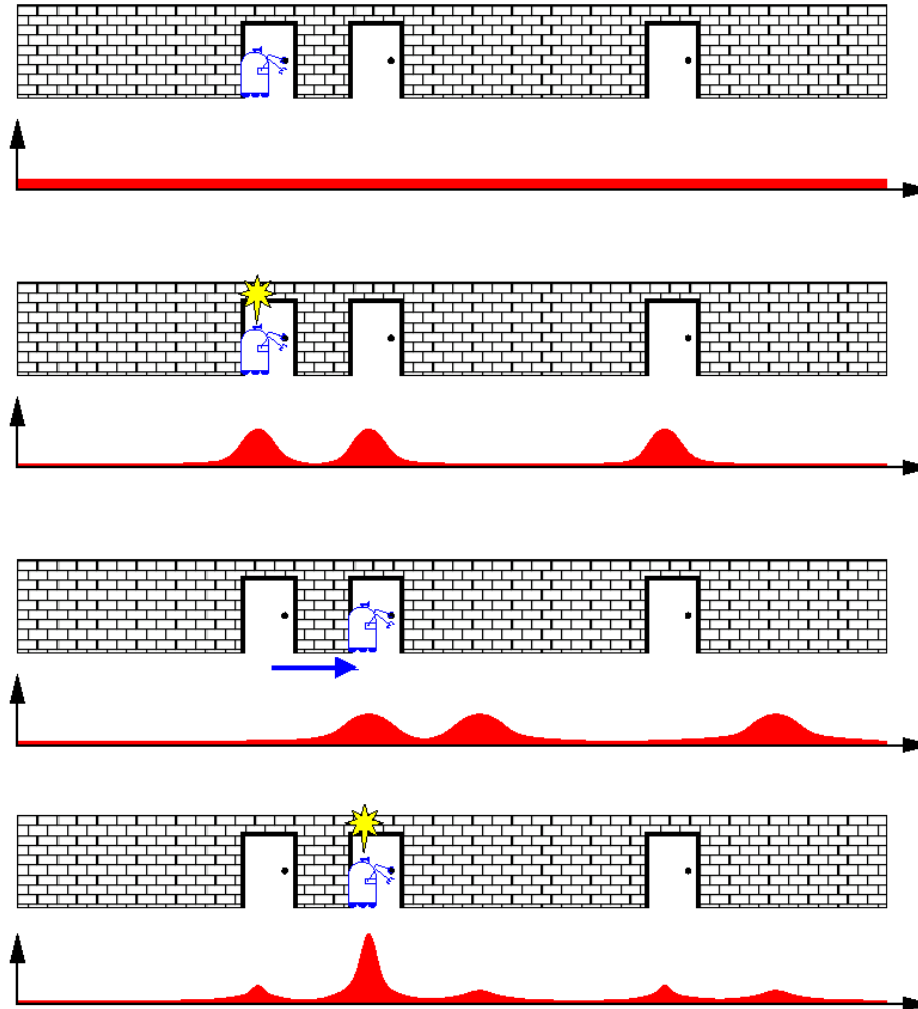


# Introduction to Mobile Robotics

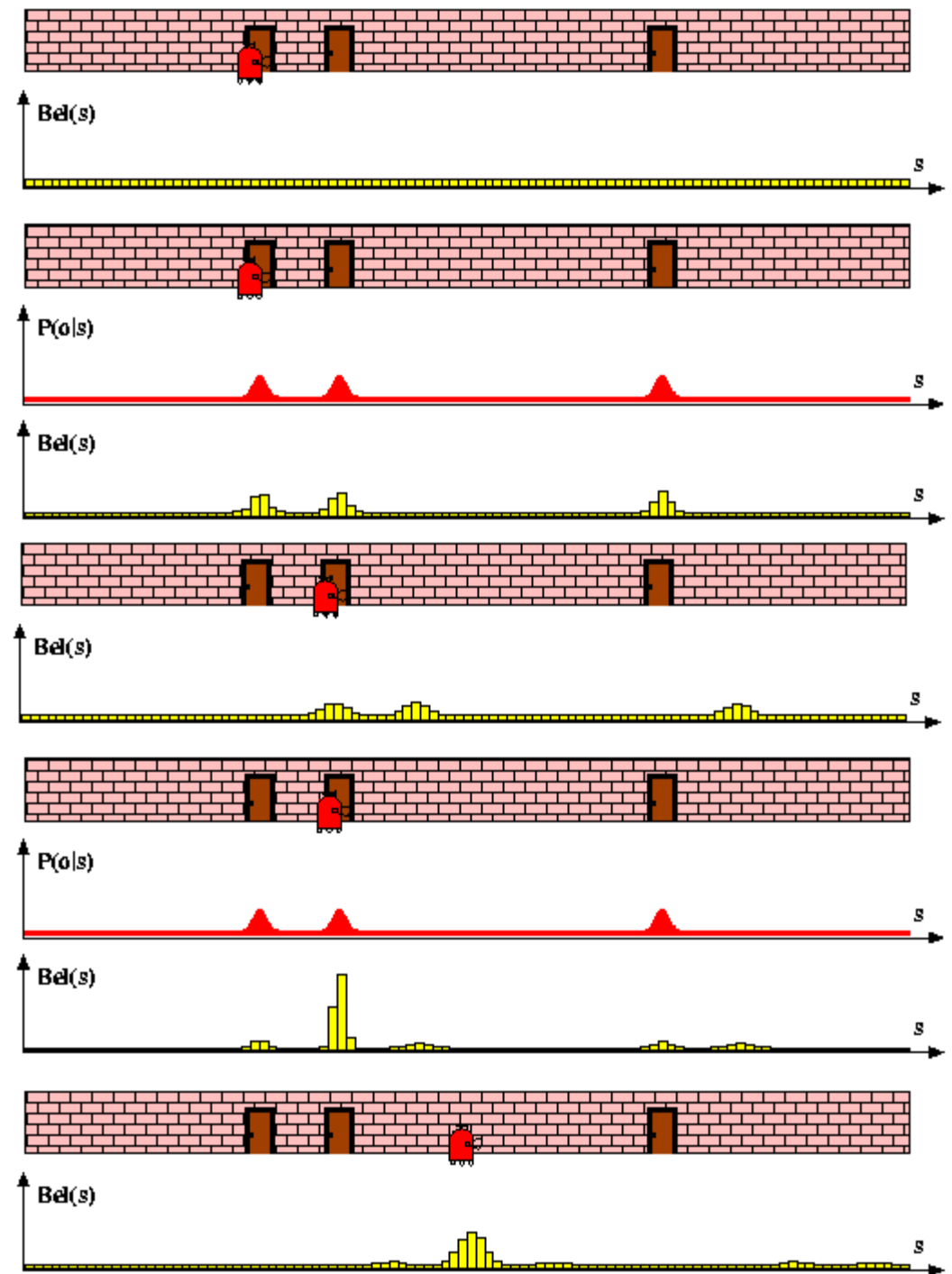
## Bayes Filter – Discrete Filters

# Probabilistic Localization

$$Bel(x \mid z, u) = \alpha p(z \mid x) \int_{x'} p(x \mid u, x') Bel(x') dx'$$



# Piecewise Constant

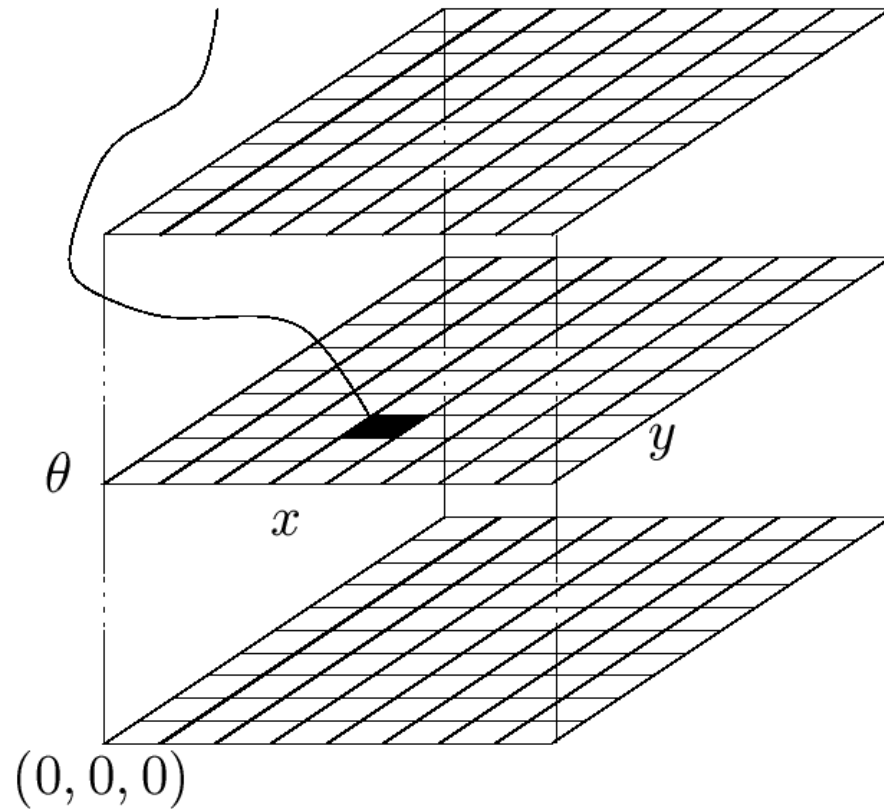


# Discrete Bayes Filter Algorithm

1. Algorithm **Discrete\_Bayes\_filter**(  $Bel(x), d$  ):
2.  $\eta = 0$
3. If  $d$  is a perceptual data item  $z$  then
  4. For all  $x$  do
  5.  $Bel'(x) = P(z | x) Bel(x)$
  6.  $\eta = \eta + Bel'(x)$
  7. For all  $x$  do
  8.  $Bel'(x) = \eta^{-1} Bel'(x)$
9. Else if  $d$  is an action data item  $u$  then
  10. For all  $x$  do
  11.  $Bel'(x) = \sum_{x'} P(x | u, x') Bel(x')$
12. Return  $Bel'(x)$

# Piecewise Constant Representation

$$Bel(x_t = \langle x, y, \theta \rangle)$$



# Implementation (1)

- To update the belief upon sensory input and to carry out the normalization one has to iterate over all cells of the grid.
- Especially when the belief is peaked (which is generally the case during position tracking), one wants to avoid updating irrelevant aspects of the state space.
- One approach is to update only sub-spaces of the state space.
- This, however, requires to monitor the relevant sub-spaces.
- To identify localization errors, a typical approach is to monitor the likelihood of the observations given the active sub-spaces.

