Laboratory Report 2:RC Circuit Analysis

Circuit Theory and Electronics Fundamentals

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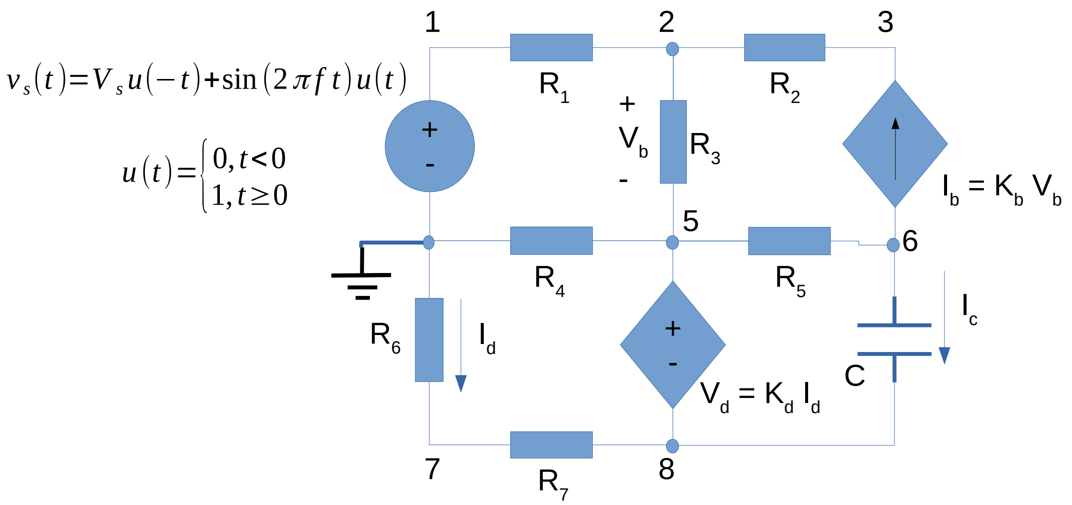
Contents

1 Introduction

The objective of this laboratory assignment is to analyze a RC circuit to find the natural and forced response as well as doing a frequency analysis. Furthermore, it is asked to run a simulation using NgSpice to detect small diferences between the different approaches and understand why said differences happen. The circuit can be seen in Figure 1.

In Section 2, a theoretical analysis of the circuit is presented. In Section 3, the circuit is analysed by simulation and the results are compared to the theoretical results obtained in Section 2. The conclusions of this study are outlined in Section 4.

Figure 1: RC Circuit with alternate voltage source (V_s), linear dependent sources (V_d-linear current controlled voltage source and I_b-linear voltage controlled current source) and capacitor C



The values given for this report can be found in table 1(Obtained with the number 95772).

Name	Values
R1	1.01949191994 KΩ
R2	2.05054429461 KΩ
R3	3.09286027724 KΩ
R4	4.12838973576 KΩ
R5	3.06635427647 KΩ
R6	2.01254230153 KΩ
R7	1.00502981701 KΩ
Va	5.24204797361 V
C	1.01905568201 uF
Kb	7.23185131759 mS
Kd	8.12820254987 KΩ

Table 1: Values obtained by using the Python program that can be found in folder $python$.

2 Theoretical Analysis

In this section we will present the answers to questions 2.1 to 2.6. For solving purposes, all the unkown currents used in the node analysis were considered to be diverging from the node. The node V_4 is connected to the ground (GND) in every exercise, the voltage in it being 0V.

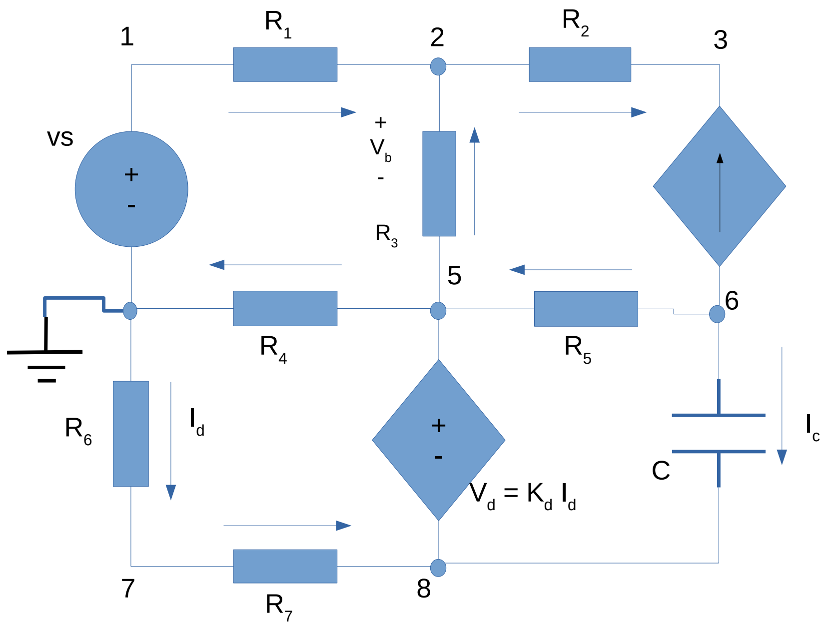


Figure 2: Representation of the current directions considered.

2.1 Exercise 1

The first thing which must be noticed is that there is an independent voltage source (V_s) and a linear current controlled voltage source (V_d) in this circuit. Knowing that a nodal analysis can't include the analysis of nodes that are connected to voltage sources, it becomes clear that is useless to analyze nodes 1 and 4 (connected to V_s) and also nodes 5 and 8 (connected to V_d) using this method.

