Integrales.pdf

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Cálculo I

1º Grado en Ciencia e Ingeniería de Datos

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Integrales primitivas

1. $\int \frac{x}{x^2+1}\,dx = \frac{1}{2}\int \frac{2x}{x^2+1} = \frac{1}{2}\log(x^2)+C$

2. $\int x^2 e^{x^3}dx = \frac{1}{3}\int 3x^2 e^{x^3}dx = \frac{1}{3}e^{x^3}+C$

3. $\int \frac{x}{(3x^2+5)^3}\,dx = \frac{1}{6}\int \frac{6x}{(3x^2+5)^3} = \begin{Bmatrix} t=3x^2+5 \\ 6x\,dx = dt \end{Bmatrix} \frac{1}{6}\int \frac{1}{t^3}\,dt = \frac{1}{6}\int t^{-3}$

$= \frac{1}{6}\cdot\frac{t^{-2}}{-2}+c = \frac{1}{6}\cdot\frac{1}{-2\cdot(3x^2+5)^2} = \frac{-1}{12}\cdot\frac{1}{(3x^2+5)^2}+c$ C. Variable

4. $\int \frac{\cos(\log(x))}{x}\,dx = \operatorname{sen}(\log(x)+C$

5. $\int \frac{x^2}{\sqrt[3]{3x^3+5}} = \frac{1}{9}\int \frac{9x^2}{\sqrt[3]{3x^3+5}}\,dx = \begin{Bmatrix} t=3x^3+5 \\ 9x^2=dt \end{Bmatrix} \frac{1}{9}\int \frac{1}{\sqrt[3]{t}}\,dt = \frac{1}{9}\int t^{-1/3}dt$

$= \frac{1}{9}\cdot\frac{t^{2/3}}{2/3}dt = \frac{1}{6}\cdot t^{2/3}\,dt = \frac{1}{6}\cdot\sqrt[3]{(3x^3+5)^2}$ => C. variable

I. por partes (polinomio . exponenencial)

6. $\int x\cdot e^x dx = \begin{Bmatrix} u=x & du=1 \\ dv=e^x & v=e^x \end{Bmatrix} = x\cdot e^x - \int 1\cdot e^x = xe^x - e^x + C$

I. partes (polinomio . f. trigonométrica)

7. $\int x\cdot\cos x\,dx = \begin{Bmatrix} u=x & du=1 \\ dv=\cos(x) & v=\operatorname{sen}(x) \end{Bmatrix} = x\cdot\operatorname{sen}(x) - \int \operatorname{sen}(x)\,dx =$

$x\operatorname{sen}(x) - (-\cos(x)) = x\cdot\operatorname{sen}(x)+\cos(x)+C$

8. $\int x\cdot\log(x)\,dx = \begin{Bmatrix} u=\log(x) & du=\frac{1}{x} \\ dv=x & v=\frac{x^2}{2} \end{Bmatrix} = \frac{x^2}{2}\cdot\log(x) - \int \frac{x^{\cancel{2}}}{2}\cdot\frac{1}{\cancel{x}}\,dx =$

$\frac{x^2}{2}\cdot\log(x) - \frac{1}{2}\cdot\frac{x^2}{2}+C = \frac{x^2}{2}\log(x) - \frac{1}{4}\cdot x^2 + C$ I. partes polinomio . logaritmo

9. $\underbrace{\int e^x \cdot sen(x)}_{I} = \left\{ \begin{matrix} u = e^x & du = e^x \\ dv = sen\, x & v = -\cos(x) \end{matrix} \right\} = -e^x \cdot \cos(x) + \int e^x \cos(x)\, dx =$

$\left\{ \begin{matrix} u = e^x & du = e^x \\ du = \cos(x) & v = sen(x) \end{matrix} \right\} = -e^x \cdot \cos(x) + e^x \cdot sen(x) - \int sen(x) \cdot e^x =$

I. por partes cíclica

$I = -e^x \cos(x) + e^x \cdot sen(x) - I \; ; \quad I = \frac{1}{2} \cdot (-e \cos(x) + e^x \cdot sen(x)) + C$

I. partes función

10. $\int \log(x) \cdot dx = \int 1 \; \log(x)\, dx = \left\{ \begin{matrix} u = \log(x) & du = \frac{1}{x} \\ dv = 1 & v = x \end{matrix} \right\} = x \cdot \log(x) - \underbrace{\int \cancel{x} \cdot \frac{1}{\cancel{x}} dx}_{x} + C$

11. $\int \frac{x^3}{x^4+5} dx = \frac{1}{4} \int \frac{4x^3}{x^4+5} dx = \frac{1}{4} \cdot \log(x^4+5) + C$

12 $\int x \cdot sen(x^2+3) = \frac{1}{2} \int 2x \cdot sen(x^2+3) = -\frac{1}{2} \cdot \cos(x^2+3) + C$

13. $\int x \cdot e^{3x^2} dx = \frac{1}{6} \int 6x \cdot e^{3x^2} dx = \frac{1}{6} \cdot e^{3x^2} + C$

14. $\int x^2 \cdot (3x^3+7)^8 dx = \frac{1}{9} \int \underbrace{9x^2}_{dt} \cdot (\underbrace{3x^3+7}_{t})^8 dx = \frac{1}{9} \cdot \int t^8 dt =$

$\frac{1}{9} \cdot \frac{t^9}{9} = \frac{1}{81} \cdot t^9 = \frac{1}{81} \cdot (3x^3+7)^9 + C$ cambio de variable

15. $\int e^x \cdot \cos(e^x)\, dx = sen(e^x) + C$

I partes (pol. exp)

16. $\int (\overset{u}{x+2}) \cdot \overset{dv}{e^x} = \left\{ \begin{matrix} u = x+2 ; du = dx \\ dv = e^x \quad v = e^x \end{matrix} \right\} = (x+2) \cdot e^x - \int e^x dx = (x+2) \cdot e^x - e^x + C$

polinomio · exponencial

I. partes (pol. f. trig)

17. $\int \overset{u}{x} \cdot \overset{dv}{sen(x)}\, dx = \left\{ \begin{matrix} u = x & du = dx \\ dv = sen(x) & v = -\cos(x) \end{matrix} \right\} = -x \cdot \cos(x) + \int \cos(x)\, dx = -x \cdot \cos(x) + sen(x) \cdot x$

polinomio · f. trigonométrica

I. partes (pol. log)

18 $\int \overset{dv}{x^2} \cdot \overset{u}{\log}\, x \cdot dx = \left\{ \begin{matrix} u = \log(x) & du = \frac{1}{x} \\ dv = x^2 & v = \frac{x^3}{3} \end{matrix} \right\} = \log(x) \cdot \frac{x^3}{3} - \int \frac{x^{\cancel{3}\,2}}{3} \cdot \frac{1}{\cancel{x}} = \log(x) \cdot \frac{x^3}{3} - \frac{1}{3} \cdot \frac{x^3}{3} + C$

polinomio · logaritmo


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19. $\int \overbrace{e^x}^{dv} \cdot \overbrace{\cos(x)}^{u}\, dx = \left\{\begin{matrix} u=\cos(x) & du=-\text{sen}(x) \\ dv=e^x & v=e^x \end{matrix}\right\} = \cos(x)\cdot e^x - \int \underbrace{e^x}_{dv} \cdot \underbrace{-\text{sen}(x)}_{u}$

exponencial · f. trigonométrica
I

$= \left\{\begin{matrix} u=-\text{sen}(x) & du=-\cos(x) \\ dv=e^x & v=e^x \end{matrix}\right\} = \cos(x)\cdot e^x - \left(-\text{sen}(x)\cdot e^x) + \int e^x \cdot \cos(x)\right) =$

$\cos(x)\cdot e^x + \text{sen}(x)\, e^x - \underbrace{\int e^x \cdot \cos(x)}_{I}$; $I = \cos(x)\cdot e^x + \text{sen}(x)\cdot e^x - I$

$I = \frac{1}{2}\cdot(\cos(x)\, e^x + \text{sen}(x)\cdot e^x) + C$ I. cíclica

21. $\int \text{arctang}(x)\, dx = \int \overset{dv}{1}\ \overbrace{\text{arctan}(x)}^{u}\, dx = \left\{\begin{matrix} u=\text{arctan} & du=\frac{1}{1+x^2} \\ dv=1 & v=x \end{matrix}\right\} =$

$\text{arctan}(x)\cdot x - \int x \cdot \frac{1}{1+x^2} = \text{arctan}(x)\cdot x - \frac{1}{2}\cdot \log(|1+x^2|) + C$ I. partes función · 1

22. $\int \frac{x^2}{x+1}\, dx$ $\quad \begin{array}{r|l} x^2 & x+1 \\ -x^2-x & x-1 \\ \hline -x & \\ +x+1 & \\ \hline 1 & \end{array}$ $= \int x-1+\frac{-1}{x+1}\, dx = \frac{x^2}{2} - x - \log(|x+1|) + C$

I. racionales (Alg de Euclides)

23. $\int \frac{x+3}{x-1}\, dx$ $\quad \begin{array}{r|l} x+3 & x-1 \\ -x+1 & 1 \\ \hline 4 & \end{array}$ $= \int 1+\frac{4}{x-1}\, dx = x + 4\cdot \log(|x-1|)\, dx$

g. numerador >= denominador

I. racional (fracciones simples)

24. $\int \frac{1}{x\cdot(x+1)}\, dx = \frac{A}{x}dx + \frac{B}{x+1}dx = A\cdot(x+1) + Bx = 1$

g. numerador < denominador

$\left\{\begin{matrix} Ax + A + Bx = 1\ ;\ x=0 \Rightarrow A=1 \\ x=1 \Rightarrow 2+B=1\ ;\ B=-1 \end{matrix}\right\} = \int \frac{1}{x}\, dx + \int -\frac{1}{x+1} =$

$\log(|x|) - \log(|x+1|) + C$

I. racional (fracciones simples)

25. $\int \frac{1}{x^2+3x+2}\, dx\ ;\ \frac{A}{x+1} + \frac{B}{x+2} = \frac{A\cdot(x+2) + B\cdot(x+1)}{(x+1)\cdot(x+2)}\ ;\ 1 = A(x+2) + B\cdot(x+1)$

g. numerador < denominador

$\left.\begin{matrix} x=-1 \Rightarrow A=1 \\ x=-2 \Rightarrow B=-1 \end{matrix}\right\}\ ;\ \int \frac{1}{x+1}dx + \int -\frac{1}{x+2}\, dx = \log(|x+1|) - \log(|x+2|) + C$

