate of the economy. Notably, the exercise shows an inverse relationship between η and γ: as informality decreases, economic growth dampens. We can use the groups defined previously to gain the intuition behind this result. Recall that the growth rate of the economy, (31), is mainly determined by the quality of interactions of formal workers. Therefore, with an increase in the cost of informality, the marginal workers who decide to formalize, *informal movers*, are less skilled than all existing formal workers, *formal stayers*. Consequently, the average quality of formal workers' interactions decreases, a lower λ_f^f, which lowers the growth rate. Similarly, as the average formal worker has a lower skill as a result of the policy, the average quality of interactions of *informal stayers* with formal workers also decreases, λ_i^f, lowering the growth rate as well. In essence, *informal movers* "crowd out" formal worker interactions. This logic illustrates the dynamic implications of worker sorting. While we only increase the static costs of being informal, the reallocation of workers affects future production by lowering the economy's growth rate.

Figure 5(c) shows the effect on consumption.[35] Higher informality costs translate into lower consumption. The primary reason is that increased informality costs act as a tax on *informal stayers*. Firms can perfectly pass on these costs to workers in the form of lower wages. Consequently, faced with higher operational costs, informal firms lower wages, which negatively affects consumption for the *informal stayers*. Similarly, *informal movers* experience a slight wage decrease when migrating to the formal sector. In contrast, wages for *formal stayers* remain unchanged. Overall, higher informality costs negatively impact consumption with lower wages of initially informal workers.

Finally, Figure 5(d) displays the policy effect on welfare. Equation (32) shows that welfare encompasses both the discounted value from consumption and the value of learning. As previously discussed, the policy has a negative effect on the discounted value of consumption, on average. Additionally, the policy negatively impacts workers' learning opportunities, resulting in a lower growth rate. Consequently, the policy has an unambiguous negative effect on welfare overall. Specifically, increasing η by 15% leads to a welfare decrease of 0.5%.

5.2 The Cost of Formalization

Other types of policies used to decrease informality rates are those aimed at reducing the costs of operating formally. In our framework, two parameters govern these costs for formal workers: the payroll tax, τ, and the registering costs, F. Since payroll taxes often serve a redistributive purpose in many developing countries, we focus on the effects of changing the fixed component

[35] Formally, when η changes, there is an instantaneous effect on the level of consumption in the new BGP, in addition to the growth effect shown in Figure 5(b).

of the hiring costs, F. This can be interpreted as the government reducing bureaucratic costs to register formal workers.

Similar to our previous counterfactual exercise, we take our estimated value for F and create a grid around it. The lower and upper bounds of the grid represent a 20% decrease and increase of the estimated parameter, respectively. By solving for the Balanced Growth Path (BGP) at each point on the grid, Figure 6 displays the results.

A reduction in registration costs leads to a reduction in the informality rate. Equation (7) shows that formal firms also pass hiring costs onto workers through wages. Thus, reducing the costs of hiring formal workers translates into higher formal wages, which incentivizes marginal workers to formalize. Hence, as expected, Figure 6(a) shows that a decrease in F leads to a reduction in the economy's informality rate. Interestingly, by comparing Figures 5(a) and 6(a), we can notice that decreasing formal worker hiring costs is much more effective at reducing the size of the informal sector than increasing informality costs. While an increase of 20% in η decreases informality by 1.5%, a reduction of 20% in F reduces the informal sector by 3%.

Figure 6(b) illustrates the negative effect of decreasing formal hiring costs on economic growth. The mechanics behind this result are similar to those observed when increasing informality costs. As marginal low-skilled workers migrate to the formal sector, a decrease in F also leads to an *informal movers* crowding-out effect in the learning opportunities of *formal stayers*. The quality of interactions for formal workers decreases, leading to a reduction in the growth rate. Therefore, a decrease in formal workers' hiring costs causes the economy to slow down.

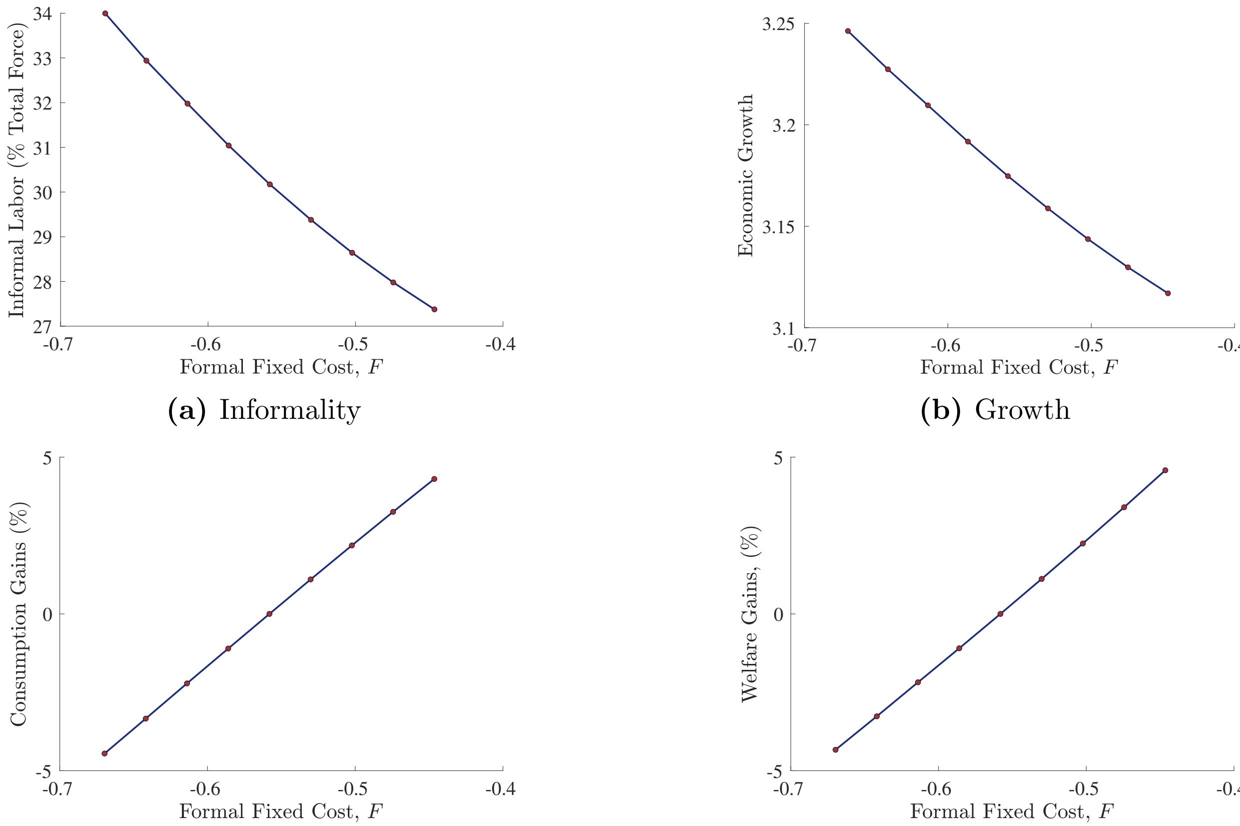
Figure 6(c) displays the policy effects on aggregate consumption. In contrast to Figure 5(c), a decrease in formality costs leads to higher aggregate consumption. Crucially, lower formal hiring costs translate into higher formal wages for both *informal movers* and *formal stayers*. The increase in formal sector wages boosts production and aggregate consumption. Therefore, although both increasing informality costs and decreasing formality costs lead to a reduction in the informal sector, the former policy has a negative effect on consumption by lowering informal workers' wages, while the latter has a positive effect by increasing formal wages.

