UNIVERSITÀ DI BOLOGNA

School of Engineering

Master Degree in Automation Engineering

Distributed Control Systems

Distributed Simplex for Cooperative Multi-robot Assignment

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Abstract

An implementation of a distributed version of the well-known simplex algorithm, is given in this report. The aim is to solve degenerate linear programs on asynchronous peer-to-peer networks with distributed information structures. So, a network of agents is considered, where every agent knows only a subset of information, and they run simplex algorithm by updating and sharing their base, until they will agree on a common solution. We show how the multi agent assignment problem can be efficiently solved by using a distributed structure, and we provide simulations in the robot operating system environment.

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Introduction

The present report provides a methodological approach to define and solve a multi-agent assignment problem. The goal is to generate a distributed algorithm, with local and sharing information structure.
In Chapter 1 we define the set-up in order to understand and implement the distributed algorithm which we tried to make as more general as possible.
In Chapter 2 we show a simulation platform by using the developed software, to simulate a fleet of wheeled-robots.

Contributions

The contribution is given by the algorithm's implementation into a distributed OS.

Chapter 1

Set-up

We are interested in a network of autonomous agents labeled $V_a = 1, 2, .., N$ which are able to receive information from their neighbours, and a set of tasks $U_a = 1, 2, .., N$. Neighbour relations between distinct pairs of agents are characterized by directed graph $\mathbb{G}_c$ with N vertices.
The assignment problem can be illustrated with a bipartite assignment graph $\mathbb{G}_a = \{V_a, U_a; E_a\}$, where an edge $(i, k) \in E_a$ exists if and only if agent i can be assigned to the task k. The cost $c_{ik} \in \mathbb{Z}$ for agent i to perform the task k is a weight on the edge (i, k). For each agent i and task k a binary decision variable $x_{ik} \in \{0, 1\}$ is introduced, which is 1 if agent i is assigned to task k and 0 otherwise.
The assignment problem corresponds to the *integer optimization problem* with the constraint that a full assignment is achived:

$$\begin{aligned}
\min_{x \geq 0} & \sum_{(i,k) \in E_a} c_{ik} x_{ik} \\
s.t. & \sum_{k|(i,k) \in E_a} x_{ik} = 1, \forall i \in 1, ..., N, \\
& \sum_{i|(i,k) \in E_a} x_{ik} = 1, \forall k \in 1, ..., N,
\end{aligned}$$

Note that the linear program has $n = |E_a|$ decision variables with $|E_a| \leq N^2$, and $d = 2N$ equality constraints.

1.1 Distributed information structure

We can consider the linear program in standard equality form:

$$\begin{aligned} &\min c^T x \\ &s.t. \quad Ax = b, \quad x \geq 0 \end{aligned}$$

Where $A \in \mathbb{R}^{d \times n}$, $b \in \mathbb{R}^d$ and $c \in \mathbb{R}^n$, are the problem data and $x \in \mathbb{R}^n$ is the vector of decision variables. A column of the linear program is a vector $h_i \in \mathbb{R}^{1+d}$ defined as

$$h_i := [c_i, A_i^T]^T$$

where $c_i \in \mathbb{R}$ is the i-th entry of c and $A_i \in \mathbb{R}^{d \times 1}$ is the i-th column of the matrix A. The set of columns is denoted by $\mathbb{H} = \{h_i\}_{i \in \{1,..,N\}}$.
We assume that every agent initially knows a subset of problem columns, called permanent columns. This means there exists a partitioning

$$\wp = \{\mathbb{P}^{[1]}, \ldots, \mathbb{P}^{[N]}\}$$

of the problem columns with

$$\mathbb{H} = \cup_{i=1}^{N} \mathbb{P}^{[i]} \quad and \quad \mathbb{P}^{[i]} \cap \mathbb{P}^{[j]} = \emptyset$$

and agent i initially knows the column set $\mathbb{P}^{[i]}$.