26. $\int \frac{1}{x^3+x}dx = \int \frac{1}{x\cdot(x^2+1)}dx$; $\frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$

grado 1 {polinomio genérico}

g. numerador < denominador

↳ grado 2

$= \frac{A(x^2+1)+x(Bx+C)}{x\cdot(x^2+1)} \rightarrow 1 = A(x^2+1)+x(Bx+C)$

$\rightarrow x=0 \Rightarrow A=1$ I. racional (f. simples)

$Ax^2+A+Bx^2+Cx = 1^{*} \Rightarrow A=1^{*}$

* termino independiente grado 0

$x^2\cdot\underbrace{(A+B)}=0 \quad 1-1=0$; $B=-1$, $C=0$

$\int \frac{1}{x} - \frac{x}{x^2+1}dx = \log|x| - \frac{1}{2}\log x^2+1c$

27. $\int \frac{1}{x^2+2x+2}dx$; $x^2+2x+2 \rightarrow (x^2+2x+1)+1 = (x+1)^2+1$; $\int \frac{1}{(x+1)^2+1}$

grado 0 < grado 2 ↳ sin raíces reales

$(x+a)^2 = x^2+2xa+a^2 = x^2+2x+1 = (x+1)^2$

$2a^2=2$; $a=1 \rightarrow$

I. racionales (cuadrados perfectos)

$\begin{Bmatrix} t=x+1 \\ dt=dx \end{Bmatrix} = \int \frac{1}{t^2+1}dt = \operatorname{arctang}(x+1)+C$

28. $\int \frac{1}{x^2+4x+8}dx$; $x^2+4x+8 = (x^2+4x+4)+4 = (x+2)^2+4$; $\int \frac{1}{(x+2)^2+4}dx$

$\rightarrow \nexists$ raíces

g. numerador < denominador (2)

$(x+a)^2 = x^2+2xa+a^2 = x^2+4x+4$

$4x=2xa$; $a=2$

$\frac{1}{4}\int \frac{1}{\frac{(x+2)^2+1}{4}} = \int \frac{1}{\left(\frac{x+2}{2}\right)^2+1}dx = \frac{1}{4}\cdot\frac{2}{1}\int \frac{1}{\left(\frac{x+2}{2}\right)^2+1}dx = \frac{1}{2}\operatorname{arctang}\left(\frac{x+2}{2}\right)^2 + C$

↳ derivada $= \frac{1}{2}$

29. $\int \frac{1}{x^2+x+2}$; $x^2+x+2 = (x^2+x+\frac{1}{4})+\frac{7}{4}$

$\rightarrow \nexists$ raíces reales

g. num < g. den (2)

$(x+a)^2 = x^2+2xa+a^2 = x^2+x+\frac{1}{4}$; $\int \frac{1}{(x+\frac{1}{2})^2+\frac{7}{4}}dx$

$x=2xa$; $a=\frac{1}{2}$

$\frac{4}{7}\int \frac{(1/7/4)}{\frac{(x+1)^2}{7/4}+1}dx = \frac{4}{7}\int \frac{1}{\left(\frac{x+1}{\frac{\sqrt{7}}{2}}\right)^2+1}dx = \frac{4}{7}\cdot\frac{\sqrt{7}}{2}\cdot\int \frac{1\cdot 2/\sqrt{7}}{\left(\frac{2\cdot(x+1)}{\sqrt{7}}\right)^2+1}dx =$

$\frac{2\sqrt{7}}{7}\cdot\operatorname{arctang}\left(\frac{2(x+1)}{\sqrt{7}}\right)+c$

si lees esto me debes un besito

I.racional (Alg. Euclides)

30. $\int \frac{x^2+3}{x+2}dx$; $\begin{array}{r|l} x^2+3 & x+2 \\ -x^2-2x & x-2 \\ 0-2x+3 & \\ -\ -2x-4 & \\ 0\quad 7 & \end{array}$; $= \int x-2+\frac{7}{x+2}dx = \frac{x^2}{2}-2x+7\log(|x+2|)+C$

g numerador >= denominador

31. $\int \frac{x^2}{x^2+1}dx$; $\begin{array}{r|l} x^2 & x^2+1 \\ -x^2-1 & 1 \\ -1 & \end{array}$; $\int 1-\frac{1}{x^2+1}dx = x - \arctan(x)+C$

g.numerador >= denominador

32. $\int \frac{x^3+3x^2+2x+5}{x+2}dx$; $\begin{array}{r|l} x^3+3x^2+2x+5 & x+2 \\ -x^3+2x^2 & x^2+x \\ 0+x^2+2x+5 & \\ -\ x^2+2x & \\ 0\quad 0\quad 5 & \end{array}$, $\frac{x^3+3x^2+2x+5}{x+2} = x^2+x+\frac{5}{x+2}$

g.numerador >= denominador

$$\int \frac{x^3+3x^2+2x+5}{x+2}dx = \int x^2+x+\frac{5}{x+2}dx = \frac{x^3}{3}+\frac{x^2}{2}+5\cdot\log|x+2|+C$$

I.racionales (f.simples)

33. $\int \frac{1}{x(x+2)}dx$; $\frac{1}{x(x+2)} = \frac{A}{x}+\frac{B}{x+2} = \frac{A(x+2)+Bx}{x\cdot(x+2)}$; $1 = A(x+2)+Bx$

g.numerador < g.denominador

$x=0 \quad 1=2A$; $A=\frac{1}{2}$

$x=-2 \quad 1=-2B$; $B=-\frac{1}{2}$

$$\int \frac{1}{x(x+2)}dx = \int \frac{1}{2x} - \frac{1}{2(x+2)}dx = \frac{1}{2}\int \frac{1}{x} - \frac{1}{2}\int \frac{1}{x+2}dx = \frac{1}{2}\cdot\log 2x - \frac{1}{2}\log|2x+2|+C$$

34. $\int \frac{1}{x^2+5+6}dx = \int \frac{1}{(x+2)\cdot(x+3)}dx$; $\frac{1}{(x+2)\cdot(x+3)} = \frac{A}{x+2}+\frac{B}{(x+3)}$

$$\frac{A\cdot(x+3)+B\cdot(x+2)}{(x+2)\cdot(x+3)} \Rightarrow 1 = A(x+3)+B(x+2)$$

$x=-3 \quad 1=-B$; $-1=B$

$x=-2$; $A=1$

$$\int \frac{1}{x^2+5+6}dx = \int \frac{1}{x+2}dx - \int \frac{1}{x+3}dx = \log|x+2| - \log|x+3|+C$$

I.racional (f.simples)

35. $\int \frac{1}{(x+2)(x^2+1)}dx$; $\frac{1}{(x+2)(x^2+1)} = \frac{A}{(x+2)}+\frac{Bx+C}{(x^2+1)}$; $\frac{A(x^2+1)+Bx+C(x+2)}{(x+2)\cdot(x^2+1)}$

$1 = A(x^2+1) + (Bx+C)(x+2); \; 1 = Ax^2 + A + Bx^2 + 2Bx + Cx + 2C$

$x=-2 \quad 1 = 5A \Rightarrow A = \frac{1}{5}$

$x=0 \quad 1 = A + C\cdot 2 \; ; \; 1 = \frac{1}{5} + 2C \; ; \; C = \frac{2}{5}$

$x=1 \quad 1 = 2A + (B+C)\cdot 3 \; ; \; 1 = \frac{2}{5} + 3B + \frac{6}{5} \qquad 1 - \frac{2}{5} - \frac{6}{5} = 3B \qquad -\frac{\cancel{3}}{5}\cdot\frac{1}{\cancel{3}} = B = -\frac{1}{5}$

$$\int \frac{1}{(x+2)(x^2+1)}\,dx = \int \frac{1/5}{x+2} + \frac{-\frac{1}{5}x + \frac{2}{5}}{(x^2+1)}\,dx = \int \frac{1/5}{x+2} - \frac{\frac{1}{5}x}{x^2+1} + \frac{\frac{2}{5}}{x^2+1}\,dx$$

$$\frac{1}{5}\int \frac{1}{x+2}\,dx - \frac{1}{5}\cdot\frac{1}{2}\int \frac{2x}{x^2+1} + \frac{2}{5}\int \frac{1}{x^2+1}\,dx =$$

$$\frac{1}{5}\log|x+2| - \frac{1}{10}\log(x^2+1) + \frac{2}{5}\arctan(x) + C$$

36. $\displaystyle\int \frac{1}{x^2+4x+5}\,dx$ ∄ raíces reales ; $x^2+4x+5 = (x^2+4x+4)+1 = (x+2)^2+1$;

g numerador < g denominador

$(x+a)^2 = x^2 + 2xa + a^2 = x^2 + 4x + 4$

$4x = 2xa \; ; \; a = 2$

I. Racionales (cuad. perf.)

$$\int \frac{1}{(x+2)^2+1}\,dx = \arctan(x+2) + C$$

37. $\displaystyle\int \frac{1}{x^2+2x+5}\,dx$ ∄ raíces reales ; $x^2+2x+5 = (x^2+2x+1)+4 = (x+1)^2+4$;

g numerador < denominador

$(x+a)^2 = x^2 + 2xa + a^2 = x^2 + 2x + 1$

$2x = 2xa \; ; \; a = 1$

$$\int \frac{1}{(x+1)^2+4}\,dx = \frac{1}{4}\int \frac{1}{\left(\frac{x+1}{2}\right)^2+1} = \frac{1}{4}\cdot\frac{2}{1}\int \frac{1/2}{\left(\frac{x+1}{2}\right)^2+1}\,dx = \frac{1}{2}\arctan\left(\frac{x+1}{2}\right)+C$$

38. $\displaystyle\int \frac{e^x}{e^{2x}+1}\,dx = \int \frac{e^x}{(e^x)^2+1}\,dx \; ; \; \begin{Bmatrix} t = e^x \\ dt = e^x dx \end{Bmatrix} ; \; \frac{1}{t^2+1}\,dt =$

$\arctan(t) + C = \arctan(e^x) + C$

39. $\int \frac{dx}{a^2e^x+b^2e^{-x}} = \int \frac{dx}{e^x a^2 + b^2 \cdot \frac{1}{e^x}} dx = \int \frac{1}{\frac{a^2(e^x)^2+b^2}{e^x}} dx = \int \frac{e^x}{a^2(e^x)^2+b^2} dx$

