

Solved selected problems of Modern Physics for Scientists and Engineers - John Taylor

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Chapter 3 - Atoms

Solution. 3.3 We know that the ratio of the masses of the H_2O atoms is

$$\frac{m_O}{2m_H} = \frac{15.999}{2 \times 1.008}$$

so we need a mass of $2 \text{ g} \times 15.999/2.016 = 15.872 \text{ g}$ of oxygen to form water with 2 g of hydrogen. \square

Solution. 3.6

(a) To form carbon monoxide CO with 1 g of carbon we need

$$1 \text{ g} \times \frac{15.999}{12.011} = 1.332 \text{ g}$$

of oxygen given that the atomic mass of carbon is 12.011.

(b) To form carbon dioxide CO_2 with 1 g of carbon we need

$$1 \text{ g} \times \frac{2 \times 15.999}{12.011} = 2.664 \text{ g}$$

of oxygen, where in this case we have 2 masses of oxygen to account for. \square

Solution. 3.7

	He	Li	Na	Ca
electrons	2	3	11	20
protons	2	3	11	20
neutrons	2	4	12	20
nucleons	4	7	23	40

□

Solution. 3.16 Correct to the nearest u these are the masses of the listed molecules

(a) H_2O :

$$(1.008 \text{ } u \times 2) + 15.999 \text{ } u = 18.015 \text{ } u \approx 18 \text{ } u$$

(b) N_2O :

$$(14.007 \text{ } u \times 2) + 15.999 \text{ } u = 44.013 \text{ } u \approx 44 \text{ } u$$

(c) O_3 :

$$15.999 \text{ } u \times 3 = 47.997 \text{ } u \approx 48 \text{ } u$$

(d) $C_6H_{12}O_6$:

$$(12.011 \text{ } u \times 6) + (1.008 \text{ } u \times 12) + (15.999 \text{ } u \times 6) = 180.156 \text{ } u \\ \approx 180 \text{ } u$$

(e) NH_3 :

$$14.007 \text{ } u + (1.008 \text{ } u \times 3) = 17.031 \text{ } u \approx 17 \text{ } u$$

(d) $CaCO_3$:

$$40.078 \text{ } u + 12.011 \text{ } u + (15.999 \text{ } u \times 3) = 100.086 \text{ } u \approx 100 \text{ } u$$

□

Solution. 3.17

- (a)
- Ne : $Z = 10 \rightarrow$ ratio: $20.180/10 = 2.018$
 - Ca : $Z = 20 \rightarrow$ ratio: $40.078/20 = 2.004$
 - Zn : $Z = 30 \rightarrow$ ratio: $65.380/30 = 2.179$
 - Zr : $Z = 40 \rightarrow$ ratio: $91.224/40 = 2.281$
 - Sn : $Z = 50 \rightarrow$ ratio: $118.711/50 = 2.374$
 - Nd : $Z = 60 \rightarrow$ ratio: $144.243/60 = 2.404$
 - Yb : $Z = 70 \rightarrow$ ratio: $173.055/70 = 2.472$
 - Hg : $Z = 80 \rightarrow$ ratio: $200.59/80 = 2.507$
 - Th : $Z = 90 \rightarrow$ ratio: $232.038/90 = 2.578$

- (b) If A is close to $2Z$ then given that $A = Z + N$ we get that $2Z \approx Z + N$ and therefore this implies that $N \approx Z$.

□

Solution. 3.24

- (a) A mole of CO_2 weights about 44 grams so 1 gram of CO_2 will contain the following amount of C atoms

$$1 \text{ gram} \times \frac{1 \text{ mol}}{44 \text{ gram}} \times \frac{6.022 \times 10^{23} \text{ } CO_2 \text{ molecules}}{1 \text{ mol}} \times \frac{1 \text{ C atom}}{1 \text{ } CO_2 \text{ molecule}} = \\ = 1.368 \times 10^{22} \text{ C atoms}$$

In the same way, there are

$$1 \text{ gram} \times \frac{1 \text{ mol}}{44 \text{ gram}} \times \frac{6.022 \times 10^{23} \text{ } CO_2 \text{ molecules}}{1 \text{ mol}} \times \frac{2 \text{ O atom}}{1 \text{ } CO_2 \text{ molecule}} = \\ = 2.737 \times 10^{22} \text{ O atoms}$$

Therefore the fraction of C atoms is

$$\frac{1.368 \times 10^{22} \text{ C atoms}}{1.368 \times 10^{22} \text{ C atoms} + 2.737 \times 10^{22} \text{ O atoms}} = 0.333$$

