

Práctica 5: Modelos de series con exógenas

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COMPLETAR PRÁCTICA 5: MODELOS DE SERIES TEMPORALES CON VARIABLES EXÓGENAS

PROCESOS ESTOCÁSTICOS Y SERIES TEMPORALES

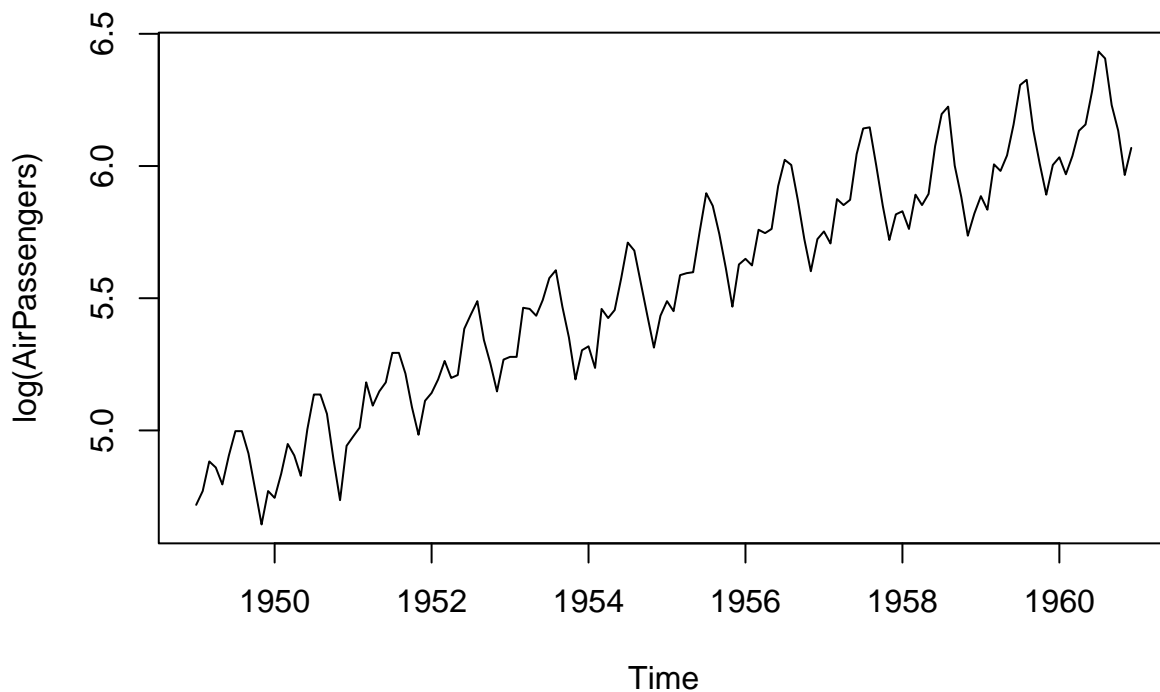
GRADO EN CIENCIA E INGENIERÍA DE DATOS

1. Modelos de Regresión para series temporales

1.1. Ajuste de un modelo de Regresión Lineal para la serie $\log(\text{AirPassengers})$

En esta sección trabajaremos con la serie transformada $\log(\text{AirPassengers})$.

```
plot(log(AirPassengers))
```



Vamos a estimar un modelo de regresión lineal múltiple para explicar el comportamiento de la serie en función de los predictores (variables exógenas) “tiempo” y “dummy estacionales”.

A) Forma 1: Ajuste RLM usando la función `lm()`

```
library(forecast)
```

```
## Registered S3 method overwritten by 'quantmod':
```

```
## method from
## as.zoo.data.frame zoo
```

Definimos los predictores:

```
tiempo <- time(log(AirPassengers)) # predictor tiempo
estacional.dummy <- seasonaldummy(log(AirPassengers)) # predictores dummy estacionales
```

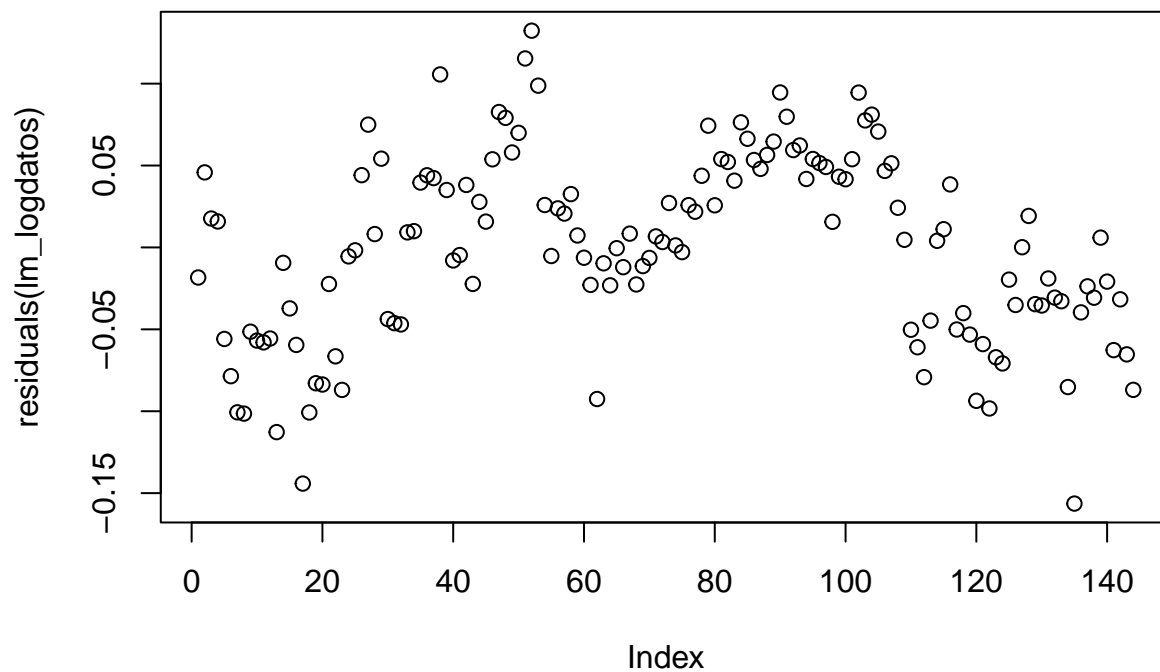
Ajustamos el modelo RLM:

```
lm_logdatos <- lm(log(AirPassengers) ~ tiempo + estacional.dummy)
summary(lm_logdatos)
```

```
##
## Call:
## lm(formula = log(AirPassengers) ~ tiempo + estacional.dummy)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.156370 -0.041016  0.003677  0.044069  0.132324
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   -2.308e+02  2.799e+00 -82.436 < 2e-16 ***
## tiempo         1.208e-01  1.432e-03  84.399 < 2e-16 ***
## estacional.dummyJan  2.132e-02  2.425e-02   0.879 0.380816
## estacional.dummyFeb -7.338e-04  2.424e-02  -0.030 0.975897
## estacional.dummyMar  1.295e-01  2.423e-02   5.343 3.92e-07 ***
## estacional.dummyApr  9.822e-02  2.423e-02   4.054 8.59e-05 ***
## estacional.dummyMay  9.585e-02  2.423e-02   3.957 0.000124 ***
## estacional.dummyJun  2.180e-01  2.422e-02   9.000 2.25e-15 ***
## estacional.dummyJul  3.219e-01  2.422e-02  13.293 < 2e-16 ***
## estacional.dummyAug  3.126e-01  2.422e-02  12.911 < 2e-16 ***
## estacional.dummySep  1.680e-01  2.421e-02   6.939 1.64e-10 ***
## estacional.dummyOct  2.985e-02  2.421e-02   1.233 0.219790
## estacional.dummyNov -1.139e-01  2.421e-02  -4.703 6.41e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.0593 on 131 degrees of freedom
## Multiple R-squared:  0.9835, Adjusted R-squared:  0.982
## F-statistic: 649.4 on 12 and 131 DF, p-value: < 2.2e-16
```

Comportamiento de los residuos del modelo:

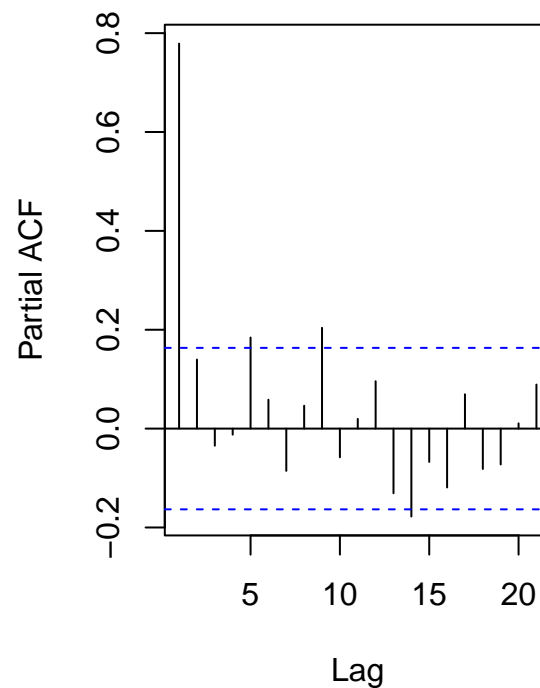
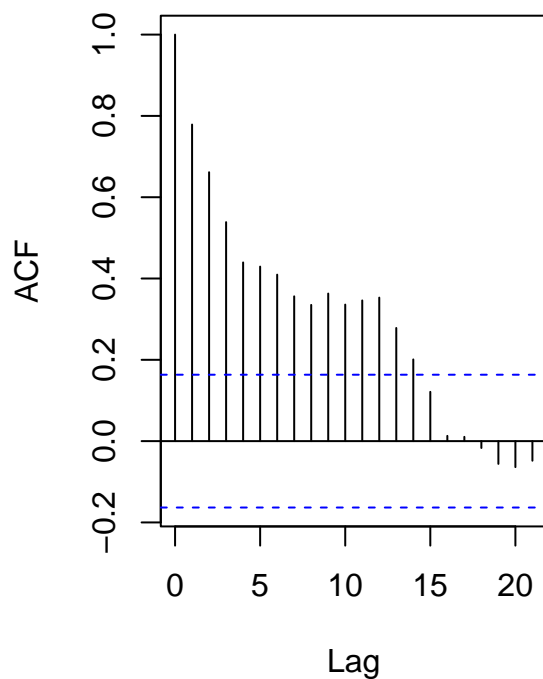
```
plot(residuals(lm_logdatos)) # gráfico residuos
```



```
par(mfrow = c(1, 2))
acf(residuals(lm_logdatos)) # correlograma simple residuos
pacf(residuals(lm_logdatos)) # correlograma parcial residuos
```

Series residuals(lm_logdatos)

Series residuals(lm_logdatos)



¿Y si usamos términos de Fourier en lugar de dummy estacionales?