Finally, 5(d) reports the effect of decreasing F on aggregate welfare. Interestingly, despite having a negative effect on economic growth, the policy has a positive effect on aggregate welfare. To understand this effect, recall that changes in welfare are caused by changes in the level of consumption in the new BGP and the changes in the rate at which the economy grows in the new BGP. As previously illustrated in 6(c), the policy has a positive effect on the consumption levels. Hence, the increase in the consumption level in the new BGP overcomes the lower growth rate, leading to an overall welfare boost. Formally, a decrease of 20% in formal workers registering costs leads to a welfare gain of 5%.

5.3 Discussion: Informality and Economic Growth

The aggregate effects of the formalization policies on economic growth are somewhat puzzling. Under both policies, a reduction in the size of the informal sector has adverse effects on growth.

Figure 6: Decrease of formal sector fixed cost, F



(a) Informality **(b)** Growth **(c)** Output **(d)** Economic Welfare

Notes: All figures depict the effect of changing formal workers' hiring costs, F, while keeping the rest of the parameters constant. We solve the model for every value of F in a grid with upper and lower bounds corresponding to a 20% increase and a 20% decrease around the estimated value of F in Table 5. Figure 6(a) shows the change in the percentage of informal workers. Figure 6(b) presents the changes in the growth rate of the economy. Figure 6(c) illustrates the instantaneous effect on consumption. Finally, Figure 6(d) depicts the welfare changes defined in (32) (consumption equivalent).

The negative relationship between informality rates and economic growth results from the interplay of three forces briefly mentioned earlier and crucially depends on the estimated meeting probabilities, π_i and π_f. First, both policies positively impact growth by improving the learning opportunities for *informal movers*. The estimated π_f and π_{if} imply that the probability of encountering a high-skill worker is significantly higher for formal workers. Hence, when *informal movers* switch to the formal sector, their chances of learning from high-skill workers increase.

Second, both policies deteriorate the learning opportunities for *informal stayers*. As the most skilled informal workers switch to the formal sector, the average skill level of informal workers decreases. Moreover, given our estimate for π_i, it is more likely for informal workers to meet other informal workers. Thus, as the skills in the informal sector decline and informal workers are more likely to learn from each other, learning opportunities for *informal stayers* decrease, negatively affecting the economy's growth rate. In other words, *informal movers* create a negative externality on *informal stayers* by lowering the sector's pool of knowledge.

Third, the policies also deteriorate the learning opportunities for *formal stayers*. As *informal movers* switch to the formal sector, the average skill level of formal workers decreases. Consequently, the quality of interactions for the most skilled workers in the economy, the *formal stayers*, declines, as it is now more likely for them to meet a low-skill worker. Thus, *informal movers* exert a negative externality on the *formal stayers* by crowding out their learning opportunities, which decreases economic growth.

In the aggregate, the policy's negative effects dominate the positive ones. The negative learning externalities that the *informal movers* exert on the *informal stayers* and *formal stayers* are greater than their improved learning opportunities. Therefore, overall, reducing the size of the informal sector dampens economic growth.

Finally, the effects of the formalization policies shed light on within-group learning dynamics. The counterfactual exercises suggest potential long-term gains from formal/informal worker segmentation. Specifically, reducing the size of the formal sector could be beneficial in the aggregate. The learning technology described in equation (25) might be the key feature driving this result. Since anyone can learn from the most skilled workers, the economy could benefit from a smaller formal sector in terms of growth rates. With fewer formal workers, the quality of interactions within the sector improves, effectively increasing the mass of workers at the far right tail of the human capital distribution. Given our estimate of π_i, informal workers frequently interact with formal workers. Consequently, they benefit from advances 'at the frontier' of the skill distribution, leading to an increase in expected human capital improvements upon meeting a formal worker. In summary, it seems that the economy could thrive in a scenario where the best workers initiate learning among themselves, expanding the knowledge frontier, and subsequently, the rest of the workers catch up.

6 Conclusion

This study sheds light on the complex dynamics of informal labor markets in developing economies, specifically in the context of Chile. By providing empirical evidence on wage differentials and growth patterns for formal and informal workers, we offer insights into the intricate relationship between human capital formation, labor market segmentation, and economic outcomes. The observed wage disparities, influenced by workers' sorting based on skill and divergent human capital accumulation over the life cycle, underscore the importance of understanding the mechanisms that drive the formal-informal wage gap. Moreover, our developed heterogeneous agent endogenous growth model offers a theoretical framework to rationalize these empirical findings, emphasizing the role of learning opportunities, fixed and variable costs, and the interplay between the formal and informal sectors in shaping economic outcomes. In the policy realm, our counterfactual analyses underscore the nuanced impacts of formalization policies. While efforts to reduce informality by either increasing the cost of informal operations or decreasing formalization costs may seem intuitive, our results caution against overlooking the intricate dynamics at play. The crowding-out effect on human capital in the formal sector, stemming from the movement of less-skilled workers, challenges the conventional wisdom regarding the benefits of formalization.