From figure 2, we can easily conclude that there are 11 unknown variables: V_b, I_b, V_d, I_d, V_1, V_2, V_3, V_5, V_6, V_7 and V_8. In this exercise, V_s is constant and the capacitor is assumed to be also constant and fully charged, meaning that the current I_c is 0 (open circuit behavior).

For the node analysis, it is necessary to consider 7 linearly independent equations to reach all the values corresponding to the voltages in each node and consequently solve the circuit. As referred above, it is possible to use the nodal method to analyze nodes 2, 3, 6 and 7.

We begin by establishing the following equations, being that equations 1 and 2 were given by the professor. Equation 3 was obtained by relating the voltage difference between nodes 2 and 5 to the voltage V_b. Finally, by using Ohm's Law for resistor R_6 and remembering that V_4 is null, we get the last equation (4) for I_d.

$$I_b = K_b V_b, \tag{1}$$

$$V_d = K_d I_d, \tag{2}$$

$$V_b = V_2 - V_5, \tag{3}$$

$$I_d = -V_7 G_6, \tag{4}$$

The equation written below (5) is a direct consequence of our choice of connecting node 4 to the ground (GND), because this choice makes evident that the value of V_4 is 0 and then:

$$V_1 - V_s = 0, \tag{5}$$

By analysing node 2 using Kirchoff's Current Law and Ohm's Law for the resistors R_1, R_2 and R_3, we get the following equation:

$$(V_3 - V_1)G_1 + (V_2 - V_3)G_2 + (V_2 - V_5)G_3 = 0, \tag{6}$$

The following equation (7), in which was also used Kirchoff's Current Law and Ohm's Law for resistor R_2, refers to node 3. Here we consider the given equation 1 and the equation 3 to substitute the current I_b.

$$(V_3 - V_2)G_2 - (V_2 - V_5)K_b = 0, \tag{7}$$

For node 6, the ensuing equation (8) was figured out by resorting to Ohm's Law for the resistor R_5 and Kirchoff's Current Law. Equations 1 and 3 were once again used to avoid using I_b. Remember that, for this exercise, I_c is null.

$$(V_6 - V_5)G_5 + (V_2 - V_5)K_b = 0, \tag{8}$$

Finally, for node 7, Kirchoff's Circuit Law and Ohm's Law (for resistors R_6 and R_7) were used to establish the following mathematical relation:

$$V_7 G_6 + (V_7 - V_8)G_7 = 0, \tag{9}$$

Since there are 7 unkown varibles, we need two more equations. The first one (10) is obtained by relating V_d to the voltage difference in nodes 5 and 8 and replacing V_d for the equations 2 and 4.

$$-V_7 G_6 K_d - (V_5 - V_8) = 0, \tag{10}$$

Ultimately, to discover the last equation, there are some theoretical concepts that must be considered. Kirchoff's Current Law implies that there is no current stuck at any node. It is also known that neither voltage sources nor resistors retain current. Then, any branch that only contains one of the said elements does not retain current as well. Merging the two branches placed on the left side of circuit (branch containing V_s with branch containing R_6) and calling Supernode to the result of this merger, it is still true that no current is retained in the Supernode. Then, considering that all unkown currents are diverging from the nodes, the resultant equation (11) is the one written right below.

$$(V_1 - V_2)G_1 - V_5 G_7 - V_7 G_6 = 0, \tag{11}$$

A system with the 7 linearly independent equations and 7 variables (regarding the voltage in each node) is, of course, possible to solve but not easy (and certainly not pratical) to deal with.

The following matrix equation (12) summarizes the 7 referred equations so it is easier to read and to instantaneously solve (by using Octave).

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -G_1 & G_1+G_2+G_3 & -G_2 & -G_3 & 0 & 0 & 0 \\ 0 & -G_2-K_b & G_2 & K_b & 0 & 0 & 0 \\ 0 & K_b & 0 & -K_b-G_5 & G_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & G_6+G_7 & -G_7 \\ G_1 & -G_1 & 0 & -G_4 & 0 & -G_6 & 0 \\ 0 & 0 & 0 & -1 & 0 & -K_dG_6 & 1 \end{bmatrix} \times \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} V_s \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{12}$$

To find all the required branches' currents we can use Ohm's Law for each resistor, which can be seen in equations 13 to 19: Now we no longer consider the currents to be diverging from the nodes, but assume the directions shown in figure 2.

$$R_1[i] = \frac{V_1 - V_2}{R_1}, \tag{13}$$

$$R_2[i] = \frac{V_2 - V_3}{R_2}, \tag{14}$$

$$R_3[i] = \frac{V_5 - V_2}{R_3}, \tag{15}$$

$$R_4[i] = \frac{V_5}{R_4}, \tag{16}$$

$$R_5[i] = \frac{V_6 - V_5}{R_5}, \tag{17}$$

$$R_6[i] = -\frac{V_7}{R_6}, \tag{18}$$

$$R_7[i] = \frac{V_7 - V_8}{R_7}, \tag{19}$$

Here we have table 2 in which the final values of the unkown variables are printed.

Node	**Voltage[V]**	**Branch**	**Current[A]**
V_b	-3.39424400e-02	I_b	-2.45466679e-04
V_d	8.01098169e+00	I_c	0.00000000e+00
V_1	5.24204797e+00	$R_6 = I_d$	9.85578501e-04
V_2	5.00298504e+00	R_1	2.34492229e-04
V_3	4.49964474e+00	R_2	2.45466679e-04
V_5	5.03692748e+00	R_3	1.09744498e-05
V_6	5.78961528e+00	R_4	1.22007073e-03
V_6	-1.98351843e+00	R_5	2.45466679e-04
V_8	-2.97405421e+00	R_7	9.85578501e-04

Table 2: Voltage and Current values(Exercise 1)

2.2 Exercise 2

To solve this exercise, we first needed to get the value of V_x, which we did by applying the equation given by the professor (25), using the values of V_6 and V_8 we got from the matrix 12. This value corresponds to the voltage in the capacitor that now functions as a voltage source. This happens because, as mentioned before, we can assume that an infinite time as passed until t=0 and it is fully charged, meaning it starts with the voltage given before we start the time.

