Laboratory Report 2:RC Circuit Analysis

Circuit Theory and Electronics Fundamentals

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1 Introduction

The objective of this laboratory assignment is to analyze a circuit using the mesh and the nodal method as well as running a simulation using NgSpice to detect small diferences between the different approaches and understand why said differences happen. The circuit can be seen in Figure 1.

In Section 2, a theoretical analysis of the circuit is presented. In Section 3, the circuit is analysed by simulation and the results are compared to the theoretical results obtained in Section 2. The conclusions of this study are outlined in Section 4.

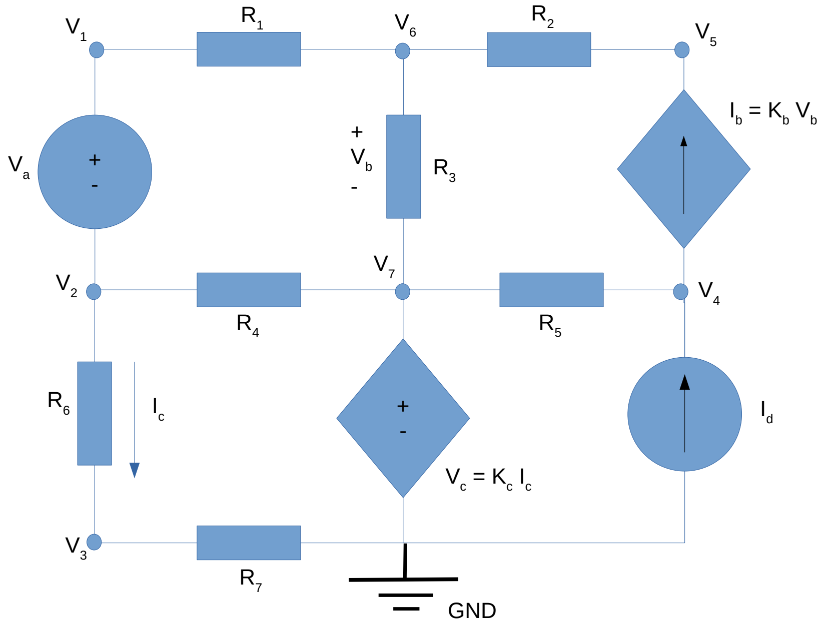

Figure 1: Circuit with an independent current and voltage source (V_a and I_d respectively) and linear dependent sources (V_c-linear current controlled voltage source and I_b-linear voltage controlled current source

The values given for this report can be found in table 1.

Name	Values
R1	1.01949191994 KΩ
R2	2.05054429461 KΩ
R3	3.09286027724 KΩ
R4	4.12838973576 KΩ
R5	3.06635427647 KΩ
R6	2.01254230153 KΩ
R7	1.00502981701 KΩ
Va	5.24204797361 V
C	1.01905568201 F
Kb	7.23185131759 mS
Kc	8.12820254987 KΩ

Table 1: Values received by the Python program that can be found in folder $python$.

