Solved selected problems of Classical Mechanics - Taylor

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Chapter 2 - Projectiles and Charged Particles

Proof. **2.1** The equation for this baseball is

$$\frac{f_{quad}}{f_{lin}} = 1.6 \times 10^3 \cdot 0.07 \cdot v$$

to find the speed at which the two forces are important we need to find a value for v such that $f_{quad}/f_{lin} = 1$ then

$$v = \frac{1}{1.6 \times 10^3 \cdot 0.07} = 0.0089\ m/s$$

In this case the velocity is so small that even though both forces are equally important, they could be neglected.
For velocities bigger than 0.0089 m/s, f_{quad} term start having more impact than the linear term so we could treat the drag force as purely quadratic.

For a beach ball of diameter 70 cm we have that

$$\frac{f_{quad}}{f_{lin}} = 1.6 \times 10^3 \cdot 0.7 \cdot v$$

then the v such that $f_{quad}/f_{lin} = 1$ we have that

$$v = \frac{1}{1.6 \times 10^3 \cdot 0.7} = 0.00089\ m/s$$

□

Proof. **2.2** The equation according Stoke's law is given by

$$f_{lin} = 3\pi\eta D v$$

if we take $\beta = 3\pi\eta$ then $b = \beta D$ we have that

$$f_{lin} = bv$$

which has the form we wanted.
On the other hand, if we take $\eta = 1.7 \times 10^{-5}\ siNs/m^2$ then

$$\beta = 1.6 \times 10^{-4}\ \mathrm{Ns/m^2}$$

which is the given value for β. □

Proof. **2.3**

(a) Given that the drag force equations for a sphere are

$$f_{lin} = 3\pi\eta Dv \qquad f_{quad} = \kappa\varrho Av^2$$

then if $A = \pi(D/2)^2$ we have that

$$\begin{aligned}\frac{f_{quad}}{f_{lin}} &= \frac{\kappa\varrho Av^2}{3\pi\eta Dv} \\ &= \frac{\kappa\varrho\pi(D/2)^2v^2}{3\pi\eta Dv} \\ &= \frac{\kappa\varrho Dv}{12\eta}\end{aligned}$$

knowing that $R = Dv\varrho/\eta$ and $\kappa = 1/4$ for a sphere then

$$\frac{f_{quad}}{f_{lin}} = \frac{R}{4\cdot 12} = \frac{R}{48}$$

(b) Given a steel ball bearing of diameter $D = 2\text{mm} = 0.002\text{m}$ moving at $v = 5\text{cm/s} = 0.05\text{m/s}$ with density $\varrho = 1.3\text{g/cm}^3 = 1300\text{kg/m}^3$ and viscosity $\eta = 12\text{Ns/m}^2$ then replacing the values in the Reynolds number equation we get that

$$R = 0.01083$$

□

Proof. **2.6**

(a) The equation $v_y = v_{ter}(1 - e^{-t/\tau})$ corresponds to the velocity of an object dropped from rest where $v_{ter} = mg/b$ and $\tau = m/b$, if we use the first two terms in the Taylor series for $e^x = 1 + x + x^2/2! + ...$ and we replace that in the equation for the velocity of an object dropped from rest we get that

$$\begin{aligned}v_y &= v_{ter}(1 - e^{-t/\tau}) \\ &= v_{ter}(1 - (1 + (-t/\tau))) \\ &= \frac{mg}{b}(1 - 1 + t/\tau) \\ &= \frac{mg}{b}\left(\frac{tb}{m}\right) \\ &= gt\end{aligned}$$

then we notice that if t is small the approximation with two terms of the series gives us a correct answer for the velocity since we expect that the velocity has a neglectable value for the air resistance at that moment.

(b) In the same way the equation $y(t) = v_{ter}t + (v_{y0} - v_{ter})\tau(1 - e^{-t/\tau})$ represents the position for a dropped object, if we replace the first three terms of the Taylor series for e^x in the equation and we consider that $v_{y0} = 0$ we have that

$$\begin{aligned} y(t) &= v_{ter}t + (0 - v_{ter})\tau(1 - e^{-t/\tau}) \\ &= v_{ter}t - v_{ter}\tau(1 - (1 + (-t/\tau) + \frac{(-t/\tau)^2}{2!})) \\ &= v_{ter}t - v_{ter}\tau(t/\tau - \frac{(-t/\tau)^2}{2}) \\ &= v_{ter}t - v_{ter}t + \frac{1}{2}\frac{v_{ter}}{\tau}t^2 \\ &= \frac{1}{2}\frac{mg}{b}\frac{1}{m/b}t^2 \\ &= \frac{1}{2}gt^2 \end{aligned}$$

then this is the position for an object falling in the vacuum where air resistance is neglected, which is the case for the first moments of the object dropped where the air resistance can be neglected.