Podemos crear manualmente los predictores correspondientes a las series de Fourier:

```
# Escribir bloque de código
n <- length(log(AirPassengers))
t <- 1:n
s1 <- sin(2 * pi * t / 12)
c1 <- cos(2 * pi * t / 12)
s2 <- sin(4 * pi * t / 12)
c2 <- cos(4 * pi * t / 12)
```

Podemos crear automáticamente los predictores series de Fourier para distintos valores de K:

```
# Para K = 6 = L/2
fourier_terms_k6 <- fourier(log(AirPassengers), K = 6)
head(fourier_terms_k6)
```

```
##           S1-12      C1-12      S2-12 C2-12 S3-12 C3-12      S4-12 C4-12
## [1,] 0.5000000 0.8660254 0.8660254 0.5      1      0 0.8660254 -0.5
## [2,] 0.8660254 0.5000000 0.8660254 -0.5      0     -1 -0.8660254 -0.5
## [3,] 1.0000000 0.0000000 0.0000000 -1.0     -1      0 0.0000000 1.0
## [4,] 0.8660254 -0.5000000 -0.8660254 -0.5      0      1 0.8660254 -0.5
## [5,] 0.5000000 -0.8660254 -0.8660254 0.5      1      0 -0.8660254 -0.5
## [6,] 0.0000000 -1.0000000 0.0000000 1.0      0     -1 0.0000000 1.0
##           S5-12      C5-12 C6-12
## [1,] 0.5000000 -0.8660254 -1
## [2,] -0.8660254 0.5000000 1
## [3,] 1.0000000 0.0000000 -1
## [4,] -0.8660254 -0.5000000 1
## [5,] 0.5000000 0.8660254 -1
## [6,] 0.0000000 -1.0000000 1
```

Ajustamos el modelo RLM:

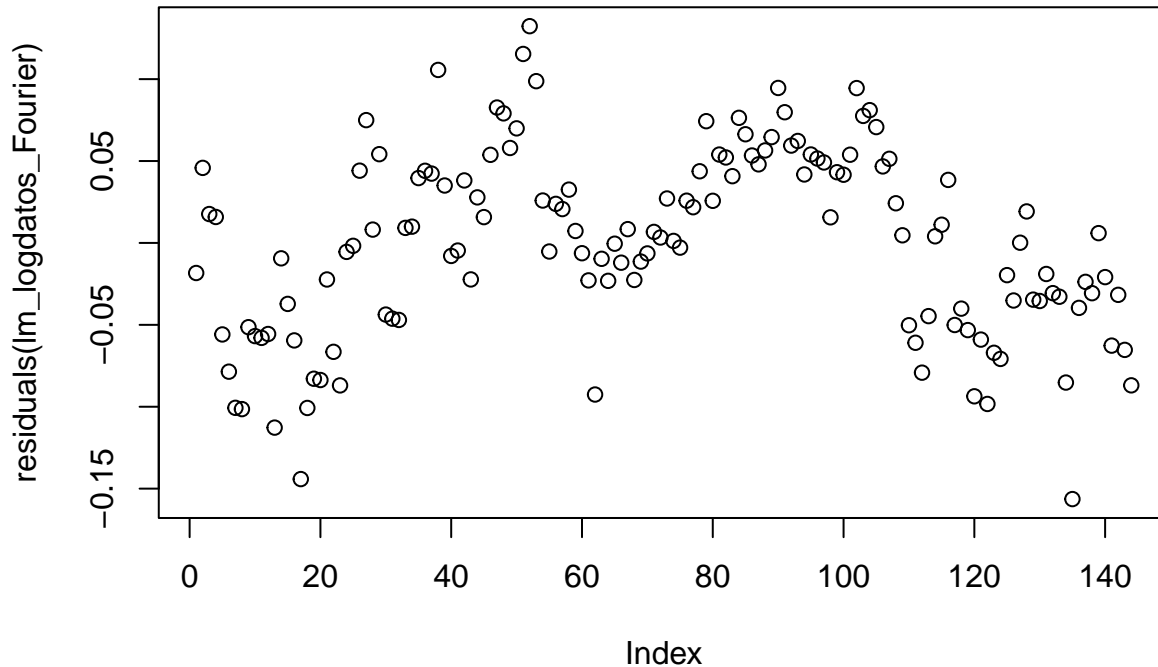
```
lm_logdatos_Fourier <- lm(log(AirPassengers) ~ tiempo + fourier_terms_k6)
summary(lm_logdatos_Fourier)
```

```
##
## Call:
## lm(formula = log(AirPassengers) ~ tiempo + fourier_terms_k6)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.156370 -0.041016  0.003677  0.044069  0.132324
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   -2.307e+02  2.799e+00 -82.419 < 2e-16 ***
## tiempo         1.208e-01  1.432e-03  84.399 < 2e-16 ***
## fourier_terms_k6S1-12 -4.936e-02  7.003e-03  -7.048 9.31e-11 ***
## fourier_terms_k6C1-12 -1.418e-01  6.990e-03 -20.287 < 2e-16 ***
## fourier_terms_k6S2-12  7.868e-02  6.992e-03  11.253 < 2e-16 ***
## fourier_terms_k6C2-12 -2.281e-02  6.990e-03  -3.264 0.001403 **
## fourier_terms_k6S3-12 -8.731e-03  6.990e-03  -1.249 0.213877
## fourier_terms_k6C3-12  2.729e-02  6.990e-03   3.904 0.000150 ***
## fourier_terms_k6S4-12  2.561e-02  6.989e-03   3.664 0.000359 ***
## fourier_terms_k6C4-12  2.215e-02  6.990e-03   3.168 0.001908 **
## fourier_terms_k6S5-12  2.137e-02  6.989e-03   3.057 0.002706 **
## fourier_terms_k6C5-12  5.515e-03  6.990e-03   0.789 0.431541
```

```
## fourier_terms_k6C6-12 2.936e-03 4.942e-03 0.594 0.553474
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.0593 on 131 degrees of freedom
## Multiple R-squared:  0.9835, Adjusted R-squared:  0.982
## F-statistic: 649.4 on 12 and 131 DF,  p-value: < 2.2e-16
```

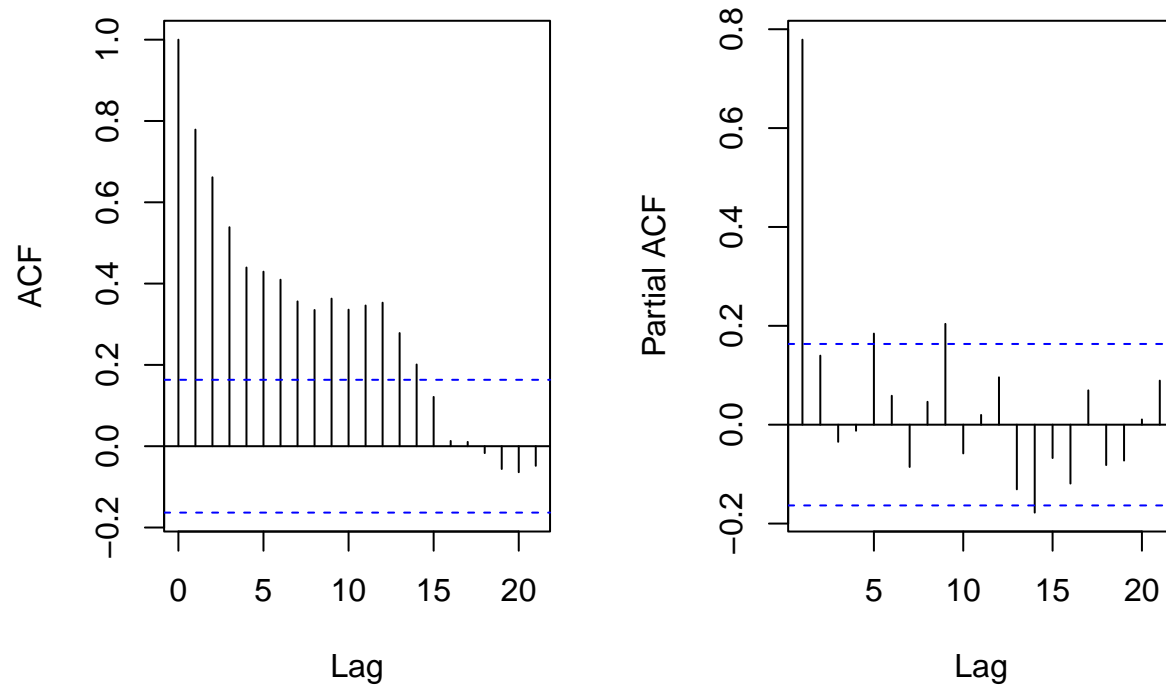
Comportamiento de los residuos del modelo:

```
plot(residuals(lm_logdatos_Fourier)) # gráfico residuos
```



```
par(mfrow = c(1, 2))
acf(residuals(lm_logdatos_Fourier)) # correlograma simple residuos
pacf(residuals(lm_logdatos_Fourier)) # correlograma parcial residuos
```

Series residuals(lm_logdatos_Fou Series residuals(lm_logdatos_Fou



B) Ajuste RLM usando la función tslm()

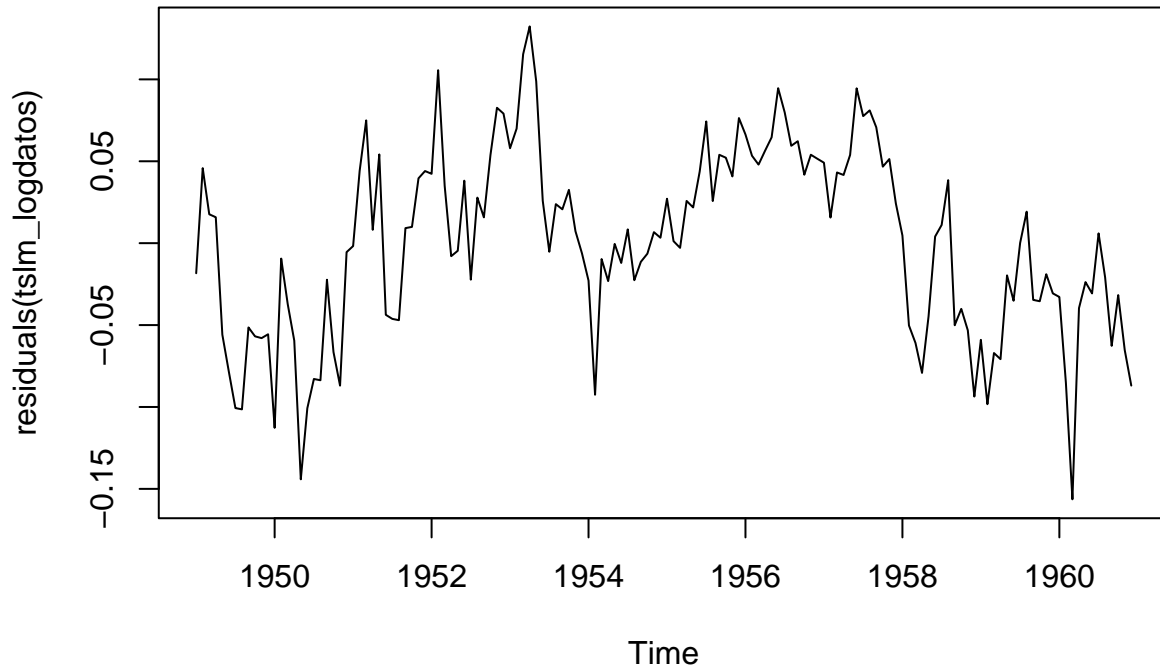
```
tslm_logdatos <- tslm(log(AirPassengers) ~ trend + season)
summary(tslm_logdatos)
```

```
##
## Call:
## tslm(formula = log(AirPassengers) ~ trend + season)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.156370 -0.041016  0.003677  0.044069  0.132324
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  4.7267804  0.0188935 250.180 < 2e-16 ***
## trend        0.0100688  0.0001193  84.399 < 2e-16 ***
## season2     -0.0220548  0.0242109  -0.911  0.36400
## season3      0.1081723  0.0242118   4.468 1.69e-05 ***
## season4      0.0769034  0.0242132   3.176  0.00186 **
## season5      0.0745308  0.0242153   3.078  0.00254 **
## season6      0.1966770  0.0242179   8.121 2.98e-13 ***
## season7      0.3006193  0.0242212  12.411 < 2e-16 ***
## season8      0.2913245  0.0242250  12.026 < 2e-16 ***
## season9      0.1466899  0.0242294   6.054 1.39e-08 ***
## season10     0.0085316  0.0242344   0.352  0.72537
## season11    -0.1351861  0.0242400  -5.577 1.34e-07 ***
## season12    -0.0213211  0.0242461  -0.879  0.38082
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.0593 on 131 degrees of freedom
## Multiple R-squared:  0.9835, Adjusted R-squared:  0.982
## F-statistic: 649.4 on 12 and 131 DF,  p-value: < 2.2e-16
```