$\left\{ \begin{matrix} e^x = t \\ e^x dx = dt \end{matrix} \right\} = \int \frac{1}{a^2 \cdot t^2 + b^2} dt = \frac{1}{b^2} \int \frac{1}{\left(\frac{a \cdot t}{b}\right)^2 + 1} dx = \frac{1}{b^2} \cdot \arctan\left(\frac{a \cdot t}{b}\right) + C$

$= \frac{1}{b^2} \cdot \arctan\left(\frac{ae^x}{b}\right) + C$

$\cos^2(x) = \frac{1+\cos(2x)}{2}$

40. $\int \cos^2 x \, dx = \int \frac{1+\cos(2x)}{2} dx = \frac{1}{2} \int 1 + \cos(2x^*) \, dx$

$\frac{1}{2} \int \left(x + \frac{sen(2x)}{2}\right) + C$

$\int \cos(2x) dx = \frac{1}{2} \int 2 \cdot \cos(2x) dx = \frac{1}{2} \int \cos(t) dt$

$t = 2x$

$2dx = dt$

$\frac{1}{2} \cdot sen\, t = \frac{1}{2} sen(2x) = \frac{sen(2x)}{2}$

41. $\int \cos^3 x \, dx = \int \cos^2 x \cdot \cos x \, dx = \int (1 - sen^2 x) \cdot \cos(x)$

$\left\{ \begin{matrix} sen(x) = t \\ \cos(x) dx = dt \end{matrix} \right\} = \int 1 - t^2 \, dt = \int t - \frac{t^3}{3} + C$

$sen(x) - \frac{sen^3(x)}{3} + C$

42. $\int \cos^4(x) \cdot sen^3(x) \, dx = \int \cos^4(x) \, sen^2(x) \cdot sen(x) dx$

$= \left\{ sen^2(x) = 1 - \cos^2(x) \right\} = \int \cos^4(x) \cdot (1 - \cos^2(x)) \cdot sen(x) dx$

$- \int \cos^4(x) \cdot (1 - \cos^2(x)) \cdot (-sen(x)) dx = \left\{ \begin{matrix} \cos(x) = t \\ dt = -sen(x) dx \end{matrix} \right.$

$- \int t^4 \cdot (1 - t^2) \, dt = + \int -t^4 + t^6 \, dt = -\frac{t^5}{5} + \frac{t^7}{7} + C$

43. $\int \sqrt{a^2 - x^2}\,dx = \left\{ \begin{array}{l} x = a\,sen(t) \\ dx = a\cos(t)\,dt \end{array} \right\} = \int \sqrt{a^2 - a^2 \cdot sen^2(t)} \cdot a\cos(t)$

$-x^2 + a^2 \rightarrow x = a \cdot sen(t)$

$\int \sqrt{a^2(1 - sen^2 t)} \cdot a\cos\,dt = \int \sqrt{a^2 \cdot \cos^2 t} \cdot a\cos t\,dt$

$= \int a \cdot \cos(t) \cdot a\cos(t)\,dt = \int a^2 \cdot \cos^2(t)\,dt =$

$\frac{1}{2}\int 2 + 2\cos(2t) = \frac{1}{2}\left(2t + \frac{sen(2t)}{2}\right) = \frac{1}{2} \cdot \left(t + sen(2t)\right)$

$a^2 \int \cos^2(t)\,dt = a^2 \cdot \int \frac{1 + \cos(2t)}{2}\,dt = \frac{a^2}{2}\left(t + \frac{sen(2t)}{2}\right) + c$

$\frac{a^2}{2}\left(t + \frac{2 \cdot sen(t) \cdot \cos(t)}{2} + c\right) \overset{*}{=} \frac{a^2}{2}\left(t + sen(t)\cos(t) + c\right)$

Tenemos que deshacer el cambio de variable, para dejarlo todo en función de x

$x = a\,sen(t)\,; \quad sen(t) = \frac{x}{a} \quad t = arcsen\left(\frac{x}{a}\right)$

$\cos^2(t) = 1 - sen^2(t) = 1 - \frac{x^2}{a^2} = \frac{-x^2 + a^2}{a^2} \Rightarrow$

$\cos(t) = \sqrt{\frac{-x^2 + a^2}{a}}$

$* \; \frac{a^2}{2}\left(arc\,sen\left(\frac{x}{a}\right) + \frac{x}{a} \cdot \frac{\sqrt{a^2 - x^2}}{a}\right) + c =$

$$\boxed{\frac{a^2}{2}\,arcsen\left(\frac{x}{a}\right) + \frac{1}{2} \cdot x \cdot \sqrt{a^2 - x^2} + C}$$

44. $\int \frac{x^7 + x^3}{x^4 - 1}\,dx$

g. numerador >= denominador

I. racionales (Algoritmo Euclides)

$$\begin{array}{r|l} x^7 + x^3 & x^4 - 1 \\ -\;x^7 \quad\; x^3 & x^3 \\ \hline \boxed{2x^3} & \end{array}$$

Aplicando A. Euclides

$\frac{D}{d} = c + \frac{R}{d}$

$$\int \frac{x^7 + x^3}{x^4 - 1} dx = \int x^3 + \frac{2x^3}{x^4-1} dx = \frac{x^4}{4} + \frac{1}{2}\log(x^4-1) + C$$

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$$\int_0^{\pi} sen(x)\cdot cos^4(x)\, dx = -\int_0^{\pi} -sen(x)\cdot cos^4(x) = \begin{cases} t = cos(x) \\ dt = -sen(x) \end{cases}$$

$$-\int_0^{\pi} t^4 \cdot dt = -\frac{t^5}{5} + C = \frac{-cos^5(x)}{5} + C = \left[\frac{-cos^5(x)}{5}\right]_0^{\pi} = \frac{1}{5} - \frac{-1}{5} = \frac{2}{5}$$

46

$$\int \frac{2x^3+3x^2-7x+14}{x^2+2x-3} dx$$

g. numerador >= g. denominador

$$\begin{array}{rrrr|l} 2x^3 & +3x^2 & -7x & +14 & x^2+2x-3 \\ -2x^3 & -4x^2 & +6x & & \overline{2x-1} \\ \hline 0 & -x^2 & -x & +14 & \\ & +x^2 & +2x & -3 & \\ \hline & 0 & x & +11 & \end{array}$$

$$= \int 2x - 1 + \frac{x+11}{x^2+2x-3} = \int 2x-1 + \underbrace{\int \frac{x+11}{x^2+2x-3}}_{I} = \frac{\cancel{2}x^2}{\cancel{2}} - x + 3\log|x-1| - 2\cdot\log|x+3|$$

$$I = \int \frac{x+11}{x^2+2x-3} = \int \frac{x+11}{(x-1)(x+3)} = \frac{A}{(x-1)} + \frac{B}{(x+3)} = \frac{A(x+3) + B(x-1)}{(x-1)\cdot(x+3)}$$

$$\Rightarrow x+11 = A(x+3) + B(x-1) = \begin{cases} x=1;\ 1+11 = A+3A;\ 12=4A \quad \boxed{A=3} \\ x=-3;\ 8=-4B;\ \boxed{B=-2} \end{cases}$$

$$\int \frac{3}{x-1} + \int \frac{-2}{x+3} = 3\int \frac{1}{x-1} + -2\int \frac{1}{x+3} = 3\log|x-1| - 2\cdot\log|x+3| + c$$

47.

$$\int \frac{e^{arcsen(x)}}{\sqrt{1-x^2}} dx = \int \frac{1}{\sqrt{1-x^2}} \cdot e^{arcsen(x)} dx = \begin{cases} arcsen(x) = t \\ \frac{1\,dx}{\sqrt{1-x^2}} = dt \end{cases}$$

$$= \int e^t\, dt = e^t + c = arcsen(x) + C$$

48 $\int \cos^2(x)\cdot \text{sen}(x)\,dx = -\int -\text{sen}(x)\cdot \cos^2(x) = \begin{cases} \cos(x)=t \\ -\text{sen}(x)=dt \end{cases}$

$-\int t^2\,dt = -\frac{t^3}{3} = \frac{-\cos^3(x)}{3} + C$

49 $\int \cos^5(x)\cdot \text{sen}^3(x)\,dx = \int \cos^5(x)\cdot \text{sen}^2(x)\cdot \text{sen}(x)$

$= \int \cos^5(x)\cdot(1-\cos^2(x))\cdot \text{sen}(x)\,dx = -\int \cos^5(x)\cdot(1-\cos^2(x))(-\text{sen}(x))$

$\left\{\begin{matrix} \cos(x)=t \\ -\text{sen}(x)=dt \end{matrix}\right\} = -\int t^5\cdot(1-t^2)\,dt = -\int t^5 - t^7 = -\frac{t^6}{6} - \frac{t^8}{8} + C$

50 $\int \text{sen}^2(x)\,dx = \int \frac{1-\cos(2x)}{2}\,dx = \frac{1}{2}\int 1-\cos(2x)\,dx = \frac{1}{2}\left(x - \frac{1}{2}\text{sen}(2x)\right)$

$-\int \cos(2x) = -\frac{1}{2}\int 2\cos(2x)\,dx = -\frac{1}{2}\text{sen}(2x)$

51 $\int \cos^4(x)\,dx = \int\left(\frac{1+\cos(2x)}{2}\right)^2 = \frac{1}{4}\int(1+\cos(2x))^2$

$\frac{1}{4}\int 1+2\cos(2x)+\cos^2(2x)\,dx = \frac{1}{4}\cdot\left(x + 2\int\cos(2x) + \int\cos^2(2x)\,dx\right)$

$\frac{1}{4}\cdot\left(x+\text{sen}(2x) + \int\frac{1+\cos(4x)}{2}\,dx\right) = \frac{1}{4}\left(x+\text{sen}(2x)+\frac{1}{2}\left(x+\frac{1}{4}\text{sen}(4x)\right)\right)$

52 $\int \text{sen}^2(x)\cdot\cos^2(x) = \int\left(\frac{1+\cos(2x)}{2}\right)\cdot\left(\frac{1-\cos(2x)}{2}\right)$

$\frac{1}{4}\int 1^2 - \cos^2(2x) = \frac{1}{4}\cdot\left(x + \left(\frac{1}{2}x + \frac{1}{8}\text{sen}(4x)\right)\right) = \frac{1}{4}x + \frac{1}{8}x + \frac{1}{32}\text{sen}(4x) + C$

