$$f(t) = j\,\frac{1}{1+t}\,e^{j\left(t+\frac{\pi}{2}\right)}\,,\quad 0 \le t \le 4\pi$$

1) Módulo y fase de $f(t)$

· Para el módulo:

$$|f(t)| = |j| \cdot \left|\frac{1}{1+t}\right| \cdot \left|e^{j\left(t+\frac{\pi}{2}\right)}\right| = 1 \cdot \frac{1}{1+t} \cdot 1 = \frac{1}{1+t}$$

• Para la fase:

$$\arg\left(f(t)\right) = \arg(j) + \arg\left(t+\frac{\pi}{2}\right) = \frac{\pi}{2} + \left(t+\frac{\pi}{2}\right) = t+\pi$$

• Para $t = 1$:

$$|f(1)| = \frac{1}{1+1} = \frac{1}{2} \qquad \arg\left(f(1)\right) = 1+\pi$$

2) Forma binómica de $f(t)$

Para encontrar la forma binómica, debemos expandir $f(t)$ en términos de parte real e imaginaria:

$$f(t) = j \cdot \frac{1}{1+t} \cdot \Big(\underbrace{\cos\left(t+\frac{\pi}{2}\right)}_{-\sin(t)} + j \cdot \underbrace{\sin\left(t+\frac{\pi}{2}\right)}_{\cos(t)}\Big)$$

$$= j \cdot \frac{1}{1+t} \cdot \left(-\sin(t) + j \cdot \cos(t)\right)$$

$$= \frac{1}{1+t} \cdot \left(-\cos(t) + j \cdot \sin(t)\right)$$

• Para $t = 1$:

$$f(1) = \frac{1}{2} \cdot \left(-\cos(1) + j \cdot \sin(t)\right)$$

$$\approx -0.2702 + 0.4207j$$

3) Inverso de $f(t)$

$$\frac{1}{f(t)} = \frac{1}{j\,\frac{1}{1+t}\,e^{j\left(t+\frac{\pi}{2}\right)}}$$

Para simplificar

$$f(t) = \frac{A+jB}{1+t} \longrightarrow \frac{1}{f(t)} = \frac{1+t}{A+jB} \cdot \frac{A-jB}{A-jB} = \frac{(1+t)\cdot(A-jB)}{A^2+B^2}$$

donde:

$$\boxed{\begin{aligned} A &= -\cos(t) \\ B &= \sin(t) \end{aligned}}$$

Para $t=1$:

$$\frac{1}{f(1)} = \frac{2\cdot(-0.5403+0.8415j)}{\underbrace{(-0.5403)^2+(0.8415)^2}_{\approx 1}} \approx -1.0806+1.683j$$

4) $f^2(t)$ y $f(t)+\frac{1}{f(t)}$

$$f^2(t) = \left(-\cos(t)+j\cdot\sin(t)\right)^2$$

Para $t=1$:

$$f^2(1) = (-0.2702+0.4207j)^2 \approx -0.104-0.2273j$$

$$f(t)+\frac{1}{f(t)} = \left(-\cos(t)+j\cdot\sin(t)\right)+\frac{(1+t)\cdot(A-jB)}{A^2+B^2}$$

Para $t=1$:

$$f(1)+\frac{1}{f(1)} = (-0.2702+0.4207j)+(-1.0806+1.683j) = -1.3508+1.2622j$$