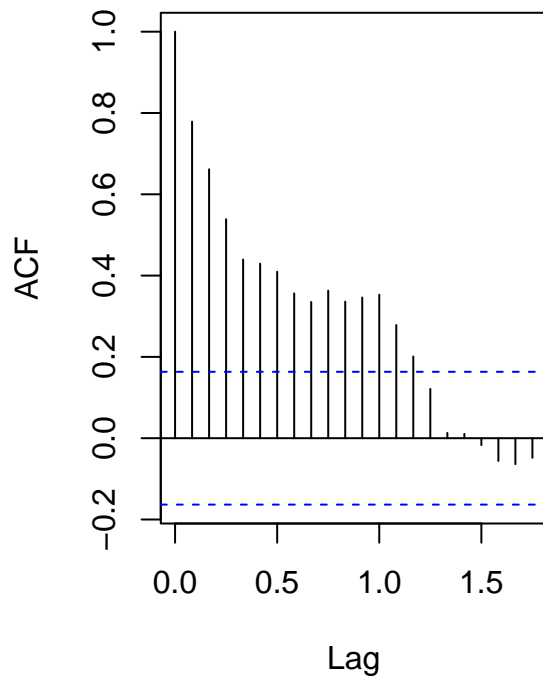
Comportamiento de los residuos del modelo:

```
plot(residuals(tslm_logdatos)) # gráfico residuos
```

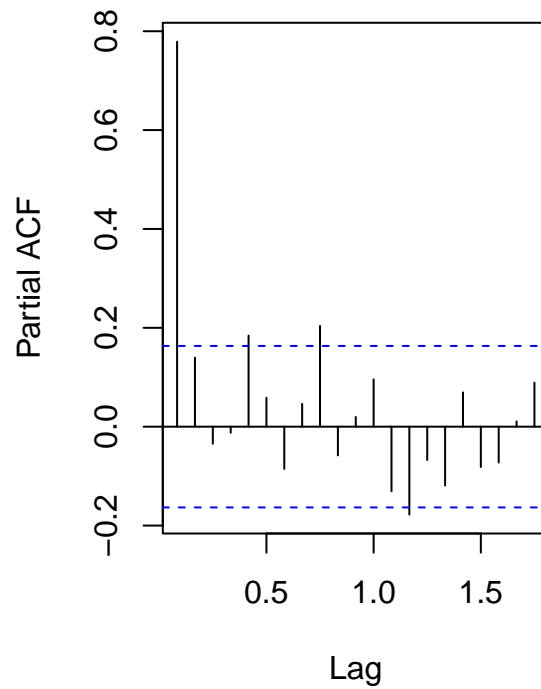


```
par(mfrow = c(1, 2))
acf(residuals(tslm_logdatos)) # correlograma simple residuos
pacf(residuals(tslm_logdatos)) # correlogerama parcial residuos
```

Series residuals(tslm_logdatos



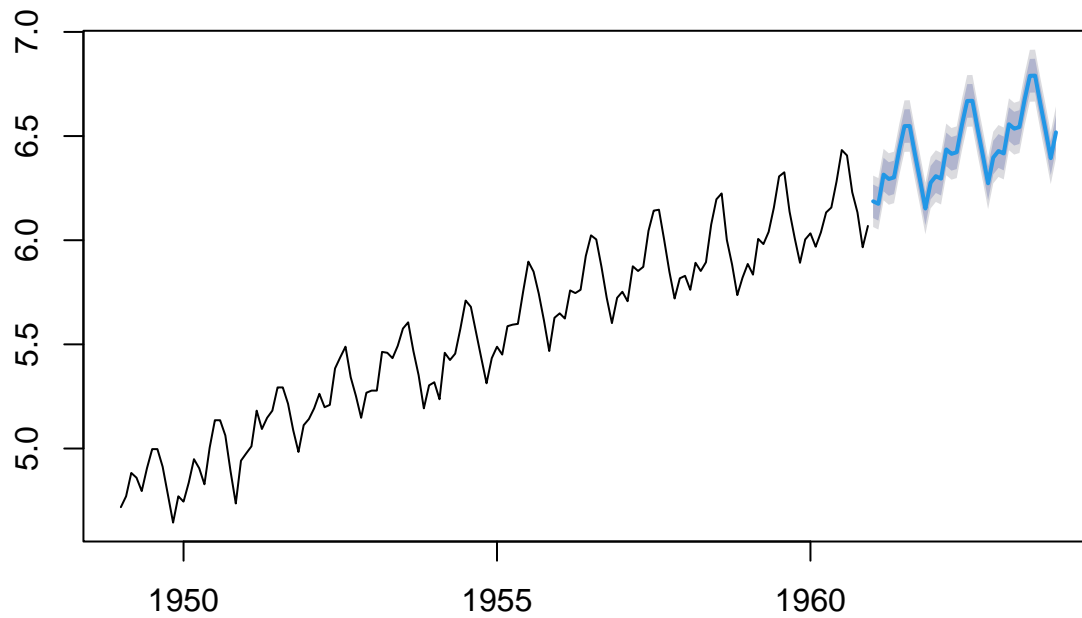
Series residuals(tslm_logdatos



Representamos la serie original y las predicciones a 3 años vista:

```
plot(forecast(tslm_logdatos, h = 36))
```

Forecasts from Linear regression model



¿Y si usamos términos de Fourier en lugar de dummy estacionales?

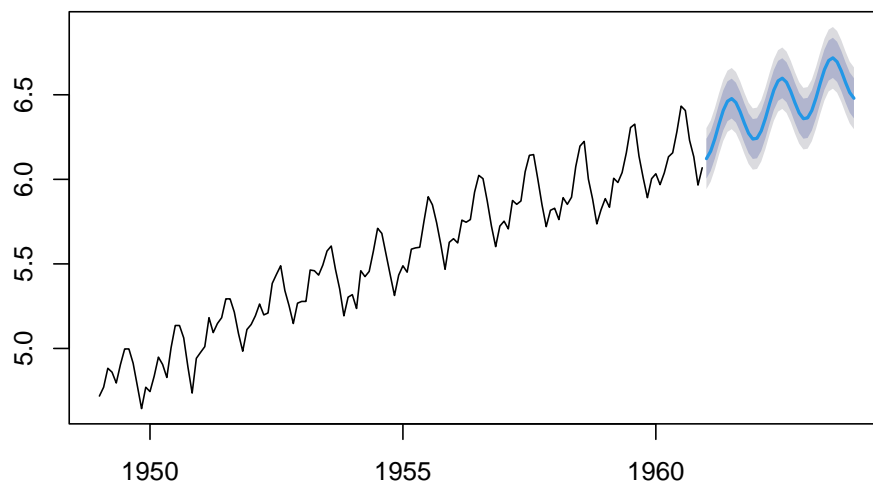
Para $K = 1$:


```
tslm_logdatos_FourierK1 <- tslm(log(AirPassengers) ~ trend + fourier(log(AirPassengers), K = 1))
summary(tslm_logdatos_FourierK1)
```

```
##
## Call:
## tslm(formula = log(AirPassengers) ~ trend + fourier(log(AirPassengers),
##      K = 1))
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.254905 -0.060940  0.004394  0.069431  0.186910
##
## Coefficients:
##                  Estimate Std. Error t value Pr(>|t|)
## (Intercept)      4.8145681   0.0150203  320.537 < 2e-16
## trend            0.0100360   0.0001798   55.809 < 2e-16
## fourier(log(AirPassengers), K = 1)S1-12 -0.0494811   0.0105698  -4.681 6.66e-06
## fourier(log(AirPassengers), K = 1)C1-12 -0.1417735   0.0105500 -13.438 < 2e-16
##
## (Intercept)          ***
## trend                ***
## fourier(log(AirPassengers), K = 1)S1-12 ***
## fourier(log(AirPassengers), K = 1)C1-12 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.08951 on 140 degrees of freedom
## Multiple R-squared:  0.9598, Adjusted R-squared:  0.9589
## F-statistic: 1113 on 3 and 140 DF, p-value: < 2.2e-16
```

```
plot(forecast(tslm_logdatos_FourierK1, newdata = data.frame(fourier(log(AirPassengers), K = 1, h = 36)))
```

Forecasts from Linear regression model



Para $K = 6$ obtenemos:

```
# Escribir bloque de código
tslm_logdatos_FourierK6 <- tslm(log(AirPassengers) ~ trend + fourier(log(AirPassengers), K = 6))
```

```
summary(tslm_logdatos_FourierK6)
```

```
##
```

```
## Call:
```

```
## tslm(formula = log(AirPassengers) ~ trend + fourier(log(AirPassengers),
```

```
##     K = 6))
```

```
##
```

```
## Residuals:
```

```
##      Min       1Q   Median       3Q      Max
```

```
## -0.156370 -0.041016  0.003677  0.044069  0.132324
```

```
##
```

```
## Coefficients:
```

```
##              Estimate Std. Error t value Pr(>|t|)
```

```
## (Intercept)      4.8121876   0.0099616 483.076 < 2e-16
```

```
## trend            0.0100688   0.0001193  84.399 < 2e-16
```

```
## fourier(log(AirPassengers), K = 6)S1-12 -0.0493586   0.0070032  -7.048 9.31e-11
```

```
## fourier(log(AirPassengers), K = 6)C1-12 -0.1418063   0.0069900 -20.287 < 2e-16
```

```
## fourier(log(AirPassengers), K = 6)S2-12  0.0786797   0.0069920  11.253 < 2e-16
```

```
## fourier(log(AirPassengers), K = 6)C2-12 -0.0228128   0.0069900  -3.264 0.001403
```

```
## fourier(log(AirPassengers), K = 6)S3-12 -0.0087308   0.0069900  -1.249 0.213877
```

```
## fourier(log(AirPassengers), K = 6)C3-12  0.0272922   0.0069900   3.904 0.000150
```

```
## fourier(log(AirPassengers), K = 6)S4-12  0.0256113   0.0069893   3.664 0.000359
```

```
## fourier(log(AirPassengers), K = 6)C4-12  0.0221473   0.0069900   3.168 0.001908
```

```
## fourier(log(AirPassengers), K = 6)S5-12  0.0213690   0.0069891   3.057 0.002706
```

```
## fourier(log(AirPassengers), K = 6)C5-12  0.0055151   0.0069900   0.789 0.431541
```

```
## fourier(log(AirPassengers), K = 6)C6-12  0.0029362   0.0049423   0.594 0.553474
```

```
##
```

```
## (Intercept)          ***
```

```
## trend                ***
```

```
## fourier(log(AirPassengers), K = 6)S1-12 ***
```

```
## fourier(log(AirPassengers), K = 6)C1-12 ***
```

```
## fourier(log(AirPassengers), K = 6)S2-12 ***
```

```
## fourier(log(AirPassengers), K = 6)C2-12 **
```

```
## fourier(log(AirPassengers), K = 6)S3-12
```

```
## fourier(log(AirPassengers), K = 6)C3-12 ***
```

```
## fourier(log(AirPassengers), K = 6)S4-12 ***
```

```
## fourier(log(AirPassengers), K = 6)C4-12 **
```

```
## fourier(log(AirPassengers), K = 6)S5-12 **
```

```
## fourier(log(AirPassengers), K = 6)C5-12
```

```
## fourier(log(AirPassengers), K = 6)C6-12
```

```
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##
```