2 Theoretical Analysis

2.1 Mesh Analysis

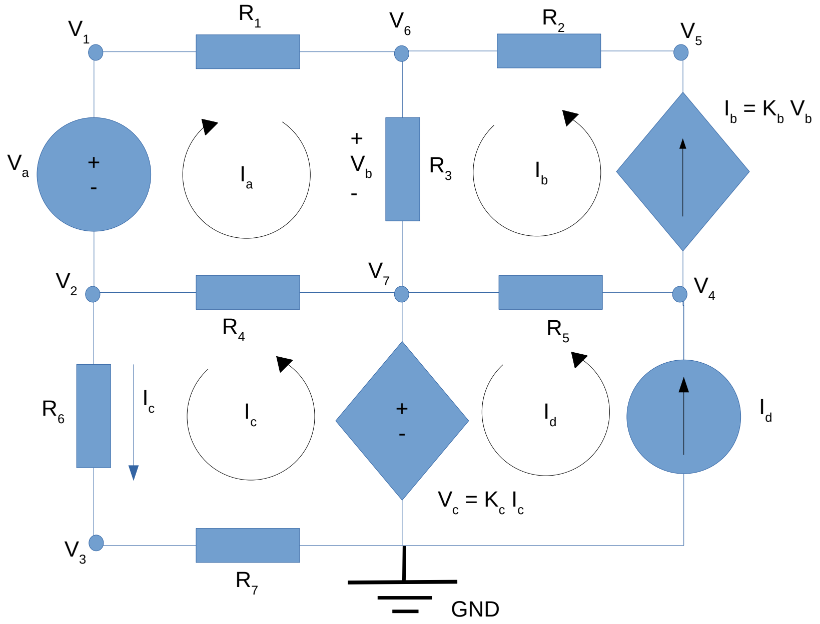

Figure 2: Representation of mesh currents in the circuit.

Figure 2 shows the mesh currents considered for the circuit analysis, with the current I_a flowing clockwise and the rest of the currents (I_b, I_c and I_d) flowing counter-clockwise. In the meshes containing I_b and I_d, the currents were considered to be the same as the current sources in said meshes.

From this circuit, there can then be extracted 3 equations to figure out the value of the components necessary for the circuit analysis.

The first one, Equation 1, was obtained by using Ohm's Law, assuming it is known the value of the voltage and the resistance in resistor 3 and that the current flowing through it is (I_a + I_b).

$$I_b = K_b(I_a + I_b)R_3, \tag{1}$$

Equation 2 was figured out by analysing the top left mesh, using Kirchoff's Voltage Law and Ohm's Law for the resistors. Since the current I_a is flowing clockwise, the voltage in V_a is negative and the currents in resistors 3 and 4 are, correspondingly, (I_a + I_b) and (I_a + I_c), as these pairs of currents are flowing the same way in said resistors.

$$-V_a + I_aR_1 + (I_a + I_b)R_3 + (I_a + I_c)R_4 = 0, \tag{2}$$

Finally, from the bottom left mesh, there is Equation 3, in which was also used Kirchoff's Voltage Law and Ohm's Law. The voltage in V_c is negative due to the current flow.

$$-K_cI_c + I_cR_6 + I_cR_7 + (I_a + I_c)R_4 = 0, \tag{3}$$

By developing these 3 equations, the matrix below (4) is achieved as to simplify the calculations. This matrix was solved in Octave, getting the values of the currents I_a, I_b and I_c that can be found in table 2. It was not necessary to solve for the value of the current in the bottom right mesh since it is already known (equivalent to I_d).

$$\begin{bmatrix} -K_bR_3 & 1-K_bR_3 & 0 \\ R_1+R_3+R_4 & R_3 & R_4 \\ R_4 & 0 & R_6+R_7-K_c+R_4 \end{bmatrix} \times \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 0 \\ V_a \\ 0 \end{bmatrix} \tag{4}$$

With these currents, it is possible to discover the values of the voltages in each node, using the equations 5 through 11 down below and knowing that I_b = K_bV_b and V_c=K_cI_c.

$$V_7 = V_c, \tag{5}$$

$$V_6 = V_7 + V_b, \tag{6}$$

$$V_5 = V_6 + R_2I_b, \tag{7}$$

$$V_4 = V_7 + R_5(I_d - I_b), \tag{8}$$

$$V_3 = V_0 + R_7I_c, \tag{9}$$

$$V_2 = V_3 + R_6I_c, \tag{10}$$

$$V_1 = V_2 + V_a, \tag{11}$$

Table 2 shows the nodes' voltages discovered by replacing the known variables in equations 5 to 11. The branch currents were obtained with the following equations 12 to 17 (resorting to figure 2).

