Fourier Transform Model of Image Formation Part 1

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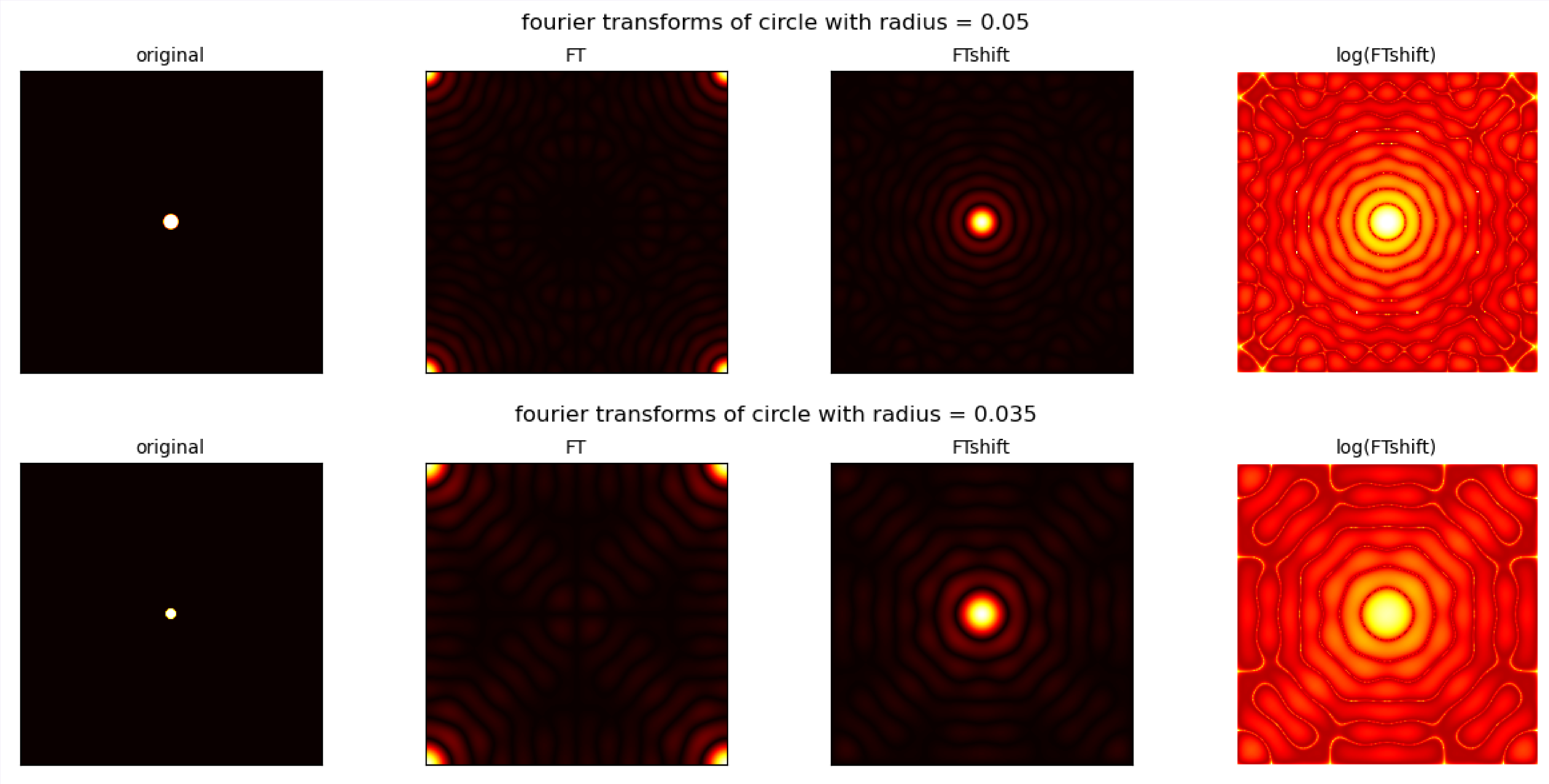
Objectives

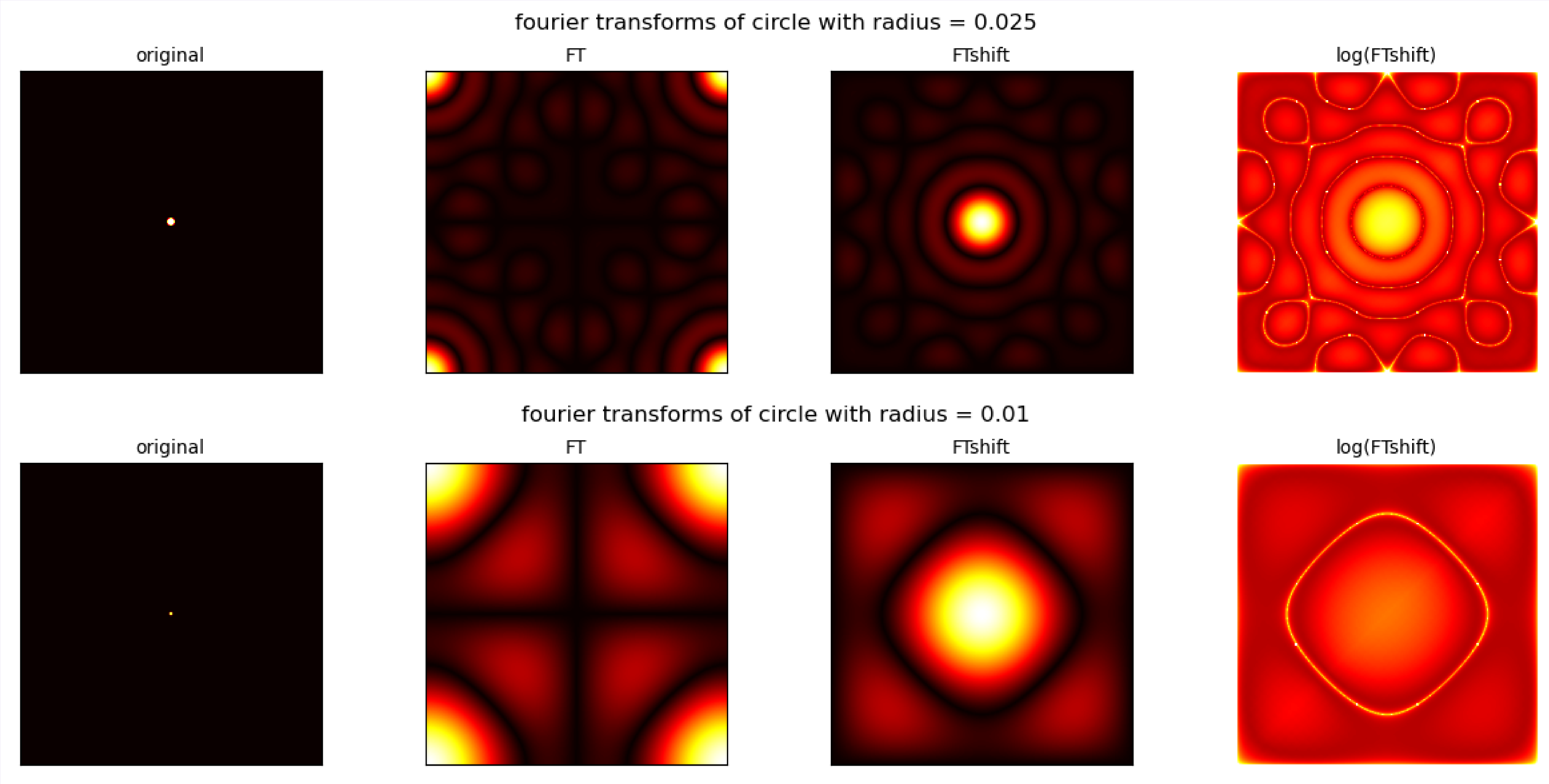
1. Get the fourier transform of synthetic images and observe the resulting image
2. Apply fourier transform and inverse fourier transform in black and white images and observe there resulting image
3. Use convolution to simulate an imaging system with a set aperture and the image it captures
4. Use varrying sizes and shapes of apertures to determine how it affects the captured image
5. Use correlation for template matching