Finally, the mass of C atoms in 1 gram of CO_2 is

$$1.368 \times 10^{22} \text{ C atoms} \times \frac{12 \text{ grams}}{6.022 \times 10^{23} \text{ C atoms}} = 0.272 \text{ grams}$$

Which implies that the fraction of mass in C is

$$\frac{0.272 \text{ C grams}}{1 \text{ } CO_2 \text{ grams}} = 0.272$$

- (b) A mole of CH_4 contains 6.022×10^{23} CH_4 molecules then

$$6.022 \times 10^{23} \text{ } CH_4 \text{ molecules} \times \frac{1 \text{ C atom}}{1 \text{ } CH_4 \text{ molecule}} = 6.022 \times 10^{23} \text{ C atoms}$$

In the same way, there are

$$6.022 \times 10^{23} \text{ } CH_4 \text{ molecules} \times \frac{4 \text{ H atoms}}{1 \text{ } CH_4 \text{ molecule}} = 2.408 \times 10^{24} \text{ H atoms}$$

Therefore the fraction of C atoms is

$$\frac{6.022 \times 10^{23} \text{ C atoms}}{6.022 \times 10^{23} \text{ C atoms} + 2.408 \times 10^{24} \text{ H atoms}} = 0.2$$

Finally, the mass of C atoms in 1 mol of CH_4 is

$$6.022 \times 10^{23} \text{ C atoms} \times \frac{12 \text{ grams}}{6.022 \times 10^{23} \text{ C atoms}} = 12 \text{ grams}$$

Which implies that the fraction of mass in C is

$$\frac{12 \text{ C grams}}{16 \text{ } CH_4 \text{ grams}} = 0.75$$

- (c) A mole of C_2H_6 contains 6.022×10^{23} C_2H_6 molecules then we have that

$$1000 \text{ } C_2H_6 \text{ mol} \times \frac{6.022 \times 10^{23} \text{ } C_2H_6 \text{ molecules}}{1 \text{ } C_2H_6 \text{ mol}} \times \frac{2 \text{ C atom}}{1 \text{ } C_2H_6 \text{ molecule}} \\ = 1.204 \times 10^{27} \text{ C atoms}$$

In the same way, there are

$$1000 \text{ } C_2H_6 \text{ mol} \times \frac{6.022 \times 10^{23} \text{ } C_2H_6 \text{ molecules}}{1 \text{ } C_2H_6 \text{ mol}} \times \frac{6 \text{ H atoms}}{1 \text{ } C_2H_6 \text{ molecule}} \\ = 3.613 \times 10^{27} \text{ H atoms}$$

Therefore the fraction of C atoms is

$$\frac{1.204 \times 10^{27} \text{ C atoms}}{1.204 \times 10^{27} \text{ C atoms} + 3.613 \times 10^{27} \text{ H atoms}} = 0.249$$

Finally, the mass of C atoms in 1000 moles of C_2H_6 is

$$1.204 \times 10^{27} \text{ C atoms} \times \frac{12 \text{ grams}}{6.022 \times 10^{23} \text{ C atoms}} = 23992.03 \text{ grams}$$

Which implies that the fraction of mass in C is

$$\frac{23992.03 \text{ C grams}}{(1000 \times 30) \text{ } C_2H_6 \text{ grams}} = 0.799$$

□

Solution. 3.25 From the ideal gas law, we know that $pV = Nk_B T$ where N is the number of molecules and k_B is Boltzmann's constant. Hence we see that $N/V = p/k_B T$ where N/V is the number of molecules per volume or the number density of molecules. □

Solution. 3.27 By multiplying the ideal gas law by the molar mass M_{He} of He we get that

$$\begin{aligned} M_{He}(pV) &= M_{He}(nRT) \\ \frac{pM_{He}}{RT} &= \frac{m_{He}}{V} \\ \rho_{He} &= \frac{pM_{He}}{RT} \end{aligned}$$

Where we used that $M_{He}n = m_{He}$ which is the mass of He also, we used that $m_{He}/V = \rho_{He}$ is the density of He . Therefore by replacing the ρ_{He} by ρ_{H_2O} and the values of $M_{He} = 4 \text{ g/mol}$ and $T = 300 \text{ K}$ we get that p is

$$\begin{aligned} p &= \frac{\rho_{H_2O}RT}{M_{He}} \\ &= \frac{997048 \text{ g/m}^3 \cdot 8.314 \text{ J/Kmol} \cdot 300 \text{ K}}{4 \text{ g/mol}} \\ &= 621709280.4 \frac{\text{J}}{\text{m}^3} \cdot \frac{1 \text{ atm}}{1.01 \times 10^5 \text{ N/m}^2} \\ &= 6155.53 \text{ atm} \end{aligned}$$