In our work in progress, we are expanding the model along two dimensions. First, we are exploring alternative learning technologies that better capture the dynamics reflected in the wage paths observed in the data. We believe that by considering these different technologies, the quantitative effects of formalization policies will become more plausible. Second, our current framework attributes all wage improvements to workers' human capital accumulation. In reality, it is plausible that firm productivity improvements over time also influence the observed wage paths. Therefore, we are working to incorporate technological changes from the firm side, both when employing formal and informal workers. Assessing the relative importance of workers' human capital accumulation and firm productivity improvements on growth and welfare, and understanding how worker sorting between formal and informal sectors affects both channels, is crucial for policy design.

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A Theory Appendix

A.1 Derivations

A.1.1 HJB equations

With this notation, the value of being a formal worker is defined as

$$V_f(z,t) = Y_f(t)\Delta + \frac{1}{1+\rho\Delta}\begin{bmatrix} \alpha\lambda_f^f\Delta \int_{\bar{z}(t+\Delta)}^{\infty} \max\Big\{V_f(\tilde{z},t+\Delta), V_f(z,t+\Delta)\Big\} k\left(\frac{\tilde{z}}{z}\right) g(\tilde{z},t)d\tilde{z} \\ +\alpha\lambda_f^i\Delta \int_0^{\bar{z}(t+\Delta)} \max\Big\{V_i\left(\tilde{z},t+\Delta\right), V_f(z,t+\Delta)\Big\} k\left(\frac{\tilde{z}}{z}\right) g(\tilde{z},t)d\tilde{z} \\ +\left(1-(\alpha+\delta)\Delta\right) V_f(z,t+\Delta) \end{bmatrix}$$

By multiplying both sides by $(1+\Delta\rho)$, dividing by Δ, and collecting terms, the Bellman equation can be rewritten as:

$$\begin{aligned} \frac{V_f(z,t) - V_f(z,t+\Delta)}{\Delta} + \rho V_f(z,t) = {} & Y_f(z)(1+\rho\Delta) - \delta V_f(z,t+\Delta) \\ & + \alpha\lambda_f^f \int_{\bar{z}_{t+\Delta}}^{\infty} \max\left\{V_f(\tilde{z},t+\Delta) - V_f(z,t+\Delta), 0\right\} k\left(\frac{\tilde{z}}{z}\right) g(\tilde{z})\mathrm{d}\tilde{z} \\ & + \alpha\lambda_f^i \int_0^{z^*_{t+\Delta}} \max\left\{V_i\left(\tilde{z},t+\Delta\right) - V_f(z,t+\Delta), 0\right\} k\left(\frac{\tilde{z}}{z}\right) g(\tilde{z})\mathrm{d}\tilde{z} \end{aligned}$$

By taking $\Delta \to 0$ and using the fact that $V_f(z,t)$ is increasing in z and for every $z \geq \bar{z}$ implies $V_f(z,t) \geq V_i(z,t)$, it follows that the Bellman equation for the formal workers is

$$(\rho+\delta)V_f(z,t) - \dot{V}_f(z,t) = Y_f(z) + \alpha\lambda_f^f \int_z^{\infty} \left(V_f(\tilde{z},t) - V_f(z,t)\right) k\left(\frac{\tilde{z}}{z}\right) g(\tilde{z})d\tilde{z} \tag{A1}$$

An analogous process yields the Bellman equation for informal workers:

$$\begin{aligned} (\rho+\delta)V_i(z,t) - \dot{V}_i(z,t) = Y_i(z) & + \alpha\lambda_i^f \int_{\bar{z}_t}^{\infty} \left(V_f(\tilde{z},t) - V_i(z,t)\right) k\left(\frac{\tilde{z}}{z}\right) g(\tilde{z})d\tilde{z} \\ & + \alpha\lambda_i^i \int_z^{\bar{z}_t} \left(V_i(\tilde{z},t) - V_i(z,t)\right) k\left(\frac{\tilde{z}}{z}\right) g(\tilde{z})d\tilde{z} \end{aligned} \tag{A2}$$

A.1.2 Kolmogorov Forward Equation

Then the cummulative distribution function after an interval Δ for any $z \leq \bar{z}$ is

$$\begin{aligned} G\left(z,t+\Delta\right) &= \int_0^z \Pr\left(\tilde{z}_{t+\Delta} \leq z \mid \tilde{z}_t = y\right) g(y,t)\mathrm{d}y \\ &= \int_0^z (1-\alpha\Delta) + \alpha\Delta\left[\mathbb{P}_i^i + \mathbb{P}_i^f - \lambda_i^i \int_z^{\bar{z}} k\left(\frac{x}{y}\right) g(x,t)\mathrm{d}x - \lambda_i^f \int_{\bar{z}}^{\infty} k\left(\frac{x}{y}\right) g(x,t)\mathrm{d}x\right] g(y,t)\mathrm{d}y(1-\delta\Delta) \\ &+ \delta\Delta \int_0^z B(y,t)\mathrm{d}y \end{aligned}$$

Note that by definition, $\mathbb{P}_i^i + \mathbb{P}_i^f = 1$ and hence the KFE for any $z \leq \bar{z}$ is

$$\begin{aligned}\frac{\partial G(z,t)}{\partial t} =& -\alpha\lambda_i^i \int_0^z \int_z^{\bar{z}} k\left(\frac{x}{y}\right) g(x,t)g(y,t)\mathrm{d}x\mathrm{d}y \\ & -\alpha\lambda_i^f \int_0^z \int_{\bar{z}}^{\infty} k\left(\frac{x}{y}\right) g(x,t)g(y,t)\mathrm{d}x\mathrm{d}y \\ & -\delta\left[G(z,t) - B(z,t)\right]\end{aligned} \tag{A3}$$