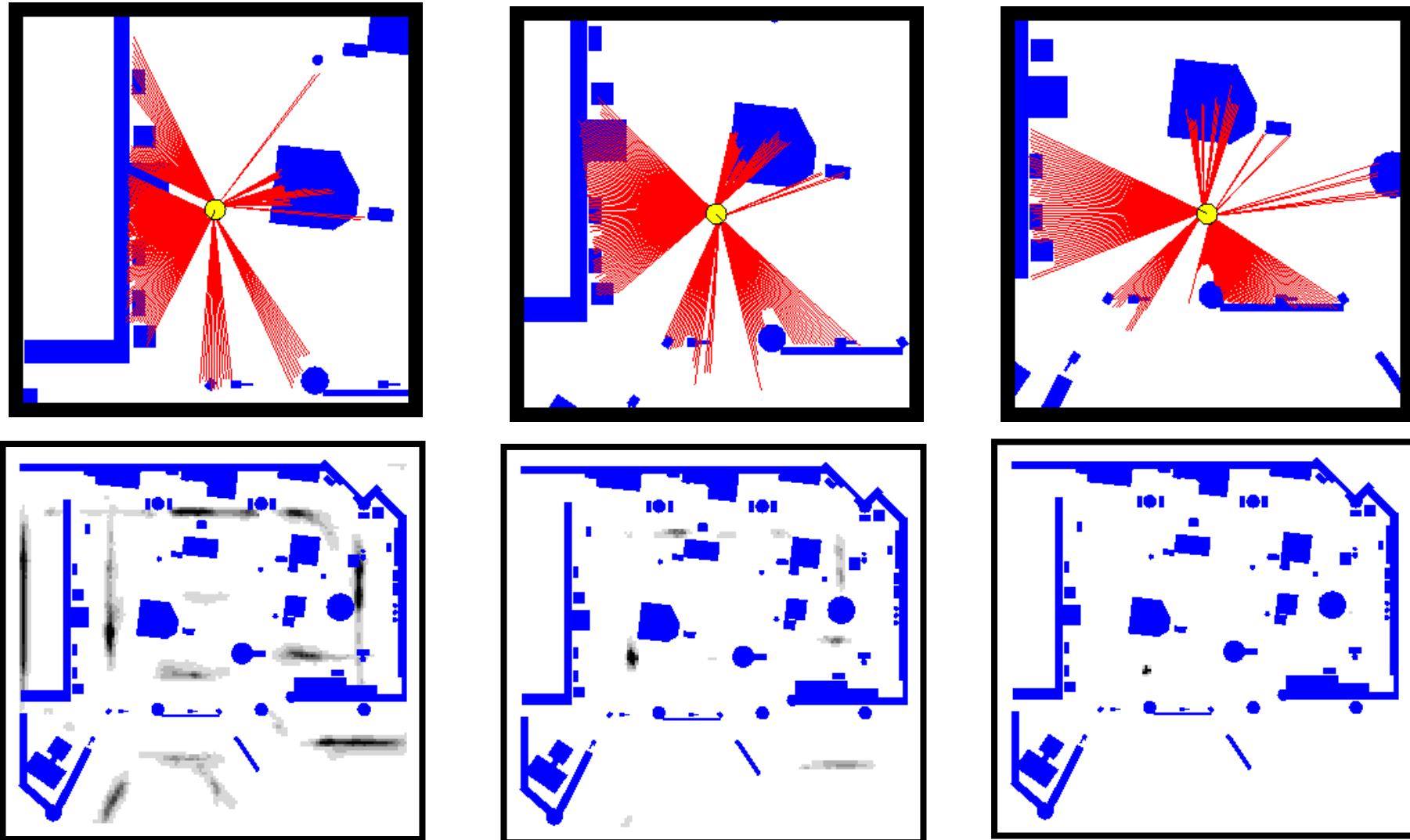
# Implementation (2)

- To efficiently update the belief upon robot motions, one typically assumes a bounded Gaussian model for the motion uncertainty.
- This reduces the update cost from  $O(n^2)$  to  $O(n)$ , where  $n$  is the number of states.
- The update can also be realized by shifting the data in the grid according to the measured motion.
- In a second step, the grid is then convolved using a separable Gaussian Kernel.
- Two-dimensional example:

$$\begin{array}{ccc} 1/16 & 1/8 & 1/16 \\ \boxed{1/8} & \boxed{1/4} & \boxed{1/8} \\ 1/16 & 1/8 & 1/16 \end{array} \cong \begin{array}{c} 1/4 \\ \boxed{1/2} \\ 1/4 \end{array} + \begin{array}{c} 1/4 \\ \boxed{1/2} \\ 1/4 \end{array}$$