$$R_1[i] = I_a, \tag{12}$$

$$R_2[i] = I_b, \tag{13}$$

$$R_3[i] = I_a + I_b, \tag{14}$$

$$R_4[i] = I_a + I_c, \tag{15}$$

$$R_5[i] = Id - Ib, \tag{16}$$

$$R_6[i] = R_7[i] = I_c, \tag{17}$$

Node	Voltage[V]	Branch	Current[A]
V_b	-3.39424400e-02	I_b	-2.45466679e-04
V_c	8.01098169e+00	I_c	9.85578501e-04
V_1	8.21610218e+00	$R_1 = I_a$	2.34492229e-04
V_2	2.97405421e+00	R_2	-2.45466679e-04
V_3	9.90535781e-01	R_3	-1.09744498e-05
V_4	1.18884552e+01	R_4	1.22007073e-03
V_5	7.47369895e+00	R_5	1.26452236e-03
V_6	7.97703925e+00	R_6	9.85578501e-04
V_7	8.01098169e+00	R_7	9.85578501e-04

Table 2: Voltage and Current values using the mesh analysis.

2.2 Nodal Analysis

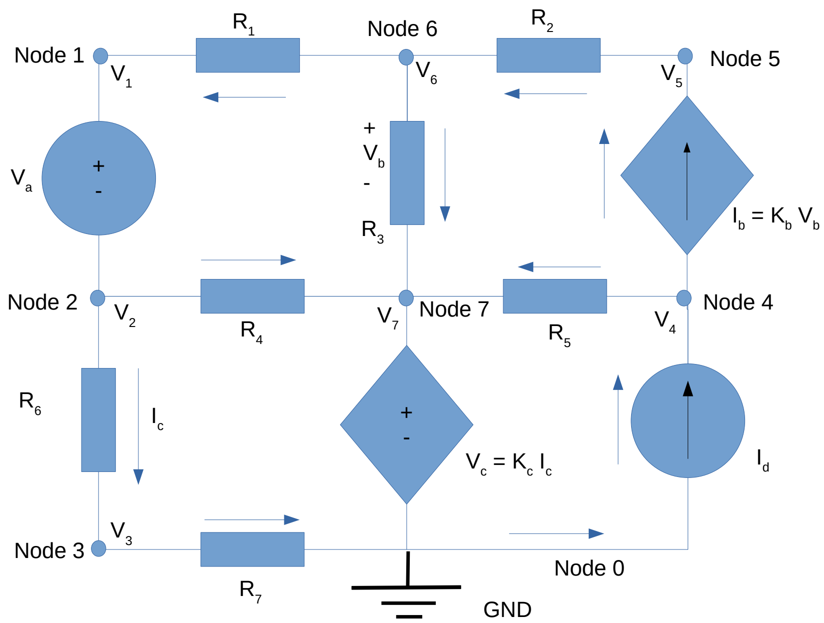

Figure 3: Representation of the circuit with emphasis on its nodes.

In order to consistantly analyze this circuit with nodal method, we had figure 1 as a reference to build figure 3, having considered the same notation for both. In the whole following analysis, we have considered that the current flows according to the direction suggested by the arrow next to each branch.

The first thing which must be noticed is that there is an independent voltage source (V_a) and a linear current controlled voltage source (V_c) in this circuit. Knowing that a nodal analysis can't include the analysis of nodes that are connected to voltage sources, it becomes clear that is useless to analyze nodes 1 and 2 (connected to V_a) and also nodes 0 and 7 (connected to V_c) using this method.

From figure 3, we can easily conclude that there are 11 unknown variables: V_b I_b V_c I_c V_1 V_2 V_3 V_4 V_5 V_6 V_7. If we look at the position we chose for the GND (Ground), we can also infer that value of V_0 is 0.

Then, it is mandatory to consider 11 linearly independent equations to reach all the values corresponding to the mentioned variables and consequently solve the circuit. There are 2 equations (given by the Professor - 18 and 19) explicitly written in figure 3 which are related to the two linear dependent sources:

$$K_b V_b - I_b = 0, \tag{18}$$

$$K_c I_c - V_c = 0, \tag{19}$$

As referred above, it is possible to use the nodal method to analyze nodes 3, 4, 5 and 6.