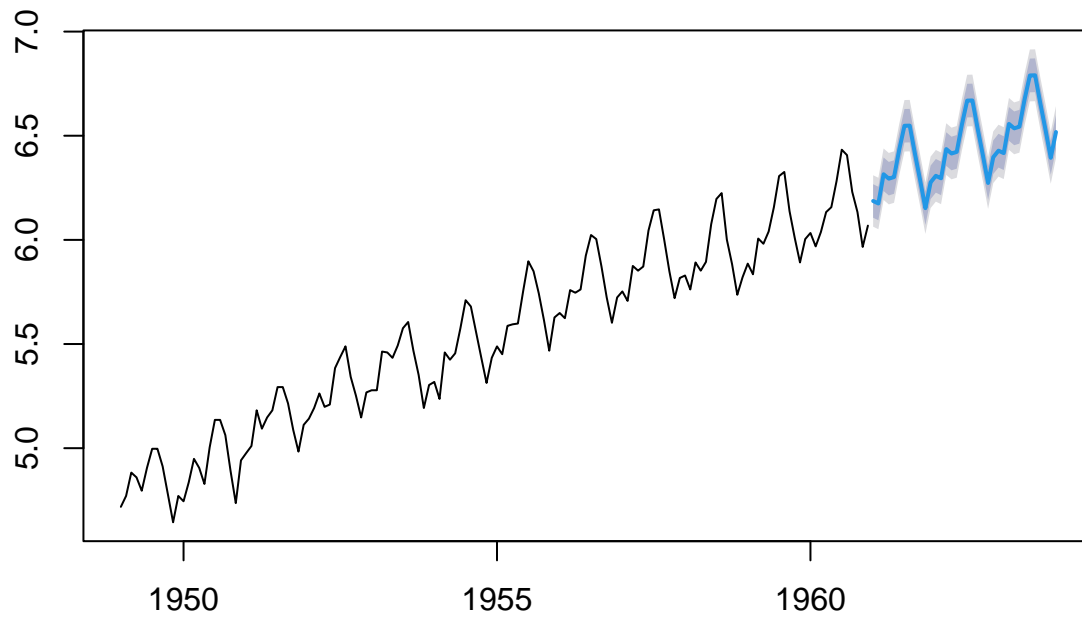
```
## Residual standard error: 0.0593 on 131 degrees of freedom
```

```
## Multiple R-squared:  0.9835, Adjusted R-squared:  0.982
```

```
## F-statistic: 649.4 on 12 and 131 DF,  p-value: < 2.2e-16
```

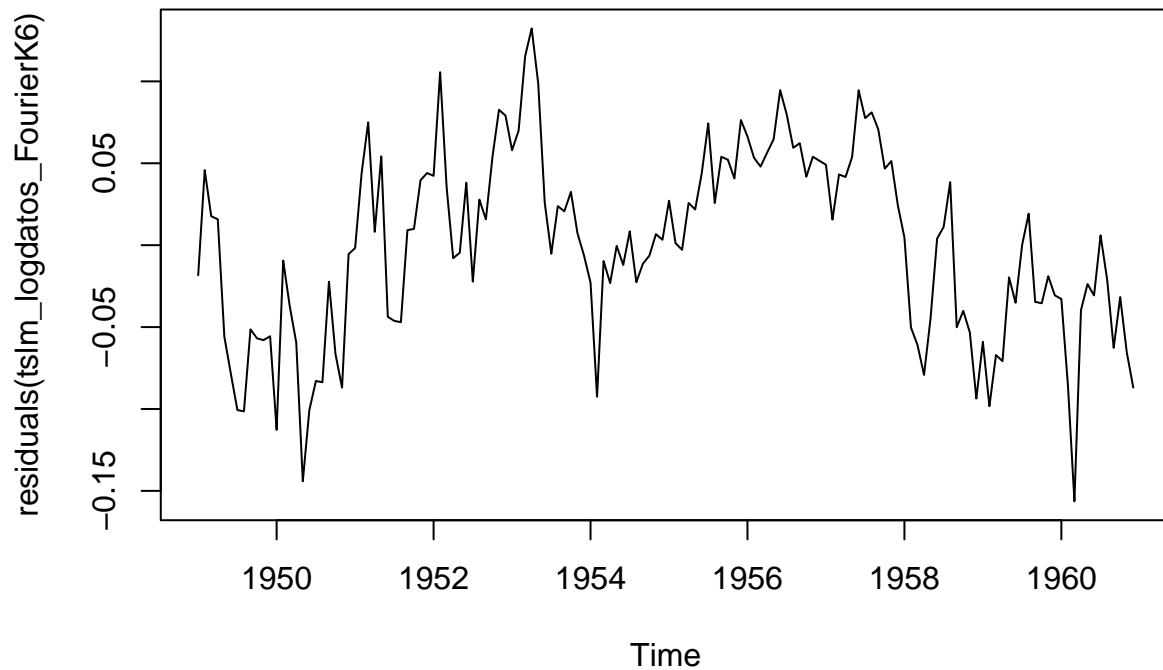
```
plot(forecast(tslm_logdatos_FourierK6, newdata = data.frame(fourier(log(AirPassengers), K = 6, h = 36)))
```

Forecasts from Linear regression model



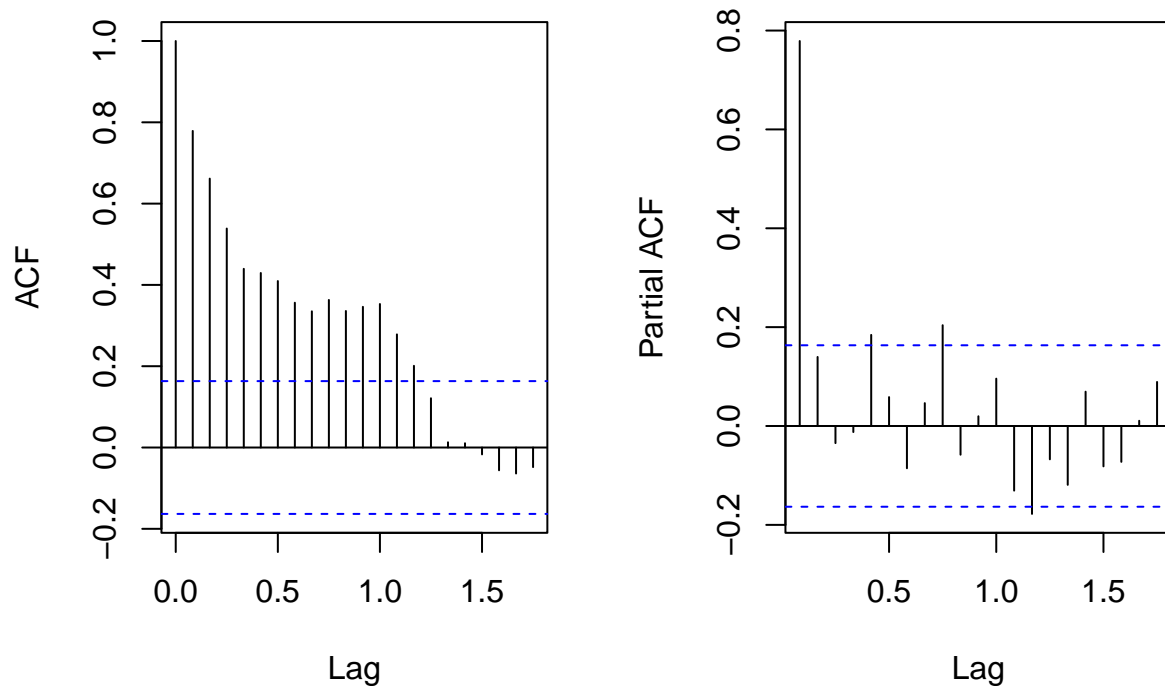
Comportamiento de los residuos del modelo:

```
# Comportamiento de los residuos del modelo:
plot(residuals(tslm_logdatos_FourierK6)) # gráfico residuos
```



```
par(mfrow = c(1, 2))
acf(residuals(tslm_logdatos_FourierK6)) # correlograma simple residuos
pacf(residuals(tslm_logdatos_FourierK6)) # correlograma parcial residuos
```

series residuals(tslm_logdatos_Fourier) residuals(tslm_logdatos_Four

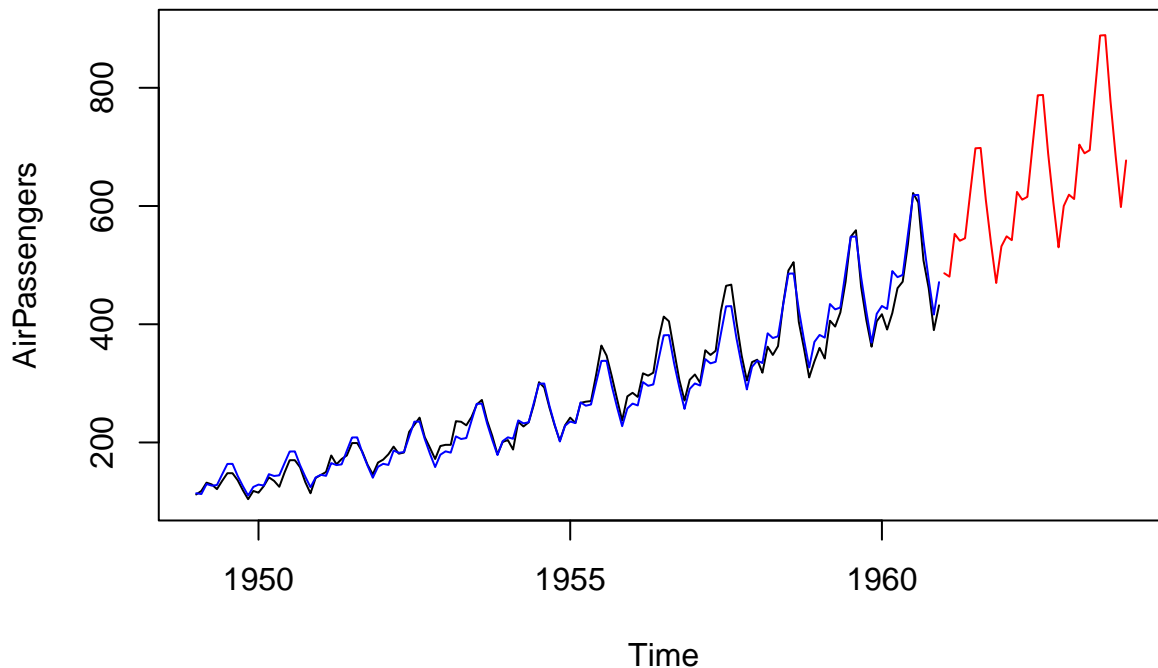


1.2. Ajuste de un modelo de Regresión (no-lineal) para la serie AirPassengers

Ahora queremos modelizar la serie AirPassengers y no su logaritmo.

ESCENARIO 1: Tomamos exponenciales sobre los resultados que obtuvimos al analizar $\log(\text{AirPassengers})$

```
plot(AirPassengers, xlim = c(1949, 1964), ylim = c(100, 900))
lines(exp(forecast(tslm_logdatos, h = 36)$fitted), col = "blue")
lines(exp(forecast(tslm_logdatos, h = 36)$mean), col = "red")
```



ESCENARIO 2: Ajuste de un modelo no-lineal para AirPassengers, que incluye interacciones entre tendencia y estacionalidad.

```
tslm_datos <- tslm(AirPassengers ~ trend * season)
summary(tslm_datos)
```

```
##
## Call:
## tslm(formula = AirPassengers ~ trend * season)
##
## Residuals:
```

