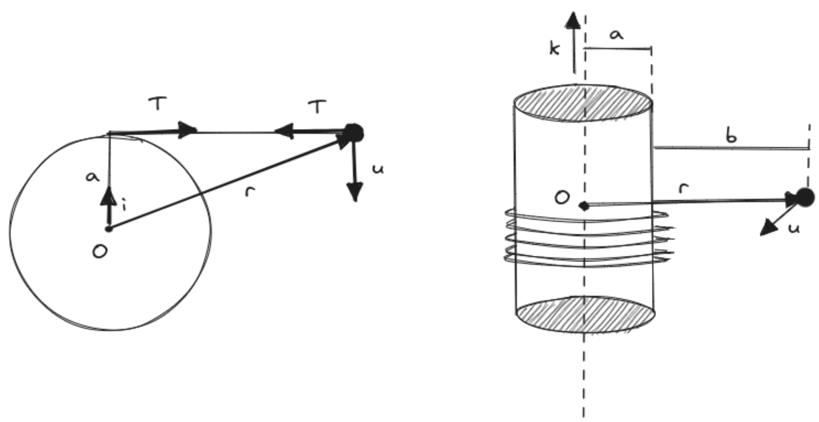
Solved selected problems of Classical Mechanics - Gregory

Franco Zacco

Chapter 11 - The angular momentum principle

Solution. 11.3 The system described looks like the following



From the diagram, the total moment of the external forces about O is

$$\boldsymbol{K_O} = [(a\boldsymbol{i}) \times \boldsymbol{T}] - [(a\boldsymbol{i}) \times \boldsymbol{T}] + [\boldsymbol{0} \times -Mg\boldsymbol{k}] + [\boldsymbol{r} \times -mg\boldsymbol{k}]$$

So we see that $\boldsymbol{K_O} \cdot \boldsymbol{k} = 0$ since the tensions on the string are opposite and the gravitational component of the particle gives us $-mg\boldsymbol{r} \cdot (\boldsymbol{k} \times \boldsymbol{k}) = 0$. Hence $\boldsymbol{L_O} \cdot \boldsymbol{k}$ the angular momentum of the system about the rotation axis is conserved.

It follows that this axial angular momentum is the same at the beginning and the end of the movement. Initially, the cylinder is at rest and the particle has a velocity $\boldsymbol{u}$ perpendicular to the string with length b describing for the first instant a circular orbit with radius b then

$$\boldsymbol{L_O} \cdot \boldsymbol{k} = mbu$$

And when the particle finally sticks to the cylinder we get that

$$\boldsymbol{L_O} \cdot \boldsymbol{k} = \left(\frac{1}{2}Ma^2\right)\Omega + ma^2\Omega$$

Since $\boldsymbol{L_O} \cdot \boldsymbol{k}$ is known to be conserved it follows that

$$\left(\frac{1}{2}Ma^2\right)\Omega + ma^2\Omega = mbu$$
$$\Omega\left(\frac{a^2}{2}(M+2m)\right) = mbu$$
$$\Omega = \frac{2mbu}{a^2(M+2m)}$$

□

Solution. 11.4 Let $\boldsymbol{k}$ be a unit vector in the direction of the rotational axis through the center of the gas sphere. Given that the cloud can move freely in space and there are no forces applied to it we have that

$$\boldsymbol{K_0} \cdot \boldsymbol{k} = 0$$

Hence $\boldsymbol{L_O} \cdot \boldsymbol{k}$ the angular momentum of the system about the rotation axis is conserved.

It follows that this axial angular momentum is the same at the beginning and the end of the movement. Initially, the cloud has the form of a uniform sphere then

$$\boldsymbol{L_O} \cdot \boldsymbol{k} = \left(\frac{2}{5}Ma^2\right)\Omega$$

Then the cloud has the form of a thin uniform circular disk hence

$$\boldsymbol{L_O} \cdot \boldsymbol{k} = \left(\frac{1}{2}Mb^2\right)\Omega'$$

Since $\boldsymbol{L_O} \cdot \boldsymbol{k}$ is known to be conserved it follows that

$$\left(\frac{1}{2}Mb^2\right)\Omega' = \left(\frac{2}{5}Ma^2\right)\Omega$$
$$\Omega' = \frac{4}{5}\frac{a^2}{b^2}\Omega$$