In terms of flows we can write the equation as

$$\begin{aligned}\frac{\partial g(z,t)}{\partial t} =& \, \alpha\lambda_i^i g(z,t) \int_0^z k\left(\frac{z}{y}\right) g(y,t)\mathrm{d}y \\ & -\alpha\lambda_i^f g(z,t) \int_{\bar{z}}^{\infty} k\left(\frac{y}{z}\right) g(y,t)\mathrm{d}y \\ & -\alpha\lambda_i^i g(z,t) \int_z^{\bar{z}} k\left(\frac{y}{z}\right) g(y,t)\mathrm{d}y \\ & -\delta(g(z,t) - b(z,t))\end{aligned}$$

Similarly the KFE for any $z \geq \bar{z}$ is

$$\begin{aligned}\frac{\partial G(z,t)}{\partial t} =& -\alpha\lambda_i^f \int_0^{\bar{z}} \int_z^{\infty} k\left(\frac{x}{y}\right) g(x,t)g(y,t)\mathrm{d}x\mathrm{d}y \\ & -\alpha\lambda_f^f \int_{\bar{z}}^z \int_z^{\infty} k\left(\frac{x}{y}\right) g(x,t)g(y,t)\mathrm{d}x\mathrm{d}y \\ & -\delta\left[G(z,t) - B(z,t)\right]\end{aligned} \tag{A4}$$

In terms of flows, the KFE can be written as

$$\begin{aligned}\frac{\partial g(z,t)}{\partial t} =& \, \alpha\lambda_i^f g(z,t) \int_0^{\bar{z}} k\left(\frac{z}{y}\right) g(y,t)\mathrm{d}y \\ & +\alpha\lambda_f^f g(z,t) \int_{\bar{z}}^{z} k\left(\frac{z}{y}\right) g(y,t)\mathrm{d}y \\ & -\alpha\lambda_f^f g(z,t) \int_{\bar{z}}^{z} k\left(\frac{y}{z}\right) g(y,t)\mathrm{d}y \\ & -\delta(g(z,t) - b(z,t))\end{aligned}$$

A.2 Proofs of propositions

A.2.1 Proof of proposition 3

Step 1: $V_i(0,t) > V_f(0,t)$ *at any time* t.

Step 2: $V_i(z,t)$ *and* $V_f(z,t)$ *are monotonically increasing in* z.

A.2.2 Proof of proposition 4

A.2.3 Proof of proposition 5

Note that using l'hospital rule on the Pareto assumption implies that

$$\lim_{x\to\infty} \frac{\Phi'(x)x}{x^{-\frac{1}{\theta}}} = \frac{k}{\theta}$$

Going back to the KFE for formal workers, dividing both sides of by $x^{-1/\theta}$ yields

$$\gamma \frac{\Phi'(x)x}{x^{-\frac{1}{\theta}}} = \alpha \left(\frac{1-\Phi(x)}{x^{-\frac{1}{\theta}}} \right) \left[\lambda_i^f(\bar{x})\Phi(\bar{x}) + \lambda_f^f(\bar{x}) \left(\Phi(x) - \Phi(\bar{x}) \right) \right]$$

Thus, by taking the limit as $x \to \infty$ and using our Pareto tail assumption 1, it follows that the growth rate of the economy is

$$\gamma = \alpha\theta \left[\lambda_i^f(\bar{x})\Phi(\bar{x}) + \lambda_f^f(\bar{x}) \left(1 - \Phi(\bar{x}) \right) \right] = \alpha\theta\psi \left(\Phi(\bar{x}) \right) \tag{A5}$$

B Empirical Appendix

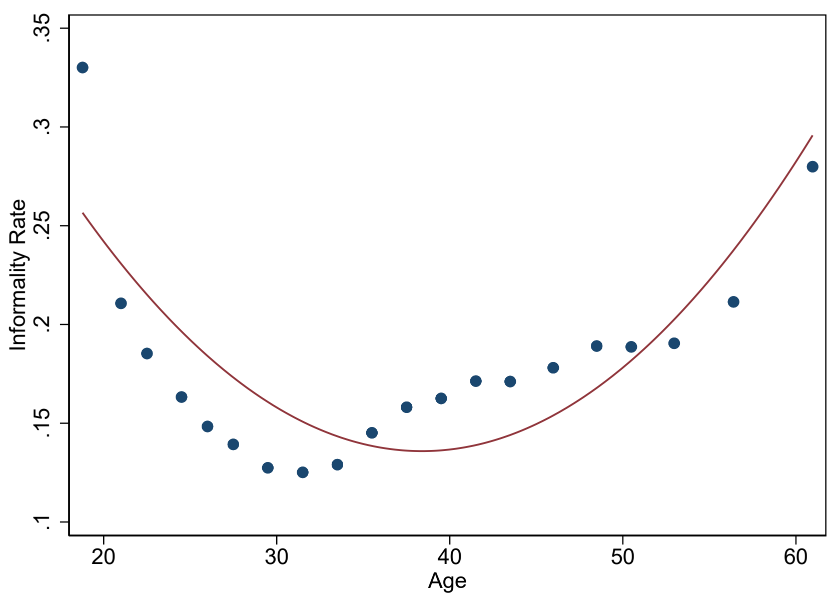
B.1 Additional Descriptive Statistics

This section reports additional descriptive statistics for informal workers in Chile. With the caveat that we only display statistics for formal and informal salaried workers, we confirm previously well-established informality patterns for the Chilean economy:[36]

1. Informality and wage follow a U-shaped pattern.
2. Informality decreases with educational attainment.
3. Informality decreases with firm size.

Then, we report informality rates by occupation and economic sector.