Results and Analysis

2.1 Familiarization with Discrete FT


fourier transforms of circle with radius = 0.05
original
FT
FTshift
log(FTshift)
fourier transforms of circle with radius = 0.035
original
FT
FTshift
log(FTshift)

Fourier Transform of Synthetic Images

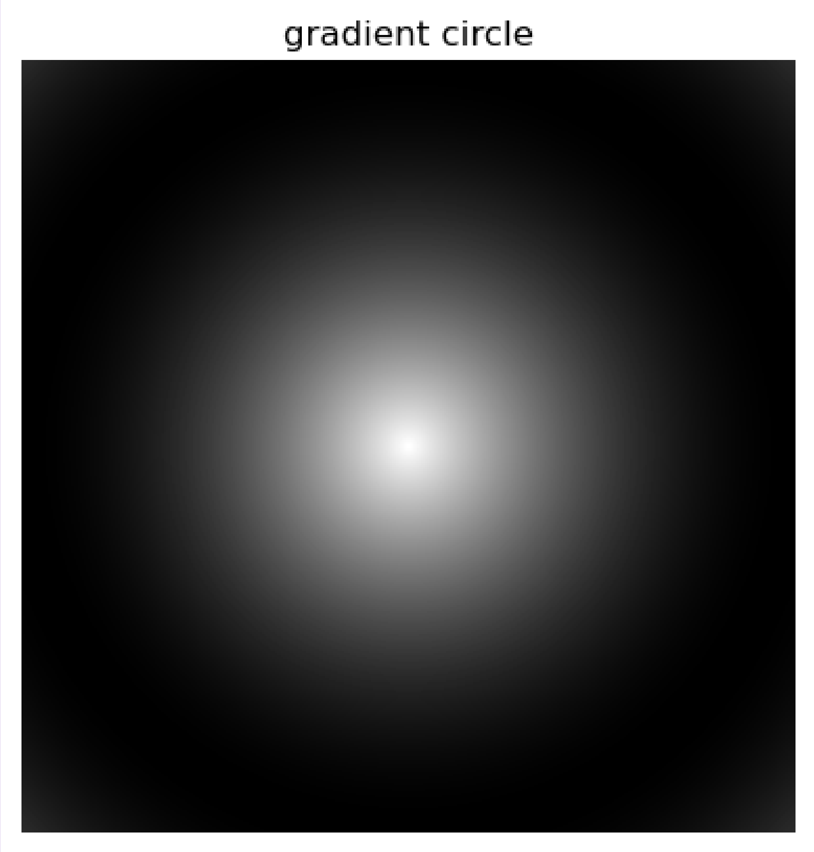


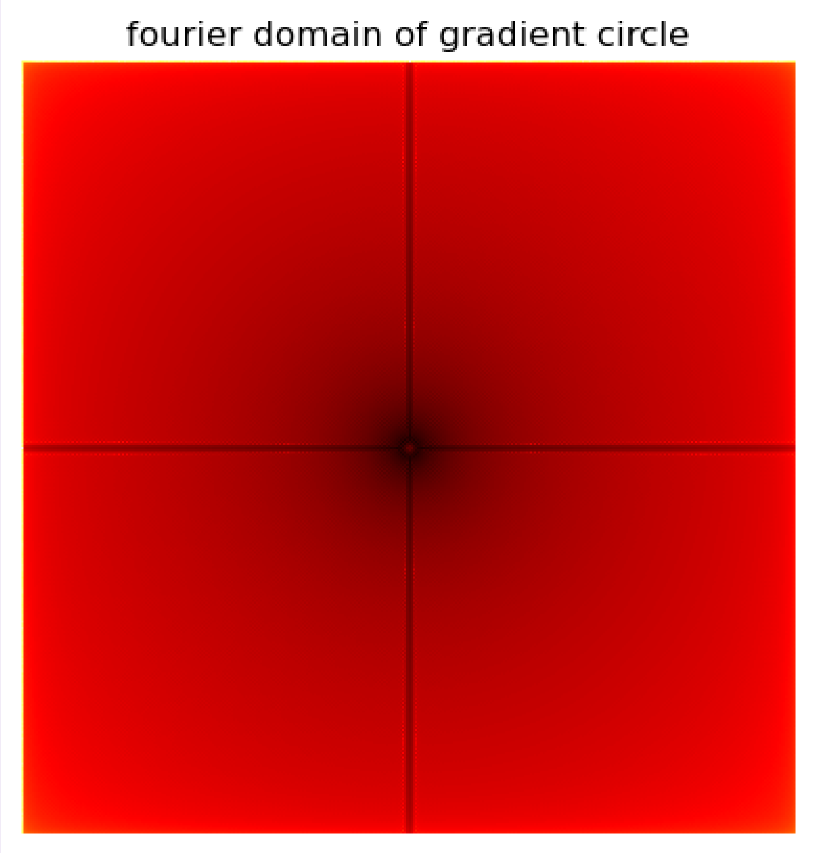
The Fourier Transform represents an signal or an image as a sum of sinusoidal waves. For an image, taking its fourier transform shows us the image's frequency domain or fourier domain. Areas with low frequency mean there is a little change or detail in that area, while a high frequency means there are important details in that area [1].