Finally, the initial and final kinetic energies of the system are

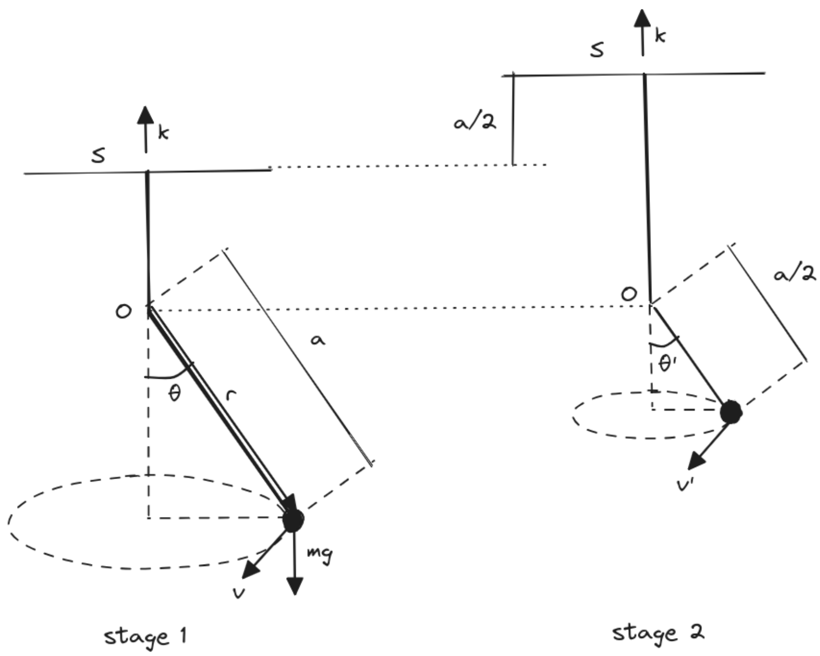
$$\frac{1}{2}\left(\frac{2}{5}Ma^2\right)\Omega^2 \quad \text{and} \quad \frac{1}{2}\left(\frac{1}{2}Mb^2\right)\Omega'^2$$

On using the value of Ω' found above and simplifying, the kinetic energy of the system is found to increase by

$$\begin{aligned}\Delta T &= \frac{1}{2}\left(\frac{1}{2}Mb^2\right)\left(\frac{4}{5}\frac{a^2}{b^2}\Omega\right)^2 - \frac{1}{2}\left(\frac{2}{5}Ma^2\right)\Omega^2 \\ &= \frac{4Ma^4}{25b^2}\Omega^2 - \frac{1}{5}Ma^2\Omega^2 \\ &= \frac{Ma^2}{25b^2}(4a^2 - 5b^2)\Omega^2\end{aligned}$$

□

Solution. 11.5 The system described in the two stages looks like the following



Let's first analyze Newton's equation of the conical pendulum shown

$$T\cos\theta - mg = 0$$
$$T\sin\theta = \frac{mv^2}{a\sin\theta}$$

Where we take into account the components of the string tension and the centripetal acceleration. Hence the tangential velocity v of the particle is given by

$$\frac{g\sin\theta}{\cos\theta} = \frac{v^2}{a\sin\theta}$$
$$v = \sqrt{\frac{ga}{\cos\theta}}\sin\theta$$

Let's analyze the moment with respect to O. From the diagram, the total moment of the external forces about O is

$$\boldsymbol{K_O} = (\mathbf{0}\times\boldsymbol{T}) + (\boldsymbol{r}\times\boldsymbol{T}) + (\boldsymbol{r}\times -Mg\boldsymbol{k})$$

So we see that $\boldsymbol{K_O}\cdot\boldsymbol{k} = 0$ since $\mathbf{0}\times\boldsymbol{T} = \mathbf{0}$, also $\boldsymbol{r}\times\boldsymbol{T} = \mathbf{0}$ since they are parallel vectors. The gravitational component of the particle gives us $-mg\boldsymbol{r}\cdot(\boldsymbol{k}\times\boldsymbol{k}) = 0$. Hence $\boldsymbol{L_O}\cdot\boldsymbol{k}$ the angular momentum of the system about the rotation axis is conserved.

It follows that this axial angular momentum is the same at the beginning and at the end of the movement. Initially, the particle is rotating on a circular path of radius $a\sin\theta$ with an tangential velocity we derived above then

$$\begin{aligned}\boldsymbol{L_0}\cdot\boldsymbol{k} &= m\cdot\sqrt{\frac{ga}{\cos\theta}}\sin\theta\cdot a\sin\theta \\ &= ma\sqrt{\frac{ga}{\cos\theta}}\sin^2\theta\end{aligned}$$