To get the value intended of R_eq, it is necessary to achieve the value of the current supplied by the new voltage source (V_x), I_x. This is done by the following node analysis.

Since V_s is now null, it can be ignored, making it so that node 1 is the same as node 4, with a voltage of 0V. This means that the node 4 that could not be used for node analysis before provides another equation. However, the voltage source V_x is connected to node 6, this one being no longer useful. For this exercise, we have 6 unkown variables for the node voltages (V_2, V_3, V_5, V_6, V_7 and V_8) and we can analyse the nodes 2, 3, 4 and 7.

These 3 equations are all obtained from the exercise above (reference to equations 6, 7 and 9) to, but assuming that V_1 is null, since in the nodes 2, 3 and 7, nothing has changed from the previous circuit, when it comes to connected components.

$$(V_3 - V_1)G_1 + (V_2 - V_3)G_2 + (V_2 - V_5)G_3 = 0, \tag{20}$$

$$(V_3 - V_2)G_2 - (V_2 - V_5)K_b = 0, \tag{21}$$

$$V_7G_6 + (V_7 - V_8)G_7 = 0, \tag{22}$$

From node 1, we can achieve the equation below by using Kirchoff's Circuit Law and Ohm's Law for resistors R_1, R_6 and R_7.

$$-V_7G_6 - V_5G_4 - V_2G_1 = 0, \tag{23}$$

The following equations are also trivial. The first one, equation 24 is explained in the first exercise (reference to equation 10) and equation 25 is given by the professor and can be used as the value of V_x is constant.

$$-V_7G_6K_d - (V_5 - V_8) = 0, \tag{24}$$

$$V_6 - V_8 = V_x, \tag{25}$$

These equations are now enough to solve for the value of V_2, V_3, V_5, V_6, V_7 and V_8, using the matrix below. This was solve in Octave as it was not pratical to solve it normally and the results are presented in table 3.

$$\begin{bmatrix} G_1+G_2+G_3 & -G_2 & -G_3 & 0 & 0 & 0 \\ -G_2-K_b & G_2 & K_b & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & G_6+G_7 & -G_7 \\ -G_1 & 0 & -G_4 & 0 & -G_6 & 0 \\ 0 & 0 & -1 & 0 & -K_dG_6 & 1 \\ 0 & 0 & 0 & 1 & 0 & -1 \end{bmatrix} \times \begin{bmatrix} V_2 \\ V_3 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ V_x \end{bmatrix} \tag{26}$$

The value of I_x is achieved by using Kirchhoff Current Law (KCL) in node 6. We considered the currents to be diverging from the node. From this, the equation below was obtained.

$$I_x + (V_2 - V_5)K_b + (V_5 - V_6)G_6 = 0, \tag{27}$$

Finally, we get the value of R_{eq}:

$$R_{eq} = \frac{V_x}{I_x}, => R_{eq} = \frac{8.76366949e+00}{2.85800945e-03} <=> R_{eq} = 3.06635428e+03\Omega \quad (28)$$

The value of τ is $R_{eq} * C$ which is equal to $3.12478575e-03\Omega * uF$.

Node	Voltage[V]	Branch	Current[A]
V_b	-0.00000000e+00	I_b	0.00000000e+00
V_d	-0.00000000e+00	$R_6 = I_d$	-0.00000000e+00
V_1	0.00000000e+00	R_1	0.00000000e+00
V_2	-0.00000000e+00	R_2	-0.00000000e+00
V_3	0.00000000e+00	R_3	0.00000000e+00
V_5	-0.00000000e+00	R_4	-0.00000000e+00
V_6	8.76366949e+00	R_5	2.85800945e-03
V_6	0.00000000e+00	R_7	0.00000000e+00
V_8	-0.00000000e+00	I_x	2.85800945e-03
V_x	8.76366949e+00		

Component	Value
R_{eq}	3.06635428e+03 Ω
τ	3.12478575e-03 $\Omega * uF$

Table 3: Voltage and Current values(Exercise 2)

2.3 Exercise 3

For this exercise, we used the capacitor voltage V_x for t<0 as the initial condition, as sugested. Using the equivalent resistance R_eq calculated in the previous exercise, we can assume that the natural solution for V_6 is like the one on equation 29, as this is a RC series.

$$V_{6n}(t) = Ae^{-\frac{t}{RC}}, \quad (29)$$

Since the amplitude for which V_6 varies is given by the initial condition V_6 = V_x + V_8, the A in the equation above can be replaced by those two voltages. The value for V_8 was the one calculated on exercise 2 (2.2). The equation for the natural solution V_6 is then represented below as the graph of V_6n in function of t, in the interval [0, 20] ms (3).