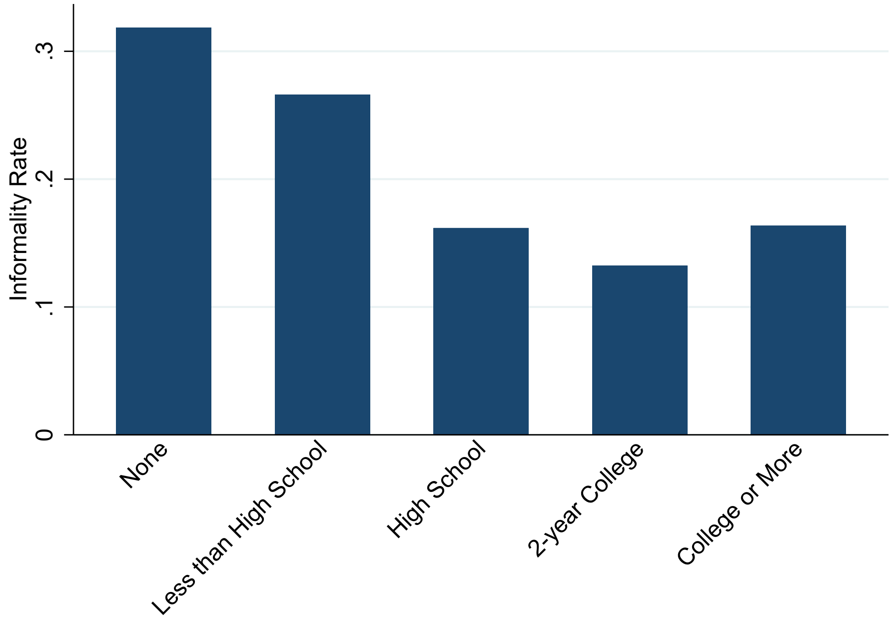
Figure B1: Informality Rate by Age



Notes: Figure B1 displays a binscatter plot of informality rates and age. We consider 20 equally-sized bins according to worker's age. Moreover, we restrict our sample to salaried workers, excluding self-employed workers, entrepreneurs, workers without remuneration, and public-sector and armed forces employees.

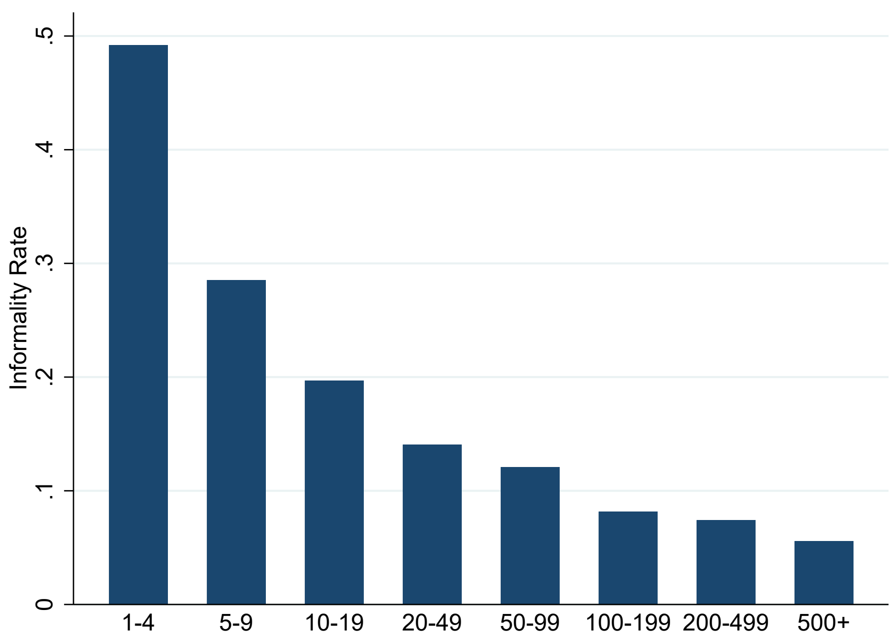
[36]See Ulyssea (2020a) for a comprehensive review of informal workers empirical regularities.

Figure B2: Informality Rate by Educational Attainment



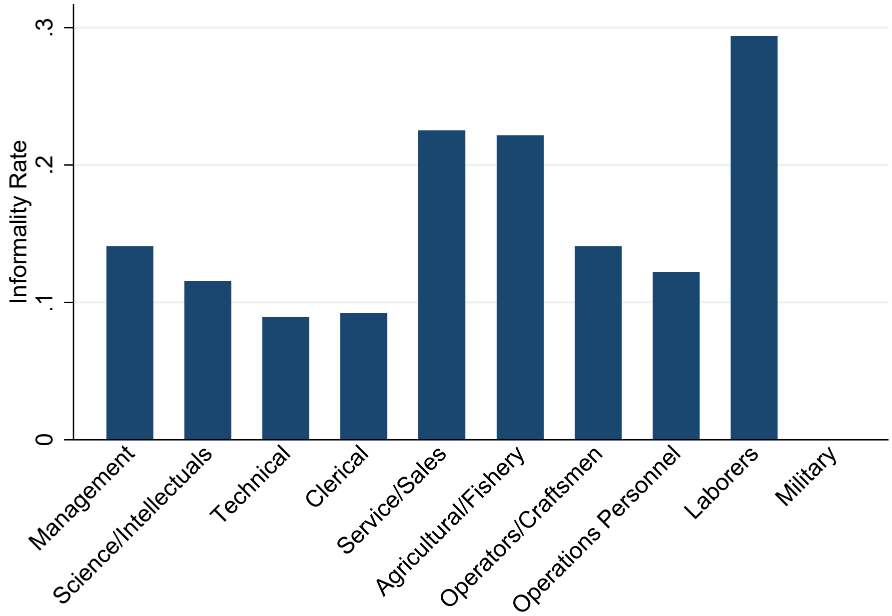
Notes: Figure B2 displays informality rates by education groups. We restrict our sample to salaried workers, excluding self-employed workers, entrepreneurs, workers without remuneration, and public-sector and armed forces employees.