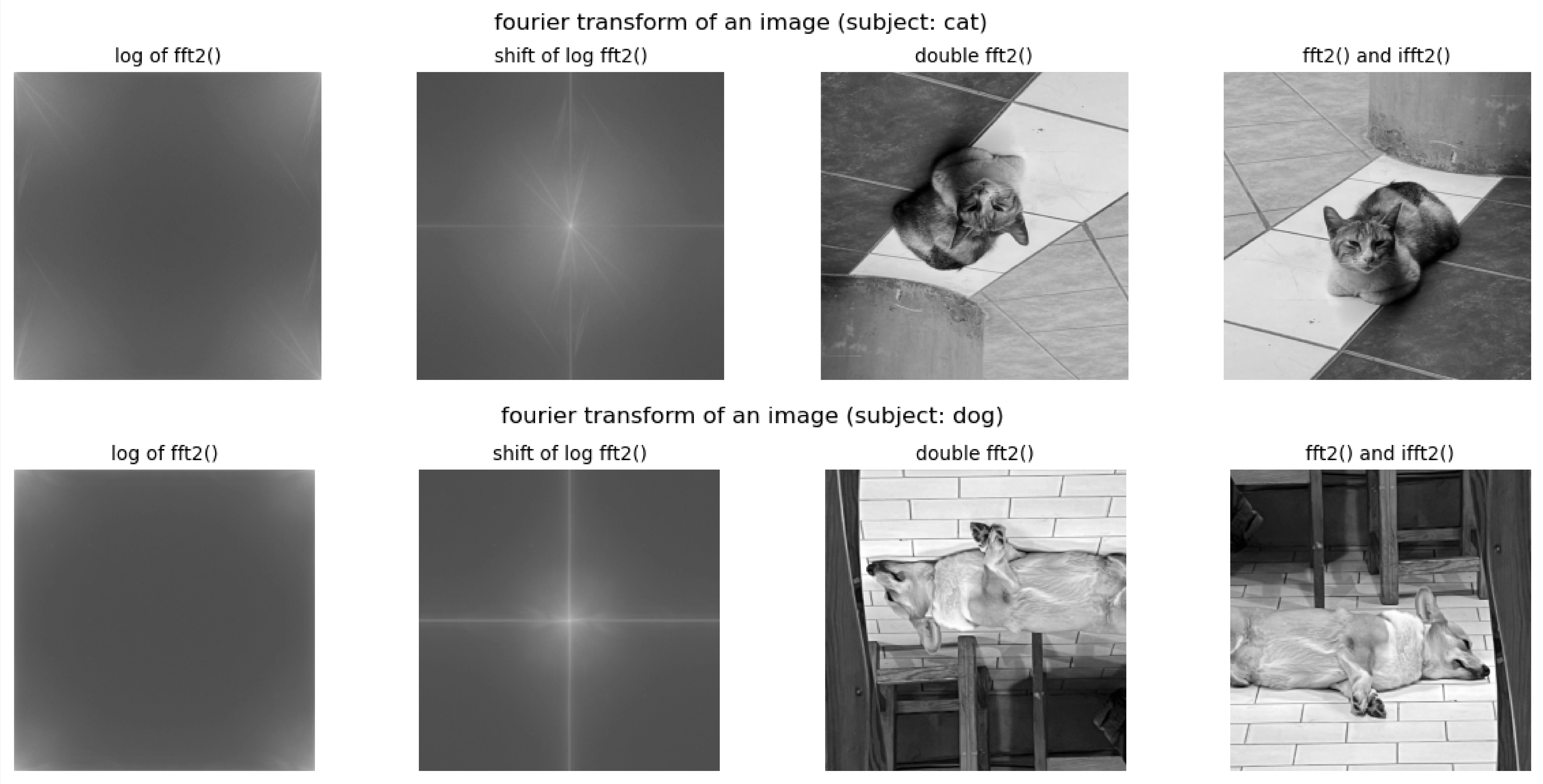
When doing the fourier transform, numpy places the zero-frequency component at the left-hand side of the image. This is why we use fftshift() to shift the zero-frequency component to the center.

Looking at the fourier domain of the circles, we see that the center of a circle has a bright spot. This is surrounded by complex pattern that is symmetrical in all four quadrants. We can also observe that as the circle in the center gets smaller, the circle in the fourier domain gets bigger and the patterns get less complex.

The patterns in the fourier domain are a result of the harsh transition between the white circle to the black background. If we look at the fourier domain of a gradient circle, we see that the patterns are not present [2].






fourier transform of an image (subject: cat)
log of fft2()
shift of log fft2()
double fft2()
fft2() and ifft2()
fourier transform of an image (subject: dog)
log of fft2()
shift of log fft2()
double fft2()
fft2() and ifft2()

In this section, we apply the fourier transform on black and white images. I've chosen photos with a cat and a dog as a subject. The results of the fourier transform in the logarithmic scale and its ftshift() are as expected form previous applications ft these functions.

Applying fft2() twice on the image results in a reconstruction of the original image but inverted horizontally, as is what happened for both the cat and the dog images.

If we apply the inverse fourier transform the the result of a fft2(), the original image is reconstructed in the correct orienation. This is because inverse fourier transforms has a flip operator, which flips the resulting image.