$$V_{6n}(t) = (V_x + V_8)e^{-\frac{t}{RC}}, \quad (30)$$

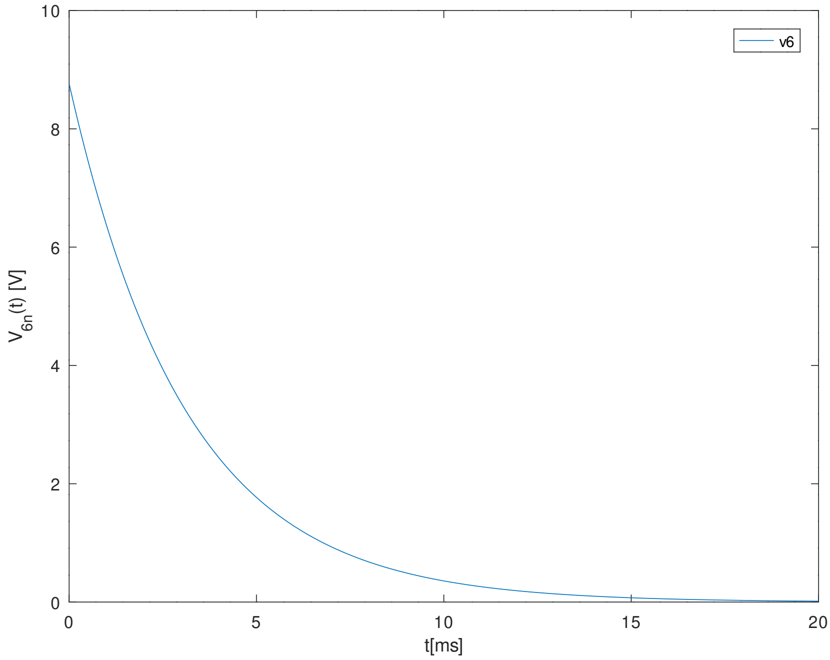
2.4 Exercise 4

For this exercise, to find the forced solution V_6f(t), we will work with the frequency f = 1000 Hz and the phasor voltage source V_s. Since the voltage source for t>0 is v_s(t) = sen(2πft), to simplify the calculations and as suggested, we will work with the absolute of V_s, which was given and it is 1V.

The node analysis for this exercise is pretty similar to the one in 2.1 as the voltage source is once again V_s and the capacitor no longer works as a voltage source. This means that we now have 7 varibles V_1, V_2, V_3, V_5, V_6, V_7 and V_8 and we can use the node method in nodes 2, 3, 6 and 7, since all the other ones are connected to voltage sources.

Equations 31 to 35 were all obtained from the node analysis in exercise 1. The first three are achieved, correspondingly, by nodes 2,3 and 7. The second to last equation refers to the current controlled voltage source V_d. Finally, the last one is the supernode that we got from merging the two branches placed on the left side of circuit.

Figure 3: Natural Response of $v_{6n}(t)$ in the interval $[0, 20]ms$ using $V_x(t < 0)$ as the initial condition



$$(V_3 - V_1)G_1 + (V_2 - V_3)G_2 + (V_2 - V_5)G_3 = 0, \tag{31}$$

$$(V_3 - V_2)G_2 - (V_2 - V_5)K_b = 0, \tag{32}$$

$$V_7G_6 + (V_7 - V_8)G_7 = 0, \tag{33}$$

$$-V_7G_6K_d - (V_5 - V_8) = 0, \tag{34}$$

$$(V_1 - V_2)G_1 - V_5G_7 - V_7G_6 = 0, \tag{35}$$

The seventh and final equation (39) necessary to complete the matrix is obtained by analysing node 6, using Kirchoff's Circuit Law and Ohm's Law for the resistor R_5. The equation for V_b was secured previously (3). Because we are working with phasors, equations 36 and 38 are used to figure out the current flowing through the capacitor, by using its impendance Z_c.

$$I_cZ_c = V_c, \tag{36}$$

$$Z_c = \frac{1}{jwC}, \tag{37}$$

$$Y_c = \frac{1}{Z_c}, \tag{38}$$

$$(V_6 - V_5)G_5 + (V_2 - V_5)K_b + (V_6 - V_8)Y_c = 0, \tag{39}$$

From this we get the following matrix. Knowing that $w = 2\pi * f$ with $f = 1kHz$, the matricial equation was solved in Octave. The results are printed on the table 4.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -G_1 & G_1+G_2+G_3 & -G_2 & -G_3 & 0 & 0 & 0 \\ 0 & -G_2-K_b & G_2 & K_b & 0 & 0 & 0 \\ 0 & K_b & 0 & -K_b-G_5 & G_5+Y_c & 0 & -Y_c \\ 0 & 0 & 0 & 0 & 0 & G_6+G_7 & -G_7 \\ G_1 & -G_1 & 0 & -G_4 & 0 & -G_6 & 0 \\ 0 & 0 & 0 & -1 & 0 & -K_dG_6 & 1 \end{bmatrix} \times \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} V_s \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{40}$$

Node	**Complex amplitude**
V_1	$(1.00000000e+00)*exp(j*-1.23188228e-18)$
V_2	$(9.54395127e-01)*exp(j*2.24403353e-16)$
V_3	$(8.58375346e-01)*exp(j*4.72719969e-15)$
V_5	$(9.60870161e-01)*exp(j*-3.16349236e-17)$
V_6	$(5.69389779e-01)*exp(j*1.49717904e-01)$
V_7	$(3.78386164e-01)*exp(j*3.66762560e-17)$
V_8	$(5.673345858e-01)*exp(j*-0.00000000e+00)$

Table 4: Complex amplitudes in all nodes(Exercise 4)

Passing the obtained values into real time function the following expression for the forced solution is obtained: $v_6(t)$ = $(-5.63020126e-01)$*sin(2*pi*1000*t)

