CBO + BFF applied to Q-control

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1 Introduction

2 Models

Bellman Residual Method (BRM)

$$\begin{aligned} J(\theta) &= \mathbb{E}[((\mathbb{T}^{\pi_*} - \mathbb{I})Q^*(s,a;\theta))^2] \\ &= \mathbb{E}[\mathbb{E}[j^{ctrl}(s_m, a_m, s_{m+1};\theta)|s_m, a_m]^2] \\ j^{\mathrm{ctrl}}(s_m, a_m, s_{m+1};\theta) &= r(s_{m+1}, s_m, a_m) + \gamma \max_{a'} Q^*(s_{m+1}, a';\theta) - Q^*(s_m, a_m;\theta) \end{aligned}$$

Using Batches, and a specific sampling method (UR, DS, BFF), at each iteration we get an estimate $\tilde{J}(\theta)$for the loss $J(\theta)$.

SGD

$M = 1000$	Batch size
epochs $= 1$	
$\gamma = 0.9$	Discount factor
$\tau_k = \max(\tau_i \cdot \tau_r^k, \tau_i \cdot \tau_f)$	Learning rate

$$\Delta\theta_k = -\tau_k \nabla_\theta \tilde{J}(\theta)$$

CBO

$N = 90$	Number of particles
$m = 1000$	Batch size
epochs $= 1$	
$\gamma = 0.9$	Discount factor
$\delta = 1 \times 10^{-5}$	Threshold of difference below which particles take a brownian motion step
$\eta_k = \max(\eta_i \cdot \eta_r^k, \eta_i \cdot \eta_f)$	Learning rate
$\tau_k = \max(\tau_i \cdot \tau_r^k, \tau_i \cdot \tau_f)$	Exploration rate
$\beta_k = \min(\beta_i \cdot \beta_r^k, \beta_i \cdot \beta_f)$	1/Characteristic energy

$$\begin{aligned} \bar{\theta}_k &= \frac{\sum_{j=1}^N \theta_k^j \exp(-\beta_k \tilde{J}(\theta))}{\sum_{j=1}^N \exp(-\beta_k \tilde{J}(\theta))} \\ \Delta\theta_k^j &= (-\eta_k I + \tau_k \sqrt{\eta_k} \cdot Z)(\theta^j - \bar{\theta}); \quad Z = \mathrm{diag}(\{z_i \sim \mathcal{N}(0,1)\}_{i=1}^N) \end{aligned}$$

3 Numerical Example

3.1 Setup

Continuous statespace.

$$\begin{aligned}
&\mathbb{S} = (0, 2\pi] \\
&\Delta s_m = a_m \epsilon + \sigma\sqrt{\epsilon} Z_m \\
&a_m \in \mathbb{A} = \{\pm 1\} \\
&a_m \sim \pi_b(\cdot|s_m) \\
&\varepsilon = \tfrac{2\pi}{32} \\
&\sigma = 0.2 \\
&r(s_{m+1}, s_m, a_m) = \sin(s_{m+1}) + 1 \\
&\pi_b(a|s) = \tfrac{1}{|\mathbb{A}|} \\
&\text{Model : ResNet}
\end{aligned}$$

Discrete statespace

$$\begin{aligned}
&\mathbb{S} = \{\tfrac{2\pi k}{n} : k \in \mathbb{Z} \cap [0, n-1]\} \\
&\Delta s_m = \tfrac{2\pi}{n} a_m \epsilon + \sigma\sqrt{\epsilon} Z_m \\
&a_m \in \mathbb{A} = \{\pm 1\} \\
&a_m \sim \pi_b(\cdot|s_m) \\
&n = 32 \\
&\varepsilon = 1 \\
&\sigma = 1 \\
&r(s_{m+1}, s_m, a_m) = \sin(s_{m+1}) + 1 \\
&\pi_b(a|s) = \tfrac{1}{2} + a\sin(s) \\
&\text{Model : Tabular}
\end{aligned}$$

- Policy is sampled using fixed behaviour policy $\pi_b : \mathbb{A} \times \mathbb{S} \to [0, 1]$ generating long and normal trajectories.
- Reference θ^* is computed by running UR SGD for 1 epoch based on longer trajectory with $\tau = \max(0.8 \cdot 0.9992^k, 0.3)$, $M = 1000$.
- Perform hyperparameter optimization, using optuna, fixing $M = 1000, \delta = 10^{-5}$, running 150 trials, and 90 particles for CBO, taking error of UR as loss. 3 Variables for SGD, 9 for CBO.
- Average 10 instances using best hyperparameters found, $\hat{\theta}$, plotting $Q(\cdot, \cdot; \theta)$, and $e_k = e(\theta_k) = \|Q(\cdot, \cdot; \theta^*) - Q(\cdot, \cdot; \theta_k)\|$.
- Visualize optimization landscape by evaluating error on affine combination of parameters of θ^* and random initialization parameters ($e(\alpha\theta^* + (1 - \alpha)\theta)$ vs $\alpha \in [0, 1]$) *since e is a sort of "distance", then we expect it to be linear near $\alpha = 1$*