1.2 Distributed simplex algorithm

The formalization of this distributed algorithm is given as follows. The state of each agent is a lex-feasibile base, $w^{[i]}(t) = \mathbb{B}^{[i]}$, so a set of basis columns sufficies to fully determine the solution of a linear program, and thus every agent keeps and updates a copy of a possible solution. To initialize the algorithm, every agent creates an artificial basis using a method known as Big-M method. So, the corresponding initial base $\mathbb{B}_M$ is as follows: it chooses $A_{B_M} = \mathbb{I}_d$ and $c_{B_M} = M \cdot \mathbf{1}$, where the artificial value M is larger than any other cost coefficient in the original problem, i.e., M can be chosen as the largest number realizable by the processor. The state transition function STF performed by each agent to update its basis $\mathbb{B}^{[i]}$ proceeds as follows. Having received columns from its in-neighbours, and having its set of permanent columns $\mathbb{P}^{[i]}$ in its memory, the agent performs two actions:

- it sorts all columns in its memory lexicographically
- it performs the Simplex algorithm, and updates its basis with the new, improved basis.

We provide a pseudo code description of the algorithm in the following table.

```
Problem data:          $\mathbb{G}_c(t), (\mathbb{H}, b), \mathbb{P})$ ;
Processor state:       $\mathbb{B}^{[i]} \subseteq$ with card $(\mathbb{H}) = d$ ;
Initialization:        $\mathbb{B}^{[i]} := \mathbb{B}_M$ ;
Function MSG($\mathbb{B}^{[i]}$,j) begin
    Result: $\mathbb{B}^{[i]}$
end
Function STF($\mathbb{B}^{[i]}, \cup_{j\in\mathbb{N}(i,t)} y_j$) begin
    if $y_j \neq$ null for all $j \in \mathbb{N}_l(i,t)$ then
        $\mathbb{H}^{tmp} \leftarrow$lexsort $\{\mathbb{P}^{[i]} \cup \mathbb{B}^{[i]} \cup (\cup_{j\in\mathbb{N}(i,t)} y_j)\}$
        $\mathbb{B}^{[i]} \leftarrow$ Simplex($\mathbb{H}^{tmp}, \mathbb{B}^{[i]}$)
    end
    else
        $\mathbb{B}^{[i]} \leftarrow$ null
    end
end
```

```
Algorithm 1: Simplex($\mathbb{H}, \mathbb{B}$)
Data: A set of columns $\mathbb{H}$, a lex-feasible basis $\mathbb{B} \in \mathbb{H}$
while $\exists e \in \mathbb{H}$ such that $[r^T_{\{B\cup e\}} c_{\{B\cup e\}}, r^T_{\{B\cup e\}}] \prec 0$ do
    $\mathbb{B} \leftarrow$ Pivot($\mathbb{B}, e$)
end
```

```
Algorithm 2: Pivot($\mathbb{B}, e$)
Data: Lex-feasible basis $\mathbb{B}$, non-basic column $e \in \mathbb{H}$
if $[r^T_{\{B\cup e\}} c_{\{B\cup e\}}, r^T_{\{B\cup e\}}] \prec 0$ then
    select leaving column $\ell_{lex}(e)$ via lex ratio test
    if $\ell_{lex}(e) \neq \emptyset$ then
        $\mathbb{B} \leftarrow (\mathbb{B} \cup e)\ \ell_{lex}(e))$ ;                // make the pivot
    else
        $\mathbb{B} \leftarrow$ null ;                         // problem is unbounded
end
```

The procedure of replacing a basic column with a non-basic one is called *pivot*, and in order to handle degeneracy a variant of the simplex is given by taking $[r^T_{\{B\cup e\}} c_{\{B\cup e\}}, r^T_{\{B\cup e\}}]$ with reduced cost defined as:

$$\bar{c}_e = c_e - A_e^T (A_B^{-1})^T c_b =: r^T_{\{B\cup e\}} c_{\{B\cup e\}}$$

where $c_e \in \mathbb{R}$ and $A_e \in \mathbb{R}^d$ refer to the problem data related to column e. Starting with a lex-feasible basis, the simplex method is defined such that after a pivot iteration the new basis is again lex-feasible. Let $\mathbb{B}$ be a lex-feasible basis and e the entering column, let the leaving column be chosen by the *lexicographic ration test*,