□

Proof. **2.9** We are requested to integrate the equation

$$\frac{m\ dv_y}{v_y - v_{ter}} = -b\ dt$$

so we integrate the equation both sides from 0 to t on the right hand side and on the left hand side the integration should be from v_{y0} to v_y, to avoid confusion with the integration limit we renamed the variable on the left hand side of the equation to v'_y.

$$\begin{aligned} \int_{v_{y0}}^{v_y} \frac{m\ dv'_y}{v'_y - v_{ter}} &= \int_0^t -b\ dt \\ m\int_{v_{y0}}^{v_y} \frac{dv'_y}{v'_y - v_{ter}} &= -b\int_0^t dt \\ m[\log(v_y - v_{ter}) - \log(v_{y0} - v_{ter})] &= -b[t - 0] \\ \log(\frac{v_y - v_{ter}}{v_{y0} - v_{ter}}) &= \frac{-b}{m}t \\ \frac{v_y - v_{ter}}{v_{y0} - v_{ter}} &= e^{-t/\tau} \\ v_y &= v_{ter} + (v_{y0} - v_{ter})e^{-t/\tau} \end{aligned}$$

Notice that $\tau = m/b$. Therefore we got the same result as with the inspection method. □

Proof. **2.11**

(a) In this case since the y-axis is looking upwards we have the following equation of motion

$$m\dot{v_y} = -bv_y - mg$$

where both the linear drag force and the gravity point downward. From now on we consider $v_y = v$ so we can also write this equation as

$$m\dot{v} = -b(v + v_{ter})$$

with $v_{ter} = mg/b$, and we know the solution for this kind of equations, so we get that

$$v(t) = (v_0 + v_{ter})e^{-t/\tau} - v_{ter}$$

Now integrating this equation, from 0 to t we get $y(t)$ as

$$\begin{aligned} y(t) &= \int_0^t v(t')\ dt' \\ &= \int_0^t [(v_0 + v_{ter})e^{-t'/\tau} - v_{ter}]\ dt' \\ &= (v_0 + v_{ter})\tau(1 - e^{-t/\tau}) - v_{ter}t \end{aligned}$$

(b) To find the moment at which the object reaches the highest point we need to find where the derivative of the position's equation is zero so, we first derivate $y(t)$ as

$$\frac{dy(t)}{dt} = (v_0 + v_{ter})e^{-t/\tau} - v_{ter}$$

then making the equation equal to 0 we find the time t at which the position of the object reaches its highest point

$$\begin{aligned} (v_0 + v_{ter})e^{-t/\tau} - v_{ter} &= 0 \\ e^{-t/\tau} &= \frac{v_{ter}}{v_0 + v_{ter}} \\ -t/\tau &= \log\left(\frac{v_{ter}}{v_0 + v_{ter}}\right) \\ t &= -\tau[\log(v_{ter}) - \log(v_0 + v_{ter})] \\ t &= \tau[\log(v_0 + v_{ter}) - \log(v_{ter})] \\ t &= \tau \log\left(\frac{v_0}{v_{ter}} + 1\right) \end{aligned}$$

now replacing this value in the equation for $y(t)$ we get that

$$\begin{aligned}
y_{max} &= (v_0 + v_{ter})\tau(1 - e^{-\log(1+\frac{v_0}{v_{ter}})}) - v_{ter}\tau \log\left(1 + \frac{v_0}{v_{ter}}\right) \\
&= (v_0 + v_{ter})\tau(1 - \frac{v_{ter}}{v_0 + v_{ter}}) - v_{ter}\tau \log\left(1 + \frac{v_0}{v_{ter}}\right) \\
&= (v_0 + v_{ter})\tau - v_{ter}\tau - v_{ter}\tau \log\left(1 + \frac{v_0}{v_{ter}}\right) \\
&= v_0\tau - v_{ter}\tau \log\left(1 + \frac{v_0}{v_{ter}}\right) \\
&= \tau\left[v_0 - v_{ter}\log\left(1 + \frac{v_0}{v_{ter}}\right)\right]
\end{aligned}$$