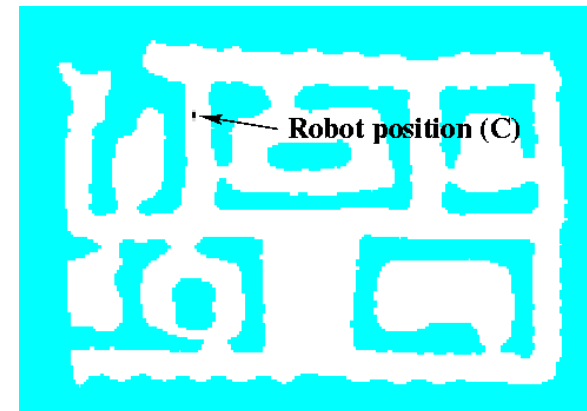
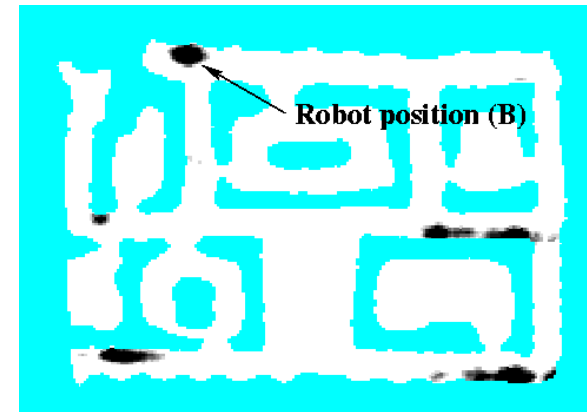
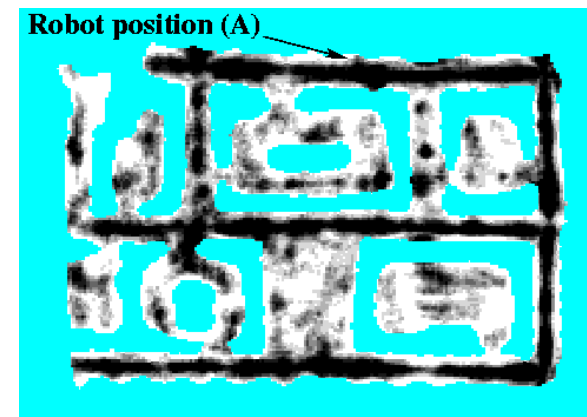
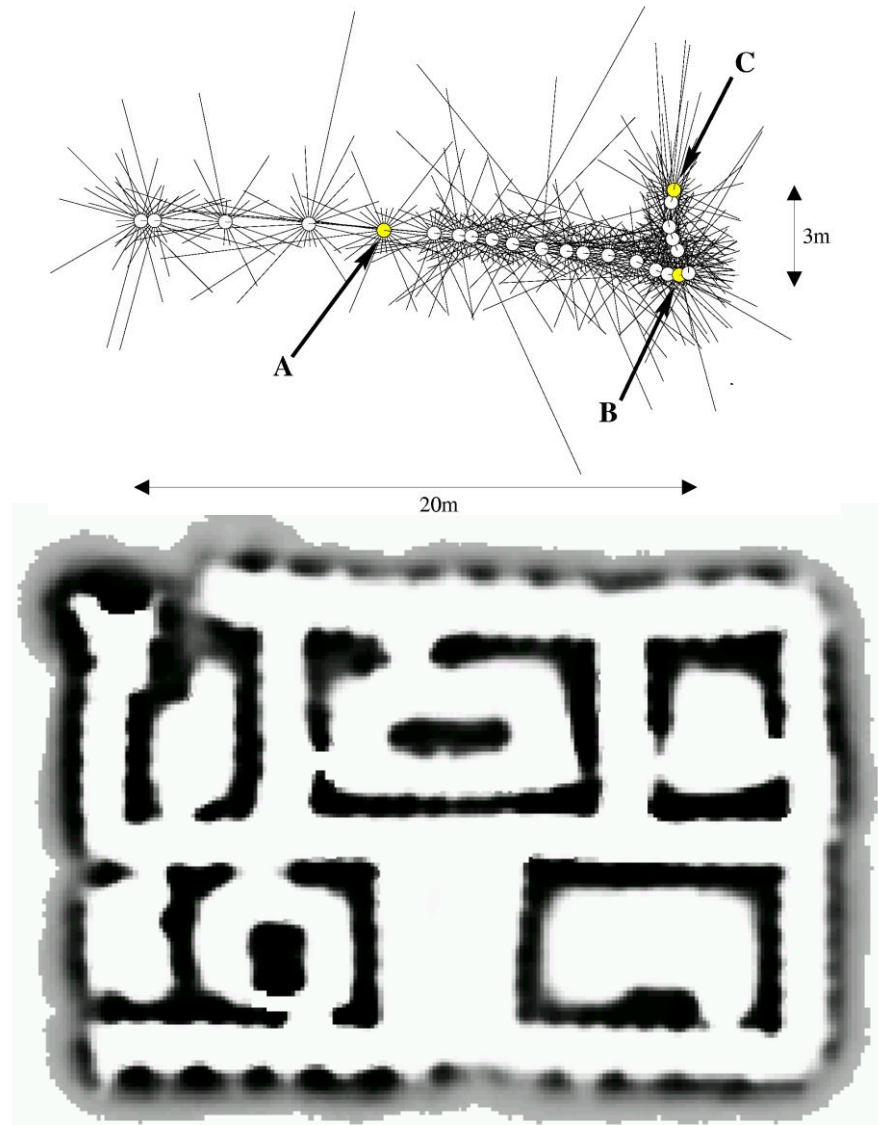
- Fewer arithmetic operations
- Easier to implement