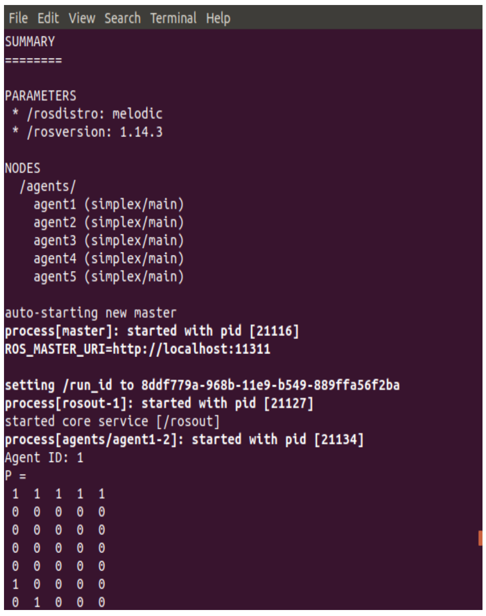
$$\ell_{lex}(e) = arg \quad lexmin[A_B^{-1}b, A_B^{-1}]_{\bullet j}/(A_B^{-1}A_e)_{\bullet j} \quad | \quad (A_B^{-1}A_e)_{\bullet j} > 0$$

where the subscript $\bullet j$ denotes selection of the j-th row of the matrix, respectively vector, then the next basis is again lex-feasible.

Citation [1]

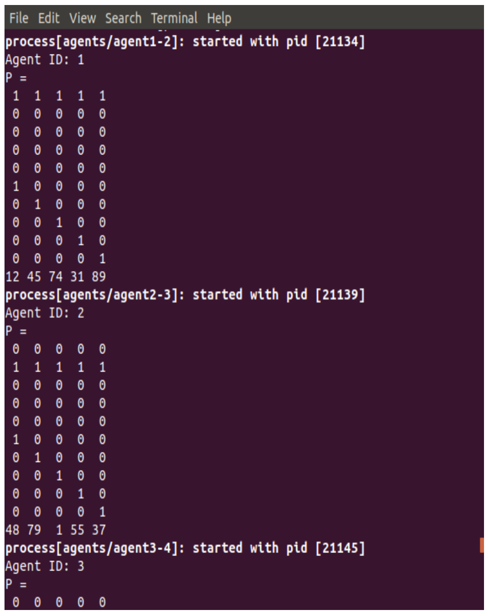
1.3 Code explanation

The implementation of distributed simplex is done by using robot operating system, and in order to establish communication between agents, we have defined a ring digraph, so the i-th agent receives the base from the (i-1)-th agent and sends its base to the (i+1)-th agent, except for the first and last agent. Let's start looking to the main program, **main.cpp**, at beginnig we receive identification number, used to distinguish between nodes, and then we create an object of *Agent's* class. Initialization of Agent's class consist of a base matrix $\mathbb{B}$, a constrained matrix $\mathbf{A}$, a cost vector inside the matrix $\mathbb{H}$, a publisher and a subscriber with the id number given at node, in order to create the ring digraph. At last element, a publisher is used to send velocity's comand to robot base, and move the robot to the desired target position.



```
SUMMARY
========

PARAMETERS
 * /rosdistro: melodic
 * /rosversion: 1.14.3

NODES
  /agents/
    agent1 (simplex/main)
    agent2 (simplex/main)
    agent3 (simplex/main)
    agent4 (simplex/main)
    agent5 (simplex/main)

auto-starting new master
process[master]: started with pid [21116]
ROS_MASTER_URI=http://localhost:11311

setting /run_id to 8ddf779a-968b-11e9-b549-889ffa56f2ba
process[rosout-1]: started with pid [21127]
started core service [/rosout]
process[agents/agent1-2]: started with pid [21134]
Agent ID: 1
P =
 1  1  1  1  1
 0  0  0  0  0
 0  0  0  0  0
 0  0  0  0  0
 0  0  0  0  0
 1  0  0  0  0
 0  1  0  0  0
```