2.5 Exercise 5

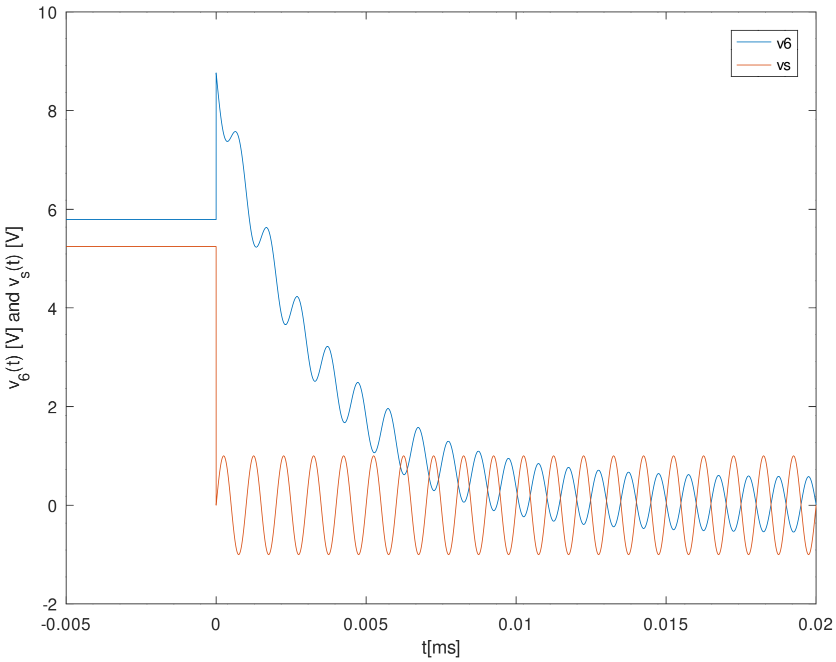
To figure out V_6, we simply had to convert the phasor to real time fuctions, being so that the value of V_6 is just the sum of the natural solution with the forced solution, calculated in exercises 3 and 4. The equation for V_s(t) is given and it is written below, as the one for V_6(t). The frequency used was 1000 Hz.

$$V_s(t) = \sin 2\pi f t, \tag{41}$$

$$V_6(t) = (V_x + V_8)e^{-\frac{t}{RC}} + V_{6f}(t), \tag{42}$$

Finally, in figure 4, we have the graph for both V_s and V_6 in function of t, in the interval [-5, 20] ms. As stated in the simultation analyis, it was expected for V_s to not show a frequency response, since it is the source of frequency change. The same is not the case for V_6 as is depends on V_s.

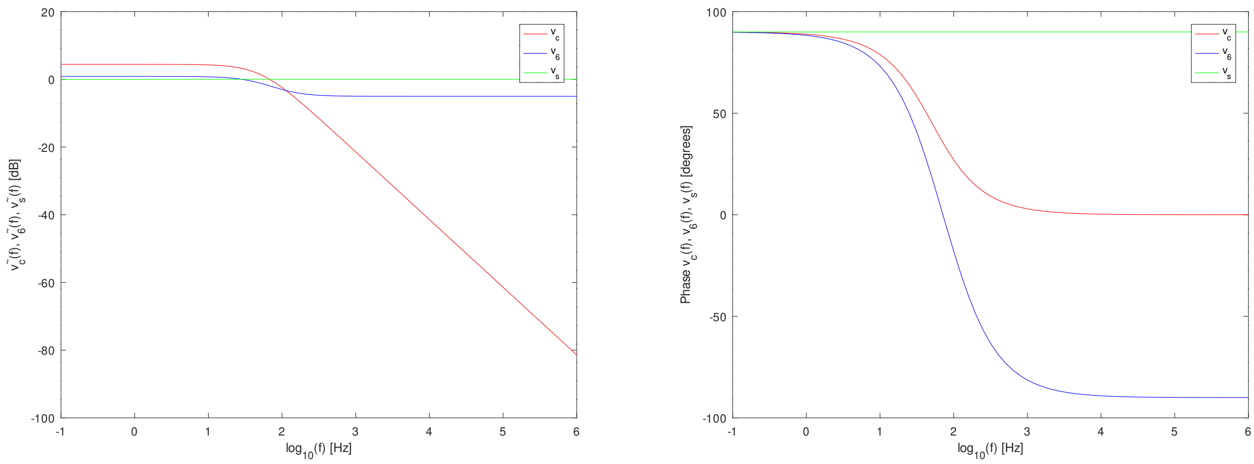
Figure 4: Final solution of $v_6(t)$ and $v_s(t)$ in the interval $[-5, 20]ms$



2.6 Exercise 6

We know that $v_c(f)$ and $v_6(f)$ only depend on frequency. Using the method applied on section 2.4 but for a vector of frequency values, allows for a determintation of all phasors in function of frequency. After determining $\tilde{V}_6$ and $\tilde{V}_8$ we can find $\tilde{V}_c$, since $\tilde{V}_c = \tilde{V}_6 - \tilde{V}_8$, getting all thdata required to plot the magnitude in dB and phase in degrees of $v_c(f)$, $v_6(f)$ and $v_s(f)$.

Figure 5: Frequency response, $v_c(f)$, $v_6(f)$, and $v_s(f)$ in V. Magnitude in dB and Phase is in degrees



The graphics above confirmed our expectations seen that, considering V_s = 1 (as suggested by the Professor) and analyzing a simple equivalent RC circuit, it becomes clear that $V_c = \frac{1}{1+jwRC}$. Having this in consideration, it is trivial to infer that V_c decreases when frequency increases. Applying the same thinking to V_6 (which is equal to $V_c + V_8$, with V_8 being independent from the frequency), this voltage must also decrease when frequency increases.

3 Simulation Analysis

3.1 Exercise 1

In this section we proceed to do the anlysis of the circuit through the use of the Ngspice simulation program. In figure 6 we have the circuit that was inputed into Ngspice (and also the considered current flows and nodes). The file can be found at the sim folder inside the $T2$ folder.

