CBO Applied to RL

V -evaluation, continuous state space (5.1.)

Outline

- Markov decision process with a continuous state space $\mathbb{S} = \{s \in (0, 2\pi]\}$.
- Dynamics described by

$$\begin{aligned} s_{m+1} &= s_m + \alpha(s_m)\epsilon + \sigma(s_m)\sqrt{\epsilon} Z_m \\ \alpha(s) &= 2\sin(s)\cos(s), \quad \sigma(s) = 1 + \cos(s)^2, \quad \epsilon = 0.1 \end{aligned}$$

- Immediate reward function, $R(s) = (\cos(2s) + 1)$.
- Discount factor $\gamma = 0.9$.
- 3 layer FCNN, $V(s;\theta)$. Two hidden layers with $\cos$ activation function, and each hidden layer contains 50 neurons.

$$\begin{aligned} &V(s;\theta) = V\left(x; \{w_i, b_i\}_{i=1}^{3}\right) = L_{w_3,b_3} \circ \cos \circ L_{w_2,b_2} \circ \cos \circ L_{w_1,b_1}((\cos s, \sin s)) \\ &L_{w_i,b_i}(x) = w_i x + b_i, \quad w_i \in \mathbb{R}^{n_{i-1} \times n_i}, \quad b_i \in \mathbb{R}^{n_i}, \quad n_0 = 2, n_1 = n_2 = 50, n_3 = 1 \end{aligned}$$

- θ^* is computed with Algorithms 1-4 based on trajectory $\{s_m\}_{m=1}^{10^6}$ with

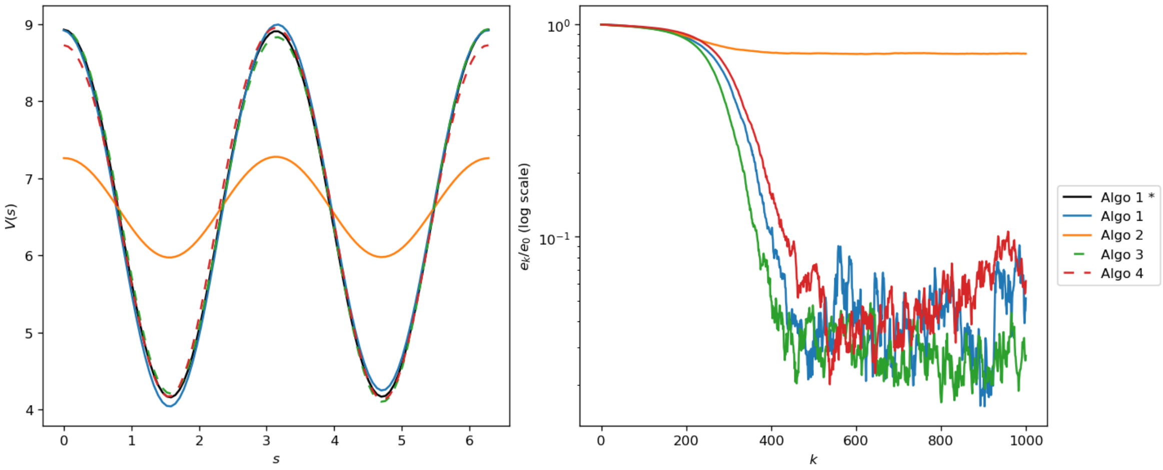
$$f\left(s_m, s_{m+1}, \theta\right) = R\left(s_m\right) + \gamma V\left(s_{m+1};\theta\right) - V\left(s_m;\theta\right), \quad \tau = 0.1, \quad M = 1000$$

- The SGD algorithm runs for a single epoch with the same initialization θ_0.
- Error at step k, e_k is defined as $e_k = \|V(\cdot,\theta_k) - V^*\|_{L^2}$.
- Reference $V^*(s)$ is computed by running Algorithm 1 for 10 epochs based on longer trajectory $\{s_m\}_{m=1}^{10^7}$, with $\tau = 0.01$, $M = 1000$.
- We visualize relative error, $\log_{10}(e_k/e_0)$.
- **NB**, I made one modification to paper:
  - Since $V(s,\theta) \mapsto V(s,\theta) + \delta$ is a symmetry in f, then a better way of measuring error, e_k, is