Figure B3: Informality Rate by Firm Size



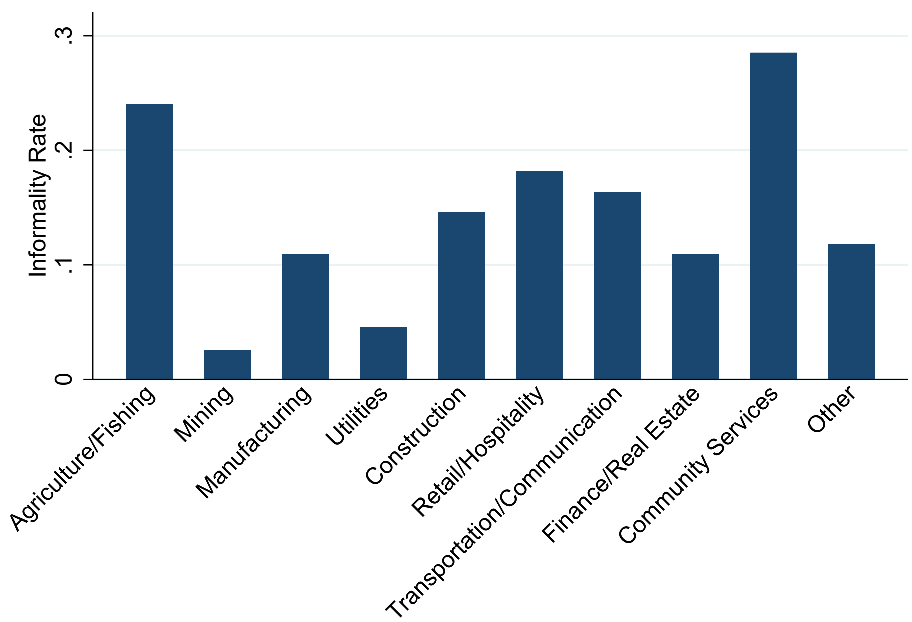
Notes: Figure B3 displays informality rates by firm size. Bars denote firm size groups, and labels denote the range of employees in each group. We restrict our sample to salaried workers, excluding self-employed workers, entrepreneurs, workers without remuneration, and public-sector and armed forces employees.

Figure B4: Informality Rate by Occupation



Notes: Figure B4 displays informality rates by occupations. We restrict our sample to salaried workers, excluding self-employed workers, entrepreneurs, workers without remuneration, and public-sector and armed forces employees.

Figure B5: Informality Rate by Sector

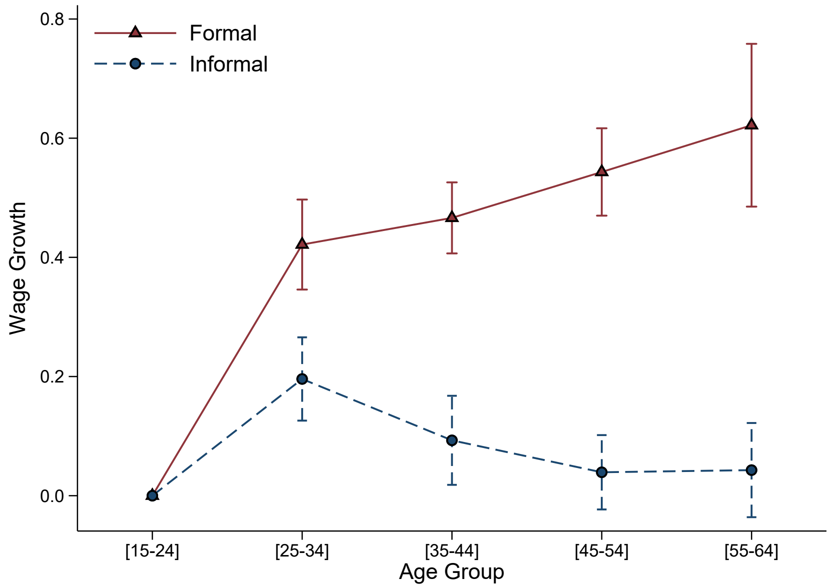


Notes: Figure B5 displays informality rates by economic sector. We restrict our sample to salaried workers, excluding self-employed workers, entrepreneurs, workers without remuneration, and public-sector and armed forces employees.

B.2 Additional Figures on Wages Over the Life Cycle

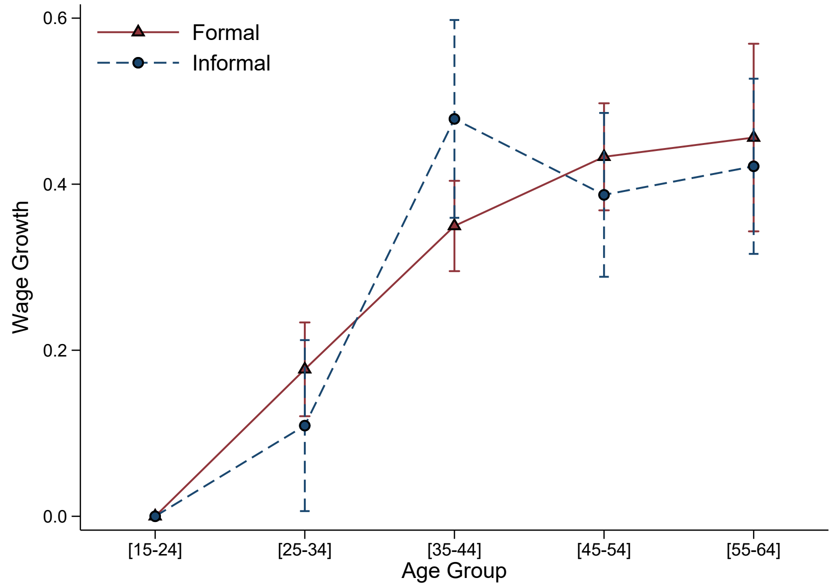
In this section, we report additional figures on wages over the life cycle for Chile. We begin by presenting wage profiles over the life cycle for all workers in the economy, including entrepreneurs, self-employed individuals, and public sector employees. Then, focusing exclusively on salaried workers, we present wage-age profiles by controlling for observable worker characteristics, as well as wage-experience profiles, both in the raw data and after controlling for observables.