(c) Given that if the drag force is very small, the terminal velocity is very big and therefore v_0/v_{ter} is very small, then we can approximate $\log(1 + v_0/v_{ter})$ by two terms of its Taylor series which give us

$$\begin{aligned}
y_{max} &= \tau\left[v_0 - v_{ter}\left(\frac{v_0}{v_{ter}} - \frac{1}{2}\left(\frac{v_0}{v_{ter}}\right)^2\right)\right] \\
&= \tau\left[v_0 - v_0 + \frac{1}{2}\frac{v_0^2}{v_{ter}}\right] \\
&= \tau\frac{1}{2}\frac{v_0^2}{v_{ter}} \\
&= \frac{1}{2}\frac{m}{b}\frac{v_0^2}{\frac{mg}{b}} \\
&= \frac{1}{2}\frac{v_0^2}{g}
\end{aligned}$$

therefore y_{max} is reduced to it's vacuum value.

$\square$

Proof. **2.17** From the equation for $x(t)$ we want to get t as a function of x then

$$\begin{aligned}
x &= v_{x0}\tau(1 - e^{-t/\tau}) \\
\frac{x}{v_{x0}\tau} &= 1 - e^{-t/\tau} \\
1 - \frac{x}{v_{x0}\tau} &= e^{-t/\tau} \\
\log(1 - \frac{x}{v_{x0}\tau}) &= -t/\tau \\
-\tau\log(1 - \frac{x}{v_{x0}\tau}) &= t
\end{aligned}$$

now we replace the equation for t in the equation of $y(t)$ to get y as a function of x

$$\begin{aligned}
y(t) &= (v_{y0} + v_{ter})\tau(1 - e^{-t/\tau}) - v_{ter}t \\
&= (v_{y0} + v_{ter})\tau(1 - e^{\log(1-\frac{x}{v_{x0}\tau})}) + v_{ter}\tau\log(1 - \frac{x}{v_{x0}\tau}) \\
&= (v_{y0} + v_{ter})\tau(1 - (1 - \frac{x}{v_{x0}\tau})) + v_{ter}\tau\log(1 - \frac{x}{v_{x0}\tau}) \\
&= (v_{y0} + v_{ter})\tau\frac{x}{v_{x0}\tau} + v_{ter}\tau\log(1 - \frac{x}{v_{x0}\tau}) \\
&= \frac{v_{y0} + v_{ter}}{v_{x0}}x + v_{ter}\tau\log(1 - \frac{x}{v_{x0}\tau})
\end{aligned}$$

which is the equation we wanted. ☐

Proof. **2.19**

(a) We know that the equations for a motion where no air resistance is considered are

$$x = v_{x0}t + x_0 \tag{1}$$

$$y = y_0 + v_{y0}t - \frac{1}{2}gt^2 \tag{2}$$

considering $x_0 = y_0 = 0$ and from (1) we write t in terms of x we get that

$$t = \frac{x}{v_{x0}}$$

then we replace this value of t in (2) to get

$$y = \frac{v_{y0}}{v_{x0}}x - \frac{1}{2}g\left(\frac{x}{v_{x0}}\right)^2$$

(b) Given that τ and v_{ter} approach infinity we can approximate $\log(1 - x/v_{x0}\tau)$ by its Taylor series as

$$\log(1 - \frac{x}{v_{x0}\tau}) = -\frac{x}{v_{x0}\tau} - \frac{1}{2}\frac{x^2}{v_{x0}^2\tau^2}$$

then the equation for y in terms of x is given by

$$
\begin{aligned}
y &= \frac{v_{y0} + v_{ter}}{v_{x0}} x + v_{ter}\tau \left[-\frac{x}{v_{x0}\tau} - \frac{1}{2}\frac{x^2}{v_{x0}^2\tau^2} \right] \\
&= \frac{v_{y0}}{v_{x0}} x + \frac{v_{ter}}{v_{x0}} x - \frac{v_{ter}}{v_{x0}} x - \frac{1}{2}\frac{v_{ter} x^2}{v_{x0}^2 \tau} \\
&= \frac{v_{y0}}{v_{x0}} x - \frac{1}{2} g \left(\frac{x}{v_{x0}} \right)^2
\end{aligned}
$$

