Solved selected problems of Lagrangian and Hamiltonian by Mann

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Solution. **5.2** From equation (5.0.26) we have that

$$\sum_{j=0}^{\infty}(j+2)(j+1)a_{j+2}x^j - \sum_{j=0}^{\infty} j(j-1)a_j x^j - 2\sum_{j=0}^{\infty} j a_j x^j + \lambda(\lambda+1)\sum_{j=0}^{\infty} a_j x^j = 0$$

Then

$$\sum_{j=0}^{\infty}\Big[(j+2)(j+1)a_{j+2} - j(j-1)a_j - 2ja_j + \lambda(\lambda+1)a_j\Big]x^j = 0$$

So, must be that

$$\begin{aligned}
(j+2)(j+1)a_{j+2} - j(j-1)a_j - 2ja_j + \lambda(\lambda+1)a_j &= 0\\
(j+2)(j+1)a_{j+2} + a_j(-j(j-1) - 2j + \lambda(\lambda+1)) &= 0\\
(j+2)(j+1)a_{j+2} + a_j(-j^2 - j - \lambda j + \lambda^2 + \lambda + \lambda j) &= 0\\
(j+2)(j+1)a_{j+2} + a_j(-j(j+1+\lambda) + \lambda(\lambda+1+j)) &= 0\\
(j+2)(j+1)a_{j+2} + a_j(\lambda - j)(j+1+\lambda) &= 0
\end{aligned}$$

Where we added and subtracted λj in the third step. Hence

$$a_{j+2} = -\frac{(\lambda-j)(\lambda+j+1)}{(j+2)(j+1)}a_j$$

Which is the expected recurrence formula. $\square$