# Grid-based Localization





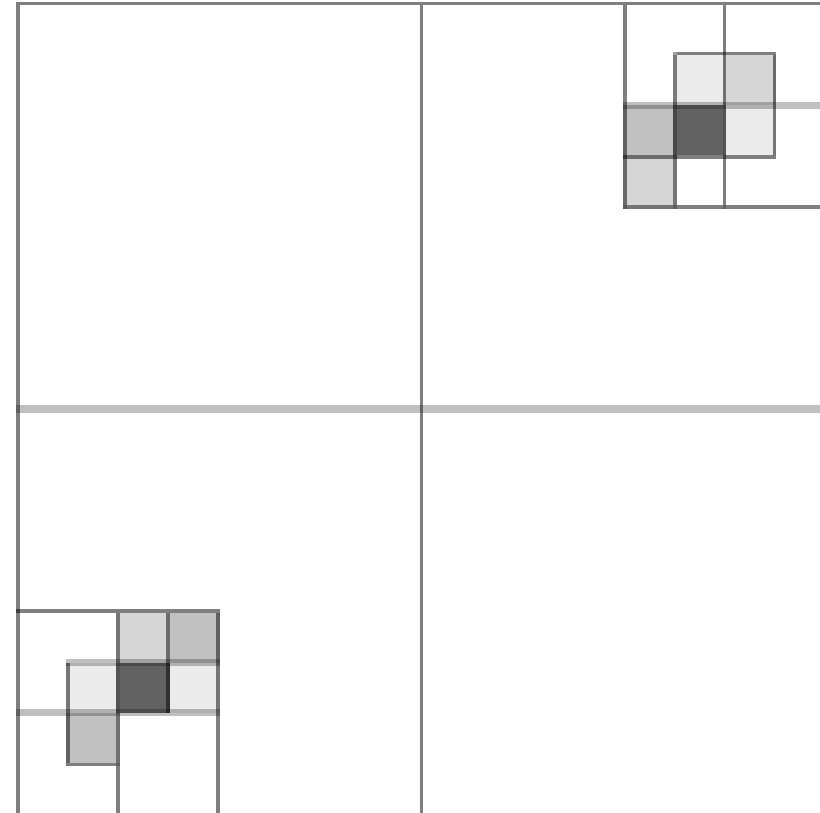
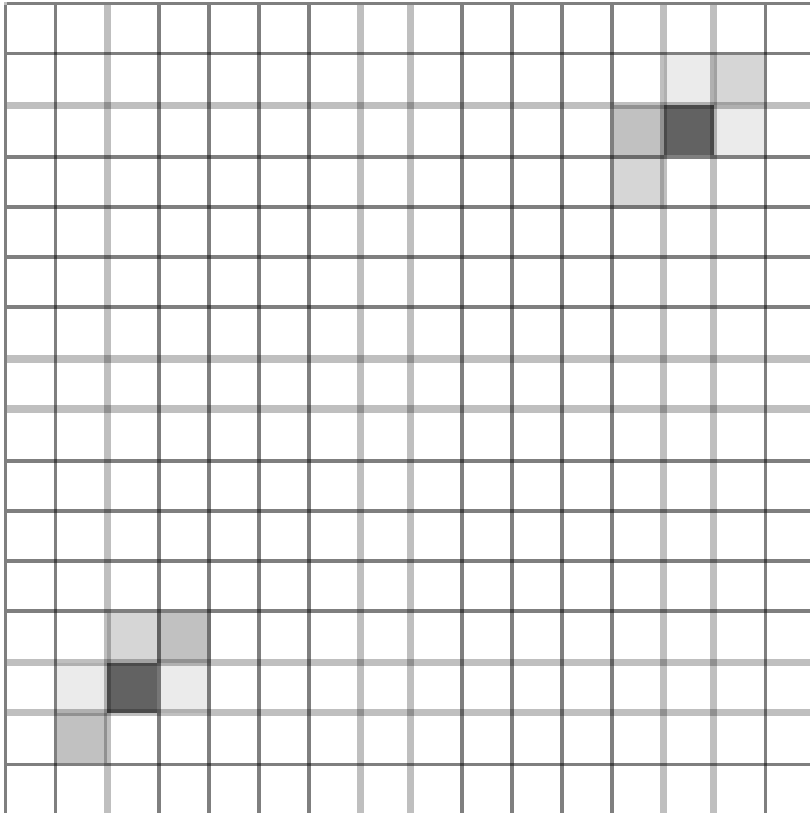
# Sonars and Occupancy Grid Map