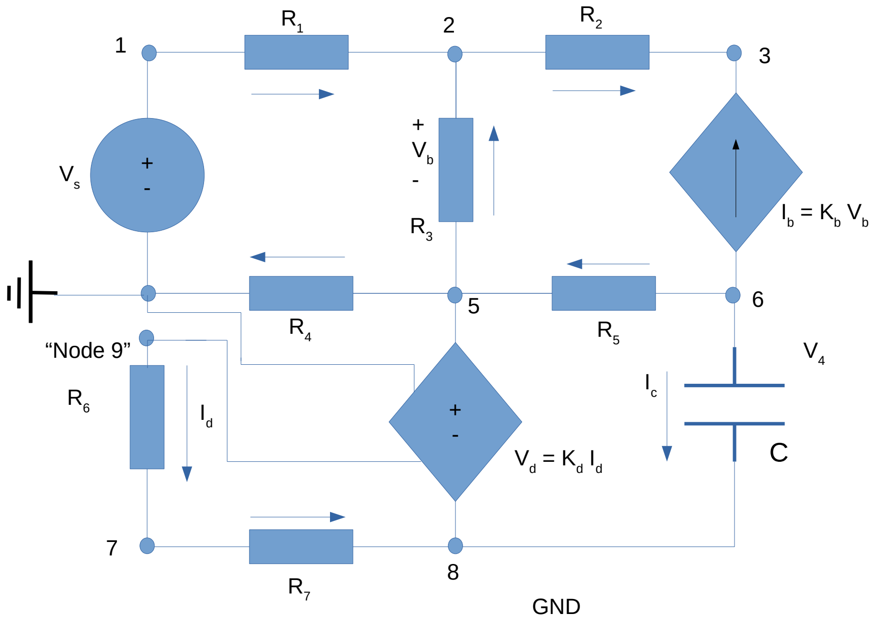


Figure 6: Considered circuit for Ngspice simulation

V_9 refers to an extra ficticious node created specifically for the Ngspice simulation, and it is below $0(GND)$ and above resistor R6 as it can be seen above in figure 6.The reason this node is necessary is because when creating a current controlled voltage source, Ngspice gets the current value by refering to a voltage source from where the current goes through. Since I_d does not go through any voltage source in the circuit (does not go through V_s) we used this extra node to create a voltage source of $0V$ (Which can be confirmed since $V_9 =$GND$= 0$) from which we are certain I_d is passing by.

Table 5 shows the simulated operating point results for the circuit under analysis given the values found on Table 1, considering t<0, which means $v_S(t)$ = V_s as seen in Figure 1. The variables representation and format are automatically determined by Ngspice.

We can get all the missing values given the voltage different of the nodes where they are defined.

$$V_b = v(v2, v5) = -3.39424e - 02V \tag{43}$$

$$V_d = v(v5, v8) = 8.010982e + 00V \tag{44}$$

Table 5: Operating point for $t < 0$. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt. (the g in "gib" refers to the Ngspice notation of a voltage controlled current source)

Name	**Value [A or V]**
@c1[i]	0.000000e+00
@gib[i]	-2.45467e-04
@r1[i]	2.344922e-04
@r2[i]	2.454667e-04
@r3[i]	1.097445e-05
@r4[i]	1.220071e-03
@r5[i]	2.454667e-04
@r6[i]	9.855785e-04
@r7[i]	9.855785e-04
v1	5.242048e+00
v2	5.002985e+00
v3	4.499645e+00
v5	5.036927e+00
v6	5.789615e+00
v7	-1.98352e+00
v8	-2.97405e+00
v9	0.000000e+00

3.2 Exercise 2

In this section, we simulate the operating point for $v_s(0) = 0$, replacing the capacitor with a voltage source $V_x = V_6 - V_8$ using the values of the respective nodes as obtained in section 3.1.

It is required to replace the capacitor since we can assume that an infinite time as passed until t=0 and it is fully charged, meaning it starts with the voltage given before we start the time.

Table 6 shows the simulated operating point results for the circuit under analysis given the values found on Table 1, considering t=0, and the above mentioned considerations.

Table 6: Operating point for $t = 0$, $v_s(0) = 0$ and capacitor replaced with $V_x = V_6 - V_8$. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt. (the g in "gib" refers to the Ngspice notation of a voltage controlled current source)

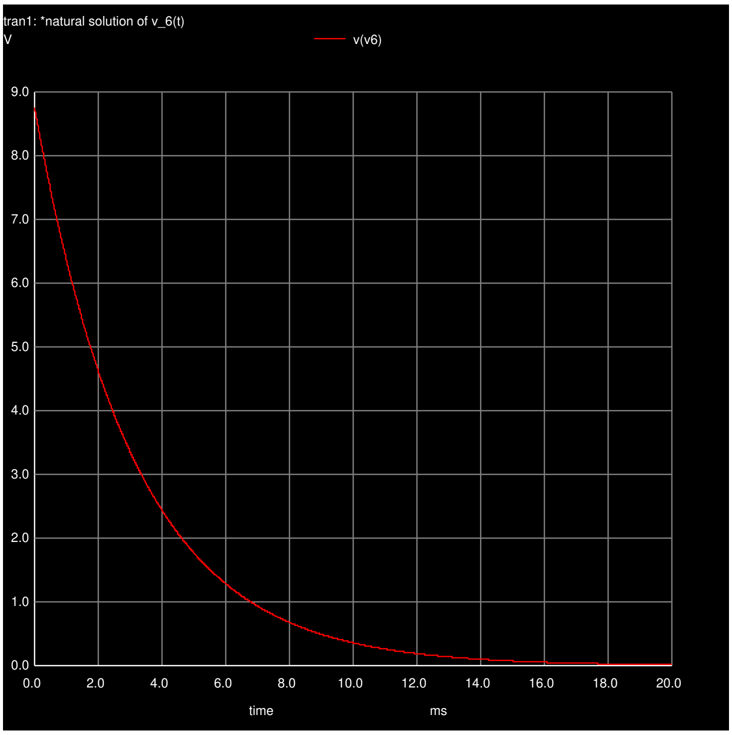
Name	**Value [A or V]**
@gib[i]	0.000000e+00
@r1[i]	0.000000e+00
@r2[i]	0.000000e+00
@r3[i]	0.000000e+00
@r4[i]	0.000000e+00
@r5[i]	2.858009e-03
@r6[i]	0.000000e+00
@r7[i]	0.000000e+00
v1	0.000000e+00
v2	0.000000e+00
v3	0.000000e+00
v5	0.000000e+00
v6	8.763669e+00
v7	0.000000e+00
v8	0.000000e+00
v9	0.000000e+00

3.3 Exercise 3

In this section we used the results obtained in section 3.2 to determine the natural solution $v_{6n}(t)$ in the interval $[0, 20]ms$ using the boundary conditions V_6 and V_8.

