CBO Applied to RL

V -evaluation, continuous state space (5.1.)

Outline

- Markov decision process with a continuous state space $\mathbb{S} = \{s \in (0, 2\pi]\}$.
- Dynamics described by

$$
\begin{aligned}
&s_{m+1} = s_m + \alpha(s_m)\epsilon + \sigma(s_m)\sqrt{\epsilon}Z_m \\
&\alpha(s) = 2\sin(s)\cos(s), \quad \sigma(s) = 1 + \cos(s)^2, \quad \epsilon = 0.1
\end{aligned}
$$

- Immediate reward function, $R(s) = (\cos(2s) + 1)$.
- Discount factor $\gamma = 0.9$.
- 3 layer FCNN, $V(s;\theta)$. Two hidden layers with $\cos$ activation function, and each hidden layer contains 50 neurons.

$$
\begin{aligned}
&V(s;\theta) = V\left(x; \{w_i, b_i\}_{i=1}^{3}\right) = L_{w_3,b_3} \circ \cos \circ L_{w_2,b_2} \circ \cos \circ L_{w_1,b_1}((\cos s, \sin s)) \\
&L_{w_i,b_i}(x) = w_i x + b_i, \quad w_i \in \mathbb{R}^{n_{i-1} \times n_i}, \quad b_i \in \mathbb{R}^{n_i}, \quad n_0 = 2, n_1 = n_2 = 50, n_3 = 1
\end{aligned}
$$

- θ^* is computed with Algorithms 1-4 based on trajectory $\{s_m\}_{m=1}^{10^6}$ with

$$
f(s_m, s_{m+1}, \theta) = R(s_m) + \gamma V(s_{m+1}; \theta) - V(s_m; \theta), \quad \tau = 0.1, \quad M = 1000
$$

- The SGD algorithm runs for a single epoch with the same initialization θ_0.
- Error at step k, e_k is defined as $e_k = \|V(\cdot, \theta_k) - V^*\|_{L^2}$.
- Reference $V^*(s)$ is computed by running Algorithm 1 for 10 epochs based on longer trajectory $\{s_m\}_{m=1}^{10^7}$, with $\tau = 0.01$, $M = 1000$.
- We visualize relative error, $\log_{10}(e_k/e_0)$.
- **NB**, I made one modification from the paper:
  - Since $V(s,\theta) \mapsto V(s,\theta) + \delta$ is a symmetry in f, then a better way of measuring error, e_k, is

$$
e_k = \|V(\cdot, \theta_k) - V^* - \mu_k\|_2, \quad \mu_k = \int V(\cdot, \theta_k) - V^*
$$