# Tree-based Representation

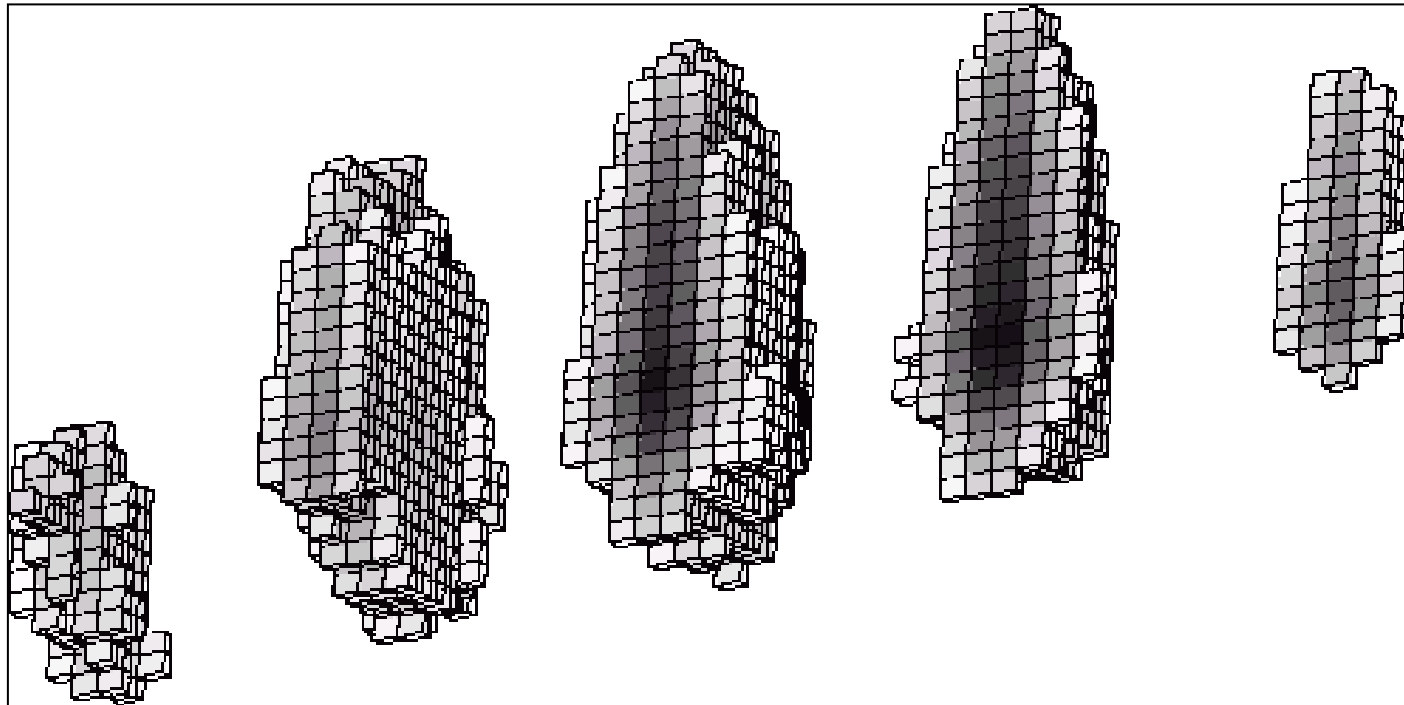
**Idea:** Represent density using a variant of octrees

**Quad-Tree:**



# Tree-based Representations

- Efficient in space and time
- Multi-resolution



# Summary

- Discrete filters are one way for implementing Bayes Filters
- They are based on histograms for representing the density.
- They have huge memory and processing requirements
- Can easily recover from localization errors
- Their accuracy depends on the resolution of the grid.
- Special approximations need to be made to make this approach having dynamic memory and computational requirements.