An image can be reconstructed by applying another fourier transform because an original wave can be reconstructed as long as we have its frequency and phase information, which the result of the first fourier transform stores.

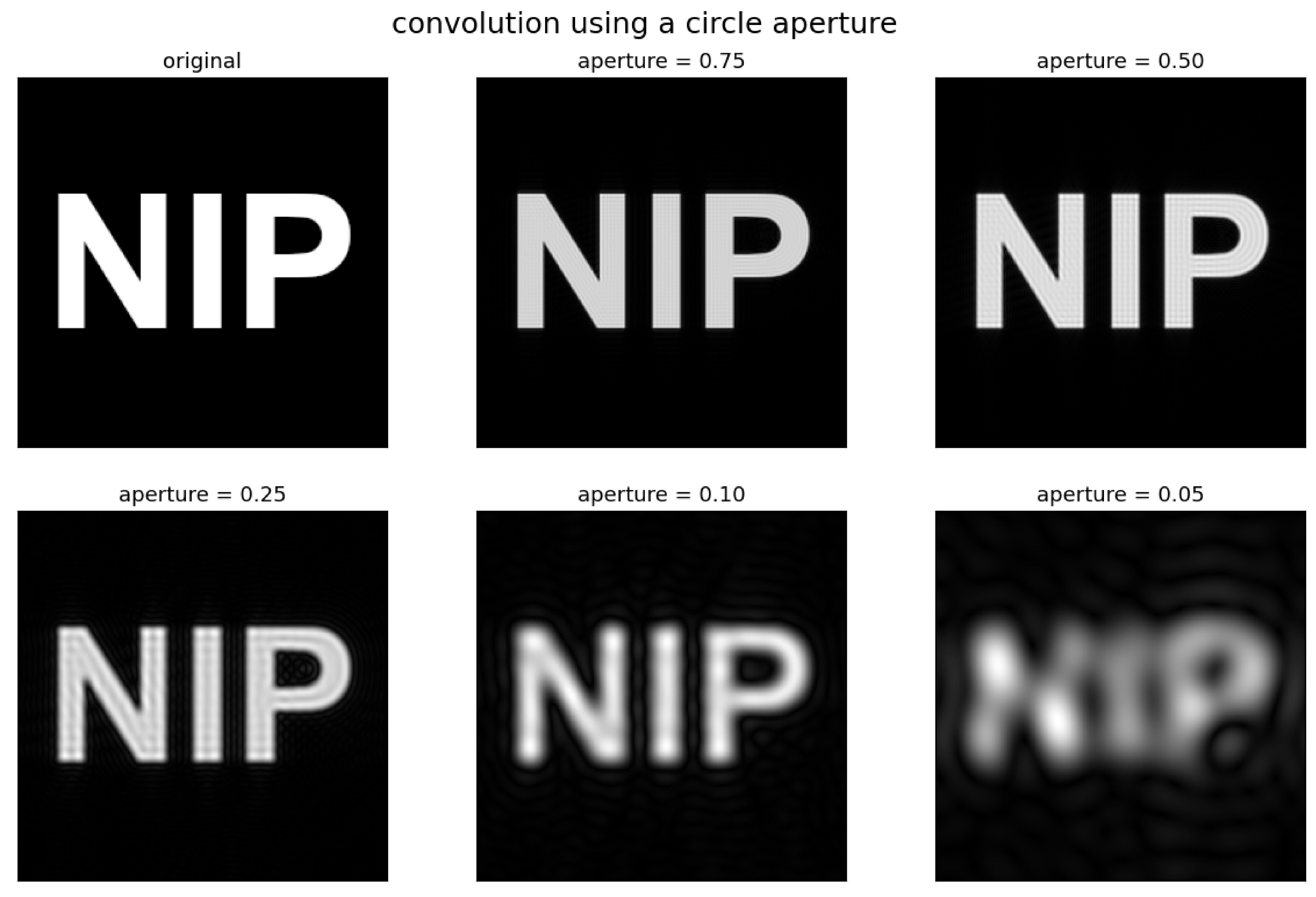
The result of the fast fourier transform on the images (without applying the logarithmic scale to it) appears like a black image!

Results and Analysis

2.2 Simulation of an Imaging System

Convolution is the smearing of one function against another. In computer simulations, this is used to simulate how a lens of a certain size would view an image. Here, we test the resulting image of different aperture sizes.