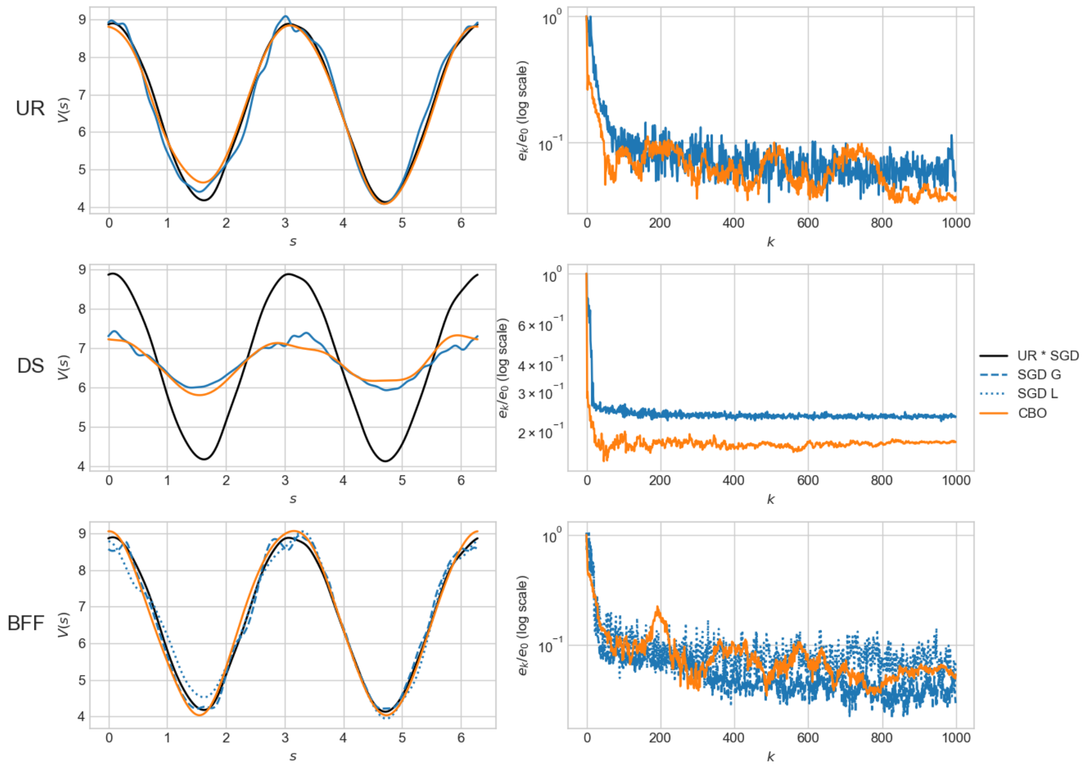
SGD vs CBO

SGD Parameters:

$$
M = 1000, \quad \text{epochs} = 1, \quad \gamma = 0.9, \quad \tau = 0.1
$$

CBO Paramaters:

$$
\begin{array}{ll}
N = 30 & \\
m = 1000 & \\
\text{epochs} = 1 & \\
\delta = 1 \times 10^{-5} & \text{Threshold of difference below which particles take a brownian motion step} \\
\eta_k = \max(0.5 \cdot 0.998^k, 0.01) & \text{Learning rate} \\
\tau_k = \max(0.1 \cdot 0.998^k, 0.01) & \text{Exploration rate} \\
\beta_k = \max(30 \cdot 1.002^k, 80) & 1/\text{Characteristic energy}
\end{array}
$$



Q-evaluation and control, continuous state space (4.1.)

Outline

- MDP with a continuous state space $\mathbb{S} = \{s \in (0, 2\pi]\}$.
- Dynamics described by

$$
\begin{aligned}
\Delta s_m &= a_m \epsilon + \sigma \sqrt{\epsilon} Z_m \\
a_m &\in \mathbb{A} = \{\pm 1\} \\
a_m &\sim \pi(\cdot | s_{m-1}) \\
\varepsilon &= \frac{2\pi}{32} \\
\sigma &= 0.2 \\
r(s_{m+1}, s_m, a_m) &= \sin(s_{m+1}) + 1
\end{aligned}
$$

- 3 layer FCNN, $Q^\pi(s, a; \theta)$. Two hidden layers with $\cos$ activation function, and each hidden layer contains 50 neurons. Output layer of size $|\mathbb{A}|$.
- Reference Q^* is computed by running UR for 10 epochs based on longer trajectory $\{s_m\}_{m=1}^{10^7}$, with $\tau = 0.01$, $M = 1000$.

Q-evaluation

Estimating Q^π for fixed policy $\pi(a|s) = 1/2 + a\sin(s)/5$.

$$
j^{eval}(s_m, a_m, s_{m+1}; \theta) = r(s_{m+1}, s_m, a_m) + \gamma \int Q^\pi(s_{m+1}, a; \theta) \pi(a|s_{m+1}) da - Q^\pi(s_m, a_m; \theta)
$$

Q-control

Fixed behavior policy to generate training trajectory, $\pi(a|s) = 1/|\mathbb{A}|$.

$$
j^{ctrl}(s_m, a_m, s_m + 1; \theta) = r(s_{m+1}, s_m, a_m) + \gamma \max_{a'} Q^\pi(s_{m+1}, a'; \theta) - Q^\pi(s_m, a_m; \theta)
$$