In stage 2, where the string has reduced to a length of $a/2$ we have that

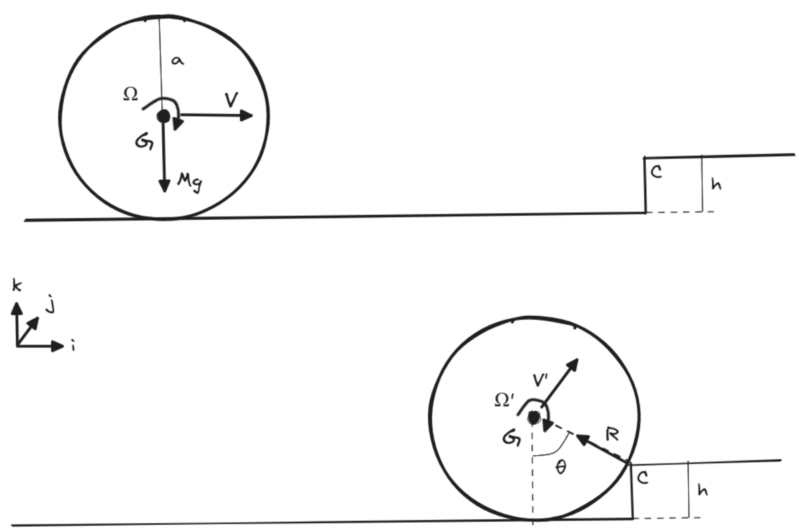
$$\begin{aligned}\boldsymbol{L_0}\cdot\boldsymbol{k} &= m\cdot\sqrt{\frac{ga}{2\cos\theta'}}\sin\theta'\cdot\frac{a\sin\theta'}{2} \\ &= \frac{ma}{2}\sqrt{\frac{ga}{2\cos\theta'}}\sin^2\theta'\end{aligned}$$

Since $\boldsymbol{L_O}\cdot\boldsymbol{k}$ is known to be conserved it follows that

$$\begin{aligned}\frac{ma}{2}\sqrt{\frac{ga}{2\cos\theta'}}\sin^2\theta' &= ma\sqrt{\frac{ga}{\cos\theta}}\sin^2\theta \\ \frac{ga}{8\cos\theta'}\sin^4\theta' &= \frac{ga}{\cos\theta}\sin^4\theta \\ \frac{\sin^4\theta'}{\cos\theta'} &= 9\end{aligned}$$

Where we used that initial angle is $\theta = 60°$. Finally, by solving the equation numerically we find that $\theta' = 83.77°$. □

Solution. 11.7 Let's consider the system described below



From the diagram, initially, the momentum around C is given by

$$\begin{aligned}\boldsymbol{L_C} &= M(a-h)V\boldsymbol{j} + Ma^2\Omega\boldsymbol{j} \\ &= M(2a-h)V\boldsymbol{j}\end{aligned}$$

where $I = Ma^2$ is the moment of inertia of the hoop with respect to G and since we assume the movement is a rotation without slipping the tangential velocity is also $V = a\Omega$.

Let's analyze now the moment right after the hoop encounters the step, then we have that

$$\begin{aligned}\boldsymbol{L_C} &= MaV'\boldsymbol{j} + Ma^2\Omega'\boldsymbol{j} \\ &= 2MaV'\boldsymbol{j}\end{aligned}$$

Since the momentum is known to be conserved we get that the instantaneous speed of the center of mass is

$$\begin{aligned}2MaV' &= M(2a-h)V \\ V' &= \left(1 - \frac{h}{2a}\right)V\end{aligned}$$

and the instantaneous angular velocity of the hoop is

$$\Omega' = \left(1 - \frac{h}{2a}\right)\frac{V}{a}$$

Now that the hoop is at the step, there are two options either the hoop goes up the step where the center of mass performs an arc trajectory and the particle C remains in contact with the step (at rest) or the particle C does not remain in contact with the step. Let's now decompose the forces in the GC direction assuming the particle C remains in contact with the step

$$\frac{MV'^2}{a} = Mg\cos\theta - R$$
$$R = Mg\frac{a-h}{a} - \frac{M}{a}\left(1-\frac{h}{2a}\right)^2 V^2$$

where we used that $\cos\theta = (a-h)/a$ and we replaced the value we got for V'. If $Mg\cos\theta < R$ then the particle C will not stay in contact with the step and this will happen if

$$\frac{M}{a}\left(1-\frac{h}{2a}\right)^2 V^2 > Mg\frac{(a-h)}{a}$$
$$V^2 > g(a-h)\left(1-\frac{h}{2a}\right)^{-2}$$