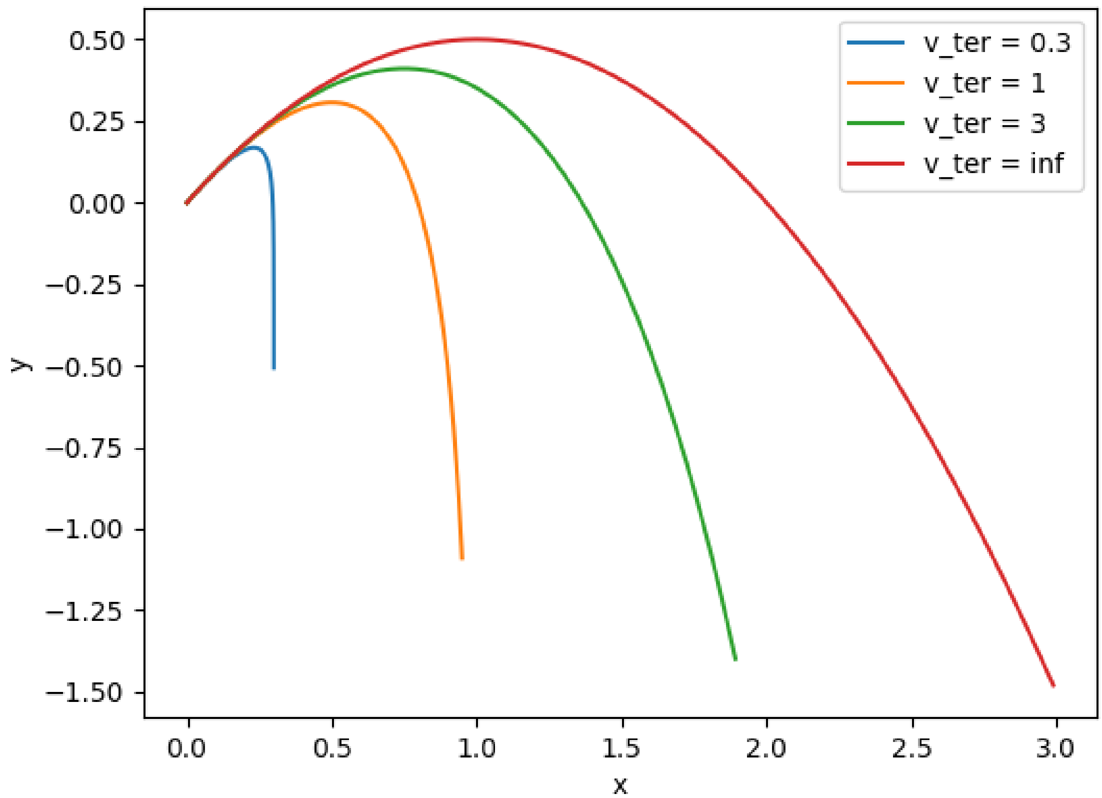
where we used that $v_{ter}/\tau = g$. Finally we get the same equation for a motion where no air resistance is considered.

$\square$

Proof. **2.20** Below we show the trajectories for t between 0 and 3 with different $v_{ter} = \tau$ which means different values for drag. We are assuming $v_{x0} = v_{y0} = 1$ and $g = 1$. Finally the trajectory shown for $v_{ter} = \infty$ is described by

$$x = v_{x0}t + x_0$$
$$y = y_0 + v_{y0}t - \frac{1}{2}gt^2$$

that was calculated in last problem.



□

Proof. **2.22**

(a) We know that for the vacuum the range equation is

$$R_{vac} = \frac{2v_{x0}v_{y0}}{g}$$

if we assume $g = 1$ and $v_0 = 1$ then $v_{x0} = \cos(\theta)$ and $v_{y0} = \sin(\theta)$ also we know that the maximum range occurs at $\theta = \pi/4$ then

$$R_{vac} = \frac{2\cos(\pi/4)\sin(\pi/4)}{1} = 1$$

(b) If we solve the equation for R in a linear medium by binary search with the code shown below