(a) Agents ID

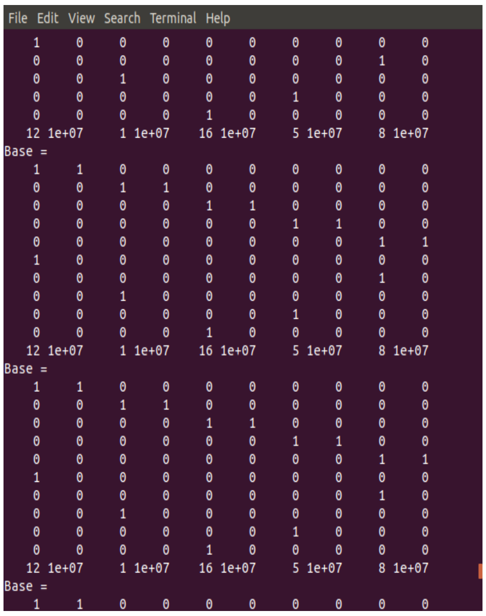


```
process[agents/agent1-2]: started with pid [21134]
Agent ID: 1
P =
 1  1  1  1  1
 0  0  0  0  0
 0  0  0  0  0
 0  0  0  0  0
 0  0  0  0  0
 1  0  0  0  0
 0  1  0  0  0
 0  0  1  0  0
 0  0  0  1  0
 0  0  0  0  1
12 45 74 31 89
process[agents/agent2-3]: started with pid [21139]
Agent ID: 2
P =
 0  0  0  0  0
 1  1  1  1  1
 0  0  0  0  0
 0  0  0  0  0
 0  0  0  0  0
 1  0  0  0  0
 0  1  0  0  0
 0  0  1  0  0
 0  0  0  1  0
 0  0  0  0  1
48 79  1 55 37
process[agents/agent3-4]: started with pid [21145]
Agent ID: 3
P =
 0  0  0  0  0
```