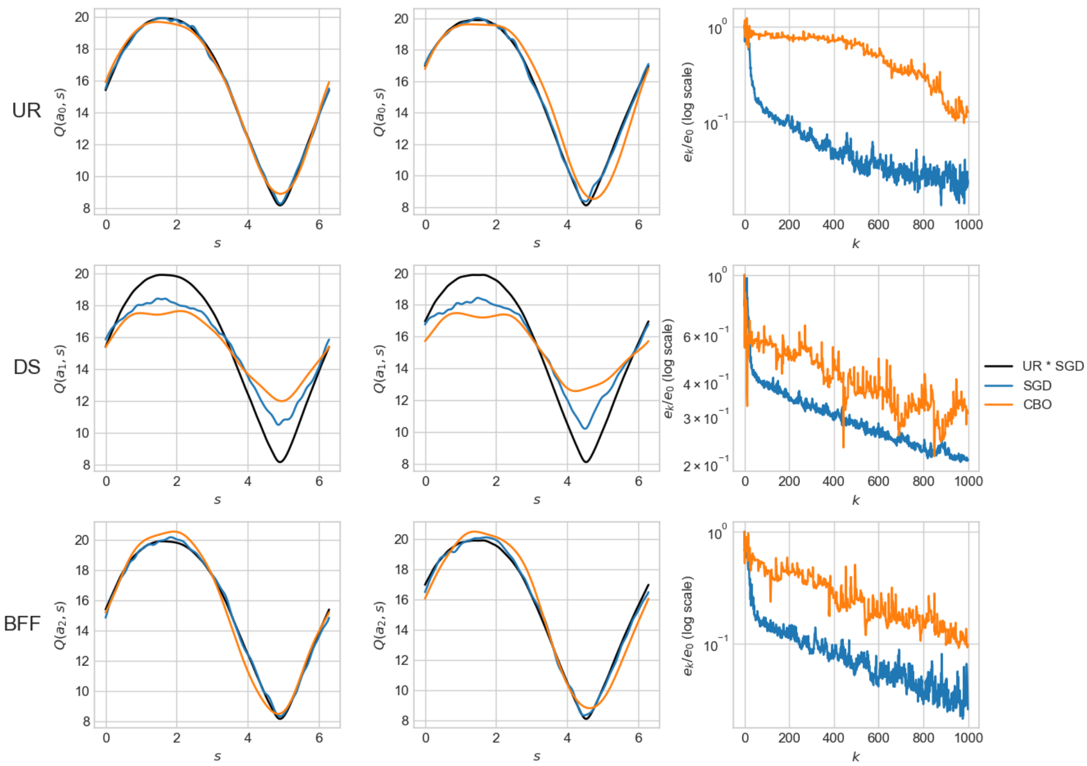
SGD vs CBO

SGD Parameters:

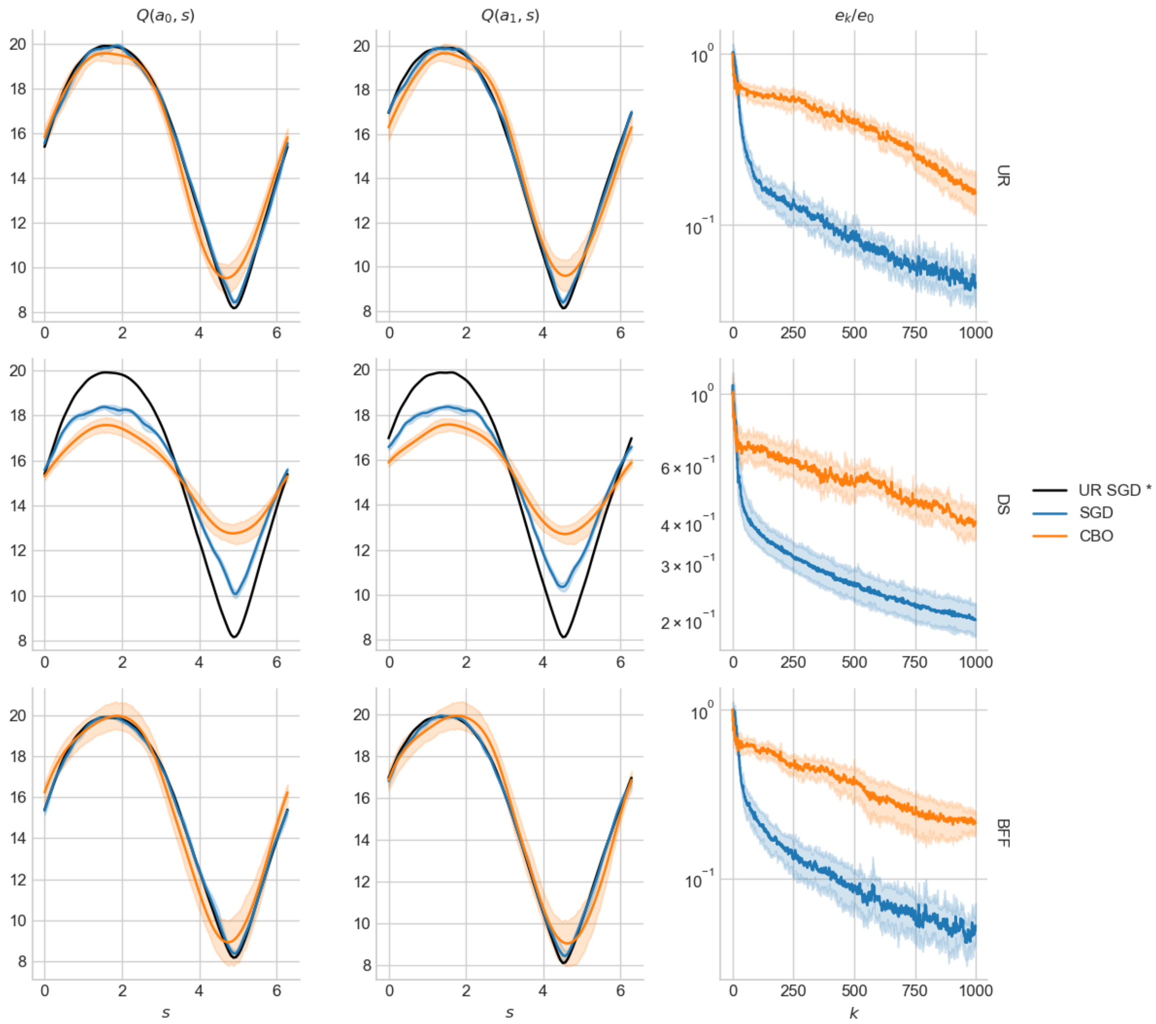
$$M = 1000, \quad \text{epochs} = 1, \quad \gamma = 0.9, \quad \tau = 0.1$$

CBO Parameters:

$N = 90$	
$m = 1000$	
$\text{epochs} = 1$	
$\delta = 1 \times 10^{-5}$	Threshold of difference below which particles take a brownian motion step
$\eta_k = \max(0.6 \cdot 0.9992^k, 0.075)$	Learning rate
$\tau_k = \max(0.8 \cdot 0.9992^k, 0.3)$	Exploration rate
$\beta_k = \max(8 \cdot 1.002^k, 20)$	1/Characteristic energy



To visualize the stability of the algorithms, we can show statistics (mean and sd) on 10 runs:



Future Work

- Applying rigorous approaches to hyper-parameter tuning
- Testing the CBO + BFF on more complex problems
  - Higher dimensional
  - Deeper neural networks
  - Open AI Gyms examples
  - Less smooth dynamics
- Visualizing behavior of particles in CBO

Summary and Conclusion

From the experiments on the simple environments, we note that

- BFF and UR have similar performance using SGD and CBO, which are superior to DS, for V-evaluation and Q-control.
- CBO and SGD have similar performances with BFF, UR, and DS in V-evaluation, however CBO performs worse than SGD in Q-control.

Thus, we don't experience a notable advantage in the combination of BFF and CBO, for V-evaluation and Q-control, however we reaffirm the validity of BFF over DS.