$-\int \cos^2(2x) = -\int\frac{1+\cos(4x)}{2} = -\frac{1}{2}\int 1+\cos(4x) = -\frac{1}{2}\left(x+\frac{1}{4}\text{sen}(4x)\right)$

53 $\int \sqrt{a^2 - x^2}\, dx = \left\{ \begin{array}{l} x = a\,sen(t) \\ dx = a\cos(t)\,dt \end{array} \right\} = \int \sqrt{a^2 - a^2 \cdot sen^2(t)} \cdot a\cos(t)\,dt$

$= \int \sqrt{a^2(1 - sen^2(t))} \cdot a\cos(t)\,dt = \int \sqrt{a^2\cos^2(t)} \cdot a\cos(t)\,dt$

$= a^2 \int \cos^2(t)\,dt = a^2 \int \frac{1 + \cos(2t)}{2}\,dt = \frac{a^2}{2} \int 1 + \cos(2t)\,dt$

$\frac{a^2}{2} \cdot \left(t + \frac{1}{2} sen(2t)\right) + C = \frac{a^2}{2} \cdot \left(t + \frac{1}{\cancel{2}} \cdot \cancel{2} sen(t) \cdot \cos(t)\right) + C =$

* $x = a\,sen(t)$; $sen(t) = \frac{x}{a}$ $\quad \cos^2(t) = 1 - sen^2(t) = 1 - \frac{x^2}{a^2} = \frac{a^2 - x^2}{a^2}$

$\cos(t) = \sqrt{\frac{a^2 - x^2}{a}}$

$\frac{a^2}{2} \cdot \left(arcsen\left(\frac{x}{a}\right) + \frac{x}{a} \cdot \frac{\sqrt{a^2 - x^2}}{a}\right) = \frac{a^2}{2} arcsen\left(\frac{x}{a}\right) + \frac{1}{2} \cdot x \cdot \sqrt{a^2 - x^2} + C$

54. $\int \frac{1}{\cos(x)} = \int \frac{\cos(x)}{\cos^2(x)}\,dx = \int \frac{\cos(x)}{1 - sen^2(x)}\,dx = \left\{ \begin{array}{l} sen(x) = t \\ \cos(x) = dt \end{array} \right\} \int \frac{1}{1 - t^2}\,dt$

$\int \frac{-1}{t^2 - 1}\,dt = \frac{A}{t+1} + \frac{B}{t-1} = \frac{A \cdot (t-1) + B \cdot (t+1)}{(t+1) \cdot (t-1)}$; $-1 = A \cdot (t-1) + B \cdot (t+1)$

$t = 1$; $-1 = 2B$; $B = -\frac{1}{2}$

$t = -1$; $-1 = -2A$; $A = \frac{1}{2}$

$\int \frac{1}{2 \cdot (t+1)} - \frac{1}{2(t-1)}\,dt = \int \frac{1/2}{t+1} - \frac{1/2}{t-1} = \frac{1}{2} \int \frac{1}{t+1} - \frac{1}{2} \int \frac{1}{t-1}\,dt$

$\frac{1}{2} \log|t+1| - \frac{1}{2} \log|t-1| + C = +\frac{1}{2} \log|sen(x) - 1| - \frac{1}{2} \log|sen(x) + 1| + C$

55. $\int \frac{sen(\sqrt{x})}{\sqrt{x}}\,dx \quad 2 \int \frac{1}{2\sqrt{x}} \cdot sen(\sqrt{x})\,dx = \left\{ \begin{array}{l} \sqrt{x} = t \\ \frac{1}{2\sqrt{x}} = dt \end{array} \right.$

$2 \int sen(t)\,dt = 2 \cdot -\cos(t)\,dt = -2 \cdot \cos(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}} + C$

Otra forma $\left\{ \begin{array}{l} x = t^2 \\ dx = 2t\,dt \end{array} \right\}$ $2 \int \frac{sen(t)}{\sqrt{t^2}} = \int \frac{sen(t)}{\cancel{t}} 2\cancel{t}$

$= -2\cos\sqrt{x} + C$

56 $\int \frac{1}{1+\sqrt[3]{x}}\,dx = \left\{\begin{array}{l} x=t^3 \\ dx=3t^2 \end{array}\right\} \int \frac{1}{1+\sqrt[3]{t^3}}\,3t^2 = \int \frac{3t^2}{1+t}$; $\begin{array}{l|l} \cancel{3}t^2 & t+1 \\ \hline -\cancel{3}t^2+3t & 3t+3 \\ \;\;-3t\;\;3 & \\ \hline \;\;\;-3 & \end{array}$

g. numerador >= denominador

$\int 3t+3+\frac{-3}{t+1} = \frac{3t^2}{2}+3t-3\log|t+1|+C = \frac{3\sqrt[3]{x^2}}{2}+3\sqrt[3]{x}-3\log|\sqrt[3]{x}-1|+C$

Integrales definidas

57 Calcular el área encerrada entre $f(x)$ y OX

$f(x) = x^3 - x$;

1. Igualamos a cero : $x^3 - x = 0 \left\{\begin{array}{l} x=0 \\ x^2-1 \; ; \; x=\pm 1 \end{array}\right.$

2. Ordenamos los puntos de corte : $\underbrace{-1 \;\text{———}\; 0}_{I_1} \underbrace{\;\text{———}\; 1}_{I_2}$

3. Sobre cada subintervalo, evaluaremos un punto interior

$I_1 \rightarrow f(-0{,}5) = (-0{,}5)^3 - 0{,}5 = 0{,}375 \Rightarrow A_1 = \int_{-1}^{0} x^3 - x\,dx = \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{-1}^{0}$

$= (0-0) - \left(\frac{1}{4} - \frac{1}{2}\right) = \frac{1}{4}$

$I_2 \rightarrow f(0{,}5) = 0{,}5^3 - 0{,}5 = \boxed{\;}\,0{,}375 < 0 \Rightarrow A_2 = -\int_{0}^{1} x^3 - x\,dx = -\left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{0}^{1}$

$= -\left[\left(\frac{1}{4} - \frac{1}{2}\right) - (0-0)\right] = \frac{1}{4}$; $A_T = A_1 + A_2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}\,u^2$

58. Calcula el área encerrada entre las curvas $f(x) = x$; $g(x) = x^2$

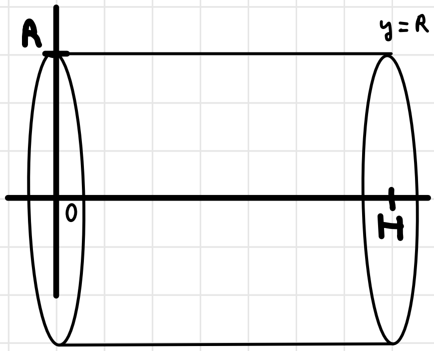
1. Igualamos ambas funciones : $x = x^2 \left\{\begin{array}{l} x=0 \\ x=1 \end{array}\right.$

2. Ordenamos los p. de corte obtenidos : $\underbrace{0 \;\text{———}\; 1}_{I_1}$

3. Tomamos un Pi del intervalo: $\left\{\begin{array}{l} f(0{,}5) = 0{,}5 \\ g(0{,}5) = 0'25 \end{array}\right\} f(0{,}5) > g(0{,}5)$

4. $A = \int_0^1 f(x) - g(x)\,dx = \int_0^1 x - x^2\,dx = \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = \left(\frac{1}{2} - \frac{1}{3}\right) - 0 = \frac{1}{6}$

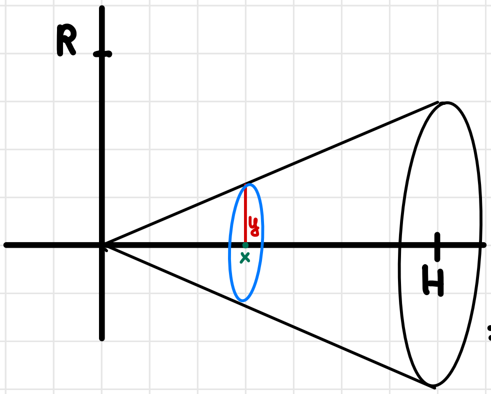
59. Calcular el volumen de un cilindro de radio R y altura H



$$V=\int_0^H \pi (\underbrace{R}_{f(x)})^2 dx = \pi R^2 [x]_0^H = \overbrace{\pi R^2 H}^{V.\ cilindro}$$

El intervalo, es entre H y 0 $[0, H]$

60. Calcular el volumen de un cono de radio R y altura H



Th. Thales: $\frac{R}{H} = \frac{y}{x} \Rightarrow \underbrace{y}_{f(x)} = \frac{R}{H}x$

$$= V_{ox} = \int_0^H \pi \left(\frac{R}{H}x\right)^2 dx = \pi \frac{R^2}{H^2}\int_0^H x^2 dx$$

$$= \left[\pi \frac{R^2}{H^2}\cdot\frac{x^3}{3}\right]_0^H = \frac{\pi \cdot R^2 \cdot H}{3} \Big\}\ \text{V. del cono}$$

61 $\int \frac{x^7+3}{x^4+x^2}dx$;

$$\begin{array}{r|l} x^7+3 & x^4+x^2 \\ \hline -x^7+x^5 & x^3-x \\ x^5+3 & \\ -x^5+x^3+3 & \\ x^3+3 & \end{array}$$

; $\int (x^3-x)+\frac{x^3+3}{x^4+x^2}dx$;

g. num < g. denom

$$= \frac{x^4}{4}-\frac{x^2}{2}+\int \frac{3}{x^2}+\frac{x-3}{x^2+1}dx = \int \frac{3}{x^2}dx + \int \frac{x}{x^2+1}dx - 3\int \frac{1}{x^2+1}dx = -\frac{3}{x}+\frac{1}{2}\log|x^2+1|-3\arctan(x)+C$$

$$\int \frac{x^3+3}{x^4+x^2}dx = \int \frac{x^3+3}{x^2(x^2+1)}dx = \frac{A}{x^2}+\frac{B}{x}+\frac{Cx+D}{x^2+1} = \frac{A(x^2+1)+Bx(x^2+1)+x^2(Cx+D)}{x^2\cdot(x^2+1)}$$

$$x^3+3 = Ax^2+Bx^3+Cx^3+Dx^2+Bx+A = x^3(B+C)+x^2(A+D)+Bx+A$$

$$\begin{cases} B+C=1 \\ A+D=0 \\ B=0 \\ A=3 \end{cases} \quad \begin{matrix} A=3 \\ B=0 \\ C=1 \\ D=-3 \end{matrix} \qquad \frac{x^3+3}{x^2(x+1)} = \frac{3}{x^2}+\frac{x}{x}+\frac{x-3}{x^2+1}$$