(b) Permanent cols

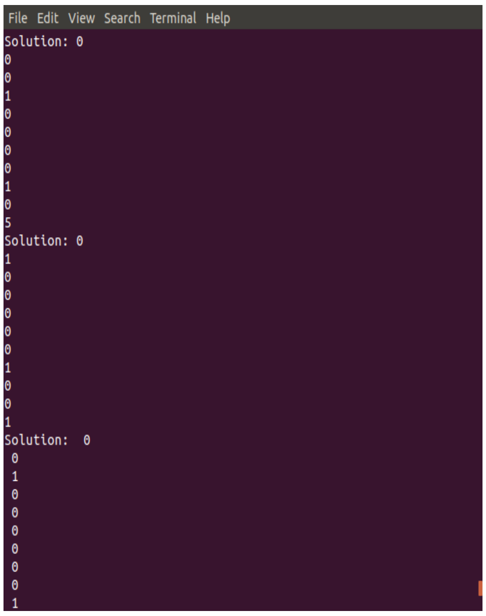
Figure 1.1: Agents initialization

The simplex algorithm is implemented inside **matrixcb** function, and starts by creating the $\mathbb{H}_{tmp}$ matrix with columns received in a base message from its neighbour and its permanent columns. So the function lexicographic sorting all the columns of the matrix $\mathbb{H}_{tmp}$, to provides a unique ordering of the columns. The lexicographic ordering ensures that all agents optimize with respect to the same lexicographic objective. Then the Simplex algorithm is defined with three parameters $\mathbb{H}_{tmp}$, $\mathbb{B}$ and $\mathbb{P}$, inside the file **Simplex.cpp**. At beginning it initializes all matrices to zero, and enters inside the while cycle until all columns of $\mathbb{H}_{tmp}$ are checked. If vector e is already inside the base $\mathbb{B}$, is skipped, else it used to check the reduced cost as described previously. If the reduced cost is lexico-negative, a leaving column is choosen by lexicographic radio test to be changed with vector e. The base obtained by simplex function, is stored to a temporary base $\mathbb{B}_{tmp}$ and it is checked with the previous one $\mathbb{B}$. If the base sended by the previous agent is changed, then all the agents are connected and I can start to converge to the common solution. After a while, if the base is not more changing then **matrixcb** function will stop to execute, and the **end** function will be used to define the solution. Inside the common base, it will be present one of the permanent columns of matrix $\mathbb{P}$, by looking only to the column related to our agent, so from the top, the columns with the ones to the same row as our agent id.



```
     1     0     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0     1     0
     0     0     1     0     0     0     0     0     0     0
     0     0     0     0     0     0     1     0     0     0
     0     0     0     0     1     0     0     0     0     0
    12 1e+07     1 1e+07    16 1e+07     5 1e+07     8 1e+07
Base =
     1     1     0     0     0     0     0     0     0     0
     0     0     1     1     0     0     0     0     0     0
     0     0     0     0     1     1     0     0     0     0
     0     0     0     0     0     0     1     1     0     0
     0     0     0     0     0     0     0     0     1     1
     1     0     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0     1     0
     0     0     1     0     0     0     0     0     0     0
     0     0     0     0     0     0     1     0     0     0
     0     0     0     0     1     0     0     0     0     0
    12 1e+07     1 1e+07    16 1e+07     5 1e+07     8 1e+07
Base =
     1     1     0     0     0     0     0     0     0     0
     0     0     1     1     0     0     0     0     0     0
     0     0     0     0     1     1     0     0     0     0
     0     0     0     0     0     0     1     1     0     0
     0     0     0     0     0     0     0     0     1     1
     1     0     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0     1     0
     0     0     1     0     0     0     0     0     0     0
     0     0     0     0     0     0     1     0     0     0
     0     0     0     0     1     0     0     0     0     0
    12 1e+07     1 1e+07    16 1e+07     5 1e+07     8 1e+07
Base =
     1     1     0     0     0     0     0     0     0     0
```

(a) Optimal Base



```
Solution: 0
0
0
1
0
0
0
0
1
0
5
Solution: 0
1
0
0
0
0
0
1
0
0
1
Solution:  0
 0
 1
 0
 0
 0
 0
 0
 0
 1
```

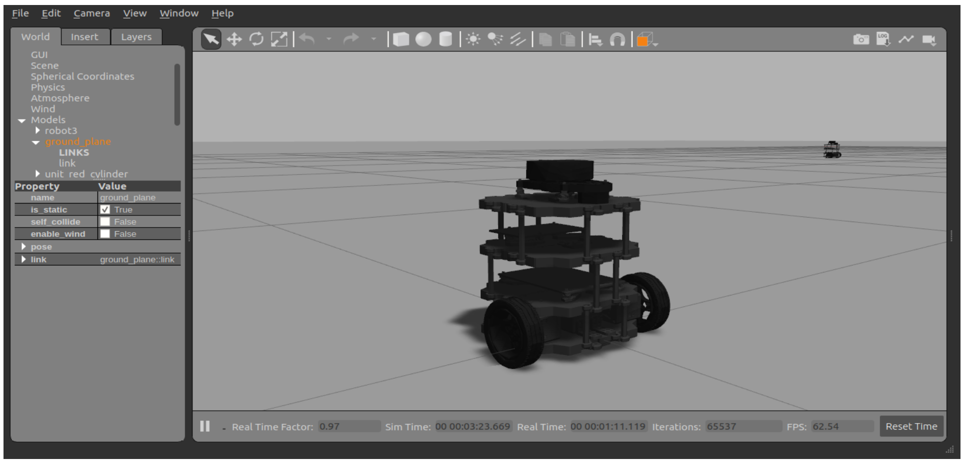
(b) Solution

Figure 1.2: End function

Chapter 2

Simulations

Gazebo is the simulation environment used to test the algorithm with a fleet of wheeled-robots. The simulations are given with a maximum of 4 agents, and we used TurtleBot as wheeled mobile robot. The generation of the constrained vector is given by calcute distance between the initial robot position and target positions. Where targets are positionated at each corner of arena, so at positions (10,10),(-10,10),(10,-10) and (-10,-10). After launching robots inside gazebo, we need to run the simplex in order to establish the communication and to select the target for each of them. The function **goalPosition** uses solution provides from **end** function, and sets the distance to cover and the angle to turn, then it sends these parameters to functions **rotate** and **move**.