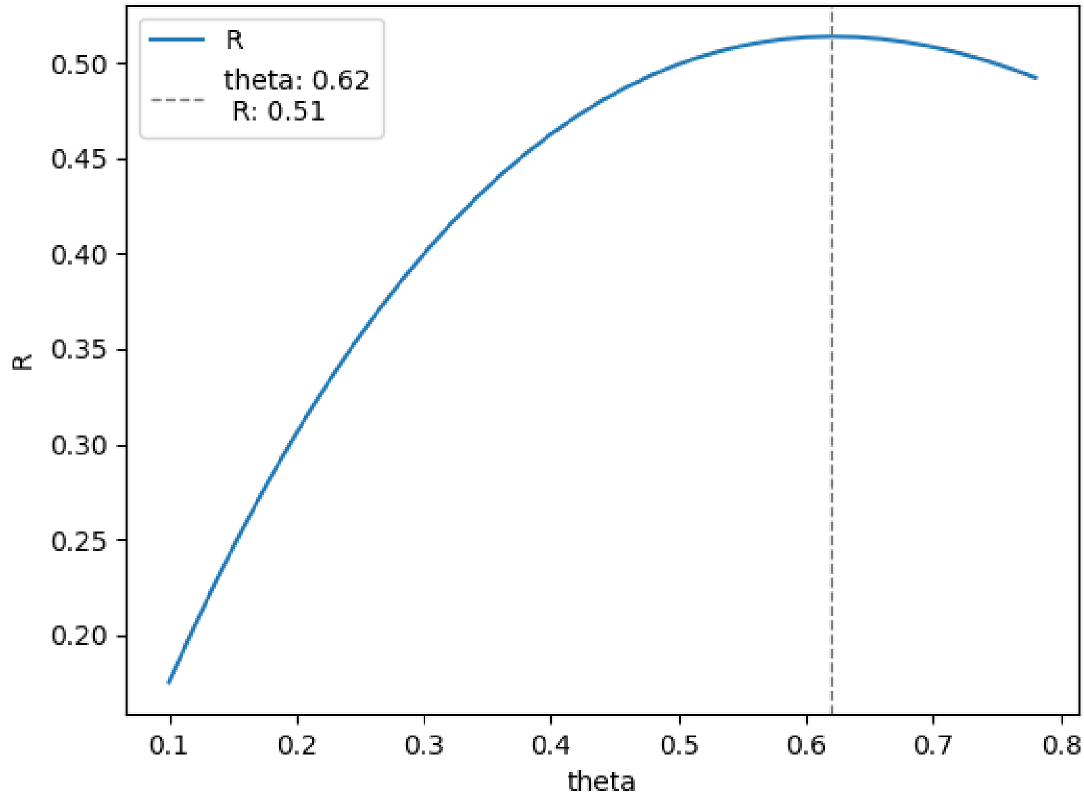
```
def binary_search(theta=0.75):
    R_max = 0.4
    R_min = 0.6
    epsilon = 0.000001
    f_mid = 1000

    while abs(f_mid) > epsilon:
        R_mid = R_min + ((R_max - R_min)/2)
        f_mid = func(R_mid, theta)
        if f_mid > 0:
            R_max = R_mid
        elif f_mid < 0:
            R_min = R_mid

    return f_mid, R_mid
```

we get that for $R = 0.4995$ the function value is less than the epsilon we set. Therefore the Range in a linear medium when $\theta = 0.75$ is $R = 0.4995$.

(c)(d) By doing binary search for different values of θ in a range between 0.1 and 0.8, we obtained the following plot



It is also plotted the θ angle for which R is maximum, which happens to be at 0.62 and we obtained an R maximum of 0.51.

□

Proof. **2.24**

(a) We defined $v_{ter} = \sqrt{mg/c}$ where $m = 4/3\pi(D/2)^3\varrho_{sph}$ which is the multiplication of the sphere volume and the density and $c = \kappa\varrho_{air}\pi(D/2)^2$ where $\kappa = 1/4$ with all that we get that

$$\begin{aligned} v_{ter} &= \sqrt{\frac{4/3\pi(D/2)^3\varrho_{sph}g}{\kappa\varrho_{air}\pi(D/2)^2}} \\ &= \sqrt{\frac{8D\varrho_{sph}g}{3\varrho_{air}}} \end{aligned}$$

(b) Given that v_{ter} is proportional to $\sqrt{\varrho_{sph}}$ then if the sphere 1 is denser than sphere 2 we have that $\varrho_{sph1} > \varrho_{sph2}$ so $\sqrt{\varrho_{sph1}} > \sqrt{\varrho_{sph2}}$ and therefore v_{ter} for sphere 1 is going to be bigger than for sphere 2.

(c) Also v_{ter} is proportional to $\sqrt{D}$ then if the sphere 1 is larger than sphere 2 we have that $D_1 > D_2$ so $\sqrt{D_1} > \sqrt{D_2}$ and therefore v_{ter} for sphere 1 is going to be bigger than for sphere 2.