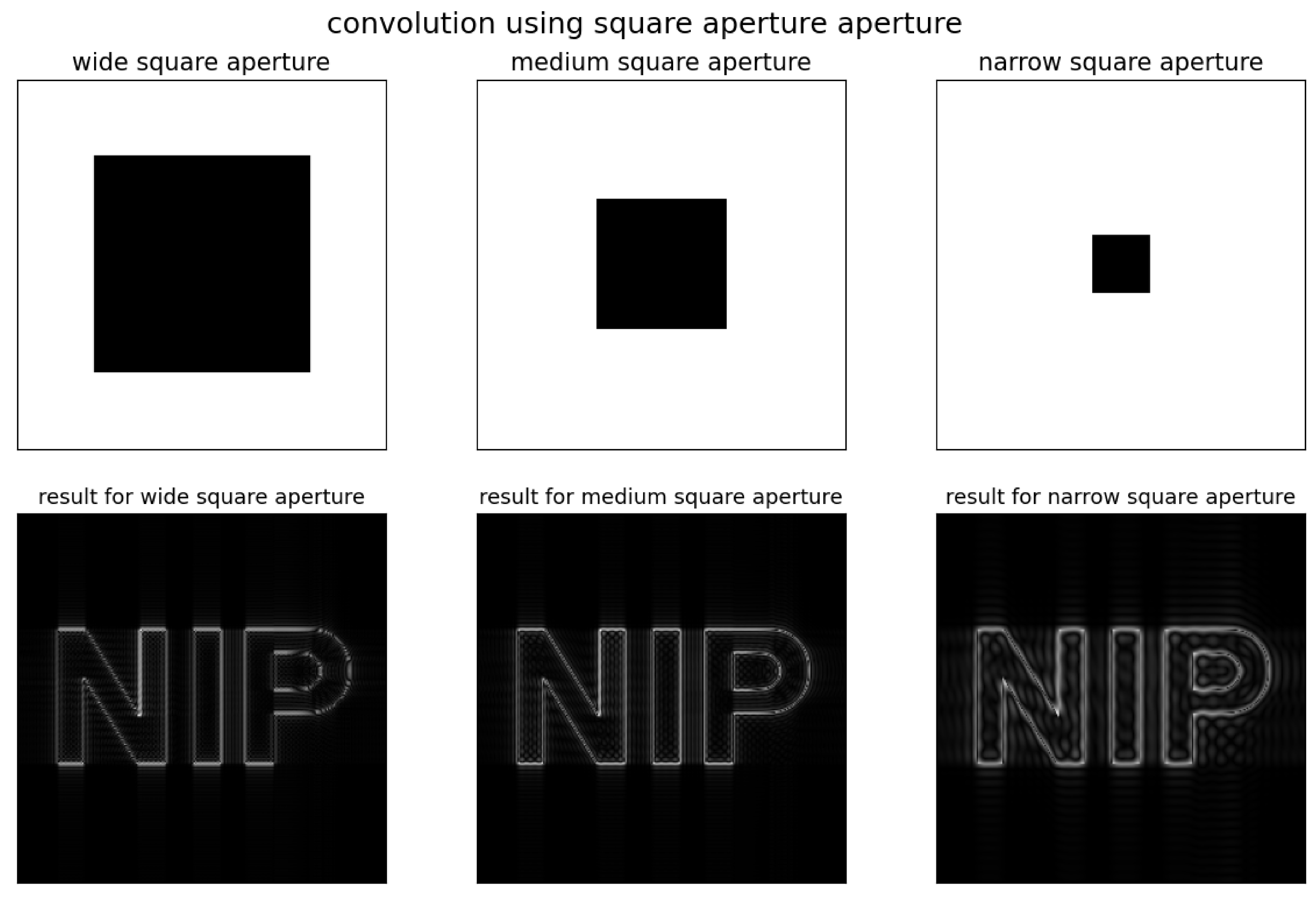
We see that as the aperture size decreases, the image becomes more blurred and distorted. This reflects actual results of camera lenses because smaller apertures can only gather smaller amounts of light.



I wanted to see the effect of different aperture shapes in viewing and image. Here, I test out a square-shaped aperture.

We can see that the edges of the text are smeared and more emphasized. This results in the text being more pronounced. The smaller squares also improve its contrast against the background.

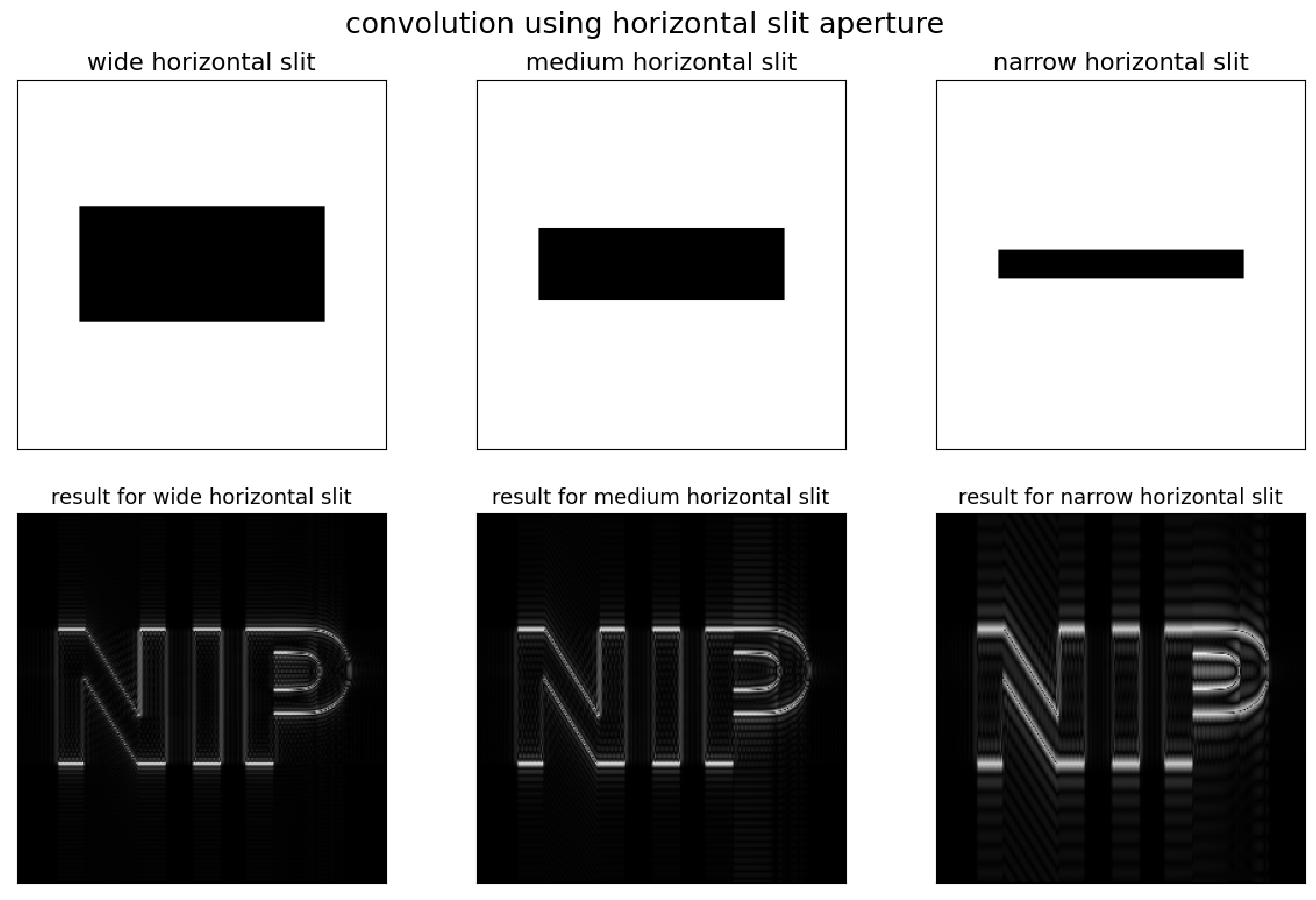
bonus!