62. $\int \frac{e^x + e^{4x}}{3+e^{2x}}dx = \frac{e^x(1+e^{3x})}{3+e^{2x}}dx = \frac{e^x(1+(e^x)^3)}{3+(e^x)^2}dx \quad \begin{cases} e^x = t \\ e^x dx = dt \end{cases}$

$e^x \to e^x \quad 4x \to 4 \cdot e^{(4x)}$

$= \int \frac{1+t^3}{3+t^2}dt$; $\begin{array}{r|l} t^3+1 & t^2+3 \\ -t^3+3t & t \\ \hline -3t+1 & \end{array}$; $\int t + \frac{-3t+1}{t^2+3} = \int t\,dx - 3\int \frac{t}{t^2+3} + \int \frac{1}{t^2+3}$

g.num >= denominador

$= \frac{t^2}{2} - \frac{3}{2} \cdot \log|t^2+3| + \int \frac{1/\sqrt{3}}{3\left(\frac{t}{\sqrt{3}}\right)^2+1} = \frac{t^2}{2} - \frac{3}{2} \cdot \log|t^2+3| + \frac{\sqrt{3}}{3}\arctan\left(\frac{t}{\sqrt{3}}\right)$

$= \frac{e^{2x}}{2} - \frac{3}{2}\log|e^{2x}+3| + \frac{\sqrt{3}}{3}\arctan\left(\frac{e^x}{\sqrt{3}}\right) + C$

63. $\int \frac{dx}{\sqrt{x}\,(1+\sqrt[3]{x})} = \begin{cases} x = t^6 \\ dx = 6t^5 dt \end{cases} \int \frac{6t^5}{t^3(1+t^2)}dt$; $\begin{array}{r|l} 6t^5 & t^5+t^3 \\ -6t^5+6t^3 & 6 \\ \hline -6t^3 & \end{array}$

g numerador >= denom

$\int 6 + \frac{-6t^3}{t^5+t^3}dt = 6t - \int \frac{6\cancel{t^3}}{\cancel{t^3}(t^2+1)} = 6t - 6\arctan(t) + C$

$= 6\sqrt[6]{x} - 6 \cdot \arctan(\sqrt[6]{x}) + C$

64. $\frac{\log(1+x)}{1+x}dx = \int \frac{1}{1+x} \cdot \log(1+x)dx = \begin{cases} \log(1+x) = t \\ \frac{1}{1+x}dx = dt \end{cases}$

$= \int t\,dt = \frac{t^2}{2} + C = \frac{(\log(1+x))^2}{2} + C$

65. $\int \frac{1}{\sqrt[3]{x} + \sqrt{x}}dx = \begin{cases} x = t^6 \\ dx = 6t^5 \end{cases} \int \frac{1}{t^2+t^3}6t^5 = \int \frac{6t^{\cancel{5}\,3}}{\cancel{t^2}(t+1)}$

$\int \frac{6t^3}{t+1}$; $\begin{array}{r|l} 6t^3 & t+1 \\ -6t^3+6t^2 & 6t^2-6t+6 \\ \hline -6t^2 & \\ -6t^2-6t & \\ \hline +6t & \\ -6t+6 & \\ \hline -6 & \end{array}$ $= \int 6t^2 - 6t + 6 + \frac{-6}{t+1}dx$

g.num >= denom

$= \frac{6t^3}{3} - \frac{6t^2}{2} + 6t - 6 \cdot \log|t+1| + C$

$= 2t^3 - 3t^2 + 6t - 6\log|t+1| + C \to$ Sustituir

66. $\int \frac{x+1}{x^2+2x}dx = \int \frac{x+1}{x(x+2)}dx = \frac{A}{x} + \frac{B}{x+2} = \frac{A(x+2)+Bx}{x(x+2)}$

$x+1 = A(x+2)+Bx$: $x=0 \Rightarrow +1 = 2A$; $A=\frac{1}{2}$; $x=-2 \Rightarrow B=\frac{1}{2}$

$\int \frac{1/2}{x} + \frac{1/2}{x+2} = \frac{1}{2}\int \frac{1}{x}dx + \frac{1}{2}\int \frac{1}{x+2}dx = \frac{1}{2}\left(\log|x| + \log|x+2|\right) + C$

67. $\int \frac{\sqrt{x}}{-1+\sqrt{x}}dx = \begin{Bmatrix} x=t^2 \\ dx=2t\,dt \end{Bmatrix} \int \frac{t}{-1+t}2t\,dt = \int \frac{2t^2}{t-1}dx$; $\begin{array}{r|l} 2t^2 & t-1 \\ -2t^2+2t & 2t+2 \\ \hline +2t & \\ -2t+2 & \\ \hline +2 & \end{array}$

$\int 2t+2+\frac{2}{t-1}dt = \frac{\cancel{2}t^2}{\cancel{2}} + 2t + 2\cdot\log|t-1| + C = x + 2\sqrt{x} + 2\log|\sqrt{x}-1| + C$

68. $\int \arctan(x)\,dx = \arctan(x)\cdot x - \int x\cdot\frac{1}{x^2+1}dx = x\arctan(x) \cdot \frac{1}{2}\log|x^2+1| + C$

$u = \arctan(x)$ $du = \frac{1}{x^2+1}dx$
$dv = dx$ $v = x$

$\int \frac{x}{x^2+1}dx = \frac{1}{2}\log|x^2+1| + C$

69. $\int \sqrt{a^2-b^2x^2}\,dx = \int \sqrt{a^2-(bx)^2}\,dx = \begin{cases} bx = a\,\mathrm{sen}(t) \\ x = \frac{a}{b}\mathrm{sen}(t) \\ dx = \frac{a}{b}\cos(t) \end{cases}$

$\int \sqrt{a^2-(a\,\mathrm{sen}(t))^2}\cdot\frac{a}{b}\cdot\cos(t)\,dt = \int \sqrt{a^2(1-\mathrm{sen}^2(t))}\cdot\frac{a}{b}\cos(t)\,dt$

$= \int \sqrt{a^2\cdot\cos^2(t)}\cdot\frac{a}{b}\cos(t)\,dt = \frac{a^2}{b}\int \cos^2(t)\,dt = \frac{a^2}{b}\int \frac{1+\cos(2t)}{2}dt$

$= \frac{a^2}{2b}\int 1+\cos(2t)\,dt = \frac{a^2}{2b}\cdot\left(t + \frac{\mathrm{sen}(2t)}{2}\right) + C$

$= \frac{a^2}{2b}\left(t + \frac{\cancel{2}\,\mathrm{sen}(t)\cos(t)}{\cancel{2}}\right) + C = \frac{a^2}{2b}\left(\arcsen\left(\frac{bx}{a}\right) + \frac{bx}{a}\cdot\frac{\sqrt{a^2-b^2x^2}}{a}\right) + C$

70. $$\int \frac{\sqrt{9-4x^2}}{x}dx = \int \frac{\sqrt{3^2-(2x)^2}}{x}dx = \left\{\begin{array}{l} 2x = 3\,sen(t) \\ x = \frac{3}{2}sen(t) \\ dx = \frac{3}{2}\cos(t)\,dt \end{array}\right\} =$$

$a^2 \quad -x^2$

$$\int \frac{\sqrt{3^2-(3sen(t))^2}}{\frac{3}{2}sen(t)} \cdot \frac{3}{2}\cos(t)\,dt = \int \frac{\sqrt{3^2(1-sen^2(t))}}{\cancel{\frac{3}{2}}\,sen(t)}\cancel{\frac{3}{2}}\cos(t)\,dt$$

$$= 3\int \frac{\cos^2(t)}{sen(t)}dt = 3\cdot\int \frac{\cos^2(t)}{sen^2(t)}\cdot sen(t)\,dt = \int \frac{\cos^2(t)}{1-\cos^2(t)}\cdot sen(t)\,dt$$

$$= 3\int \frac{\cos^2(t)}{\cos^2(t)-1}(-sen(t))\,dt = \left\{\begin{array}{l} \cos(t) = u \\ -sen(t)\,dt = du \end{array}\right\} \; 3\int \frac{u^2}{u^2-1}du$$

$$\begin{array}{r|l} u^2 & u^2-1 \\ \hline -u^2+1 & 1 \\ +1 & \end{array}$$

$$= \int 1 + \frac{1}{u^2-1}du = 3\left(u + \int \frac{1}{(u-1)\cdot(u+1)}du\right)^{*}$$

$$2x = 3\,sen(t) \longrightarrow sen(t) = \frac{2x}{3}$$
$$\cos(t) = \sqrt{1-sen^2(t)} = \sqrt{1-(\tfrac{2x}{3})^2} = \sqrt{\frac{9-4x^2}{9}} = \frac{\sqrt{9-4x^2}}{3}$$

$$= 3\cdot\left(u + \frac{1}{2}\log|u-1| - \frac{1}{2}\log|u+1|\right) + C$$

$$= 3\cdot\left(\cos(t) + \frac{1}{2}\log|\cos(t)-1| - \frac{1}{2}\log|\cos(t)+1|\right) + C$$

$$= 3\left(\frac{\sqrt{9-4x^2}}{3} + \frac{1}{2}\log\left|\frac{\sqrt{9-4x^2}}{3}-1\right| - \frac{1}{2}\log\left|\frac{\sqrt{9-4x^2}}{3}+1\right|\right) + C$$

$$* \quad \frac{1}{(u-1)\cdot(u+1)}du = \frac{A}{(u-1)} + \frac{B}{u+1} = \frac{A\cdot(u+1)+B\cdot(u-1)}{(u-1)\cdot(u+1)}du$$

$$1 = A\cdot(u+1)+B\cdot(u-1) \quad \left\{\begin{array}{ll} u=1 & 1=2A\,;\; A=\frac{1}{2} \\ u=-1 & 1=-2B\,;\; B=-\frac{1}{2} \end{array}\right.$$

$$\frac{1}{2}\int \frac{1}{u-1}du - \frac{1}{2}\int \frac{1}{u+1}du = \frac{1}{2}\log|u-1| - \frac{1}{2}\log|u+1| + C$$