Property	Value
name	ground_plane
is_static	✓ True
self_collide	False
enable_wind	False
link	ground_plane::link

Real Time Factor: 0.97 Sim Time: 00 00:03:23.669 Real Time: 00 00:01:11.119 Iterations: 65537 FPS: 62.54

Figure 2.1: Turtlebot

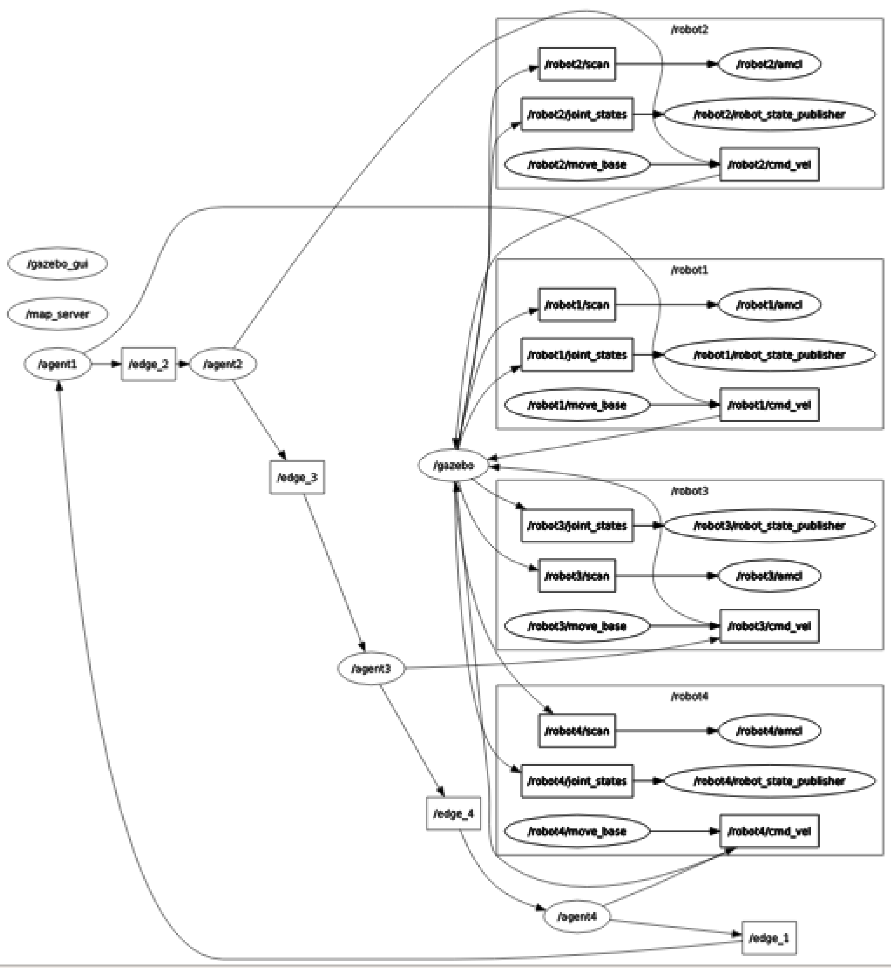


Figure 2.2: Gazebo graph

Bibliography

[1] Mathias Bürger, Giuseppe Notarstefano, Francesco Bullo and Frank Allgöwer. *A distributed simplex algorithm for degenerate linear programs and multi-agent assignments.* Automatica, 2012.