3.2 Results

Continuous

SGD	τ
i	0.08001579582322532
f	0.9835361410049764
r	0.950886309322411

CBO	η	τ	β
i	0.27998130431694734	0.45180905444083275	8.51669145194007
f	0.5194195083343263	0.4287097014952387	1.7500535407136808
r	0.9698276350455655	0.9578569049519733	1.0213109427307054

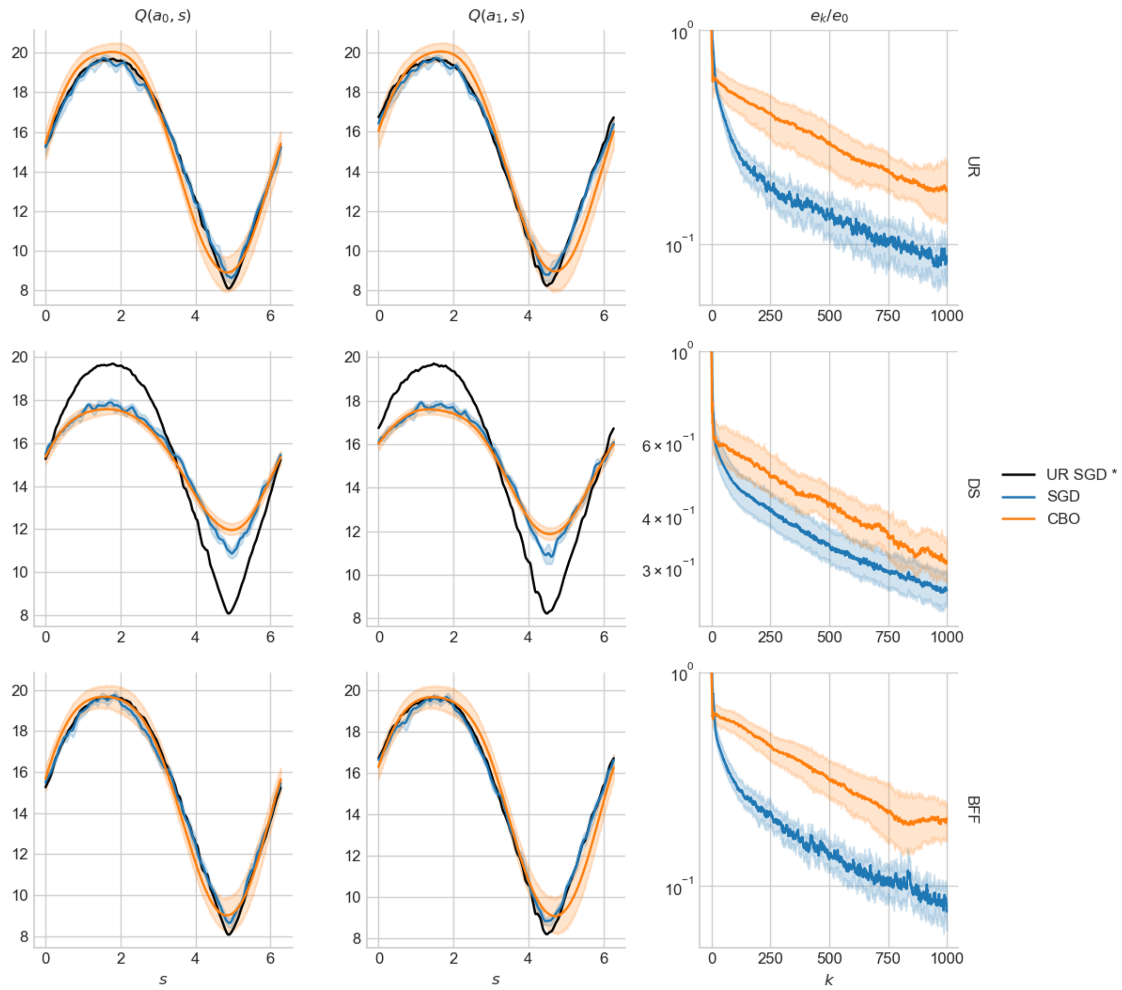
Discrete

SGD	τ
i	4.703979337147098
f	0.654883440315296
r	0.9730502549231891

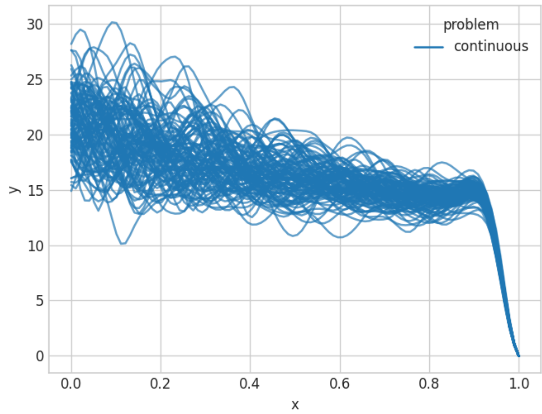
CBO	η	τ	β
i	0.9785432879550536	0.893733865582011	12.055703082474112
f	0.8242055859864529	0.3129317992204857	2.6535471522264684
r	0.9827373829994701	0.999130685913801	1.019158705231164

4 Conclusion

An important note is that the differences noted above could be related to the number of variables in the hyper-parameter search (3 for SGD, 9 for CBO), as well as the complexity/convexity of the problem, ie. if we performed more trials, it might be possible for CBO > SGD and BFF > UR in the continuous case, although we have not observed this fact. This is also based on the fact that CBO > SGD for V-eval without hyperparameter search.

