University of Technology Nuremberg
Lecture: Introduction to Mobile Robotics
Held at the Technical University of Munich
Winter term 2022/23

Department of Engineering

Prof. Dr. Wolfram Burgard
Michael Krawez
Reihaneh Mirjalili

Sheet 1

Topic: Linear Algebra

Exercise 1: Linear Algebra

(a) Consider the matrices

$$\mathbf{A} = \begin{pmatrix} 0.25 & 0.1 \\ 0.2 & 0.5 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 0.25 & -0.3 \\ -0.3 & 0.5 \end{pmatrix}.$$

Are they positive definite?

(b) For

$$\mathbf{C} = \begin{pmatrix} -3 & 0 \\ 0 & 1 \end{pmatrix},$$

find the largest value for $\mu \in \mathbb{R}$ for which $C + \mu I$ is not positive definite.

(c) Write a program in Python that determines whether a matrix is orthogonal.

(d) Use this program to investigate whether

$$\mathbf{D} = \frac{1}{3} \begin{pmatrix} 2 & 2 & -1 \\ 2 & -1 & 2 \\ -1 & 2 & 2 \end{pmatrix}$$

is orthogonal.

Exercise 1: 2D Transformations as Affine Matrices

Transformations between coordinate frames play an important role in robotics. As background, please refer to the linear algebra slides on affine transformations and transformation combination.

Considering a robot moving on a plane, its pose w.r.t. a global coordinate frame is commonly written as $\mathbf{x} = (x, y, \theta)^T$, where (x, y) denotes its position in the xy-plane and θ its orientation. The homogeneous transformation matrix that represents a pose $\mathbf{x} = (x, y, \theta)^T$ w.r.t. to the origin $(0, 0, 0)^T$ of the global coordinate system is given by

$$T = \begin{pmatrix} \mathbf{R}(\theta) & \mathbf{t} \\ 0 & 1 \end{pmatrix}, \mathbf{R}(\theta) = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix}, \mathbf{t} = \begin{pmatrix} x \\ y \end{pmatrix}$$

(a) While being at pose $\mathbf{x_1} = (x_1, y_1, \theta_1)^T$, the robot senses a landmark l at position $\mathbf{l} = (l_x, l_y)$ w.r.t. to its local frame. Use the matrix T_1 to calculate the coordinates of $\mathbf{l}$ w.r.t. the global frame.

(b) Now imagine that you are given the landmark's coordinates w.r.t. to the global frame. How can you calculate the coordinates that the robot will sense in its local frame?

(c) The robot moves to a new pose $\mathbf{x_2} = (x_2, y_2, \theta_2)^T$ w.r.t. the global frame. Find the transformation matrix T_{12} that represents the new pose w.r.t. to $\mathbf{x_1}$. *Hint:* Write T_{12} as a product of homogeneous transformation matrices.

(d) The robot is at position $\mathbf{x_2}$. Where is the landmark $\mathbf{l} = (l_x, l_y)$ w.r.t. the robot's local frame now?