□

Proof. **2.27** The Newton's second law equation in this case assuming the positive x-axis is in the direction of the incline is given by

$$m\frac{dv}{dt} = -mg\sin(\theta) - cv^2$$

if we define $v_{ter} = \sqrt{mg\sin\theta/c}$ we have that

$$\begin{aligned}
\frac{dv}{dt} &= g\sin(\theta)\left(-1 - \frac{v^2}{v_{ter}^2}\right) \\
\int_{v_0}^{v} \frac{dv}{\left(-1 - \frac{v^2}{v_{ter}^2}\right)} &= \int_0^t g\sin(\theta)dt \\
v_{ter}\left(-\arctan(\frac{v}{v_{ter}}) + \arctan(\frac{v_0}{v_{ter}})\right) &= g\sin(\theta)t \\
-\arctan(\frac{v}{v_{ter}}) &= \frac{g\sin(\theta)t}{v_{ter}} - \arctan(\frac{v_0}{v_{ter}}) \\
v &= \tan(\arctan(\frac{v_0}{v_{ter}}) - \frac{g\sin(\theta)t}{v_{ter}})
\end{aligned}$$

To find the time it takes the upward journey we need to solve $v(t) = 0$ since the velocity should be 0 when the puck is at the top.

$$\begin{aligned}
0 &= \tan(\arctan(\frac{v_0}{v_{ter}}) - \frac{g\sin(\theta)t_{top}}{v_{ter}}) \\
\frac{g\sin(\theta)t_{top}}{v_{ter}} &= \arctan(\frac{v_0}{v_{ter}}) \\
t_{top} &= \frac{v_{ter}}{g\sin(\theta)}\arctan(\frac{v_0}{v_{ter}})
\end{aligned}$$

□

Proof. **2.32**

(a) If we think of the boundary case where there is no air resistance then the trajectory is parabolic, then the velocity must be it's derivative therefore a linear equation which never stops growing therefore never finding its v_{ter}.

(b) If we write f/mg for linear and quadratic drag cases we get that

$$\frac{f_{lin}}{mg} = \frac{v}{v_{ter}} \qquad \frac{f_{quad}}{mg} = \frac{v^2}{v_{ter}^2}$$

given that the velocity is much less than v_{ter} when small air resistance is present, we see from the equations that for the quadratic equation the effect is bigger since if $v/v_{ter} < 1$ then $v^2/v_{ter}^2 \ll 1$.

$\square$

Proof. **2.35**

(a) From the equation of motion $m\dot{v} = mg - cv^2$ and defining $v_{ter} = \sqrt{\frac{mg}{c}}$ we get that

$$\begin{aligned}
m\dot{v} &= mg - cv^2 \\
\dot{v} &= g\left(1 - \frac{v^2}{v_{ter}^2}\right) \\
\int_0^v \frac{dv'}{\left(1 - \frac{v'^2}{v_{ter}^2}\right)} &= \int_0^t g dt' \quad \text{(variable replacement)} \\
v_{ter} \tanh^{-1}(\frac{v}{v_{ter}}) &= gt \\
\frac{v}{v_{ter}} &= \tanh(\frac{gt}{v_{ter}}) \\
v &= v_{ter} \tanh(\frac{gt}{v_{ter}})
\end{aligned}$$

to find the position we need to solve the following

$$\begin{aligned}
\frac{dy}{dt} &= v_{ter} \tanh(\frac{gt}{v_{ter}}) \\
\int_0^y dy' &= \int_0^t v_{ter} \tanh(\frac{gt'}{v_{ter}}) dt' \quad \text{(variable replacement)} \\
y &= \frac{v_{ter}^2}{g} \log(\cosh(\frac{gt}{v_{ter}}))
\end{aligned}$$

(b) Given that $\tau = v_{ter}/g$ we can write the above equations as

$$\begin{aligned}
v &= v_{ter} \tanh(\frac{t}{\tau}) \quad \text{and} \\
y &= \tau v_{ter} \log(\cosh(\frac{t}{\tau}))
\end{aligned}$$

if $t = \tau$ then

$$v = v_{ter} \tanh(1) = v_{ter} 0.7615$$

which means that the velocity reaches 76% of its terminal velocity when $t = \tau$.
For $t = 2\tau$ we have that $v = v_{ter}0.964$, which is the 96% of v_{ter} and for $t = 3\tau$ we have that $v = v_{ter}0.995$ which is the 99% of v_{ter}.