Considering the current I_c flowing from node 2 to node 3 and the current passing through R_7 diverging from node 3, the equation below (20) was figured out by using Kirchoff's Current Law for node 3 and Ohm's Law for the resistors R_6 and R_7. To correctly use the Ohm's Law for the resistor R_7, it is important to remember that the value we assigned to V_0 is 0.

$$(V_2 - V_3)G_6 - V_3 G_7 = 0, \tag{20}$$

The following equation (21), in which was also used Kirchoff's Current Law and Ohm's Law for resistor R_4, refers to node 4. It was considered I_d is converging to the mentioned node, contrarily to I_b and to the current at R_5 which are both diverging.

$$(V_4 - V_7)G_5 + K_b V_b = I_d, \tag{21}$$

Since I_b is flowing from node 4 to 5 and the current measured at R_2 is flowing from node 5 to 6, Kirchoff's Circuit Law (for node 5) and Ohm's Law (for R_2) were used to establish the following mathematical relation:

$$K_b V_b - (V_5 - V_6)G_2 = 0, \tag{22}$$

Considering node 6 as a spatial reference, we assumed there were two divergent currents (one flowing to node 1 and the other to node 7) and a current converging from node 5. Knowing that, the ensuing equation (23) was figured out by resorting to Ohm's Law for resistors R_1, R_2 and R_3 and Kirchoff's Current Law for node 6.

$$K_b V_b - (V_6 - V_1)G_1 - (V_6 - V_7)G_3 = 0, \tag{23}$$

After the previous analysis there are only 6 equations available to work with. This leads us to the inevitability of considering 5 additional equations.

The equation written below (24) is a direct consequence of our choice of connecting node 0 to the ground (GND), because this choice makes evident that the value of V_0 is 0 and then:

$$V_7 - V_c = 0, \tag{24}$$

By observing the branches which contain (respectively) the independent voltage source V_a and the linear dependent voltage source V_c, it is also trivial that:

$$V_1 - V_2 = V_a, \tag{25}$$

$$V_b - V_6 + V_7 = 0, \tag{26}$$

It is mandatory to relate I_c with other unknown variables. So, this equation (27) was obtained by looking at the bottom left mesh and using Ohm's Law for resistor R_7.

$$V_3 G_7 - I_c = 0, \tag{27}$$

Ultimately, to discover the last equation, there are some theoretical concepts that must be considered.

Kirchoff's Current Law implies that there is no current stuck at any node. It is also known that neither voltage sources nor resistors retain current. Then, any branch that only contains one of the said elements does not retain current as well. Merging the two branches placed on the left side of circuit (branch containing V_a with branch containing R_6) and calling Supernode to the result of this merger, it is still true that no current is retained in the Supernode. Then, considering that there is current converging from node 6 and assuming that current is diverging from Supernode to nodes 0 and 7, the resultant equation (28) is the one written right below.

$$(V_2 - V_7)G_4 + V_3G_7 - (V_6 - V_1)G_1 = 0, \tag{28}$$

A system with 11 linearly independent equations and 11 variables is of course possible to solve but not easy (and certainly not pratical) to deal with. The following matrix equation (29) summarizes the 11 referred equations so it is easier to read and to instantaneously solve (by using Octave).

$$\begin{bmatrix} K_b & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & K_c \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & G_7 & 0 & 0 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & G_6 & -G_6 - G_7 & 0 & 0 & 0 & 0 & 0 & 0 \\ K_b & 0 & 0 & 0 & 0 & G_5 & 0 & 0 & -G_5 & 0 & 0 \\ K_b & 0 & 0 & 0 & 0 & 0 & -G_2 & G_2 & 0 & 0 & 0 \\ K_b & 0 & G_1 & 0 & 0 & 0 & 0 & -G_1 - G_3 & G_3 & 0 & 0 \\ 0 & 0 & G_1 & G_4 & G_7 & 0 & 0 & -G_1 & -G_4 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} V_b \\ V_c \\ V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ V_a \\ 0 \\ 0 \\ 0 \\ I_d \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{29}$$

To find all the required branches' currents we can use Ohm's Law for each resistor, which can be seen in equations 30 to 35

$$R_1[i] = \frac{V_6 - V_1}{R_1}, \tag{30}$$

$$R_2[i] = I_b, \tag{31}$$

$$R_3[i] = \frac{V_b}{R_3}, \tag{32}$$

$$R_4[i] = \frac{V_2 - V_7}{R_4}, \tag{33}$$

$$R_5[i] = \frac{V_4 - V_7}{R_5}, \tag{34}$$

$$R_6[i] = R_7[i] = I_c, \tag{35}$$

The result of this analysis is reflected in table 3, which establishes a mathematical relation between every mentioned variables and its respective values.