I wanted to see the effect of different aperture shapes in viewing and image. Here, I test out apertures in the shape of horizontal slits.

We see a similar effect to that of the square aperture where the edges of the subject are more obvious. But here, that only happens to the horizontal lines in the image. This could be because the smeared shape is also horizontal. Narrow slits also result in more smearing.

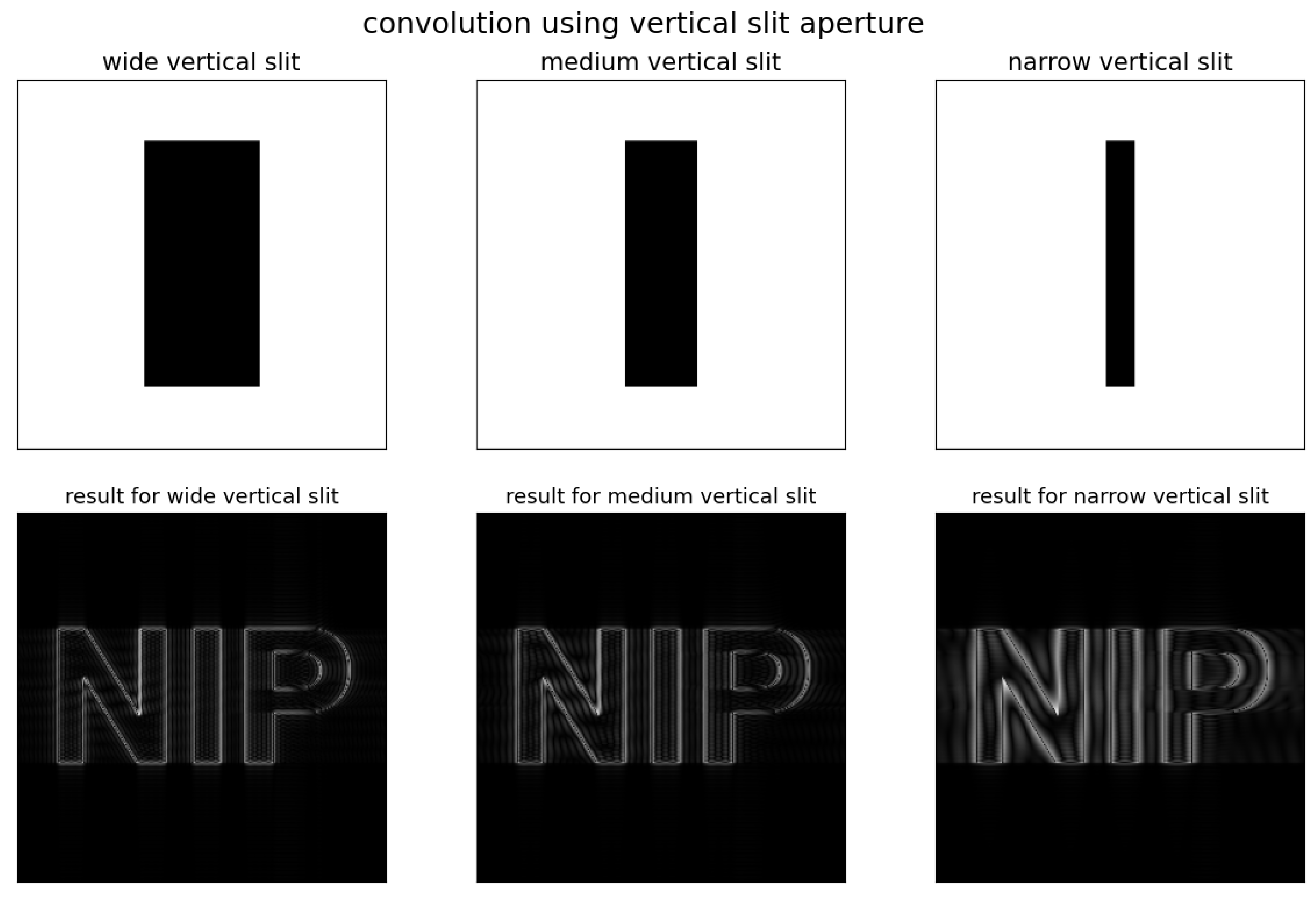
bonus!



I wanted to see the effect of different aperture shapes in viewing and image. Here, I test out apertures in the shape of vertical slits.