Suppose now that the particle C does remain on the edge of the step. Then for the hoop to mount the step is necessary some energy to raise it there, by applying the conservation of energy we have that

$$\frac{1}{2}MV'^2 + \frac{1}{2}(Ma^2)\left(\frac{V'}{a}\right)^2 + Mga > Mg(a+h)$$
$$V'^2 + ga > g(a+h)$$
$$V^2 > gh\left(1-\frac{h}{2a}\right)^{-2}$$

If we want the hoop to stay still in contact with the step and to mount it we need both the following conditions at the same time

$$V^2 < g(a-h)\left(1-\frac{h}{2a}\right)^{-2}$$

and that

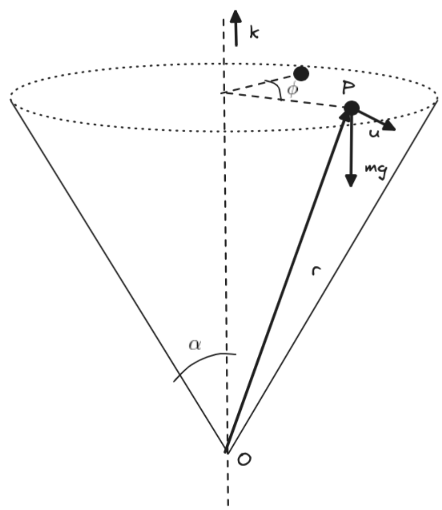
$$V^2 > gh\left(1-\frac{h}{2a}\right)^{-2}$$

This implies that

$$gh\left(1-\frac{h}{2a}\right)^{-2} < g(a-h)\left(1-\frac{h}{2a}\right)^{-2}$$
$$h < a/2$$

Therefore if the step is higher than $a/2$ the hoop cannot mount the step and stay in touch with it. $\square$

Solution. 11.8 Let's consider the system described below



Let's analyze the moment with respect to the vertical axis through O. From the diagram, the total moment of the external forces about O is

$$\boldsymbol{K_O} = \boldsymbol{r} \times -mg\boldsymbol{k} + \boldsymbol{r} \times \boldsymbol{N}$$

So we see that $\boldsymbol{K_O} \cdot \boldsymbol{k} = 0$ since the gravitational component of the particle gives us $-mg\boldsymbol{r} \cdot (\boldsymbol{k} \times \boldsymbol{k}) = 0$ and $(\boldsymbol{r} \times \boldsymbol{N}) \cdot \boldsymbol{k} = 0$ because the vectors are all in the same plane. Hence $\boldsymbol{L_O} \cdot \boldsymbol{k}$ the angular momentum of the system about the vertical axis is conserved.

Initially, the particle is a distance a from O and is horizontally projected with a velocity u hence

$$\boldsymbol{L_0} \cdot \boldsymbol{k} = m(a \sin \alpha)u$$

Later the momentum in the vertical direction has a tangential velocity of $(r \sin \alpha)\dot{\phi}$ where $\dot{\phi}$ is the angular velocity in the azimuthal direction then

$$\boldsymbol{L_0} \cdot \boldsymbol{k} = m(r \sin \alpha)(r \sin \alpha)\dot{\phi}$$

And since the momentum in the vertical direction is known to be conserved we have that

$$\begin{aligned} m(r \sin \alpha)^2 \dot{\phi} &= m(a \sin \alpha)u \\ r^2 \dot{\phi} \sin \alpha &= au \\ \dot{\phi} &= \frac{au}{r^2 \sin \alpha} \end{aligned}$$

Also, given that the cone is smooth (hence the constraint force does not work) and the gravitational force acting on the particle is a conservative force then the energy is conserved. Initially, the particle has an energy given by

$$E = \frac{1}{2}mu^2 + mg(a\cos\alpha)$$

Later the particle would have an energy of

$$E = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}m(r\sin\alpha\dot{\phi})^2 + mg(r\cos\alpha)$$

And since we know that energy is conserved we have that

$$\begin{aligned}
\frac{1}{2}m\dot{r}^2 + \frac{1}{2}m(r\sin\alpha\dot{\phi})^2 + mg(r\cos\alpha) &= \frac{1}{2}mu^2 + mg(a\cos\alpha) \\
\dot{r}^2 + (r\sin\alpha\dot{\phi})^2 + 2g(r\cos\alpha) &= u^2 + 2g(a\cos\alpha) \\
\dot{r}^2 &= u^2 - (r\sin\alpha\dot{\phi})^2 + 2g\cos\alpha(a-r)
\end{aligned}$$