	Min	1Q	Median	3Q	Max
	-36.652	-9.904	0.737	7.761	34.051

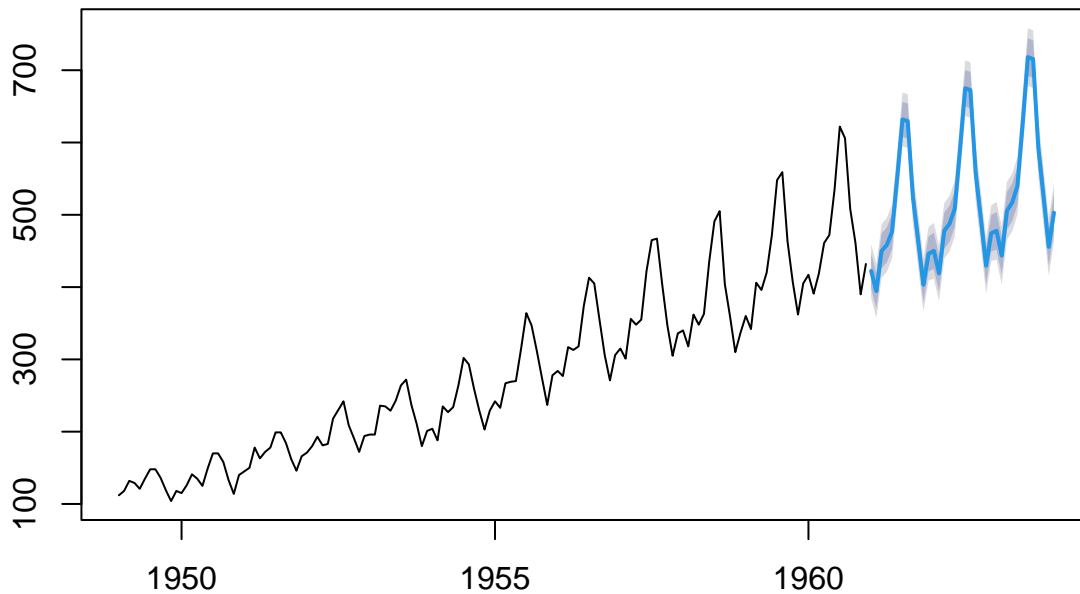
```
##
## Coefficients:
```

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	86.607517	8.794167	9.848	< 2e-16 ***
trend	2.315559	0.111641	20.741	< 2e-16 ***
season2	9.380828	12.504294	0.750	0.4546
season3	24.328380	12.572386	1.935	0.0553 .
season4	9.004371	12.641097	0.712	0.4777
season5	-1.293998	12.710418	-0.102	0.9191
season6	3.310897	12.780338	0.259	0.7960
season7	2.164044	12.850847	0.168	0.8666
season8	0.150058	12.921937	0.012	0.9908
season9	2.544289	12.993596	0.196	0.8451
season10	-13.057984	13.065816	-0.999	0.3196
season11	-21.998543	13.138589	-1.674	0.0967 .
season12	-9.092366	13.211903	-0.688	0.4927
trend:season2	-0.271270	0.157884	-1.718	0.0883 .
trend:season3	-0.007867	0.157884	-0.050	0.9603
trend:season4	0.134033	0.157884	0.849	0.3976
trend:season5	0.311480	0.157884	1.973	0.0508 .

```
## trend:season6    0.764277    0.157884    4.841 3.89e-06 ***
## trend:season7    1.281177    0.157884    8.115 4.81e-13 ***
## trend:season8    1.256410    0.157884    7.958 1.10e-12 ***
## trend:season9    0.527972    0.157884    3.344 0.0011 **
## trend:season10   0.224359    0.157884    1.421 0.1579
## trend:season11  -0.130828    0.157884   -0.829 0.4090
## trend:season12   0.047494    0.157884    0.301 0.7641
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 16.02 on 120 degrees of freedom
## Multiple R-squared:  0.985, Adjusted R-squared:  0.9822
## F-statistic: 343.4 on 23 and 120 DF, p-value: < 2.2e-16
```

```
plot(forecast(tslm_datos, h = 36))
```

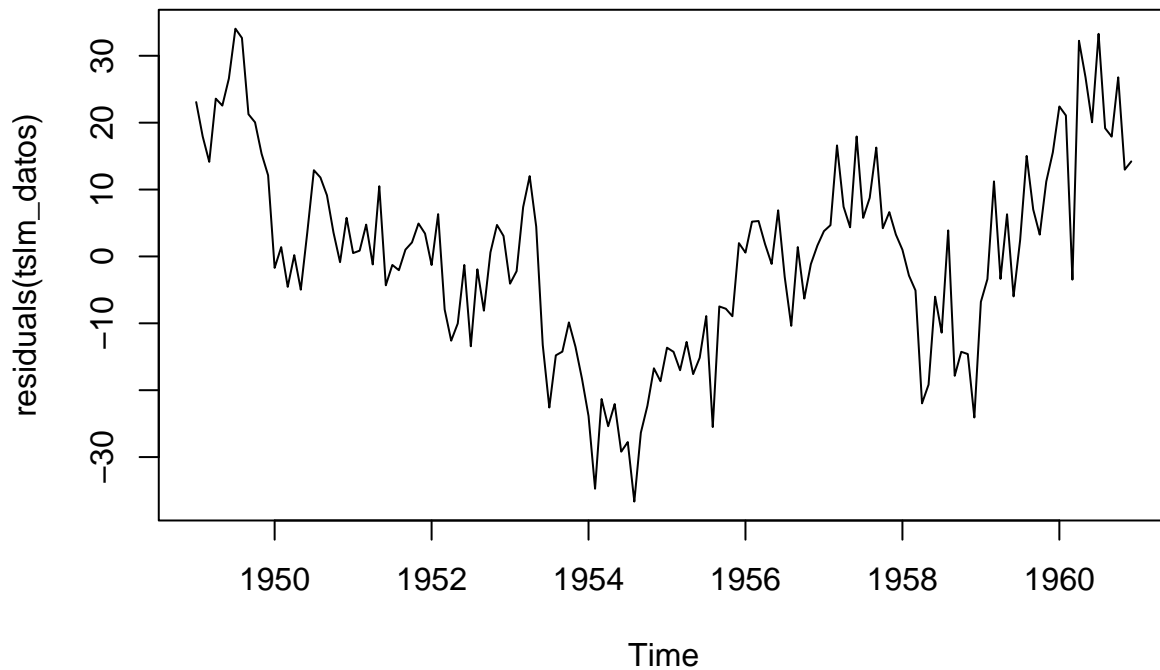
Forecasts from Linear regression model



tamiento de los residuos del modelo:

```
plot(residuals(tslm_datos)) # gráfico residuos
```

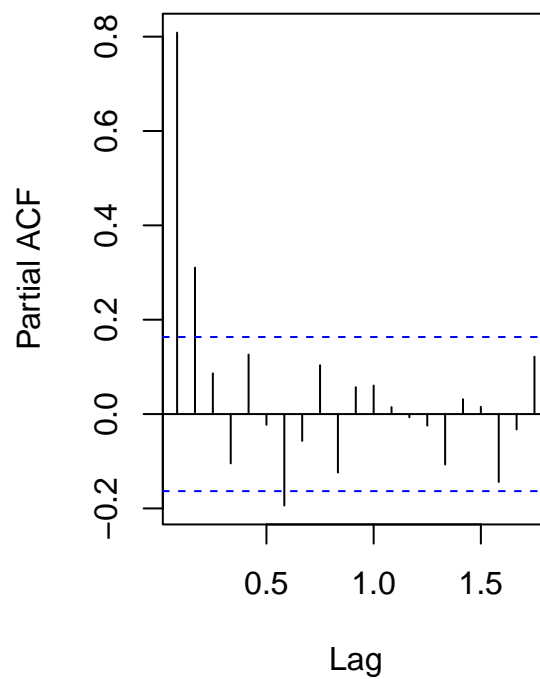
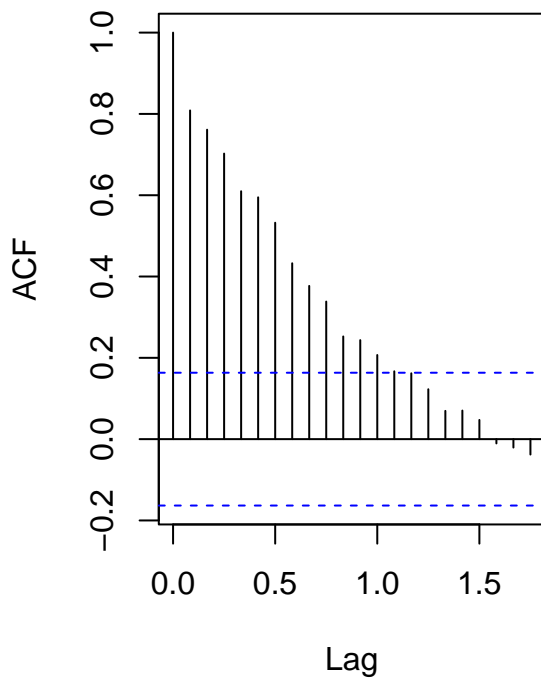
Compor-



```
par(mfrow = c(1, 2))
acf(residuals(tslm_datos)) # correlograma simple residuos
pacf(residuals(tslm_datos)) # correlograma parcial residuos
```

Series residuals(tslm_datos)

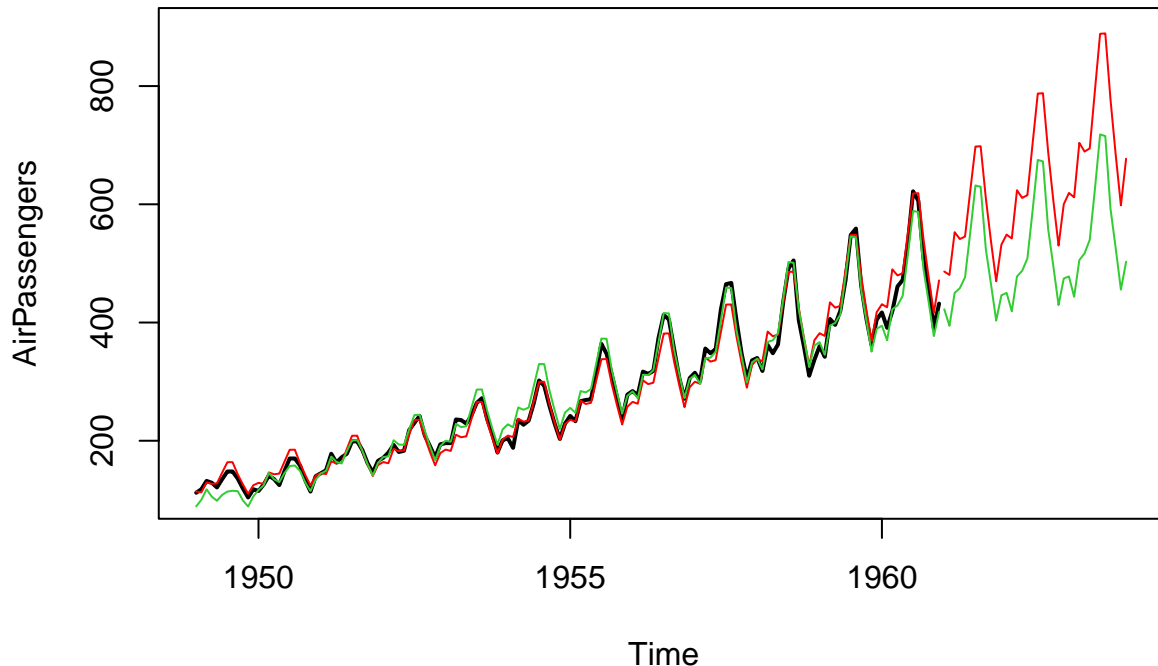
Series residuals(tslm_datos)



Comparación de predicciones en los dos escenarios:

```
plot(AirPassengers, xlim = c(1949, 1964), ylim = c(100, 900), lwd = 2)
lines(exp(forecast(tslm_logdatos, h = 36)$fitted), col = "red")
```

```
lines(exp(forecast(tslm_logdatos, h = 36)$mean), col = "red")
lines(forecast(tslm_datos, h = 36)$fitted, col = "limegreen")
lines(forecast(tslm_datos, h = 36)$mean, col = "limegreen")
```



Intervalos de predicción:

```
pred_log <- forecast(tslm_logdatos, h = 36)
pred_datos <- forecast(tslm_datos, h = 36)
```

¿Y si usamos términos de Fourier en lugar de dummy estacionales?

```
# Modelo con Fourier K=6 para AirPassengers (sin logaritmo)
tslm_datos_Fourier <- tslm(AirPassengers ~ trend * fourier(AirPassengers, K = 6))
summary(tslm_datos_Fourier)
```

```
##
## Call:
## tslm(formula = AirPassengers ~ trend * fourier(AirPassengers,
##       K = 6))
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -36.652  -9.904   0.737   7.761  34.051
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    87.06085     2.69334   32.325 < 2e-16
## trend          2.66033     0.03223  82.547 < 2e-16
## fourier(AirPassengers, K = 6)S1-12    9.69250     3.78872    2.558 0.01176
## fourier(AirPassengers, K = 6)C1-12   -6.43731     3.82908   -1.681 0.09533
## fourier(AirPassengers, K = 6)S2-12    5.63509     3.78909    1.487 0.13959
## fourier(AirPassengers, K = 6)C2-12   -7.65950     3.82872   -2.001 0.04770
## fourier(AirPassengers, K = 6)S3-12   -0.54060     3.78913   -0.143 0.88679
## fourier(AirPassengers, K = 6)C3-12    0.07139     3.82867    0.019 0.98515
```