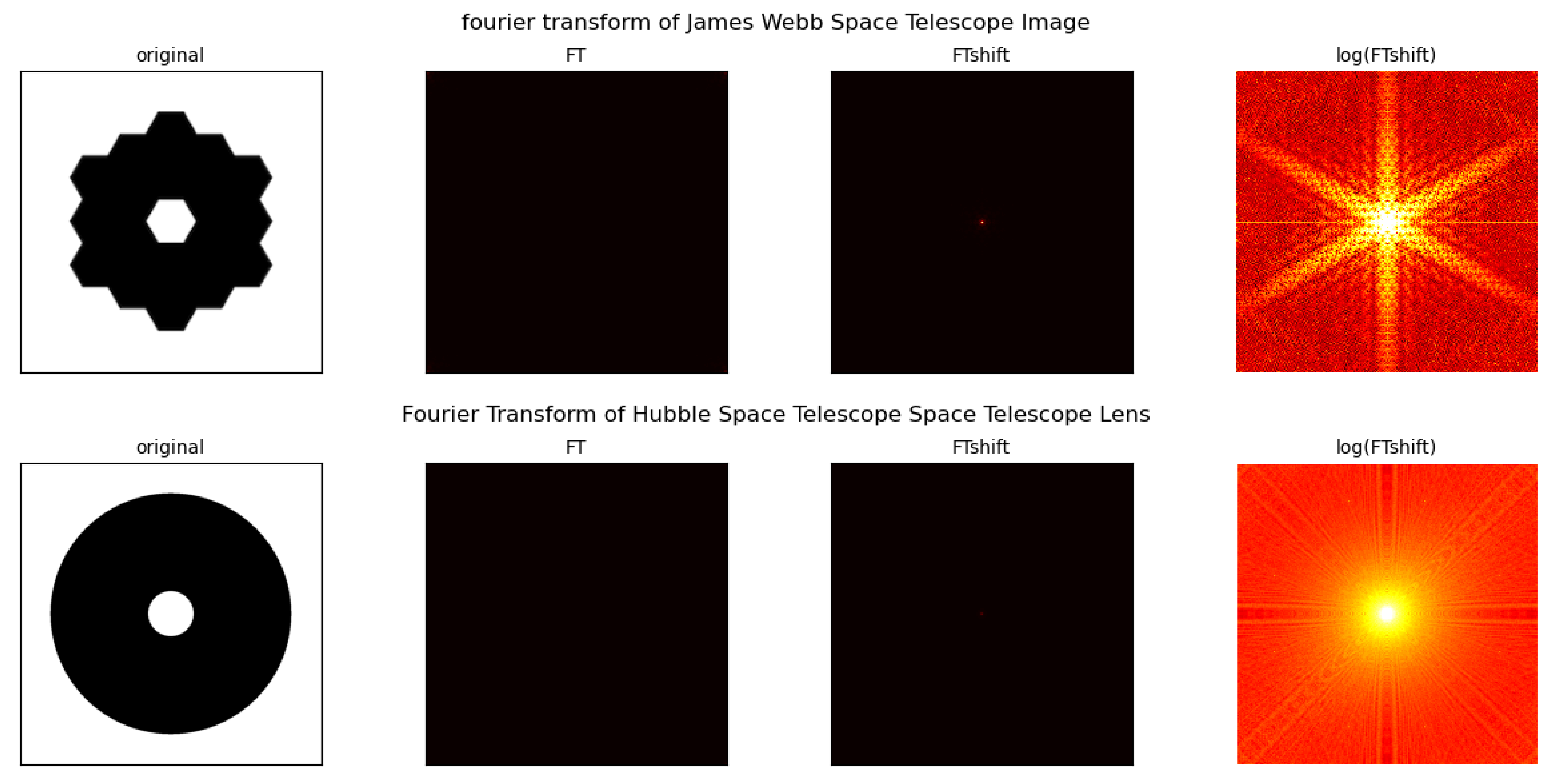
Like the results of the horizontal slit apertures, we see a similar effect where the edges of the shape are smeared and emphasized. But since we used vertical slits in this example, only the vertical lines are emphasized. The narrower slits also result in more smearing of the image.

bonus!

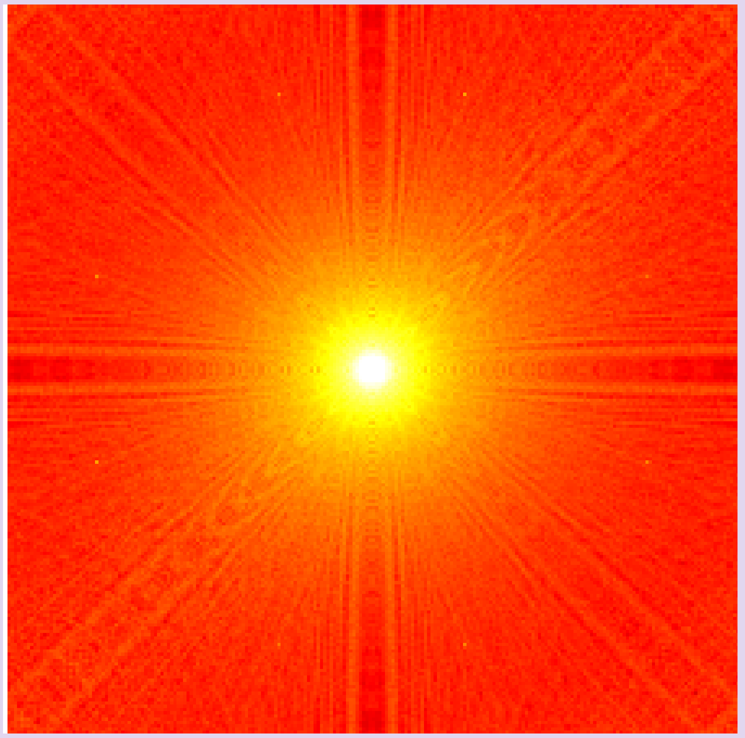


Results

Convolution of JWST and Hubble Lens



Stars can be considered point objects. So to get a simulation of what a star would look like through a telescope lens, we just have to get the fourier transform of the lens system. Here, I've simulated what a star would look like as captured by the Hubble Space Telescope and the James Webb Space Telescope. Comparing these results to actual star images taken by these telescopes, we can see the similarity in their results. For the JWST, we see the signature six-pointed ray of the star. For Hubble, we see the signature four-pointed ray with a noticeable circular haze around the star.