Now replacing the value we have for $\dot{\phi}$ we get the value for $\dot{r}^2$ that we want

$$\begin{aligned}
\dot{r}^2 &= u^2 - \frac{a^2u^2}{r^2} + 2g\cos\alpha(a-r) \\
\dot{r}^2 &= \frac{u^2}{r^2}(r^2 - a^2) + 2g\cos\alpha(a-r) \\
\dot{r}^2 &= (r-a)\left[\frac{u^2}{r^2}(r+a) - 2g\cos\alpha\right]
\end{aligned}$$

Case A. In the absence of gravity ($g = 0$), we get that r is given by

$$\begin{aligned}
\dot{r}^2 &= \frac{u^2}{r^2}(r^2 - a^2) \\
\frac{dr}{dt} &= \frac{u}{r}\sqrt{r^2 - a^2} \\
\int \frac{r}{\sqrt{r^2 - a^2}}dr &= u\int dt \\
\sqrt{r^2 - a^2} &= ut \\
r &= \sqrt{(ut)^2 + a^2}
\end{aligned}$$

And from the value we have for $\dot{\phi}$ by replacing r we can determine ϕ as follows

$$\begin{aligned}
\frac{d\phi}{dt} &= \frac{au}{((ut)^2 + a^2)\sin\alpha} \\
\int d\phi &= \frac{au}{\sin\alpha}\int \frac{dt}{(ut)^2 + a^2} \\
\phi &= \frac{au}{\sin\alpha}\frac{\arctan(ut/a)}{au} \\
\phi &= \frac{\arctan(tu/a)}{\sin\alpha}
\end{aligned}$$

Case B. If $\alpha = \pi/3$ we get that $\dot{r}^2$ is

$$\dot{r}^2 = (r-a)\left[\frac{u^2}{r^2}(r+a) - g\right]$$

We want for r to oscillate between a and $2a$ and this would happen when $\dot{r} = 0$ hence we want that

$$(r-a)\left[\frac{u^2}{r^2}(r+a) - g\right] = 0$$

This implies that $\dot{r} = 0$ when $r = a$ (which is one part of what we want) and when $r^2 = (u^2/g)(r+a)$. From the last equation, we want to find u for the case $r = 2a$ then $u = \sqrt{\frac{4}{3}ga}$.

Finally, we want to find the time it takes for P to return to $r = a$ using the velocity u we computed. By replacing u we have that

$$\begin{aligned}
\left(\frac{dr}{dt}\right)^2 &= (r-a)\left[\frac{4ga}{3r^2}(r+a) - g\right] \\
\left(\frac{dr}{dt}\right)^2 &= \frac{(r-a)g}{3r^2}(4a^2 + 4ar - 3r^2) \\
\left(\frac{dr}{dt}\right)^2 &= \frac{g(r-a)(2a-r)(2a+3r)}{3r^2} \\
t &= \int_a^{2a} \frac{dr}{\sqrt{\frac{g(r-a)(2a-r)(2a+3r)}{3r^2}}} \\
t &= \sqrt{3}\frac{1}{\sqrt{g}}\int_a^{2a} \frac{rdr}{\sqrt{(r-a)(2a-r)(2a+3r)}} \\
t &= \sqrt{3}\frac{1}{\sqrt{g}}\int_a^{2a} \frac{rdr}{\sqrt{a^3((r/a)-1)(2-(r/a))(2+3(r/a))}}
\end{aligned}$$

Let us now call $\varepsilon = r/a$ so the integral limits change to 1 and 2 and then

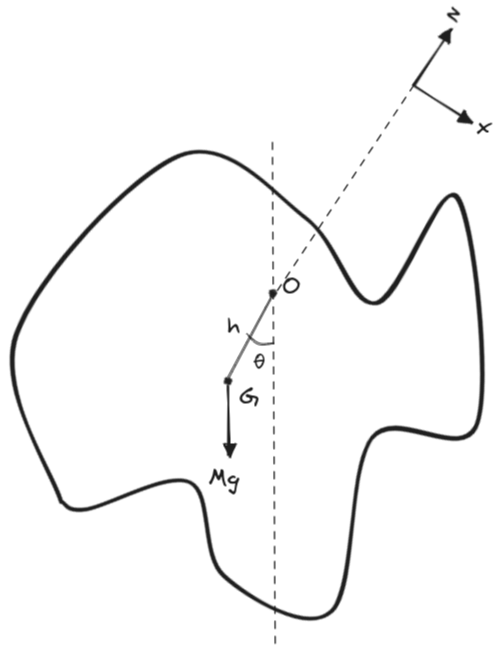
$$\begin{aligned}
t &= \sqrt{3}\frac{1}{\sqrt{g}}\int_1^2 \frac{a\varepsilon d\varepsilon}{\sqrt{a(\varepsilon-1)(2-\varepsilon)(2+3\varepsilon)}} \\
t &= \sqrt{3}\sqrt{\frac{a}{g}}\int_1^2 \frac{\varepsilon d\varepsilon}{\sqrt{(\varepsilon-1)(2-\varepsilon)(2+3\varepsilon)}}
\end{aligned}$$