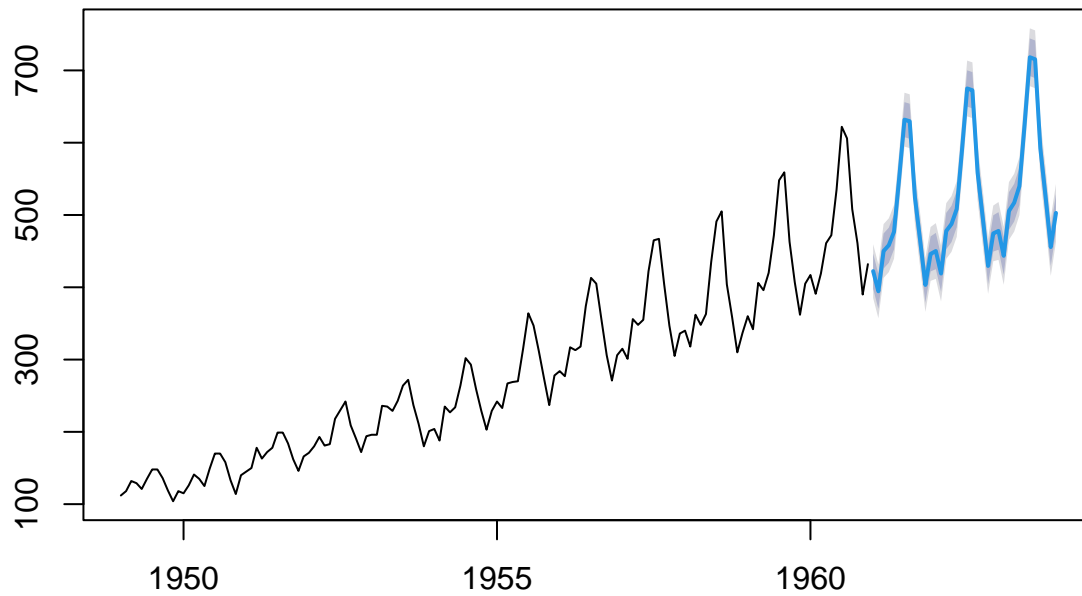
```

## fourier(AirPassengers, K = 6)S4-12      1.71359      3.78909      0.452      0.65191
## fourier(AirPassengers, K = 6)C4-12      4.81947      3.82872      1.259      0.21056
## fourier(AirPassengers, K = 6)S5-12      0.65895      3.78872      0.174      0.86222
## fourier(AirPassengers, K = 6)C5-12      0.16429      3.82908      0.043      0.96585
## fourier(AirPassengers, K = 6)C6-12     -0.50403      2.69334     -0.187      0.85187
## trend:fourier(AirPassengers, K = 6)S1-12 -0.39275      0.04558     -8.617      3.26e-14
## trend:fourier(AirPassengers, K = 6)C1-12 -0.48801      0.04558    -10.707      < 2e-16
## trend:fourier(AirPassengers, K = 6)S2-12  0.24931      0.04558      5.470      2.50e-07
## trend:fourier(AirPassengers, K = 6)C2-12  0.05847      0.04558      1.283      0.20201
## trend:fourier(AirPassengers, K = 6)S3-12 -0.05051      0.04558     -1.108      0.27003
## trend:fourier(AirPassengers, K = 6)C3-12  0.12009      0.04558      2.635      0.00952
## trend:fourier(AirPassengers, K = 6)S4-12  0.06838      0.04558      1.500      0.13614
## trend:fourier(AirPassengers, K = 6)C4-12 -0.01180      0.04558     -0.259      0.79614
## trend:fourier(AirPassengers, K = 6)S5-12  0.07433      0.04558      1.631      0.10556
## trend:fourier(AirPassengers, K = 6)C5-12  0.00952      0.04558      0.209      0.83490
## trend:fourier(AirPassengers, K = 6)C6-12  0.01445      0.03223      0.448      0.65476
##
## (Intercept)                                ***
## trend                                      ***
## fourier(AirPassengers, K = 6)S1-12          *
## fourier(AirPassengers, K = 6)C1-12          .
## fourier(AirPassengers, K = 6)S2-12
## fourier(AirPassengers, K = 6)C2-12          *
## fourier(AirPassengers, K = 6)S3-12
## fourier(AirPassengers, K = 6)C3-12
## fourier(AirPassengers, K = 6)S4-12
## fourier(AirPassengers, K = 6)C4-12
## fourier(AirPassengers, K = 6)S5-12
## fourier(AirPassengers, K = 6)C5-12
## fourier(AirPassengers, K = 6)C6-12
## trend:fourier(AirPassengers, K = 6)S1-12 ***
## trend:fourier(AirPassengers, K = 6)C1-12 ***
## trend:fourier(AirPassengers, K = 6)S2-12 ***
## trend:fourier(AirPassengers, K = 6)C2-12
## trend:fourier(AirPassengers, K = 6)S3-12
## trend:fourier(AirPassengers, K = 6)C3-12 **
## trend:fourier(AirPassengers, K = 6)S4-12
## trend:fourier(AirPassengers, K = 6)C4-12
## trend:fourier(AirPassengers, K = 6)S5-12
## trend:fourier(AirPassengers, K = 6)C5-12
## trend:fourier(AirPassengers, K = 6)C6-12
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 16.02 on 120 degrees of freedom
## Multiple R-squared:  0.985, Adjusted R-squared:  0.9822
## F-statistic: 343.4 on 23 and 120 DF, p-value: < 2.2e-16

# Predicción
fourier_futuro <- fourier(AirPassengers, K = 6, h = 36)
plot(forecast(tslm_datos_Fourier, newdata = data.frame(fourier_futuro)))

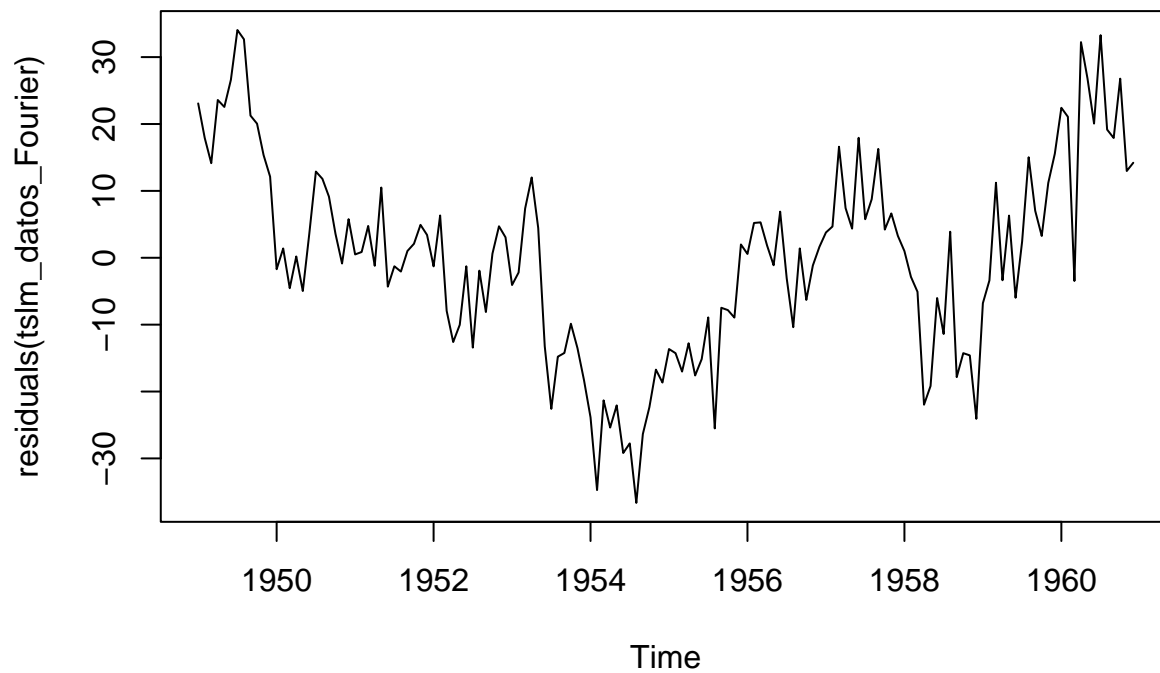
```

Forecasts from Linear regression model



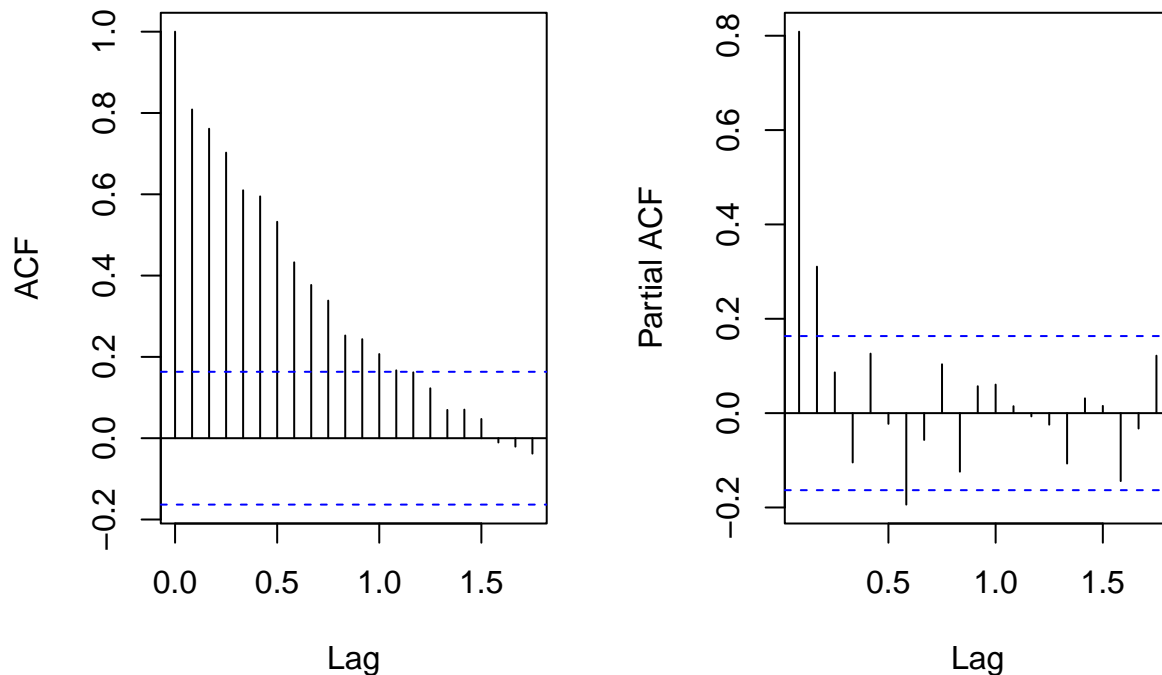
Comportamiento de los residuos

```
plot(residuals(tslm_datos_Fourier)) # gráfico residuos
```



```
par(mfrow = c(1, 2))  
acf(residuals(tslm_datos_Fourier)) # correlograma simple residuos  
pacf(residuals(tslm_datos_Fourier)) # correlograma parcial residuos
```

Series residuals(tslm_datos_Four Series residuals(tslm_datos_Four



2. Modelos de Regresión Dinámica para series temporales

Forma 1: Ajuste de regresión y ARIMA conjunto.

Forma 2: Ajuste de regresión y ajuste ARIMA de los residuos por separado.