Figure 7 shows the plot of the required result that was obtained using the transient analysis.

Figure 7: Natural solution, $v_{6n}(t)$ in V in the interval $[0, 20]ms$

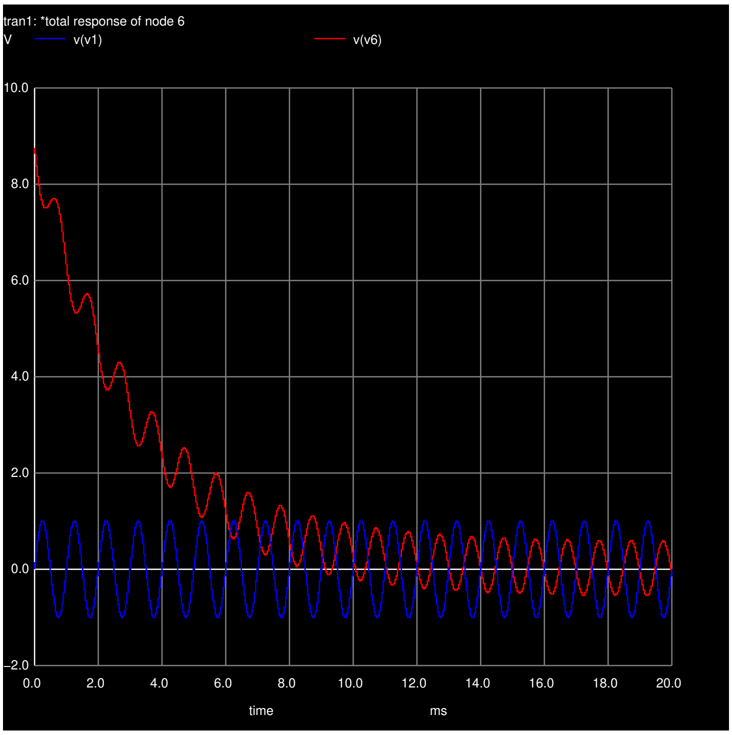


3.4 Exercise 4

In this section we are required to simulate the total response in node 6 $v_6(t)$ by repeating the procedure of 3.3 with the initially given $v_s(t)$ (shown in 1, with a frequency of $1000Hz$.

Figure 8 shows the plot of the required result that was obtained using the transient analysis as well as the stimulus (shown as $v(V_1)$ since $v_1(t) = v_s(t)$

Figure 8: Total response, $v_6(t)$ in V, and stimulus $v_s(t)$, in V in the interval $[0, 20]ms$



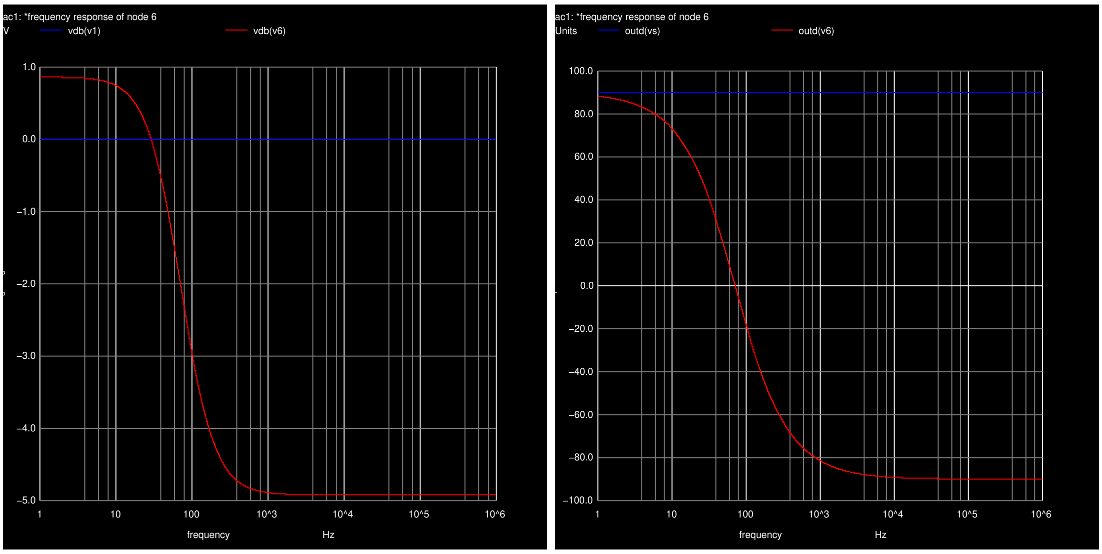
3.5 Exercise 5

In this section we are required to simulate the frequency response in node 6 $v_6(t)$ with the frequency in logscale, magnitude in dB and phase in degrees for a range of $0.1Hz$ to $1MHz$.

Figure 9 shows the plot of the required result, $v_6(f)$ as well as $v_s(f)$.