71. $\int \frac{1}{1+\text{sen}} dx$; Aplicamos el cambio universal

$$\left\{ \begin{array}{l} tg\frac{x}{2}=t \\ sen(x)=\frac{2t}{1+t^2} \\ dx=\frac{2}{1+t^2}dt \end{array} \right\} \int \frac{1}{1+\frac{2t}{1+t^2}} \cdot \frac{2}{1+t^2} dt = \int \frac{2}{1+t^2+2t} dt$$

$$\int \frac{2}{(t+1)^2} dt = \int 2\cdot(t+1)^{-2} = 2\cdot\frac{(t+1)^{-1}}{-1} = -2\frac{1}{t+1}+C = \frac{-2}{1+tg(\frac{x}{2})}+C$$

72. $\int \frac{\cos(x)}{sen(x)+\cos(x)} dx$; Aplicamos el cambio universal

$$\left\{ \begin{array}{l} sen(x)=\frac{2t}{t^2+1} \\ \cos(x)=\frac{1-t^2}{1+t^2} \\ dx=\frac{2}{1+t^2}dt \end{array} \right\} \int \frac{\frac{1-t^2}{1+t^2}}{\frac{2t}{1+t^2}+\frac{1-t^2}{1+t^2}} \frac{2}{1+t^2} dt = \int \frac{\frac{2(1-t^2)}{\cancel{1+t^2}}}{\frac{2t+1-t^2}{\cancel{1+t^2}}} dt$$

$t^2+1 \,\underline{|2t}$
$\frac{1}{2}t$

$$= \int \frac{2(t^2-1)}{(t-(1+\sqrt{2}))\cdot(t+(1+\sqrt{2}))\cdot(t^2+1)} dt$$

$$\frac{2(t^2-1)}{(t-(1+\sqrt{2}))\cdot(t+(1+\sqrt{2}))(t^2+1)} dt = \frac{A}{t-(1+\sqrt{2})} + \frac{B}{t+(1+\sqrt{2})} + \frac{Ct+D}{t^2+1}$$

$$\frac{A\cdot(t+(1+\sqrt{2}))\cdot(t^2+1)+B(t-(1+\sqrt{2}))(t^2+1)+(Ct+D)\cdot(t-(1+\sqrt{2}))\cdot(t+(1+\sqrt{2}))}{(t-(1+\sqrt{2}))\cdot(t+(1+\sqrt{2}))(t^2+1)}$$

$$\left\{ \begin{array}{l} t=1+\sqrt{2} \; ; \; 2((1+\sqrt{2})^2-1) = A(2\sqrt{2})((1+\sqrt{2})^2+1) \\ \qquad ; \; 2+2\sqrt{2} = A(2\sqrt{2})(4+2\sqrt{2}) \; ; \qquad A=\frac{1+\sqrt{2}}{\sqrt{2}(2+\sqrt{2})} \\ t=1-\sqrt{2} \; ; \; 2((1-\sqrt{2})^2-1) = B(-2\sqrt{2})\cdot((1-\sqrt{2})^2-1) \\ \qquad ; \; 2(2-2\sqrt{2}) = B(-2\sqrt{2})(4-2\sqrt{2}) \longrightarrow B=\frac{-(1-\sqrt{2})}{\sqrt{2}(2-\sqrt{2})} \end{array} \right.$$

$$\begin{cases} t=0 \quad -2=A(-1+\sqrt{2})+B(-1-\sqrt{2})+D(-1) \\ D=\dfrac{1+\sqrt{2}}{\sqrt{2}(2+\sqrt{2})}(-1+\sqrt{2})+\dfrac{-(1-\sqrt{2})}{\sqrt{2}(2-\sqrt{2})}(-1-\sqrt{2})+2=\dfrac{-1}{\sqrt{2}(2-\sqrt{2})}+2 \\ =3 \\ t=1 \quad 0=A(\sqrt{2})\cdot 2+B(-\sqrt{2})\cdot 2+(C+3)(-\sqrt{2})(\sqrt{2}) \\ 0=\dfrac{1+\sqrt{2}}{\cancel{\sqrt{2}}(2+\sqrt{2})}2\cancel{\sqrt{2}}+2\cancel{\sqrt{2}}\dfrac{(1-\sqrt{2})}{\cancel{\sqrt{2}}(2-\sqrt{2})}-2(C+3) \\ 2C=1+\sqrt{2}(2-\sqrt{2})+(1-\sqrt{2})(2+\sqrt{2})-6 = \Rightarrow C=-3 \end{cases}$$

$$\int \frac{\frac{1+\sqrt{2}}{\sqrt{2}(2+\sqrt{2})}}{t-(1+\sqrt{2})}-\frac{\frac{1-\sqrt{2}}{\sqrt{2}(2-\sqrt{2})}}{t-(1-\sqrt{2})}+\frac{-3t+3}{t^2+1}\,dt$$

$$=\frac{1+\sqrt{2}}{\sqrt{2}(2+\sqrt{2})}\int\frac{1}{t-(1+\sqrt{2})}dt-\frac{1-\sqrt{2}}{\sqrt{2}(2-\sqrt{2})}dt-3\int\frac{t}{t^2+1}dt+3\int\frac{1}{t^2+1}dt$$

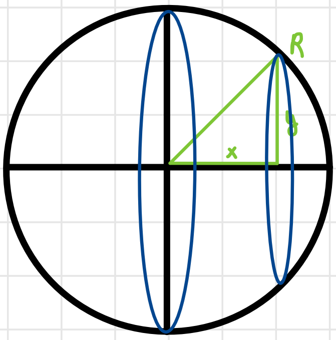
$$=\left(\frac{1+\sqrt{2}}{\sqrt{2}(2+\sqrt{2})}\right)\cdot\log|t-(1+\sqrt{2})|-\frac{1-\sqrt{2}}{\sqrt{2}(2-\sqrt{2})}\cdot\log|t-(1-\sqrt{2})|$$

$$-\frac{3}{2}\log|t^2+1|+3\arctan(t)+C$$

$$\left(\frac{1+\sqrt{2}}{\sqrt{2}(2+\sqrt{2})}\right)\log\left|\operatorname{tg}\frac{x}{2}-(1+\sqrt{2})\right|-\frac{1-\sqrt{2}}{\sqrt{2}(2-\sqrt{2})}\cdot\log\left|\operatorname{tg}\frac{x}{2}-(1-\sqrt{2})\right|$$

$$-\frac{3}{2}\log\left|\operatorname{tg}^2\frac{x}{2}+1\right|+\frac{3}{2}x+C$$

73. Calcular el volumen de una esfera de radio R



$$x^2+y^2=R^2;\quad y=\sqrt{R^2-x^2}$$

$$V.\,esfera = 2V$$

$f(x)$ ya que me da el radio que revoluciono

$$V=$$

$$V_{\frac{1}{2}\,esfera}=\int_0^R \pi(y)^2=\int_0^R \pi\cdot\left(\cancel{\sqrt{R^2-x^2}}\right)^{\cancel{2}}dx$$

$$=\pi\cdot\int_0^R R^2-x^2\,dx=\pi\left[R^2x-\frac{x^3}{3}\right]_0^R$$

$$=\pi\left(R^3-\frac{R^3}{3}\right)=\frac{2}{3}\pi R^3 \rightarrow V_{esf}=\frac{4}{3}\pi R^3$$

74 Aplica el teorema fundamental del cálculo, halla los valores de las constantes a, b, c, d sabiendo que:

$$\int_0^x (t^3-t+1)e^t\,dt=(ax^3+bx^2+cx+d)e^x$$

Como sabemos, que estas dos expresiones son equivalentes, entonces sus derivadas también deben ser iguales.

$$T(x)=\int_0^x (t^3-t+1)e^t\,dt=(ax^3+bx^2+cx+d)e^x$$

$$T'(x)=\left(\int_0^x (t^3-t+1)e^t\,dt\right)'=(ax^3+(3a+b)x^2+(2b+c)x+(c+d))e^x$$

$$T'(x)=(x^3-x+1)\cancel{e^x}=(ax^3+(3a+b)+(2b+c)x+(c+d))\cancel{e^x}$$

$\boxed{a=1}$; $-x=2b+c=-6+c=-1$; $\boxed{c=5}$

$3a+b=0$; $3+b=0$; $\boxed{b=-3}$; $c+d=1$; $5+d=1$; $\boxed{d=-4}$

75 Halla los puntos donde se anula la derivada de la función

$$f(x) = -2x + \int_0^{2x} e^{(t^2-10t+24)}dt$$

$$f'(x) = -2 + \left(\int_0^{2x} e^{(t^2-10t+24)}dt\right)$$

$$= -2 + e^{(4x^2-20+24)} \cdot 2 - \cancel{\left(e^{(24)} \cdot 0\right)} = -2+2e^{(4x^2-20+24)}$$

$$f'(x) = -2+2e^{(4x^2-20+24)} = 0 \rightarrow e^{(4x^2-20+24)} = 1 = e^0$$

$$(4x^2-20x+24)=0$$

$$\frac{+20 \pm \sqrt{20^2-4\cdot4\cdot24}}{8} = \frac{20 \pm 4}{8} < \begin{matrix} \frac{24}{8}=3 \; : x=3 \\ \frac{16}{8}=2 \; ; x=2 \end{matrix} \Big\} \text{ se anula}$$

76. $\displaystyle\int\sqrt{9-4x^2}\,dx = \int\underbrace{\sqrt{3^2-(2x)^2}}_{\sqrt{a^2-x^2}}\,dx = \begin{cases} 2x = 3\,\text{sen}(t) \\ x = \frac{3}{2}\text{sen}(t) \\ dx = \frac{3}{2}\cos(t)dt \end{cases}$

$$= \int\sqrt{3^2-(3\,\text{sen}(t))^2}\;\frac{3}{2}\cos(t)\,dt$$

$$= \int\sqrt{9\cdot(1-\text{sen}^2(t))}\;\frac{3}{2}\cos(t)\,dt = \int 3\sqrt{\cos^2(t)}\cdot\frac{3}{2}\cos(t)\,dt$$

$$= \int 3\cos(t)\cdot\frac{3}{2}\cdot\cos(t)\,dt = \frac{9}{2}\int\cos^2(t) = \frac{9}{2}\int\frac{1+\cos(2t)}{2}\,dt$$

$$= \frac{9}{4}\int 1+\cos(2t)\,dt = \frac{9}{4}\left(t+\frac{\text{sen}(2t)}{2}\right)+C = \{\text{sen}(2t) = 2\,\text{sen}(t)\cos(t)$$