2.1. Ajuste de regresión y ARIMA conjunto.

COMPLETAR

```
# Crear variables exógenas: tendencia y términos de Fourier
xreg_train <- cbind(
  trend = 1:length(log(AirPassengers)),
  fourier(log(AirPassengers), K = 6)
)

# Ajuste conjunto regresión + ARIMA
modelo_dinamico <- auto.arima(log(AirPassengers), xreg = xreg_train)
summary(modelo_dinamico)
```

```
## Series: log(AirPassengers)
## Regression with ARIMA(2,0,0)(1,0,0)[12] errors
##
## Coefficients:
##          ar1      ar2      sar1  intercept   trend    S1-12    C1-12    S2-12
##          0.6281  0.1781  0.2768     4.8197  0.0099  -0.0515  -0.1416   0.0773
## s.e.      0.0831  0.0830  0.0876     0.0344  0.0004   0.0089   0.0088   0.0051
##          C2-12    S3-12    C3-12    S4-12    C4-12    S5-12    C5-12    C6-12
##          -0.0223  -0.009   0.0278   0.0256   0.0220   0.0206   0.0062   0.0035
## s.e.      0.0051   0.004   0.0040   0.0036   0.0036   0.0036   0.0036   0.0026
```

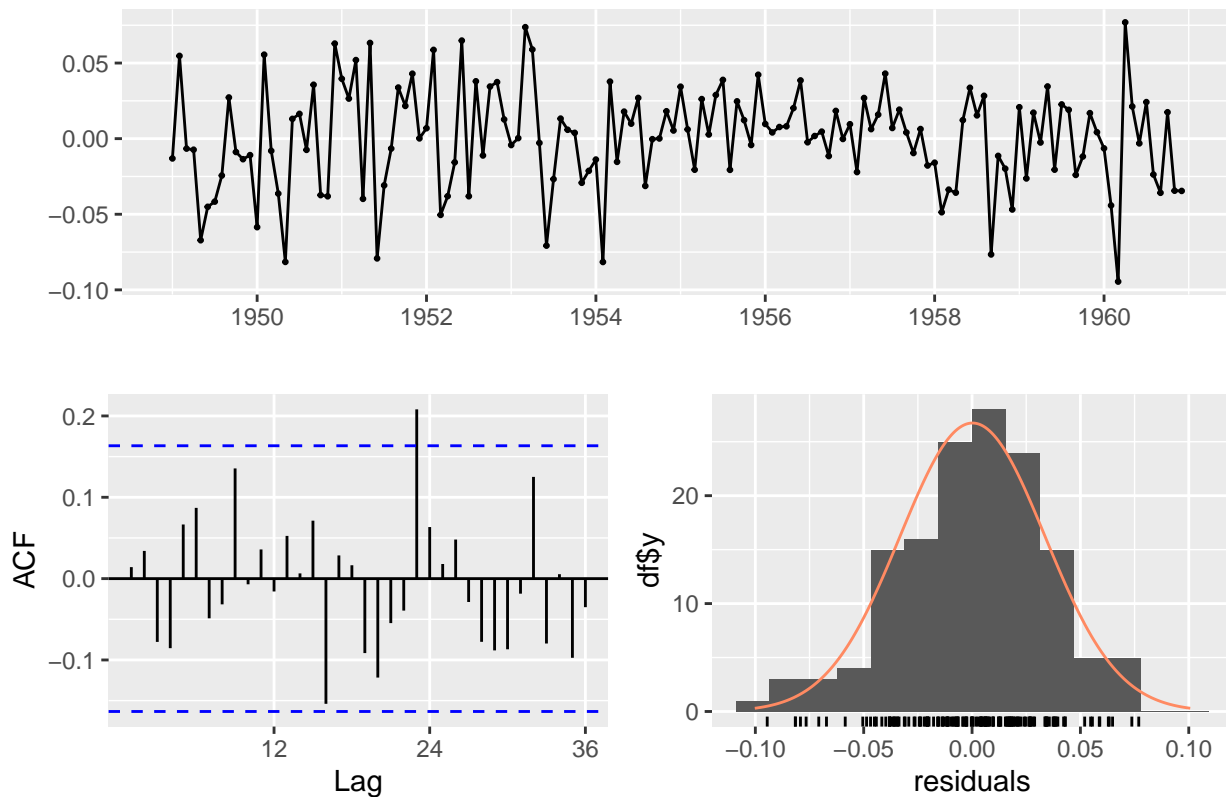
```
##
## sigma^2 = 0.001253: log likelihood = 284.27
## AIC=-534.54 AICc=-529.69 BIC=-484.06
##
## Training set error measures:
##           ME      RMSE      MAE      MPE      MAPE      MASE
## Training set 0.0002146134 0.03337733 0.02621857 0.00022698 0.4816838 0.2166072
##           ACF1
## Training set 0.01412906
```

¿El modelo es válido? Análisis de los residuos

COMPLETAR

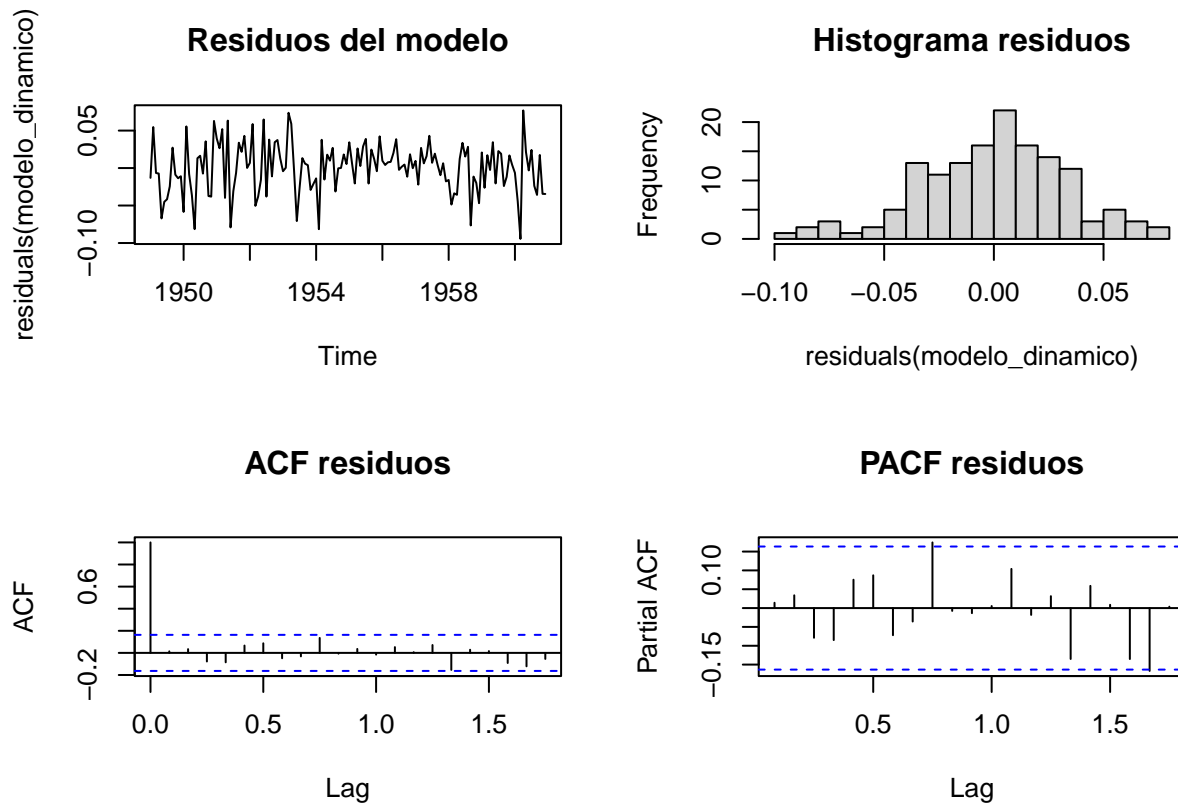
```
# Análisis completo de residuos
checkresiduals(modelo_dinamico)
```

Residuals from Regression with ARIMA(2,0,0)(1,0,0)[12] errors



```
##
## Ljung-Box test
##
## data: Residuals from Regression with ARIMA(2,0,0)(1,0,0)[12] errors
## Q* = 25.922, df = 21, p-value = 0.2094
##
## Model df: 3. Total lags used: 24
##
## Graficos adicionales
par(mfrow = c(2, 2))
plot(residuals(modelo_dinamico), main = "Residuos del modelo")
hist(residuals(modelo_dinamico), main = "Histograma residuos", breaks = 20)
```

```
acf(residuals(modelo_dinamico), main = "ACF residuos")
pacf(residuals(modelo_dinamico), main = "PACF residuos")
```



```
# Test de Ljung-Box
Box.test(residuals(modelo_dinamico), lag = 24, type = "Ljung-Box")
```

```
##
## Box-Ljung test
##
## data: residuals(modelo_dinamico)
## X-squared = 25.922, df = 24, p-value = 0.3571
```

Predicciones de la serie estacionaria y de la serie original

COMPLETAR

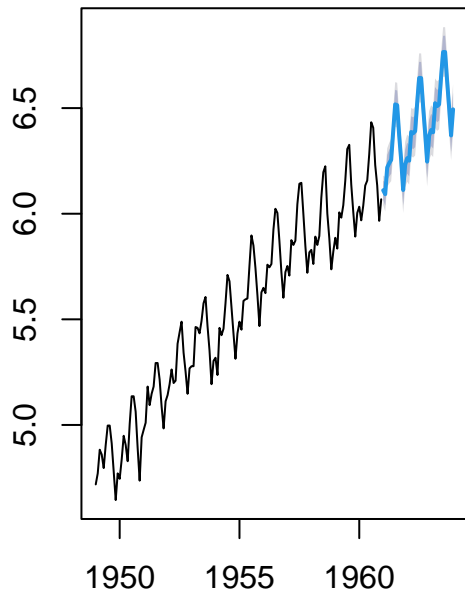
```
# Crear variables exógenas para predicción
n <- length(log(AirPassengers))
xreg_futuro <- cbind(
  trend = (n + 1):(n + 36),
  fourier(log(AirPassengers), K = 6, h = 36)
)

# Predicciones en escala logarítmica
pred_dinamico <- forecast(modelo_dinamico, xreg = xreg_futuro, h = 36)

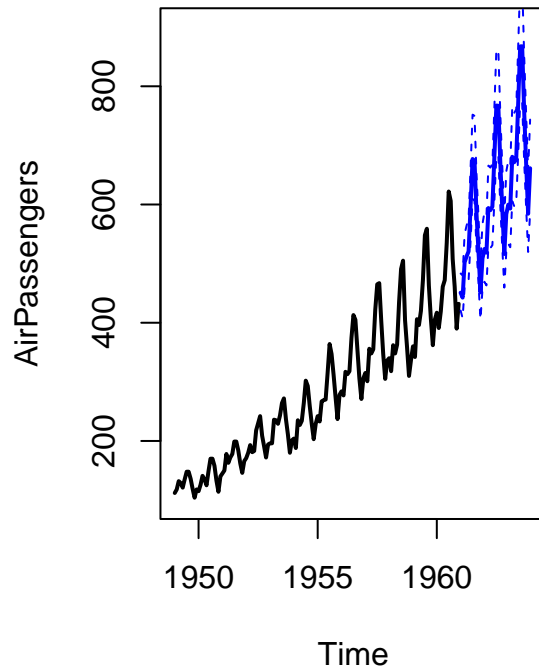
par(mfrow = c(1, 2))
# Serie log(AirPassengers)
plot(pred_dinamico, main = "Predicción log(AirPassengers)")
```

```
# Serie original AirPassengers
plot(AirPassengers,
     xlim = c(1949, 1964), ylim = c(100, 900),
     main = "Predicción AirPassengers", lwd = 2
)
lines(exp(pred_dinamico$mean), col = "blue", lwd = 2)
lines(exp(pred_dinamico$lower[, 2]), col = "blue", lty = 2)
lines(exp(pred_dinamico$upper[, 2]), col = "blue", lty = 2)
```

Predicción log(AirPassengers)