Node	Voltage[V]	Branch	Current[A]
V_b	-3.39424400e-02	I_b	-2.45466679e-04
V_c	8.01098169e+00	I_c	9.85578501e-04
V_1	8.21610218e+00	R_1	-2.34492229e-04
V_2	2.97405421e+00	R_2	-2.45466679e-04
V_3	9.90535781e-01	R_3	-1.09744498e-05
V_4	1.18884552e+01	R_4	-1.22007073e-03
V_5	7.47369895e+00	R_5	1.26452236e-03
V_6	7.97703925e+00	R_6	9.85578501e-04
V_7	8.01098169e+00	R_7	9.85578501e-04

Table 3: Voltage and Current values using the nodal analysis.

3 Simulation Analysis

In this section we proceed to do the anlysis of the circuit through the use of the Ngspice simulation programm. In figure 4 we have the circuit that was inputed into Ngspice (and also the considered current flows and nodes). The file can be found at the sim folder inside the $T1$ folder.

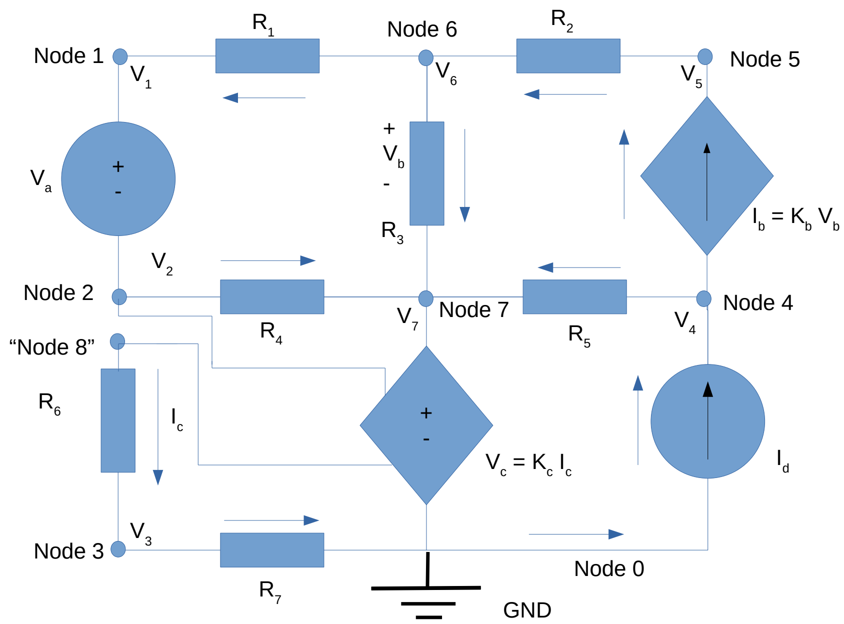

Figure 4: Considered circuit for Ngspice simulation

V_8 refers to an extra ficticious node created specifically for the Ngspice simulation, and it is below V_2 and above resistor R6 as it can be seen above in figure 4.The reason this node is necessary is because when creating a current controlled voltage source, Ngspice gets the current value by refering to a voltage source from where the current goes through. Since I_c does not go through any voltage source in the circuit (does not go through V_a) we used this extra node to create a voltage source of $0V$ (Which can be confirmed since $V_8 - V_2 = 0$) from which we are certain I_c is passing by.

Table 4 shows the simulated operating point results for the circuit under analysis given the values found on Table 1. The variables representation and format are automatically determined by Ngspice.