This would be the time for r to go from a to $2a$ then the time to go from a to $2a$ and back to a is twice this time. Therefore

$$t = 2\sqrt{3}\sqrt{\frac{a}{g}}\int_1^2 \frac{\varepsilon d\varepsilon}{\sqrt{(\varepsilon-1)(2-\varepsilon)(2+3\varepsilon)}}$$

□

Solution. **11.10** Let's consider the system as described below



Where G is the center of mass of the body and θ is the angular displacement from the vertical line.

We can treat this system as a planar rigid body hence it satisfies the Planar rigid body equations so the equations in the x and z direction give us

$$M\ddot{\theta}h = Mg\sin\theta \quad \text{and} \quad 0 = Mg\cos\theta$$

and from the momentum equation, we get that

$$I\ddot{\theta} = hMg\sin\theta$$

by replacing $Mg\sin\theta$ we get that

$$I\ddot{\theta} = M\ddot{\theta}h^2$$
$$h = \frac{I}{Mh}$$

Finally, we know that the period of small oscillations in a pendulum where l is the length of the pendulum is given by

$$\tau = 2\pi\sqrt{\frac{l}{g}}$$

Therefore, in this case, we get that

$$\tau = 2\pi\sqrt{\frac{I}{Mgh}}$$

On the other hand, the period of small oscillations of a uniform rod of length $2a$, pivoted about a horizontal axis perpendicular to the rod and distance b from its center can be computed from the above formula but first we need to compute the moment of inertial for the given system hence

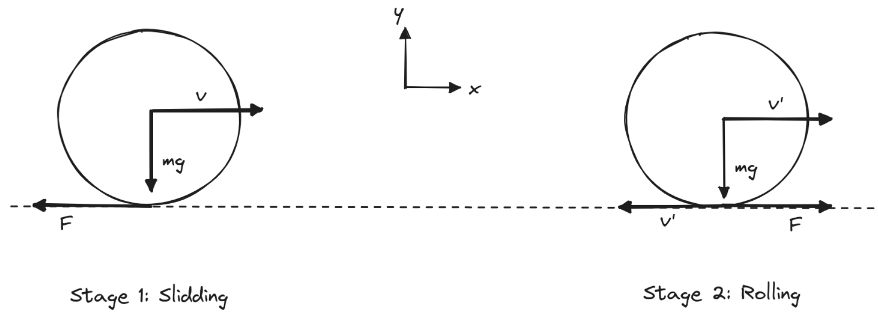
$$I = \int_{b-a}^{b+a} r^2 \frac{M}{2a} dr$$
$$I = \frac{M}{2a} \left[\frac{r^3}{3} \right]_{b-a}^{b+a}$$
$$I = \frac{M}{2a} \left[\frac{(b+a)^3}{3} - \frac{(b-a)^3}{3} \right]$$
$$I = M \left[\frac{a^2}{3} + b^2 \right]$$

Therefore the period of small oscillations is given by

$$\tau = 2\pi \sqrt{\frac{M(a^2/3 + b^2)}{Mgb}}$$
$$\tau = 2\pi \sqrt{\frac{a^2 + 3b^2}{3gb}}$$

$\square$

Solution. 11.11 Let's consider the two stages of the system, shown below



In stage 1 Newton's law in x direction says that

$$m\frac{dv}{dt} = -F$$

And in stage 2 the rotation equation says that

$$I\frac{d\omega}{dt} = rF$$

By replacing the moment of inertia of the ball $I = 2/5mr^2$ and the value of F we got from the first equation we see that

$$\begin{aligned}\frac{2}{5}mr^2\frac{d\omega}{dt} &= -rm\frac{dv}{dt} \\ \frac{2}{5}r\frac{d\omega}{dt} &= -\frac{dv}{dt} \\ \frac{2}{5}r\frac{d\omega}{dt} + \frac{dv}{dt} &= 0\end{aligned}$$

Now we integrate with respect to t then

$$\frac{2}{5}r\omega + v = C$$

Also, we know that initially $v = V$ and $\omega = 0$ so $C = V$ hence

$$\frac{2}{5}r\omega + v = V$$

If we now suppose that the ball rolls with speed V' we know that $\omega = V'/r$ hence we get that