Figure B6: Wages Over the Life Cycle for Formal and Informal Workers, All Workers



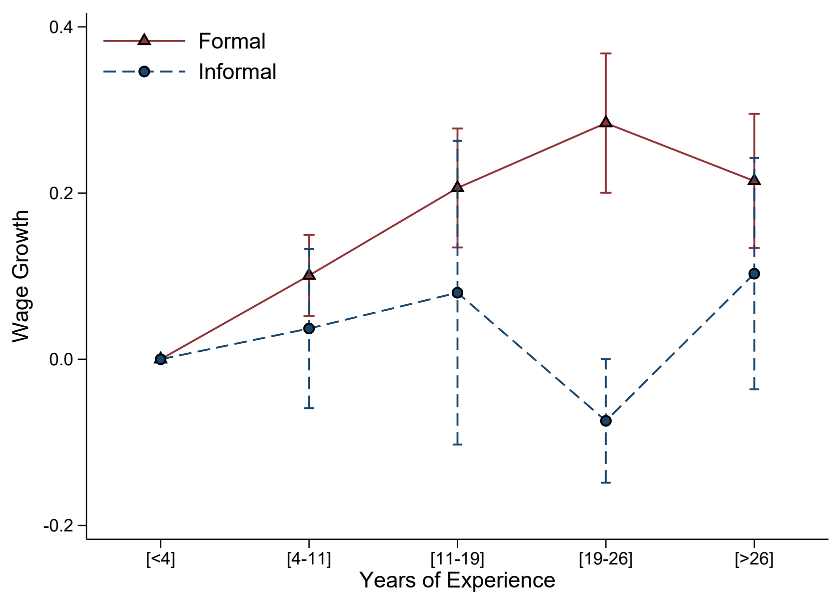
Notes: Figure B6 displays wage paths over the life cycle for formal and informal workers. We consider all workers in the economy, including self-employed workers, entrepreneurs, workers without remuneration, and public-sector and armed forces employees. We compute the average wage for each 10-year age bin relative to the average wage for workers less than 24 years. Vertical dashed lines denote 90 percent confidence intervals.

Figure B7: Wages Over the Life Cycle for Formal and Informal Workers, Controls



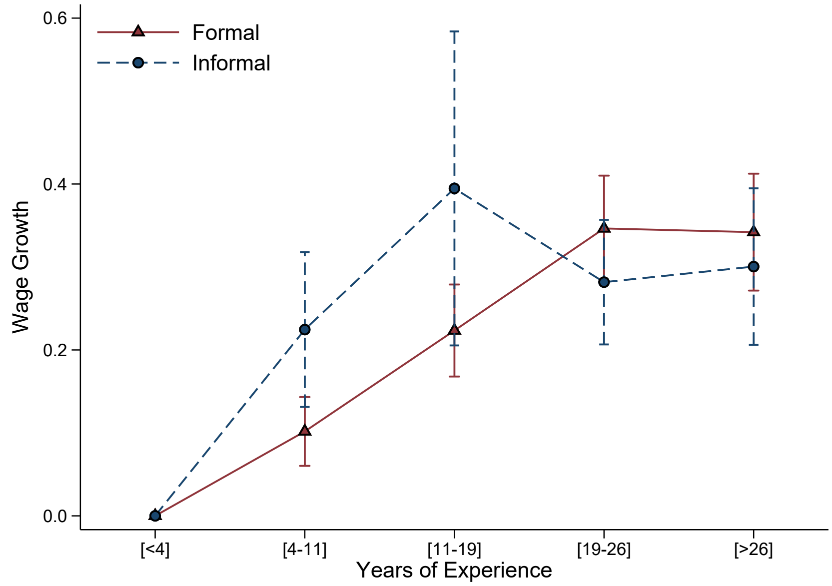
Notes: Figure B7 displays wage paths over the life cycle for formal and informal workers while residualizing wages by: education, gender, occupation, sector, region, firm size, and year fixed effects, respectively. We restrict our sample to salaried workers, excluding self-employed workers, entrepreneurs, workers without remuneration, and public-sector and armed forces employees. We compute the average wage for each 10-year age bin relative to the average wage for workers less than 24 years. Vertical dashed lines denote 90 percent confidence intervals.

Figure B8: Wages-Experience Profiles



Notes: Figure B8 displays wage-experience paths for formal and informal workers. We restrict our sample to salaried workers, excluding self-employed workers, entrepreneurs, workers without remuneration, and public-sector and armed forces employees. We compute the average wage for each experience bin relative to the average wage for workers less than 4 years of experience. Vertical dashed lines denote 90 percent confidence intervals.

Figure B9: Wages-Experience Profiles, Controls



Notes: Figure B9 displays wage-experience paths for formal and informal workers, while residualizing wages by: education, gender, occupation, sector, region, firm size, and year fixed effects, respectively. We restrict our sample to salaried workers, excluding self-employed workers, entrepreneurs, workers without remuneration, and public-sector and armed forces employees. We compute the average wage for each experience bin relative to the average wage for workers less than 4 years of experience. Vertical dashed lines denote 90 percent confidence intervals.

B.3 Evidence from Colombia

For Colombia, we utilize the *Encuesta Longitudinal Colombiana* (ELCO). The ELCO is a longitudinal survey that tracks approximately 10,000 Colombian households in rural and urban areas every three years. The objective of the survey is to observe and analyze the same households over a 12-year span, with the survey currently covering three waves: 2010, 2013, and 2016. Importantly, ELCO follows a set of individuals over time, allowing the creation of a panel data set that provides a dynamic perspective on the social and economic changes experienced by these individuals and their households. We divide our sample into two for our analysis: *(i)* Cross-sectional sample, where we combine repeated cross-sectional observations with the panel observations, and *(ii)* Panel sample, a subset of our cross-sectional sample, considering only individuals tracked over the three waves. For our analysis, we focus on urban households and exclude self-employed workers from the sample. Table B1 presents descriptive statistics for both samples.

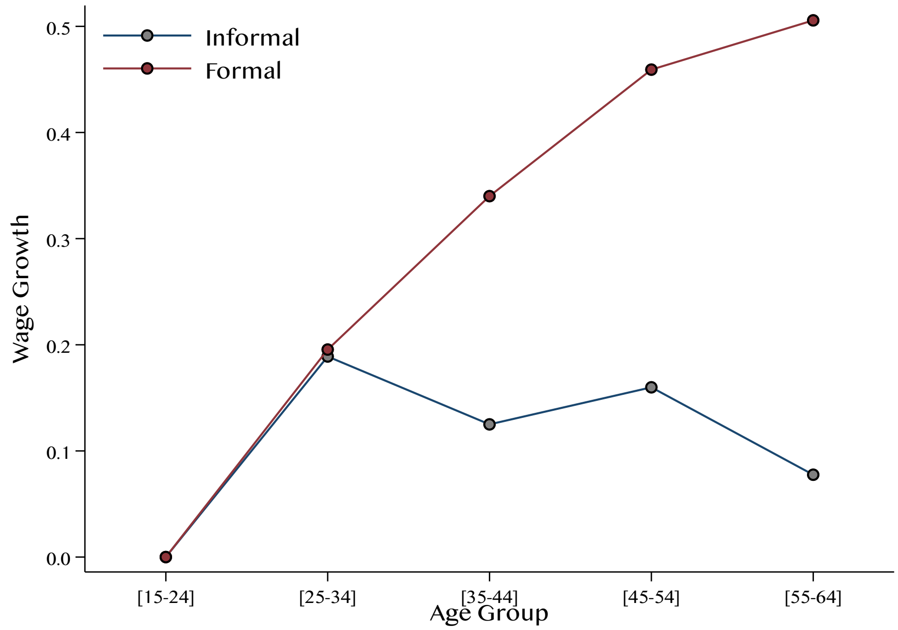
We present life cycle wage profiles for formal and informal workers in Colombia. Figure B10 displays the raw wage profiles for formal and informal workers. Figure B11 presents wage profiles after residualizing wages by education and worker fixed effects. Both figures demonstrate that wages for formal workers in Colombia increase significantly more over the life cycle than wages for informal workers.