(c) When $t \gg \tau$ we can approximate cosh from its definition as $\cosh(z) \approx e^z/2$ then

$$\begin{aligned}
y &= \tau v_{ter} \log(\cosh(\frac{t}{\tau})) \\
&\approx \tau v_{ter} \log(\frac{e^{\frac{t}{\tau}}}{2}) \\
&\approx \tau v_{ter} \left(\frac{t}{\tau} - \log(2)\right) \\
&\approx v_{ter} t - v_{ter} \tau \log(2)
\end{aligned}$$

(d) When t is small we can approximate the logarithm and the hyperbolic cosine functions by their Taylor series as $\log(1+t) = t - t^2/2$ and $\cosh(x) = 1 + x^2/2!$ then

$$\begin{aligned}
y &= \tau v_{ter} \log(\cosh(\frac{t}{\tau})) \\
&\approx \tau v_{ter} \log(1 + \frac{(t/\tau)^2}{2}) \\
&\approx \tau v_{ter} \left(\frac{(t/\tau)^2}{2} - \frac{((t/\tau)^2/2)^2}{2}\right) \\
&\approx \tau v_{ter} \left(\frac{(t/\tau)^2}{2} - \frac{(t/\tau)^4}{8}\right) \\
&\approx \frac{v_{ter}}{\tau} \frac{t^2}{2} \\
&\approx g \frac{t^2}{2}
\end{aligned}$$

given that t is already small we removed the term $(t/\tau)^4/8$ from the equation since its negligible.

$\square$

Proof. **2.38**

(a) The Newton's second law equation in this case assuming the positive y-axis is in an upward direction is given by

$$m\frac{dv}{dt} = -mg - cv^2$$

if we define $v_{ter} = \sqrt{mg/c}$ we have that

$$\begin{aligned}
\frac{dv}{dt} &= g\left(-1 - \frac{v^2}{v_{ter}^2}\right)\\
\int_{v_0}^{v} \frac{dv}{\left(-1 - \frac{v^2}{v_{ter}^2}\right)} &= \int_0^t gdt\\
v_{ter}\left(-\arctan(\frac{v}{v_{ter}}) + \arctan(\frac{v_0}{v_{ter}})\right) &= gt\\
-\arctan(\frac{v}{v_{ter}}) &= \frac{gt}{v_{ter}} - \arctan(\frac{v_0}{v_{ter}})\\
v &= \tan\left(\arctan(\frac{v_0}{v_{ter}}) - \frac{gt}{v_{ter}}\right)
\end{aligned}$$

(b) To find the time it takes the upward journey we need to solve $v(t) = 0$ since the velocity should be 0 when the projectile is at the top, then

$$\begin{aligned}
0 &= \tan\left(\arctan(\frac{v_0}{v_{ter}}) - \frac{gt_{top}}{v_{ter}}\right)\\
\frac{gt_{top}}{v_{ter}} &= \arctan(\frac{v_0}{v_{ter}})\\
t_{top} &= \frac{v_{ter}}{g}\arctan(\frac{v_0}{v_{ter}})
\end{aligned}$$

(c) For a baseball with $v_{ter} = 35$m/s we have that
$v_0 = 1$m/s then $t_{top} = 0.102$s and in vacuum $t_{top} = 0.102$s
$v_0 = 10$m/s then $t_{top} = 0.993$s and in vacuum $t_{top} = 1.020$s
$v_0 = 20$m/s then $t_{top} = 1.854$s and in vacuum $t_{top} = 2.040$s
$v_0 = 30$m/s then $t_{top} = 2.53$s and in vacuum $t_{top} = 3.061$s
$v_0 = 40$m/s then $t_{top} = 3.042$s and in vacuum $t_{top} = 4.081$s