$\text{sen}(t) = \frac{2x}{3} \Rightarrow t = \text{arcsen}\left(\frac{2x}{3}\right)$

$$= \frac{9}{4}\left(t+\frac{\cancel{2}\,\text{sen}(t)\cdot\cos(t)}{\cancel{2}}\right)+C = \frac{9}{4}\left(\text{arcsen}\left(\frac{2x}{3}\right)+\frac{2x}{3}\cdot\frac{\sqrt{9-4x^2}}{3}\right)+C$$

$\cos(t) = \sqrt{1-\text{sen}^2(t)} = \sqrt{1-\left(\frac{2x}{3}\right)^2} = \sqrt{\frac{9-4x^2}{9}} = \frac{\sqrt{9-4x^2}}{3}$

$$= \frac{9}{4}\text{arcsen}\left(\frac{2x}{3}\right)+\frac{1}{2}x\sqrt{9-4x^2}+C$$

77. $\int_0^{2} \frac{1}{\sqrt{4-x^2}}dx = \lim_{t\to 2^-} \int_0^t \frac{1}{\sqrt{4-x^2}}dx$ * (p. conflictivo; 2^2-x^2; $a^2-x^2 \to x=a\,sen(t)$)

$\begin{cases} x = 2\,sen(t) \\ dx = 2\cos(t) \end{cases}$ $\int \frac{1}{\sqrt{4-(2sen(t))}} 2\cos(t)\,dt$ $\quad t = arcsen\left(\frac{x}{2}\right)$

$$= \int \frac{1}{\sqrt{4(1-sen^2(t))}} \cos(t)\,dt = \frac{2}{2}\int \frac{1}{\cos(t)} \cdot \cos(t)\,dt = \int 1\,dt = t$$

$$^{*}\lim_{t\to 2^-}\int_0^t \frac{1}{\sqrt{4-x^2}}dx = \lim_{t\to 2^-}\left[arcsen\left(\frac{x}{2}\right)\right]_0^t = arcsen\left(\frac{t}{2}\right) - \cancel{arcosen\left(\frac{0}{2}\right)}^{\,0}$$

$$\lim_{t\to 2^-} arcsen\left(\frac{t}{2}\right) = arcosen(1) = \frac{\pi}{2}$$

78. $\int \frac{e^x}{(e^x+5)(e^x-2)}dx = \begin{cases} e^x = t \\ e^x\,dx = dt \end{cases}$ $\int \frac{1}{(t+5)\cdot(t-2)}dx$

$$^{*}\ \frac{A}{(t+5)} + \frac{B}{(t-2)} = \frac{A(t-2)+B(t+5)}{(t-2)(t+5)};\quad A(t-2)+B(t+5) = 1$$

$t=2;\ 7B = 1 \Rightarrow B = \frac{1}{7}$

$t=-5;\ -7A = 1 \Rightarrow A = -\frac{1}{7}$

$$\int \frac{-1/7}{(t+5)} + \frac{1/7}{(t-2)} = -\frac{1}{7}\int \frac{1}{t+5}dt + \frac{1}{7}\int \frac{1}{t-2}dt = -\frac{1}{7}\log|t+5| + \frac{1}{7}\log|t-2| + C$$

$$-\frac{1}{7}\log|e^x+5| + \frac{1}{7}\log|e^x-2| + C = -\frac{1}{7}\cancel{\log(e^x)}\cdot\log(5) + \frac{1}{7}\frac{\cancel{\log e^x}}{\log(2)} = -\frac{1}{7}(\log(5)) + \frac{1}{7}\left(\frac{1}{\log(2)}\right) + C$$

79. $\int \frac{5}{(x+2)(x-1)(x^2+1)} = \frac{A}{(x+2)} + \frac{B}{(x-1)} + \frac{Cx+D}{(x^2+1)}$

$$\frac{A((x-1)(x^2+1)) + B((x+2)(x^2+1)) + (Cx+D)((x+2)(x-1))}{(x+2)(x-1)(x^2+1)}$$

$$5 = A((x-1)(x^2+1)) + B((x+2)(x^2+1)) + (Cx+D)((x+2)(x-1))$$

$x=1 \quad B=\frac{5}{6} \qquad x=0 \quad D=-\frac{3}{2}$

$x=2 \quad A=-\frac{1}{3} \qquad x=-1 \quad C=-\frac{1}{2}$

$$\int \frac{-1/3}{x+2}dx + \frac{5/6}{x+1} + \frac{-1/2\,x - 3/2}{x^2+1}$$

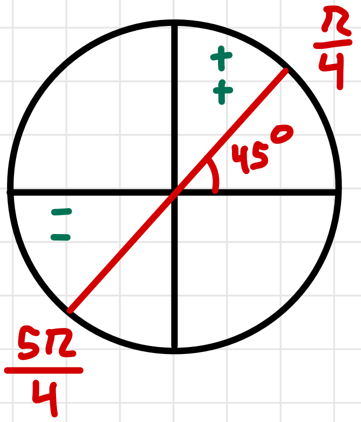
$$80 \quad \int_0^{+\infty \to p.\text{conflictivo}} x e^{-x}\,dx \; ; \; \lim_{t\to+\infty}\int_0^t x e^{-x}\,dx \; ; \; \text{calculo primitiva} \; \left\{\begin{matrix} u=x & du=1 \\ dv=e^{-x} & v=-e^{-x}\end{matrix}\right\} = x\cdot e^{-x} + \int e^{-x}$$

$$= -x\cdot e^{-x} - e^{-x} = -(x+1)e^{-x} \; ; \; \lim_{t\to+\infty}\left[-\frac{(x+1)}{e^{x}}\right]_0^t = \lim_{t\to+\infty}\left(-\frac{t+1}{e^t}+1\right) = 1 \text{ convergente}$$

$$\lim_{t\to\infty}\frac{t+1}{e^t}\left(\frac{\infty}{\infty}\right) \overset{\hat{H} \to \frac{f'(x)}{g'(x)}}{=} \lim_{t\to\infty}\frac{1}{e^t} = \frac{1}{+\infty} = 0$$

$$81. \quad \int_1^{+\infty \to p.\text{crítico}} \frac{1}{x}\,dx = \lim_{t\to+\infty}\int_1^t \frac{1}{x} = \lim_{t\to+\infty}\left[\log|x|\right]_1^t = \lim_{t\to+\infty} \log|t| - \log(1) = +\infty \text{ diverge}$$

82. Determina el área del recinto limitado por las curvas $y = sen(x)$; $y = cos(x)$



Igualamos ambas curvas para obtener p.corte

$$f(x) = g(x) \longrightarrow sen(x) = cos(x) \begin{cases} x = \frac{\pi}{4} \\ x = \frac{5\pi}{4} \end{cases}$$

$$I = \left[\frac{\pi}{4} + \frac{5\pi}{4}\right] \; ; \; \text{Punto interior: } x_i = \frac{3\pi}{4}$$

$$f\left(\frac{3\pi}{4}\right) = sen\left(\frac{3\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

$$g\left(\frac{3\pi}{4}\right) = cos\left(\frac{3\pi}{4}\right) = \frac{-\sqrt{2}}{2}$$

$$A = \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} sen(x) - cos(x)\,dx = \left[-cos(x) - sen(x)\right]_{\frac{\pi}{4}}^{\frac{5\pi}{4}} = \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) - \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right) = 2\sqrt{2}$$

ángulo α	Sen α	Cos α	tg α
0º	0	1	∄
30º	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45º	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60º	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90º	1	0	1
180º	0	-1	∄
270º	-1	0	-1
360º	0	1	∄

83. $\int_{\frac{\sqrt{3}}{8}}^{\frac{1}{4}} \sqrt{1-16x^2}\,dx$ $\quad 1^2-(4x)^2$

$$\left\{\begin{array}{l} 4x = sen(t) \\ x = \frac{1}{4} sen(t) \\ x = \frac{\sqrt{3}}{8} \to sen(t) = 4 \cdot \frac{\sqrt{3}}{8} = \frac{\sqrt{3}}{2} \to t = \frac{\pi}{3} \\ x = \frac{1}{4} \to sen(t) = 4 \cdot \frac{1}{4} = 1 = t = \frac{\pi}{2} \end{array}\right.$$

Puedo cambiar los límites de integración

$$\int_{\pi/3}^{\pi/2} \sqrt{1-sen^2(t)} \cdot \frac{1}{4} Cos(t)\,dt = \frac{1}{4}\int_{\pi/3}^{\pi/2} Cos(t) \cdot cos(t)\,dt = \frac{1}{4}\int_{\pi/3}^{\pi/2} cos^2(t)\,dt$$

$$= \frac{1}{4}\int_{\pi/3}^{\pi/2} \frac{1+cos(2t)}{2} = \frac{1}{8}\int_{\pi/3}^{\pi/2} 1+cos(2x) = \left[\frac{1}{8}t + \frac{1}{16}sen(2t)\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}} =$$

$$\frac{1}{8}\left(\frac{\pi}{2} + \cancel{\frac{sen(\pi)}{2}} - \frac{\pi}{3} - \frac{sen\left(\frac{2\pi}{3}\right)}{2}\right) = \frac{1}{8}\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right)$$

84. $\int_{-a}^{a} \sqrt{a^2-x^2}\,dx = \left\{\begin{array}{l} x = a\,sen(t) \\ dx = a\cos(t) \\ x = a \Rightarrow sen(t) = 1 ;\ t = \frac{\pi}{2} \\ x = -a \Rightarrow sen(t) = -1 ;\ t = -\frac{\pi}{2} \end{array}\right\}$

$$\int_{-\pi/2}^{\pi/2} \sqrt{a^2 - a^2 sen^2(t)}\quad a\cos(t)\,dt = \int_{-\pi/2}^{\pi/2} a\sqrt{1-sen^2(t)} \cdot a\cos(t)\,dt$$

$$a^2\int_{-\pi/2}^{\pi/2} cos(t) \cdot cos(t)\,dt = a^2\int_{-\pi/2}^{\pi/2} \frac{1+cos(2x)}{2}\,dt = \frac{a^2}{2}\int_{-\pi/2}^{\pi/2} 1+cos(2t)\,dt$$

$$= \frac{a^2}{2}\left(t + \frac{sen(2t)}{2}\right) = \frac{a^2}{2}\left[t + \frac{sen(2t)}{2}\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{\pi}{2} + \frac{sen(-\frac{\pi}{2})}{2} - \left(-\frac{\pi}{2}\right) - \frac{sen(-\frac{\pi}{2})}{2}$$

$$= \frac{\pi}{2} - \cancel{\frac{1}{2}} + \frac{\pi}{2} + \cancel{\frac{1}{2}} = \frac{a^2}{2}\pi$$