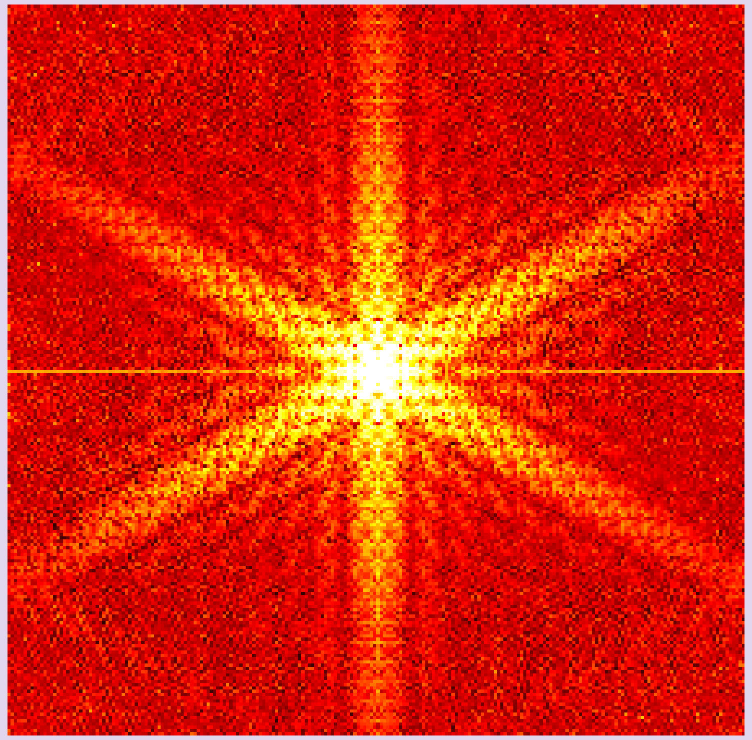


simulated

actual image

hubble space telescope

actual image



simulated

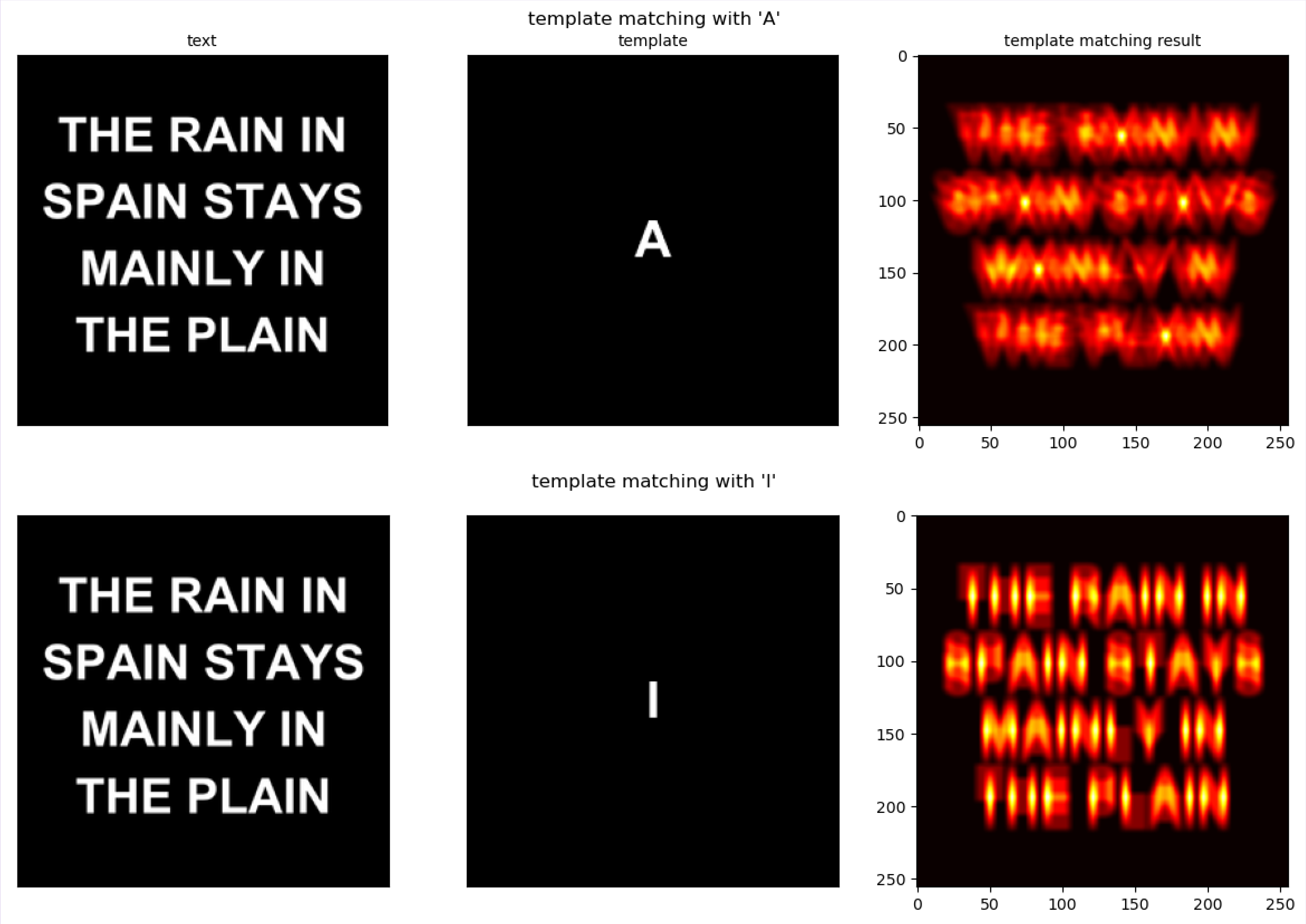
james webb space telescope

Results and Analysis

2.3 Template Matching Using Correlation

Correlation is the process of finding similarities between two different functions. In the case of Image Processing, it is finding an element in one photo that bears resemblance to a chosen image. This application is called template matching.

Here, we've tested out template matching with text. The result shows that the brightest areas are where the image perfectly matches. But we can see that in the case of A, letters with slant still have bright areas in them, denoting that they are similar to the template. Same goes for I, where all horizontal lines in the text are said to be a match.



Reflection

In this experiment, I found myself getting more comfortable working with images in Python. I'm more used to working with data in csv files and dataframes, so I think it's nice that I am able to expand my skillset.

It's also very gratifying that the results I got with my code matches with the expected result. This is why I decided to give myself a full score on technical correctness. All though I do think the star image simulation using the Hubble Space Telescope lens could be better. For me, it just didn't simulate the star as well as the JWST lens did.

But my favorite part of this activity is definitely the convolution technique. I used to do photography as a hobby, so seeing the affect of apertures on images reflected on computer simulations felt super comforting since it was familiar. This is also why I felt motivated to do extra work, which is gave myself 5 points for initiative.

Self-grading

- Technical Correctness = 35
- Quality of Presentation = 35
- Self-reflecton = 30
- Initiative = 5

Total: 105

References

I would also like to cite and thank the following sources for helping me understand the concepts in this activity.

[1] MathematicalOrchid (https://math.stackexchange.com/users/29949/mathematicalorchid), What does the Fourier Transform mean in the context of images?, URL (version: 2012-04-25): https://math.stackexchange.com/q/136826

[2] Dr. Rashi Agarwal. (2014, September 6). Fourier Transformation of a circle to show the ringing effect [Video]. YouTube. https://www.youtube.com/watch?v=H1spd9L0wRc

[3] Soriano, Maricor. (2023). Activity 2. Fourier Transform Model of Image Formation (Part 1 of 2).