□

Solution. 3.30 Let us consider a container of oxygen gas at $T = 237 \text{ K}$ and $p = 1 \text{ atm} = 1.01 \times 10^5 \text{ N/m}^2$

(a) First, we need to determine the O_2 molecule mass in kg

$$m = \frac{32 \text{ g/mol}}{6 \times 10^{23} \text{ molecules/mol}} = 5.33 \times 10^{-23} \text{ kg/molecule}$$

The collision cross-section in this case is given by

$$\sigma = \pi(2R)^2 = 4\pi(0.15 \times 10^{-9} \text{ m})^2 = 2.8 \times 10^{-19} \text{ m}^2$$

and the number density is

$$n = \frac{N}{V} = \frac{p}{kT} = \frac{1.01 \times 10^5 \text{ N/m}^2}{(1.38 \times 10^{-23} \text{ J/K})(237 \text{ K})} = 3.088 \times 10^{25} \text{ m}^{-3}$$

Finally, by solving equation (3.34) for time t we get that

$$\begin{aligned} t &= (D_{rms})^2 n \sigma \sqrt{\frac{m}{3kT}} \\ &= 1^2 \cdot (3.088 \times 10^{25}) \cdot (2.8 \times 10^{-19}) \cdot \sqrt{\frac{5.33 \times 10^{-23}}{3 \cdot (1.38 \times 10^{-23}) \cdot (237)}} \\ &= 637271.628 \text{ s} \approx 7 \text{ days} \end{aligned}$$

(b) At a pressure of 0.01 atm we have a number density of

$$n = \frac{N}{V} = \frac{p}{kT} = \frac{1013.25 \text{ N/m}^2}{(1.38 \times 10^{-23} \text{ J/K})(237 \text{ K})} = 3.098 \times 10^{23} \text{ m}^{-3}$$

Hence it would take the molecule the following time to reach 1 m distance from its starting point.

$$\begin{aligned} t &= (D_{rms})^2 n \sigma \sqrt{\frac{m}{3kT}} \\ &= 1^2 \cdot (3.098 \times 10^{23}) \cdot (2.8 \times 10^{-19}) \cdot \sqrt{\frac{5.33 \times 10^{-23}}{3 \cdot (1.38 \times 10^{-23}) \cdot (237)}} \\ &= 6393.35 \text{ s} = 1.77 \text{ hours} \end{aligned}$$

(c) knowing that $n = p/kT$ and $D_{rms} = \sqrt{Nd}$ then by replacing these values in the equation we have for time t we get that

$$\begin{aligned} Nd^2 \frac{p\sigma}{kT} \sqrt{\frac{m}{3kT}} &= t \\ Nd^2 \frac{p\sigma}{kT} &= t \sqrt{\frac{3kT}{m}} \\ p &= \frac{tkT}{Nd^2\sigma} \sqrt{\frac{3kT}{m}} \end{aligned}$$

□

Solution. 3.36 Let us consider a cubical container filled with a gas of molecules with number density n . Let us consider two sets of molecules, one set of molecules is traveling to the right (in the positive x -direction) and the other set to the left (in the negative x -direction) both with the same velocity $|v_x|$. Let us consider in particular the molecules which are at a distance Δx from the wall where $\Delta x \ll \lambda$ where λ is the mean free path. In this situation, the molecules will not collide with any other molecules before hitting the wall, hence given that every molecule is moving with the same velocity the change in momentum will be $\Delta p_x = p_x^{final} - p_x^{initial} = -2mv_x$ so the average force applied to the wall by one molecule is the change in momentum between collisions divided by the average time Δt thus

$$F_x = \frac{N}{2} \frac{|\Delta p_x|}{\Delta t} = \frac{N}{2} \frac{2m|v_x|}{\Delta x/|v_x|} = N \frac{mv_x^2}{\Delta x}$$

Where we multiplied by the number of molecules $N/2$ to account for every molecule hitting the wall in this direction, the other half N hits the opposite wall. The average pressure on the wall is then obtained as follows

$$p = \frac{F_x}{A} = \frac{Nmv_x^2}{A\Delta x} = nmv_x^2$$

Where we used that $n = N/(A\Delta x)$ is the number density. Also, if we compute in the same way the pressure for the rest of the walls where we take $\Delta y = \Delta z = \Delta x$ and $v_x^2 = v_y^2 = v_z^2$ we get that

$$\begin{aligned} p &= 2 \frac{F_x}{A} + 2 \frac{F_y}{A} + 2 \frac{F_z}{A} \\ &= \frac{2Nmv_x^2}{A\Delta x} + \frac{2Nmv_y^2}{A\Delta y} + \frac{4Nmv_z^2}{A\Delta z} \\ &= 6nmv_x^2 \end{aligned}$$