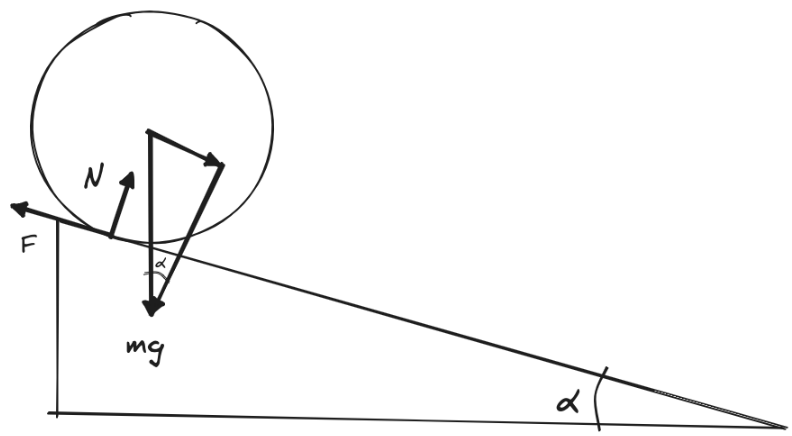
$$\begin{aligned}\frac{2}{5}V' + V' &= V \\ V' &= \frac{5}{7}V\end{aligned}$$

Let us now compute the final kinetic energy of the ball assuming it is rolling (and maybe sliding) with velocity V' hence

$$
\begin{aligned}
T' &= \frac{1}{2}mV'^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{V'^2}{r^2}\right) \\
T' &= \frac{7}{10}mV'^2 \\
T' &= \frac{7}{10}m\left(\frac{25}{49}V^2\right) \\
T' &= \frac{1}{2}m\frac{5}{7}V^2 = \frac{5}{7}T
\end{aligned}
$$

Where we used the value for V' we computed earlier and we replaced $T = 1/2\ mV^2$. Therefore the ball lost 2/7 of it's initial kinetic energy. $\square$

Solution. 11.12 Let's suppose the ball is rolling as shown below



where F is the friction force. Then the rolling equation where we are considering the positive rotation in the clockwise direction would be

$$
\begin{aligned}
I\frac{d\omega}{dt} &= rF \\
\frac{2}{5}mr^2\frac{d\omega}{dt} &= r\mu mg\cos\alpha \\
\frac{2}{5}r\frac{d\omega}{dt} &= g\mu\cos\alpha \\
\frac{2}{5}\frac{dv}{dt} &= g\mu\cos\alpha
\end{aligned}
$$

We are replacing $F = \mu N = \mu(mg\cos\alpha)$, $d\omega/dt = 1/r \cdot dv/dt$ and $I = 2/5mr^2$.

From Newton's equation in the x direction, we have the following

$$
\begin{aligned}
m\frac{dv}{dt} &= mg\sin\alpha - F \\
\frac{dv}{dt} &= g(\sin\alpha - \mu\cos\alpha)
\end{aligned}
$$

Where we have the value for the acceleration in the case of a sliding ball since the equation is the same in a sliding situation. By replacing dv/dt in the rolling equation we get that

$$
\begin{aligned}
\frac{2}{5}g(\sin\alpha - \mu\cos\alpha) &= g\mu\cos\alpha \\
\frac{2}{5}\sin\alpha &= \frac{7}{5}\mu\cos\alpha \\
\frac{2}{7}\tan\alpha &= \mu
\end{aligned}
$$

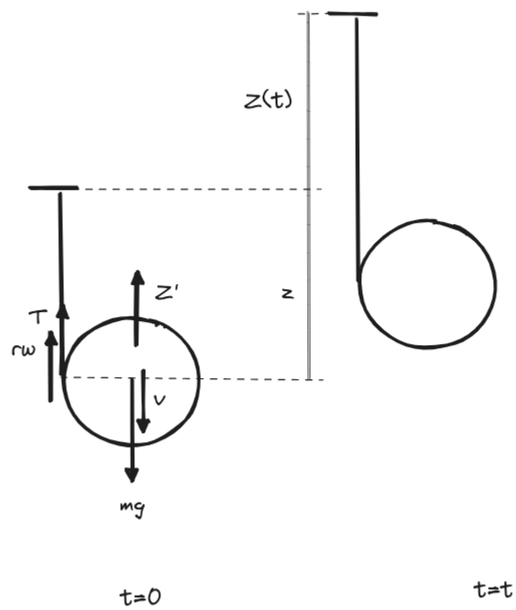
This implies that if $\mu < \frac{2}{7}\tan\alpha$ then rolling would be impossible and the ball would slide, and if $\mu \geq \frac{2}{7}\tan\alpha$ then the ball would roll.