□

Proof. **2.41** The Newton's second law equation in this case assuming the positive y-axis is in an upward direction is given by

$$m\dot{v} = -mg - cv^2$$

if we define $v_{ter} = \sqrt{mg/c}$ and we take mg as factor we have that

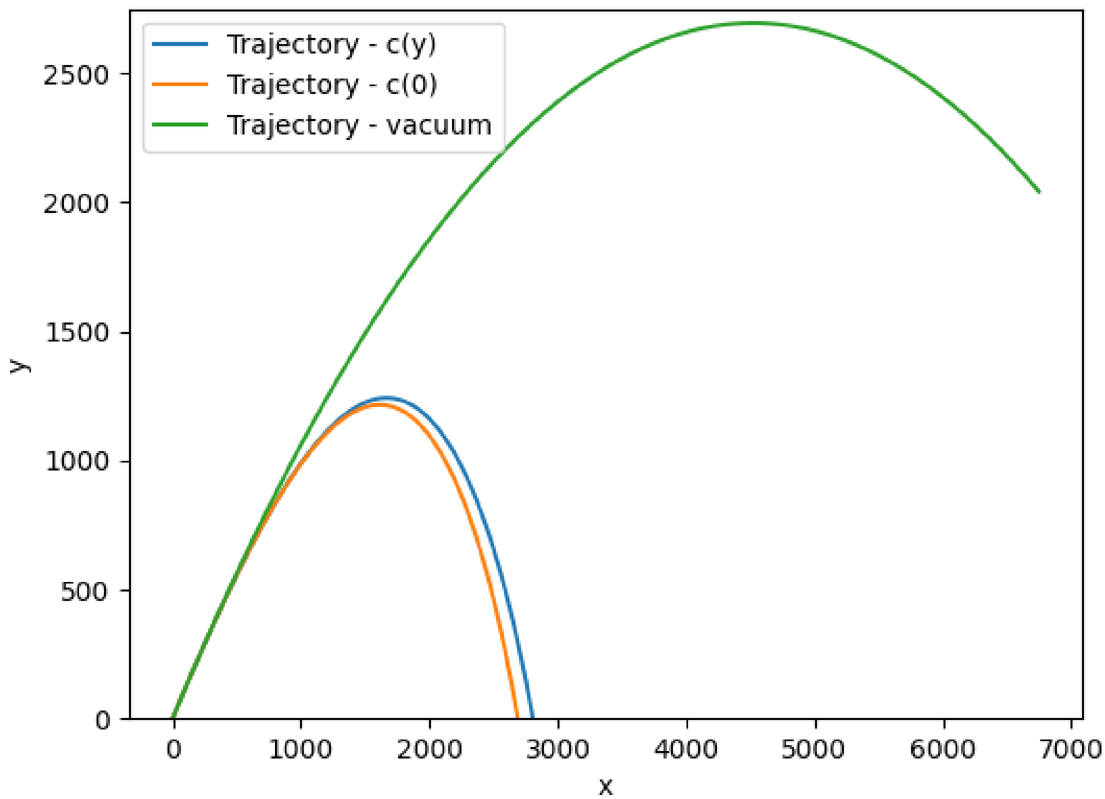
$$\begin{aligned}
\dot{v} &= g\left(-1 - \frac{v^2}{v_{ter}^2}\right) \\
\dot{v} &= -g\left(1 + \frac{v^2}{v_{ter}^2}\right) \\
v\frac{dv}{dy} &= -g\left(1 + \frac{v^2}{v_{ter}^2}\right) \\
\int_{v_0}^{v} \frac{vdv}{\left(1 + \frac{v^2}{v_{ter}^2}\right)} &= -\int_0^y gdy \\
\frac{v_{ter}^2}{2}\left(\log(v_{ter}^2 + v^2) - \log(v_{ter}^2 + v_0^2)\right) &= -gy \\
\frac{v_{ter}^2}{2g}\log\left(\frac{v_{ter}^2 + v_0^2}{v_{ter}^2 + v^2}\right) &= y
\end{aligned}$$

Since the maximum height occurs when the $v = 0$ then

$$y_{max} = \frac{v_{ter}^2}{2g}\log\left(\frac{v_{ter}^2 + v_0^2}{v_{ter}^2}\right)$$

If $v_0 = 20\text{m/s}$ and $v_{ter} = 35\text{m/s}$ then $y_{max} = 17.66\text{m}$ and in the vacuum $y_{max} = 20.41\text{m}$ $\square$

Proof. **2.44**



□

Proof. **2.52** We know that

$$\eta = v_x + iv_y$$

but also

$$\eta = Ae^{-i\omega t} = ae^{i(\delta - \omega t)}$$

then we can write η as

$$\eta = a\cos(\delta - \omega t) + ai\sin(\delta - \omega t)$$

so making both expressions equal we get the values for each component

$$v_x = a\cos(\delta - \omega t) \qquad v_y = a\sin(\delta - \omega t)$$

□