Where we multiplied by 2 each term to account for the 2 walls in each direction. In addition, we need to consider that the average v^2 is $v^2 = 3v_x^2$ so we get that

$$p = 2nmv^2 = \frac{2Nmv^2}{V}$$

So if we assume that the average kinetic energy of a molecule is related to the absolute temperature T by

$$\frac{1}{2}mv^2 = \frac{1}{4}kT$$

We get that

$$pV = NkT$$

Which is the ideal gas law. □

Solution. 3.37 From the Einstein average displacement formula, we see for the first particle that

$$\begin{aligned}\sqrt{\frac{kT}{3\pi\eta}} &= D_{rms}\sqrt{\frac{a_1}{t}} \\ &= 100 \mu m \sqrt{\frac{d_1}{2 \cdot 60 \text{ s}}}\end{aligned}$$

Where we used that a_1 is the radius of the first particle and that $a_1 = d_1/2$. Since the second particle moves in the same fluid and at the same temperature then the constant $\sqrt{\frac{kT}{3\pi\eta}}$ match for both particles then we have that

$$\begin{aligned}D_{rms}\sqrt{\frac{d_2}{2 \cdot 60 \text{ s}}} &= 100 \mu m \sqrt{\frac{d_1}{2 \cdot 60 \text{ s}}} \\ D_{rms}\sqrt{\frac{d_1}{60 \text{ s}}} &= 100 \mu m \sqrt{\frac{d_1}{2 \cdot 60 \text{ s}}} \\ D_{rms}\sqrt{\frac{1}{60 \text{ s}}} &= 100 \mu m \sqrt{\frac{1}{120 \text{ s}}} \\ D_{rms} &= \frac{100}{\sqrt{2}} \mu m = 70.71 \mu m\end{aligned}$$

Where d_2 is the diameter of the second particle and $d_2 = 2 \cdot d_1$. □

Solution. 3.40 The force applied to the electron by the electric field is given by the Lorentz force $F = eE$. But also we know by Newton's second law that $F = ma$ hence the acceleration of the electron is $a = eE/m$.

The time it takes for the electron to cross the field is $t = l/v$ so applying now the kinematic equation $v_y = v_{y0} + at$ we can compute the y -component of the velocity as follows

$$v_y = \frac{eE}{m} \frac{l}{v}$$

Where es used that $v_{y0} = 0$. Also, we know that $v_y = v \sin \theta$ so by approximating $\sin \theta \approx \theta$ for small angles we have that

$$\begin{aligned} v \sin \theta &= \frac{eEl}{mv} \\ \theta &\approx \frac{eEl}{mv^2} \end{aligned}$$

□

Solution. 3.45

- (a) By recalling that $v_d = l/t_d$ and $v_u = l/t_u$ and adding both equations from (3.67) we get that

$$\begin{aligned}\frac{l}{t_d} + \frac{l}{t_u} &= \frac{Mg}{6\pi r \eta} + \frac{qE - Mg}{6\pi r \eta} \\ \frac{1}{t_d} + \frac{1}{t_u} &= \frac{qE}{6\pi r \eta l}\end{aligned}$$

- (b) Let us compute $(1/t_d) + (1/t_u)$ for each set of measurements in Table 3.7 then we see that

t_d	t_u	$(1/t_d) + (1/t_u)$	n	$((1/t_d) + (1/t_u))/n$
15.0	6.4	0.2	2	0.1
15.0	6.3	0.2	2	0.1
15.0	6.1	0.2	2	0.1
15.0	24.4	0.1	1	0.1
15.0	24.2	0.1	1	0.1
15.0	3.7	0.3	3	0.1
15.0	3.6	0.3	3	0.1
15.0	1.8	0.6	6	0.1
15.0	2.0	0.6	6	0.1
15.0	1.9	0.6	6	0.1

Where we found an integer n for each case such that dividing $(1/t_d) + (1/t_u)$ by this number gives us always the same number so $(1/t_d) + (1/t_u)$ is always an integer multiple of one fixed number.