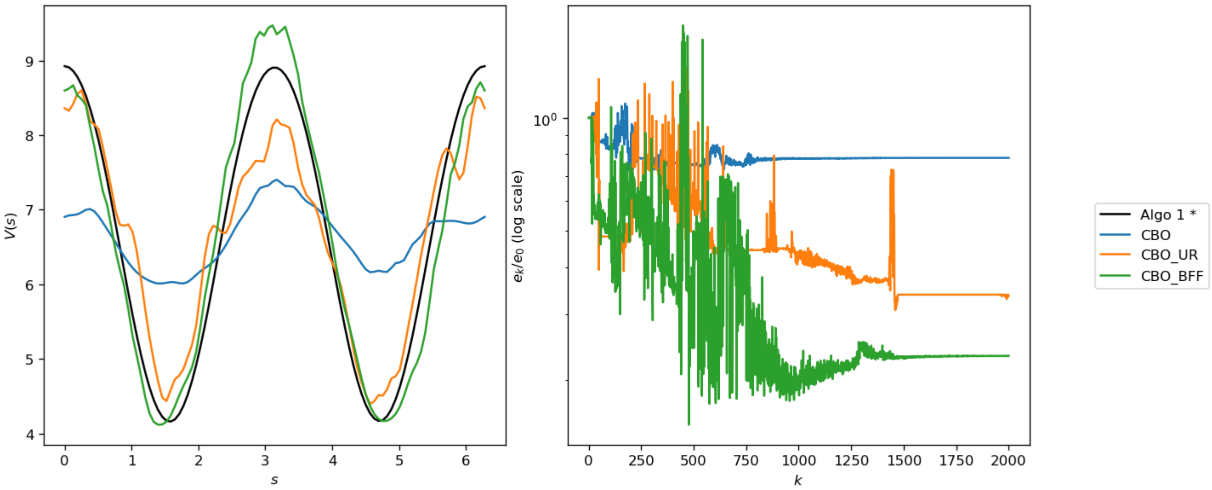
$$e_k = \|V(\cdot,\theta_k) - V^* - \mu_k\|_2, \quad \mu_k = \int V(\cdot,\theta_k) - V^*$$

SGD

- Algo 1*: Unrealistic, with 10 x longer sample trajectory
- Algo 1: Unrealistic
- Algo 2: Double Sampling
- Algo 3: BFF - gradient
- Algo 4: BFF - loss



CBO



Q-evaluation and control, continuous state space (4.1.)

Outline

- MDP with a continuous state space $\mathbb{S} = \{s \in (0, 2\pi]\}$.
- Dynamics described by

$$
\begin{aligned}
\Delta s_m &= a_m \epsilon + \sigma \sqrt{\epsilon} Z_m \\
a_m &\in \mathbb{A} = \{\pm 1\} \\
a_m &\sim \pi(\cdot | s_{m-1}) \\
\varepsilon &= \frac{2\pi}{32} \\
\sigma &= 0.2 \\
r(s_{m+1}, s_m, a_m) &= \sin(s_{m+1}) + 1
\end{aligned}
$$

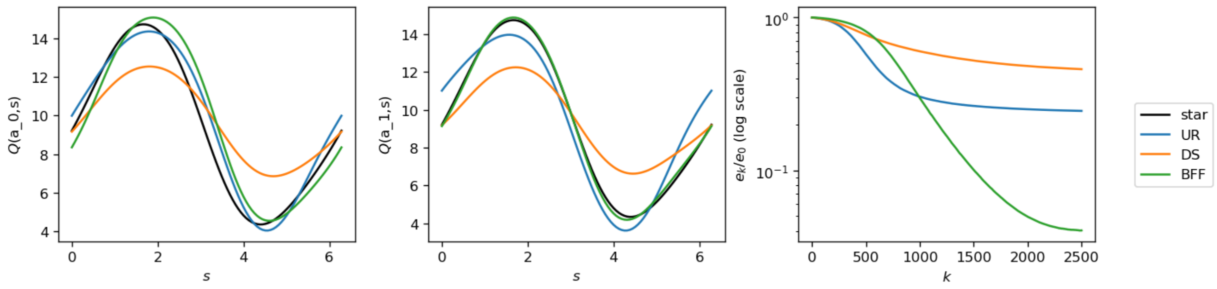
- 3 layer FCNN, $Q^\pi(s, a; \theta)$. Two hidden layers with $\cos$ activation function, and each hidden layer contains 50 neurons. Output layer of size $|\mathbb{A}|$.