Table B1: Descriptive Statistics - ELCO

	2010	2013	2016	Overall	Obs
			Panel A: Cross-Sectional Sample		
Informal Status	0.592	0.658	0.633	0.631	19,120.000
	(0.491)	(0.474)	(0.482)	(0.483)	
Age	32.985	35.887	36.722	35.267	37,543.000
	(13.606)	(13.917)	(14.147)	(13.988)	
Wage (Real)	17,187.658	18,922.483	18,117.710	18,057.430	8,661.000
	(16,040.282)	(17,991.025)	(17,082.616)	(17,050.210)	
Female	0.563	0.546	0.544	0.550	37,543.000
	(0.496)	(0.498)	(0.498)	(0.497)	
Less High School	0.777	0.756	0.711	0.744	25,298.000
	(0.416)	(0.430)	(0.453)	(0.436)	
High School	0.050	0.058	0.070	0.061	25,298.000
	(0.218)	(0.234)	(0.255)	(0.239)	
2 Year College	0.086	0.113	0.134	0.115	25,298.000
	(0.281)	(0.316)	(0.341)	(0.319)	
4 Year College	0.087	0.073	0.085	0.080	25,298.000
	(0.281)	(0.261)	(0.279)	(0.272)	
			Panel B: Panel Sample		
Informal Status	0.597	0.643	0.615	0.621	16,206.000
	(0.491)	(0.479)	(0.487)	(0.485)	
Age	33.297	36.626	39.623	36.561	27,398.000
	(13.023)	(13.559)	(13.163)	(13.504)	
Wage (Real)	17,224.568	19,451.756	18,912.667	18,509.575	7,387.000
	(15,943.761)	(18,258.968)	(17,594.064)	(17,300.497)	
Female	0.581	0.563	0.567	0.570	27,398.000
	(0.493)	(0.496)	(0.496)	(0.495)	
Less High School	0.783	0.740	0.707	0.737	20,602.000
	(0.412)	(0.439)	(0.455)	(0.440)	
High School	0.048	0.061	0.070	0.062	20,602.000
	(0.213)	(0.240)	(0.256)	(0.241)	
2 Year College	0.089	0.119	0.135	0.118	20,602.000
	(0.285)	(0.324)	(0.341)	(0.323)	
4 Year College	0.080	0.080	0.088	0.083	20,602.000
	(0.272)	(0.271)	(0.284)	(0.276)	

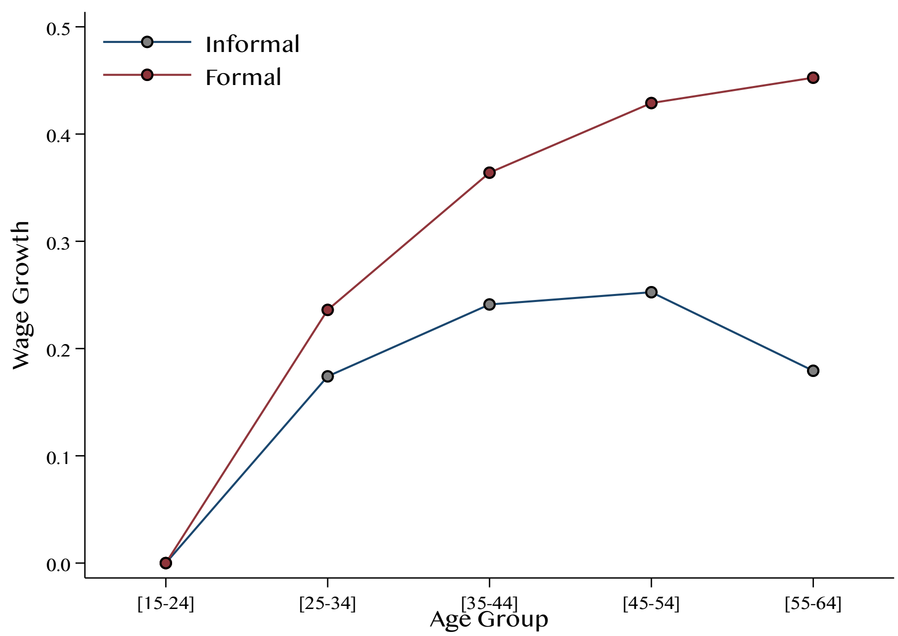
Notes: Table B1 shows the descriptive statistics of the key variables of the ELCA database (longitudinal survey of Colombia), for the years 2010, 2013 and 2016. Panel A is the sample of cross-sectional people who for who we have full information for the variables of informality status, age, education, and wages. Panel B is the sample of people followed in the three periods of analysis and with full information.

Figure B10: Wages Over the Life Cycle for Formal and Informal Workers in Colombia



Notes: Figure B10 displays wage paths over the life cycle for formal and informal workers in Colombia. We compute the average wage for each 10-year age bin relative to the average wage for workers less than 24 years.

Figure B11: Residualized Wages Over the Life Cycle for Formal and Informal Workers in Colombia



Notes: Figure B11 displays wage paths over the life cycle for formal and informal workers in Colombia. We residualized wages by controlling for education and worker fixed effects. We compute the average wage for each 10-year age bin relative to the average wage for workers less than 24 years.