- (c) Based on the information in Table 3.8 we compute v_d as follows

$$v_d = \frac{l}{t} = \frac{8.3 \times 10^{-4} \text{ m}}{15.0 \text{ s}} = 5.5 \times 10^{-5} \text{ m/s}$$

So using the equation for v_d

$$v_d = \frac{2r^2 \rho g}{9\eta}$$

we can compute the radius r of the droplet

$$\begin{aligned}r &= \sqrt{\frac{9\eta v_d}{2\rho g}} \\ &= \sqrt{\frac{9 \cdot 1.60 \times 10^{-5} \cdot 5.5 \times 10^{-5}}{2 \cdot 839 \cdot 9.8}} \\ &= 6.94 \times 10^{-7} \text{ m} = 69.4 \text{ } \mu\text{m}\end{aligned}$$

(d) Finally, we can compute q using equation (3.68) as follows

$$q = \left(\frac{6\pi r \eta l}{E} \right) \left(\frac{1}{t_d} + \frac{1}{t_u} \right)$$

We see from Table 3.7 that $(1/t_d) + (1/t_u)$ takes 4 values so replacing these values in the equation for q and knowing that

$$\begin{aligned} \frac{6\pi r \eta l}{E} &= \frac{6 \cdot \pi \cdot 6.94 \times 10^{-7} \cdot 1.60 \times 10^{-5} \cdot 8.3 \times 10^{-4}}{1.21 \times 10^5} \\ &= 1.43 \times 10^{-18} \text{ s/C} \end{aligned}$$

We have that

$$\begin{aligned} q &= 1.43 \times 10^{-18} \cdot 0.2 = 2.86 \times 10^{-19} \text{ C} \\ &= 1.43 \times 10^{-18} \cdot 0.1 = 1.43 \times 10^{-19} \text{ C} \\ &= 1.43 \times 10^{-18} \cdot 0.3 = 4.29 \times 10^{-19} \text{ C} \\ &= 1.43 \times 10^{-18} \cdot 0.6 = 8.58 \times 10^{-19} \text{ C} \end{aligned}$$

Therefore if we divide each computation by 1.43×10^{-19} we get an integer, this implies that our best estimate for e is $1.43 \times 10^{-19} \text{C}$.

□

Solution. 3.46 Suppose we perform the Rutherford scattering experiment and we get 80 counts/min for an angle of 10° then the number of particles per minute we send out according to the Rutherford formula is

$$N = k n_{sc}(10^\circ) \sin^4(10^\circ/2)$$

where $k = 4s^2E^2/ntZ^2k^2e^4$. Then for an angle of $\theta = 150^\circ$ the number of particles per minute we send out doesn't change so it must happen that

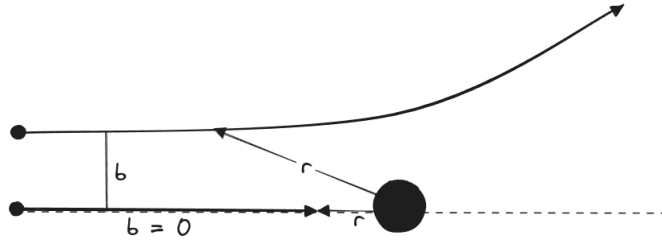
$$k n_{sc}(10^\circ) \sin^4(10^\circ/2) = k n_{sc}(150^\circ) \sin^4(150^\circ/2)$$

Therefore

$$\begin{aligned} n_{sc}(150^\circ) &= n_{sc}(10^\circ) \frac{\sin^4(10^\circ/2)}{\sin^4(150^\circ/2)} \\ &= 80 \left(\frac{0.087}{0.965} \right)^4 \text{ counts/min} \\ &= 0.0052 \text{ counts/min} \end{aligned}$$

□

Solution. 3.48 Let us define r as the distance from the nuclear center to the alpha particles as shown below



Let us consider the moment when the alpha particles are closest to the nuclear center. Then at this moment, the alpha particle where $b \neq 0$ has a kinetic energy K and a potential energy of $U = 2Zke^2/r_b$ hence

$$E = \frac{2Zke^2}{r_b} + K$$

So for this alpha particle, the closest distance to the nucleus will be at

$$r_b = \frac{2Zke^2}{E - K}$$

In the case where $b = 0$, the alpha particle will get close to the nucleus and for a moment this alpha particle will come to rest where $K = 0$. This is the moment where the alpha particle will be the closest to the nucleus hence at this moment we will have that

$$E = \frac{2Zke^2}{r_0}$$

Thus

$$r_0 = \frac{2Zke^2}{E}$$

Finally, we see that $r_0 < r_b$ since $K \geq 0$ and therefore the the alpha particle that approaches the nucleus head-on gets closer to the nucleus than any other. \square