Name	Value [A or V]
@gib[i]	-2.45467e-04
@r1[i]	-2.34492e-04
@r2[i]	-2.45467e-04
@r3[i]	-1.09744e-05
@r4[i]	-1.22007e-03
@r5[i]	2.454667e-04
@r6[i]	9.855785e-04
@r7[i]	9.855785e-04
v1	5.242048e+00
v2	5.002985e+00
v3	4.499645e+00
v5	5.036927e+00
v6	5.789615e+00
v7	-1.98352e+00
v8	-2.97405e+00
v9	0.000000e+00

Table 4: Operating point. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt. (the g in "gib" refers to the Ngspice notation of a voltage controlled current source)

We can get all the missing values given the equations showed in section 2.

$$V_c = V_7 \tag{36}$$

$$V_b = V_6 - V_7 => V_b = -3.3943 * 10^{-2} V \tag{37}$$

From Table 4 we can directly get the value of I_c:

$$I_c = @r6[i] \tag{38}$$

With this we have finalized the circuit anlysis through simulation with Ngspice.

4 Conclusion

By analysing the circuit theoretically, with both the mesh and the node methods, and then simulating the circuit using Ngspice, we can verify that the values of the unknown components match almost perfectly and all approaches agree on the final currents' directions across the circuit's branches (which can be seen below in figure 5).

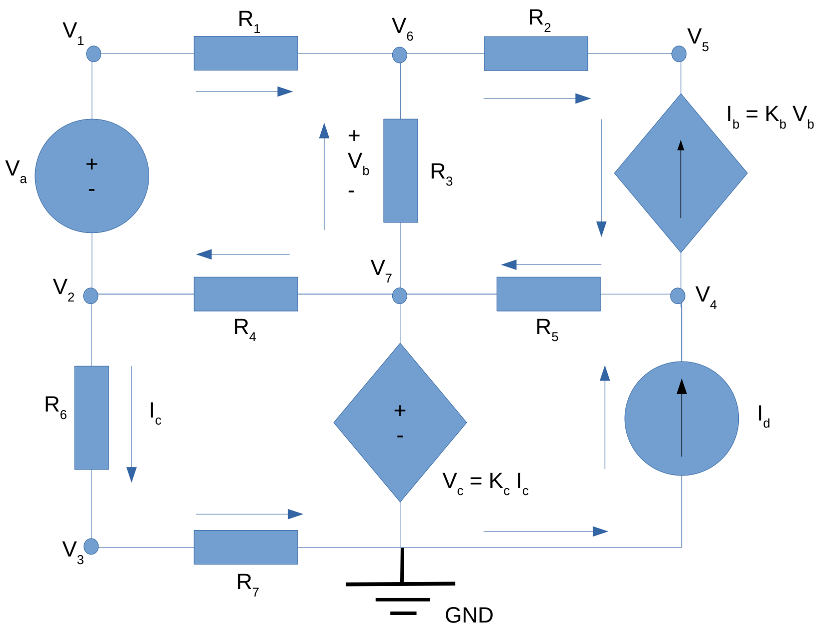

Figure 5: Representation of the final circuit with all the correct directions for the currents.

Some small discrepancies like the value of V_b obtained in section 3 differing from the ones found on sections 2.1 and 2.2 are due to the small number of decimal places considered by Ngspice leading to slight inaccuracies. However, considering that the circuit complexity is still not considered, the differences are negligible.

In any case, the node analysis uses the Kirchoff Current Law while the Mesh Analysis uses the Kirchoff Voltage Law. This means that if a circuit contains more voltage sources than current sources, the mesh method is going to be more exact, as it is the other way around.

In this circuit, both types of sources have the same number of components; a fact that might help justify the accuracy of the results. Even more, it only includes linear components, making it so that the theoretical values are expected to be almost the same as the ones obtained in the simulation. Furthermore, as already mentioned, the low complexity of the circuit makes it that both methods are still extremely reliable.

All of this leads to the conclusion that the making of this laboratory assignment was coherent and that the main goal was attained: to achieve the circuit analysis through 3 different methods (mesh, nodal and simulated analysis).