Predicción AirPassengers



2.2. Ajuste de regresión y ajuste ARIMA por separado

COMPLETAR

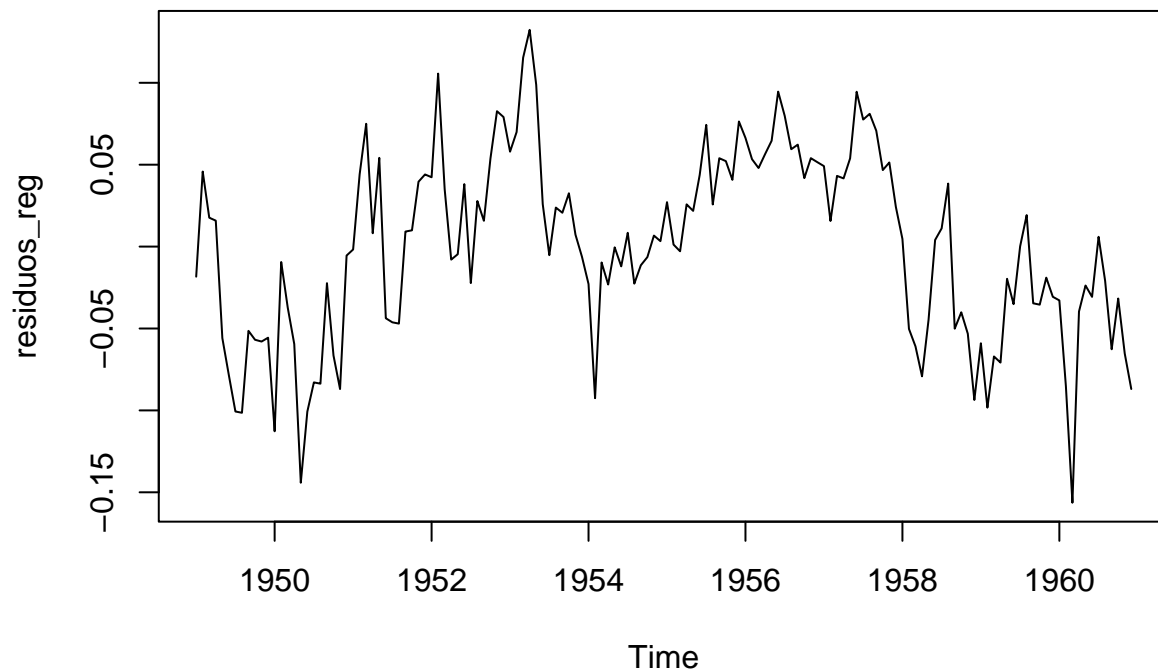
```
# Paso 1: Ajustar modelo de regresión
reg_modelo <- tslm(log(AirPassengers) ~ trend + season)
summary(reg_modelo)

##
## Call:
## tslm(formula = log(AirPassengers) ~ trend + season)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.156370 -0.041016  0.003677  0.044069  0.132324
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  4.7267804  0.0188935 250.180 < 2e-16 ***
## trend        0.0100688  0.0001193  84.399 < 2e-16 ***
## season2     -0.0220548  0.0242109  -0.911  0.36400
```

```
## season3      0.1081723  0.0242118   4.468 1.69e-05 ***
## season4      0.0769034  0.0242132   3.176  0.00186 **
## season5      0.0745308  0.0242153   3.078  0.00254 **
## season6      0.1966770  0.0242179   8.121 2.98e-13 ***
## season7      0.3006193  0.0242212  12.411 < 2e-16 ***
## season8      0.2913245  0.0242250  12.026 < 2e-16 ***
## season9      0.1466899  0.0242294   6.054 1.39e-08 ***
## season10     0.0085316  0.0242344   0.352  0.72537
## season11     -0.1351861 0.0242400  -5.577 1.34e-07 ***
## season12     -0.0213211 0.0242461  -0.879  0.38082
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.0593 on 131 degrees of freedom
## Multiple R-squared:  0.9835, Adjusted R-squared:  0.982
## F-statistic: 649.4 on 12 and 131 DF, p-value: < 2.2e-16
```

```
# Paso 2: Obtener residuos de la regresión
residuos_reg <- residuals(reg_modelo)
plot(residuos_reg, main = "Residuos del modelo de regresión")
```

Residuos del modelo de regresión



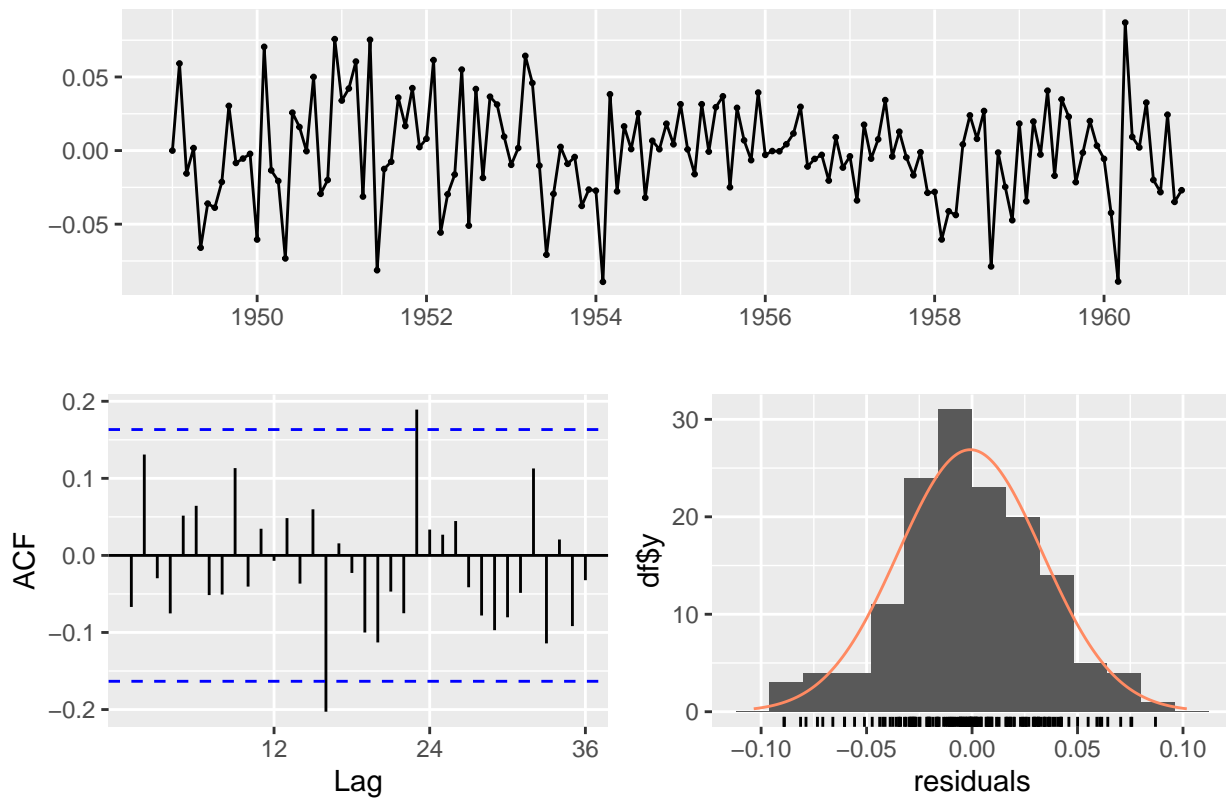
```
# Paso 3: Ajustar modelo ARIMA a los residuos
arima_residuos <- auto.arima(residuos_reg)
summary(arima_residuos)
```

```
## Series: residuos_reg
## ARIMA(1,1,1)(1,0,0)[12]
##
## Coefficients:
##      ar1      ma1      sar1
##  0.6063  -0.877   0.2266
```

```
## s.e. 0.2054 0.146 0.0871
##
## sigma^2 = 0.001196: log likelihood = 279.22
## AIC=-550.44 AICc=-550.15 BIC=-538.59
##
## Training set error measures:
##           ME      RMSE      MAE      MPE      MAPE      MASE
## Training set -0.0008472376 0.03410461 0.02610686 156.5361 323.236 0.5456
##           ACF1
## Training set -0.06686943
```

```
# Verificar residuos del ARIMA
checkresiduals(arima_residuos)
```

Residuals from ARIMA(1,1,1)(1,0,0)[12]



```
##
## Ljung-Box test
##
## data: Residuals from ARIMA(1,1,1)(1,0,0)[12]
## Q* = 28.133, df = 21, p-value = 0.1364
##
## Model df: 3. Total lags used: 24
# Paso 4: Predicciones combinadas
# Predicción de la parte de regresión
pred_reg <- forecast(reg_modelo, h = 36)

# Predicción de la parte ARIMA (residuos)
pred_arima_res <- forecast(arima_residuos, h = 36)
```



```

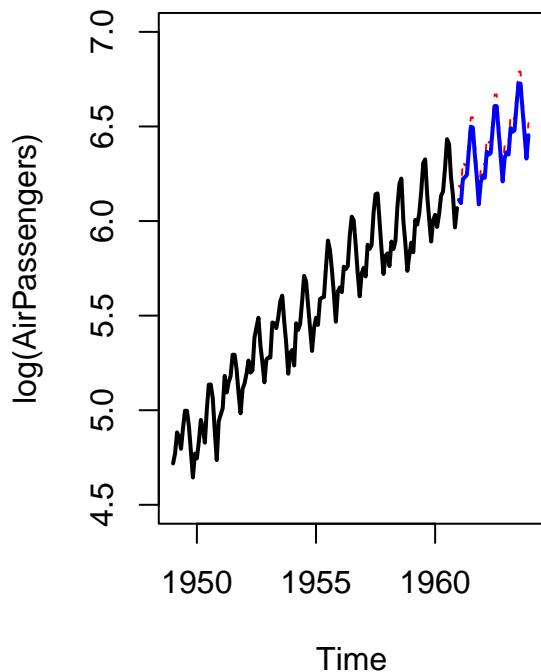
# Predicción combinada = regresión + ARIMA residuos
pred_combinada <- pred_reg$mean + pred_arima_res$mean

# Visualización
par(mfrow = c(1, 2))
plot(log(AirPassengers),
     xlim = c(1949, 1964), ylim = c(4.5, 7),
     main = "Predicción log(AirPassengers)", lwd = 2
)
lines(pred_reg$mean, col = "red", lty = 2) # Solo regresión
lines(pred_combinada, col = "blue", lwd = 2) # Regresión + ARIMA

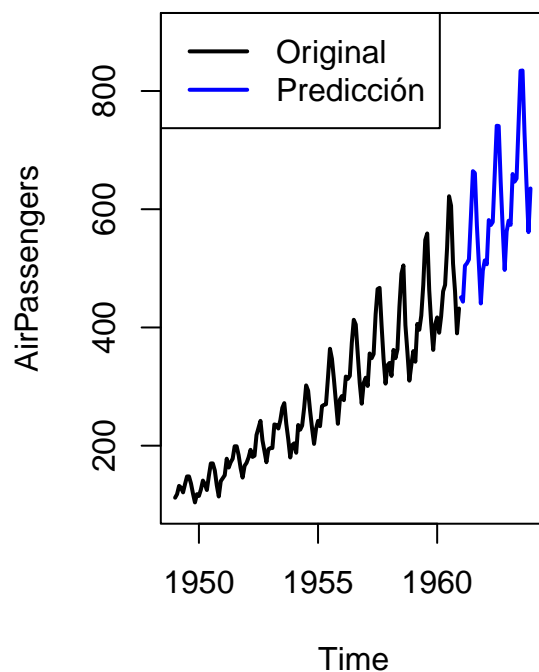
plot(AirPassengers,
     xlim = c(1949, 1964), ylim = c(100, 900),
     main = "Predicción AirPassengers", lwd = 2
)
lines(exp(pred_combinada), col = "blue", lwd = 2)
legend("topleft",
      legend = c("Original", "Predicción"),
      col = c("black", "blue"), lwd = 2
)

```

Predicción log(AirPassengers)



Predicción AirPassengers



```

# Comparación de ambos métodos
par(mfrow = c(1, 1))
plot(AirPassengers,
     xlim = c(1949, 1964), ylim = c(100, 900),
     main = "Comparación de métodos de Regresión Dinámica", lwd = 2
)
lines(exp(pred_dinamico$mean), col = "red", lwd = 2) # Método conjunto

```

```

lines(exp(pred_combinada), col = "blue", lwd = 2) # Método separado
legend("topleft",
      legend = c("Original", "Conjunto (auto.arima+xreg)", "Separado (tslm+ARIMA)"),
      col = c("black", "red", "blue"), lwd = 2
)

```

Comparación de métodos de Regresión Dinámica