Since the source of of frequency change is v_s itself, it is expected to not to show a frequency response since it changes alongside frequency (and in $log(1) = 0$), where as v_6 is dependant of the value that v_s takes.

Figure 9: Frequency response, $v_6(f)$ in V, and $v_s(f)$, in V. Phase is in degrees and Magnitude in dB



4 Conclusion

By analysing the circuit theoretically and then simulating the circuit using Ngspice, we can verify that the values of the unknown components match almost perfectly and all approaches agree on the final currents' directions across the circuit's branches (which can be seen below in figure 10).

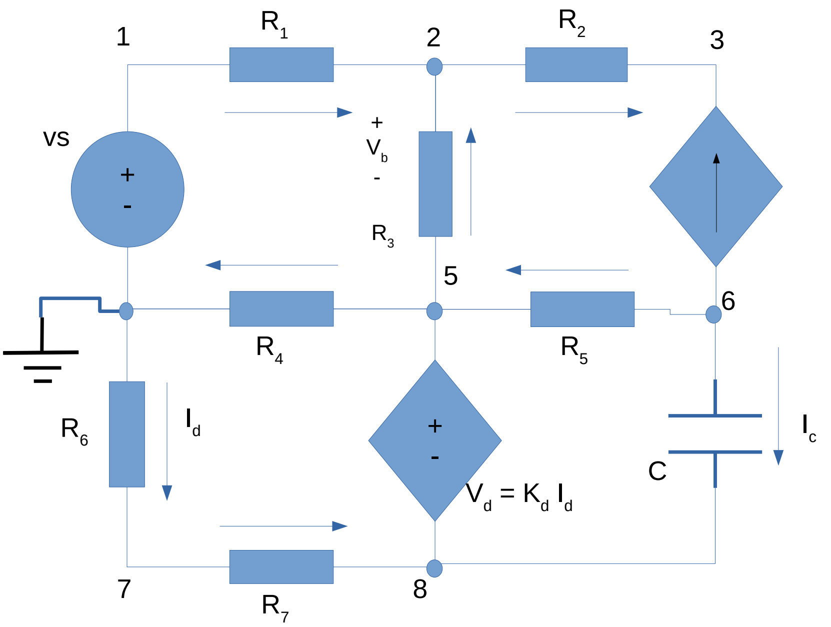


Figure 10: Representation of the final circuit with all the correct directions for the currents.

To better understand the discrepancies and compare results we present the several tables obtained side by side on octave and ngspice, respectively, in tables 7 to 8.

Name	Value [A or V]
@c1[i]	0.000000e+00
@gib[i]	-2.45467e-04
@r1[i]	2.344922e-04
@r2[i]	2.454667e-04
@r3[i]	1.097445e-05
@r4[i]	1.220071e-03
@r5[i]	2.454667e-04
@r6[i]	9.855785e-04
@r7[i]	9.855785e-04
v1	5.242048e+00
v2	5.002985e+00
v3	4.499645e+00
v5	5.036927e+00
v6	5.789615e+00
v7	-1.98352e+00
v8	-2.97405e+00
v9	0.000000e+00

Node	Voltage[V]	Branch	Current[A]
V_b	-3.39424400e-02	I_b	-2.45466679e-04
V_d	8.01098169e+00	I_c	0.00000000e+00
V_1	5.24204797e+00	$R_6 = I_d$	9.85578501e-04
V_2	5.00298504e+00	R_1	2.34492229e-04
V_3	4.49964474e+00	R_2	2.45466679e-04
V_5	5.03692748e+00	R_3	1.09744498e-05
V_6	5.78961528e+00	R_4	1.22007073e-03
V_6	-1.98351843e+00	R_5	2.45466679e-04
V_8	-2.97405421e+00	R_7	9.85578501e-04

Table 7: Comparison of exercises 1 (Upper-Ngspice, Lower-Octave)

Some small discrepancies are due to the different number of decimal places considered by Octave and Ngspice leading to slight inaccuracies. However, considering that the circuit complexity is still not considered, the differences are negligible. Furthermore, any differences in the order of 10^{-15} (or lower), are very likely related to the way the computer programs deal with mathematical operations (seen that 10^{-15} is extremely close to the precision of a double's mantissa). Note that the format of the data presented in the Ngspice tables are automatically chosen by the program.

All of this leads to the conclusion that the making of this laboratory assignment was coherent and that the main goal was attained: to achieve the circuit analysis through a theoretical and simulated approach.

Name	Value [A or V]
@gib[i]	0.000000e+00
@r1[i]	0.000000e+00
@r2[i]	0.000000e+00
@r3[i]	0.000000e+00
@r4[i]	0.000000e+00
@r5[i]	2.858009e-03
@r6[i]	0.000000e+00
@r7[i]	0.000000e+00
v1	0.000000e+00
v2	0.000000e+00
v3	0.000000e+00
v5	0.000000e+00
v6	8.763669e+00
v7	0.000000e+00
v8	0.000000e+00
v9	0.000000e+00

Node	Voltage[V]	Branch	Current[A]
V_b	-0.00000000e+00	I_b	0.00000000e+00
V_d	-0.00000000e+00	$R_6 = I_d$	-0.00000000e+00
V_1	0.00000000e+00	R_1	0.00000000e+00
V_2	-0.00000000e+00	R_2	-0.00000000e+00
V_3	0.00000000e+00	R_3	0.00000000e+00
V_5	-0.00000000e+00	R_4	-0.00000000e+00
V_6	8.76366949e+00	R_5	2.85800945e-03
V_6	0.00000000e+00	R_7	0.00000000e+00
V_8	-0.00000000e+00	I_x	2.85800945e-03
V_x	8.76366949e+00		

Table 8: Comparison of exercises 2 (Upper-Ngspice, Lower-Octave)