Finally, if we replace the value of μ in the last form of the rolling equation we get that

$$\frac{dv}{dt} = \frac{5}{2}g\left(\frac{2}{7}\tan\alpha\right)\cos\alpha$$
$$\frac{dv}{dt} = \frac{5}{7}g\sin\alpha$$

$\square$

Solution. 11.14 Let's consider the system at two times $t = 0$ and $t = t$ as shown below



The planar equations applied to the yo-yo give us

$$I\dot{\omega} = Tr \tag{1}$$
$$-m\dot{v} = T - mg \tag{2}$$

Where we are considering the clockwise direction as the positive rotation direction and r as the radius of the yo-yo. Also, knowing that $I = 1/2mr^2$ and replacing the value of T we got from (1) into (2) we get that

$$\frac{1}{2}r\dot{\omega} + \dot{v} = g \tag{3}$$

On the other hand, from the no-slipping relation, we know that the velocity of the yo-yo where it touches the string is the same as the velocity of the string, hence $\dot{Z} = r\omega - v$ which implies that

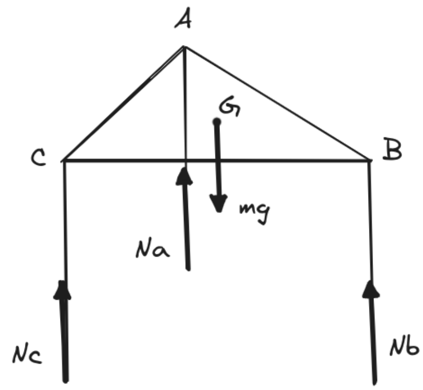
$$\ddot{Z} = r\dot{\omega} - \dot{v}$$

Then by replacing $r\dot{\omega} = \ddot{Z} + \dot{v}$ into (3) we get that

$$\frac{1}{2}(\ddot{Z} + \dot{v}) + \dot{v} = g$$
$$\frac{3}{2}\dot{v} + \frac{1}{2}\ddot{Z} = g$$
$$\dot{v} = \frac{2}{3}g - \frac{1}{3}\ddot{Z}$$

Finally, if the support moves upward with acceleration $\ddot{Z} = 2g$ then $\dot{v} = 0$ which implies that the yo-yo moves with a constant velocity. □

Solution. 11.18 Let us consider the following unsymmetrical triangular table



We now apply the equilibrium conditions. The condition $\boldsymbol{F} = 0$ give us

$$N_a\boldsymbol{k} + N_b\boldsymbol{k} + N_c\boldsymbol{k} - mg\boldsymbol{k} = 0$$

Let us assume the vertex C is at the origin of the xy plane, then the position vector of the center of mass is given by $\boldsymbol{CG} = (\boldsymbol{CA} + \boldsymbol{CB})/3$. If we take moments about C, the normal reaction $N_c\boldsymbol{k}$ makes no contribution and the condition $\boldsymbol{K_A} = 0$ give us

$$\begin{aligned}
\boldsymbol{CA} \times N_a\boldsymbol{k} + \boldsymbol{CB} \times N_b\boldsymbol{k} - \boldsymbol{CG} \times mg\boldsymbol{k} &= 0 \\
N_a(\boldsymbol{CA} \times \boldsymbol{k}) + N_b(\boldsymbol{CB} \times \boldsymbol{k}) - \frac{mg}{3}(\boldsymbol{CB} \times \boldsymbol{k}) - \frac{mg}{3}(\boldsymbol{CA} \times \boldsymbol{k}) &= 0 \\
\left(N_a - \frac{mg}{3}\right)(\boldsymbol{CA} \times \boldsymbol{k}) + \left(N_b - \frac{mg}{3}\right)(\boldsymbol{CB} \times \boldsymbol{k}) &= 0 \\
\left(\left(N_a - \frac{mg}{3}\right)\boldsymbol{CA} + \left(N_b - \frac{mg}{3}\right)\boldsymbol{CB}\right) \times \boldsymbol{k} &= 0
\end{aligned}$$

The last equation implies that

$$\left(N_a - \frac{mg}{3}\right)\boldsymbol{CA} + \left(N_b - \frac{mg}{3}\right)\boldsymbol{CB} = 0$$

Hence it must happen that both $N_a - \frac{mg}{3} = 0$ and $N_b - \frac{mg}{3} = 0$ so we get that $N_a = N_b = mg/3$ and from the equation for the first condition we get that also $N_c = mg/3$. Therefore each leg bears one-third of the table's weight. $\square$