Q-evaluation

Estimating Q^π for fixed policy $\pi(a|s) = 1/2 + a\sin(s)/5$.

$$j^{eval}(s_m, a_m, s_{m+1}; \theta) = r(s_{m+1}, s_m, a_m) + \gamma \int Q^\pi(s_{m+1}, a; \theta)\pi(a|s_{m+1})da - Q^\pi(s_m, a_m; \theta)$$

SGD

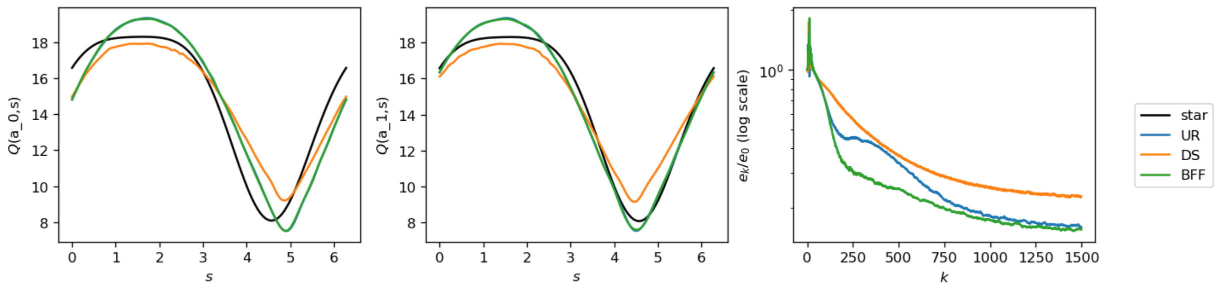


Q-control

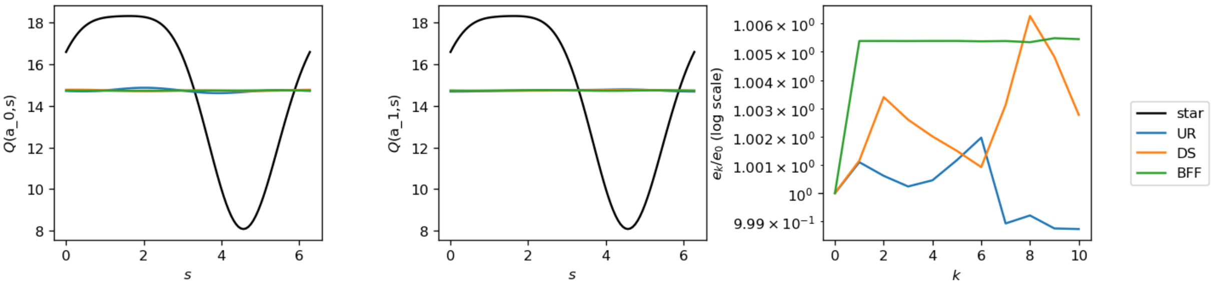
Fixed behavior policy to generate training trajectory, $\pi(a|s) = 1/|\mathbb{A}|$.

$$j^{ctrl}(s_m, a_m, s_m + 1; \theta) = r(s_{m+1}, s_m, a_m) + \gamma \max_{a'} Q^\pi(s_{m+1}, a'; \theta) - Q^\pi(s_m, a_m; \theta)$$

SGD



CBO



Q(a_0,s)
18
16
14
12
10
8
0
2
4
6
s
Q(a_1,s)
18
16
14
12
10
8
0
2
4
6
s
e_k/e_0 (log scale)
1.006×10^0
1.005×10^0
1.004×10^0
1.003×10^0
1.002×10^0
1.001×10^0
10^0
9.99×10^{-1}
0
2
4
6
8
10
k
star
UR
DS
BFF