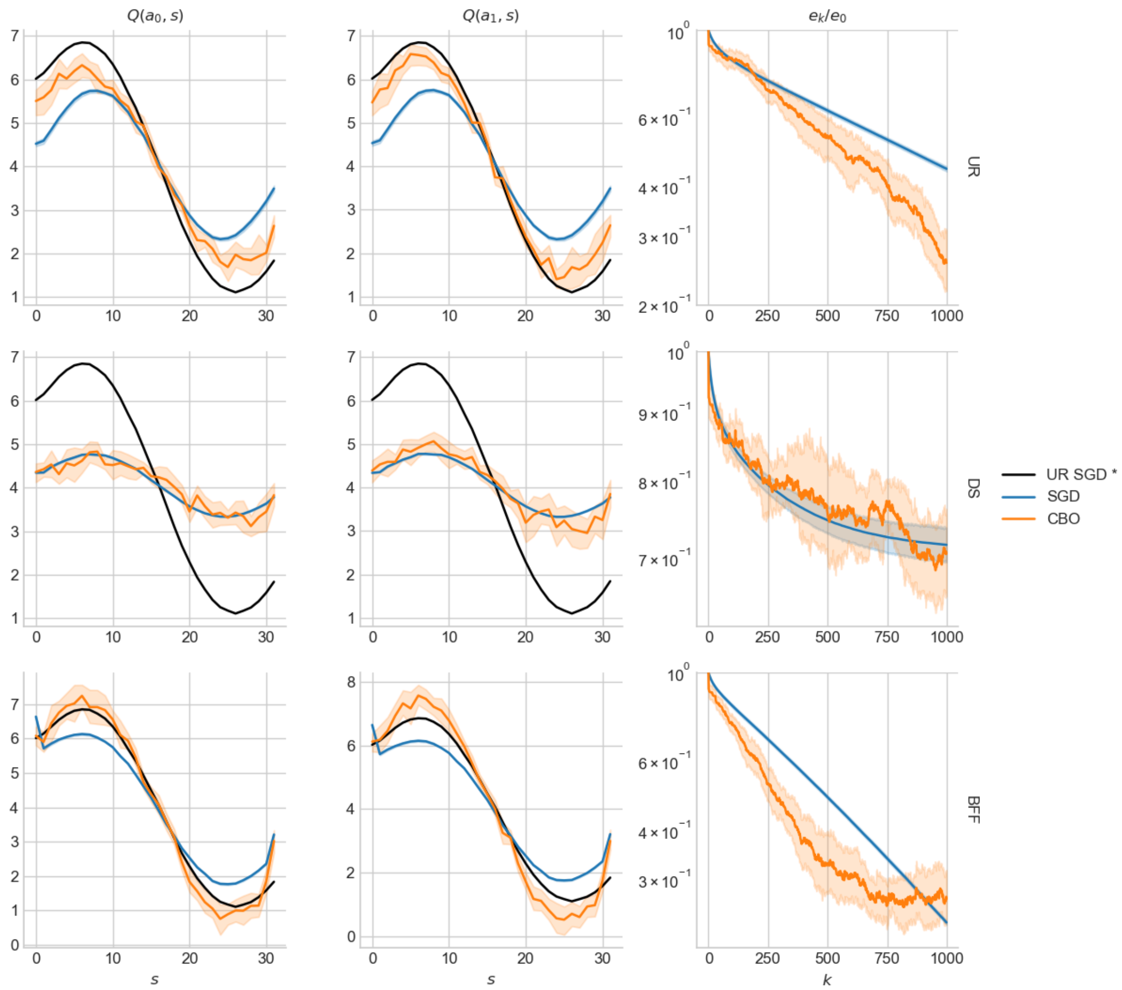


(a) Summary

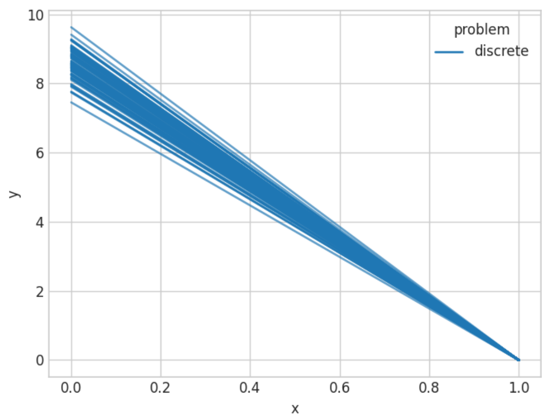


(b) Error landscape

Figure 1: Continuous case



(a) Summary



(b) Error landscape

Figure 2: Discrete case