85 $\displaystyle\int_{-\infty}^{+\infty \to \text{p.crítico}} \frac{1}{x^2-4x+8}\,dx = \lim_{t\to+\infty}\int_{-t}^{t}\frac{1}{x^2-4x+8}$ * ; Calculamos su primitiva: $\displaystyle\int \frac{1}{x^2-4x+8}$ como este polinomio no se puede factorizar, entonces debemos buscar el cuadrado perfecto

$-\infty$ p.crítico; no hay punto donde se anula el denominador

g.num < denom

$x^2-4x+8 = (x^2-4x+4)+4 = (x-2)^2+4$

$(x-a)^2 = x^2-2ax+a^2 = (x-2)^2$

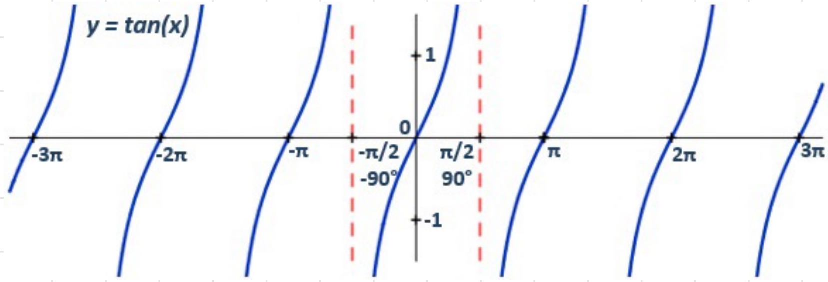
$$\int \frac{1}{(x-2)^2+4}\,dx = \frac{1}{4}\int \frac{1}{\left(\frac{x-2}{2}\right)^2+1}$$

$$= \frac{1}{4}\int \frac{1/2}{\left(\frac{x-2}{2}\right)+1}\,dx = \frac{1}{4}\cdot 2\int \frac{1/2}{\left(\frac{x-2}{2}\right)+1} = \frac{1}{2}\arctan\left(\frac{x-2}{2}\right)$$

$$* \lim_{t\to+\infty}\left[\frac{1}{2}\arctan\left(\frac{x-2}{2}\right)\right]_{-t}^{t} = \lim_{t\to+\infty}\frac{1}{2}\left(\arctan\left(\frac{t-2}{2}\right) - \arctan\left(\frac{-t-2}{2}\right)\right)$$

$$\frac{1}{2}\left(\arctan(+\infty) - \arctan(-\infty)\right) = \frac{1}{2}\left(\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right) = \frac{1}{2}\pi = \frac{\pi}{2}$$

x	$-\infty$	$-\sqrt{3}$	$\frac{-\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$	$+\infty$
$\arctan x$ (grados sexagesimales)	-90º	-60º	-30º	0º	30º	60º	90º
$\arctan x$ (radianes)	$-\frac{1}{2}\pi$	$-\frac{1}{3}\pi$	$-\frac{1}{6}\pi$	0	$\frac{1}{6}\pi$	$\frac{1}{3}\pi$	$\frac{1}{2}\pi$



86 $\displaystyle\int \frac{1}{\sqrt{x-1}+1}\,dx = \left\{\begin{array}{l} x-1 = t^2 \\ dx = 2t \end{array}\right\} = \int \frac{2t}{t+1}\,dt$; $\begin{array}{r|l} 2t & t+1 \\ -2t+2 & 2 \\ \hline 0-2 & \end{array}$; $\displaystyle\int 2 - \frac{2}{t+1}\,dt$

g.num >= denominador

$$= 2t - 2\log|t+1| + C = 2\sqrt{x-1} - 2\log\left|\sqrt{x-1}+1\right| + C$$

87 $\displaystyle\int \frac{4\cos x}{\operatorname{sen}^2(x)-4} = \left\{\begin{array}{l} t = \operatorname{sen}(x) \\ dt = \cos(x) \end{array}\right\} \int \frac{4}{t^2-4}\,dt = 4\int \frac{1}{t^2-2^2}\,dt$

$$= \int \frac{4}{(t-2)\cdot(t+2)} \;*\; ; \quad \frac{A}{(t-2)} + \frac{B}{(t+2)} = \frac{A(t+2)+B(t-2)}{(t-2)\cdot(t+2)} = \frac{4}{(t-2)\cdot(t+2)}$$

$t=-2$; $-4B=4$; $B=-1$ $\qquad t=2 \quad 4A=4$; $A=1$

$$* = \int \frac{1}{(t-2)} + \frac{-1}{t+2}\,dx = \log|t-2| - \log|t+2| + C = \log|\mathrm{sen}(x)-2| - \log|\mathrm{sen}(x)+2| + C$$

p.conflictivo

88 $\displaystyle\int_{-\infty}^{+\infty} \frac{1}{(x-2)^2+36}\,dx = \lim_{t\to+\infty}\int_{-t}^{t} \frac{1}{(x-2)^2+36}\,dx \;*$; calculamos la primitiva ; $\displaystyle\int \frac{1}{(x-2)^2+36}\,dx$

$$= \frac{1}{36}\int \frac{1}{\left(\frac{x-2}{6}\right)^2+1}\,dx = \frac{6}{36}\int \frac{1/6}{\left(\frac{x-2}{6}\right)^2+1} = \frac{1}{6}\arctan\left(\frac{x-2}{6}\right) \; ; \; * \lim_{t\to+\infty}\left[\frac{1}{6}\arctan\left(\frac{x-2}{6}\right)\right]_{-t}^{t}$$

$$\lim_{t\to+\infty} \frac{1}{6}\left(\arctan\left(\frac{t-2}{6}\right) - \arctan\left(\frac{-t-2}{2}\right)\right) = \frac{1}{6}\left(\arctan(+\infty) - \arctan(-\infty)\right)$$

$$\frac{1}{6}\left(\frac{\pi}{2} - \left(\frac{-\pi}{2}\right)\right) = \frac{1}{6}\cdot\frac{2\pi}{2} = \frac{\pi}{6}$$

89 $\displaystyle\int 6x e^{3x}\,dx = \left\{\begin{matrix} u=6x & du=6\,dx \\ dv=e^{3x} & v=\frac{1}{3}e^{3x}\end{matrix}\right\} 6x\,\frac{1}{3}e^{3x} \qquad \int \frac{1}{3}e^{3x}\cdot 6$

$$= 2x e^{3x} - 2\int e^{3x} = 2x e^{3x} - \frac{2}{3}e^{3x} + C$$

90 $\displaystyle T(x) = \int_0^{x^2-3x+2} e^{t^2}\,dt$

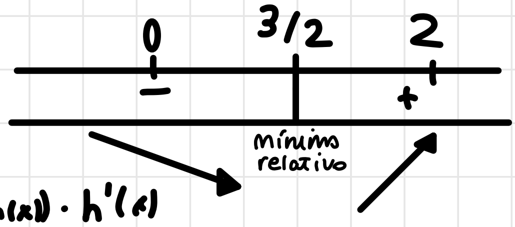
tenemos que calcular la derivada de manera que las soluciones son los p.críticos, los candidatos a ser max o min. Para clasificar dichos ptos críticos podemos estudiar el signo de $T'(x)$ o utilizar $T''(x)$

$$\left(\int_{h(x)}^{g(x)} f(t)\right)' = f(g(x))\cdot g'(x) - f(h(x))\cdot h'(x)$$

$$T'(x) = \left(\int_0^{x^2-3x+2} e^{t^2}\,dt\right)' = e^{(x^2-3x+2)^2}\cdot(2x-3) - \cancel{e^0\cdot 0}$$

$T'(x)=0 ; (2x-3)\cdot \underbrace{e^{(x^2-3x+2)^2}}_{\text{no puede dar } 0} = 0 \Rightarrow 2x-3=0 ; x=\frac{3}{2}$ (p.crític

$T'(0)$



91. Calcula $f'(x) = f(g(x))\cdot g'(x) - f(h(x))\cdot h'(x)$

$f(x)=\int_{\sqrt{x}}^{2\sqrt{x}} sen(t^2)\,dt ; \; f'(x) = sen(4x)\cdot \frac{\cancel{2}}{\cancel{2}\sqrt{x}} - sen(x)\cdot\frac{1}{2\sqrt{x}}$

$f'(x) = sen(4x)\cdot\frac{1}{\sqrt{x}} - \frac{sen(x)}{2\sqrt{x}} = \frac{1}{\sqrt{x}}\left(sen(4x)-\frac{1}{2} sen(x)\right)$

92 Calcula $f(4)$ si:

$$\int_0^{f(x)} t^2\,dt = x\cos(\pi x)$$

$$\int_0^{f(x)} t^2\,dt = \left[\frac{t^3}{3}\right]_0^{f(x)} = \frac{f^3(x)}{3} = x\cos(\pi x)$$

$x=4: \frac{f^3(4)}{3} = 4\cos(4\pi) ; \; \frac{f^3(4)}{3} = 4 ; \; f(4)=\sqrt[3]{12}$

93. La transformada de Laplace de una función $f(x)$ viene dada por:

p.crítico

$\mathcal{L}|f|(s) = \int_0^\infty f(x)e^{-sx}\,dx ; \; \mathcal{L}|sen(ax)|(s) = \int_0^\infty sen(ax)e^{-sx}\,dx$

$\lim_{t\to+\infty}\int_0^t sen(ax)e^{-sx}\,dx ;$ calculamos su primitiva $\underbrace{\int sen(ax)e^{-sx}\,dx}_{I} \rightarrow$ I. por partes

si lees esto me debes un besito

$$\left\{\begin{array}{ll} u = e^{-sx} & du = -s e^{-sx} \\ dv = \text{sen}(ax) & v = \dfrac{-\cos(ax)}{a} \end{array}\right\} \quad e^{-sx} \cdot \frac{-\cos(ax)}{a} - \int \frac{-\cos(ax)}{a} (-s \cdot e^{-sx})$$

$$= e^{-sx} \cdot \frac{-\cos(ax)}{a} - \frac{s}{a} \int \cos(ax) \cdot e^{-sx} dx = \left\{\begin{array}{ll} u = e^{-sx} & du = -s e^{-sx} \\ dv = \cos(ax) & v = \dfrac{\text{sen}(ax)}{a} \end{array}\right.$$

$$e^{-sx} \cdot \frac{-\cos(ax)}{a} - \frac{s}{a} \left(e^{-sx} \cdot \frac{\text{sen}(ax)}{a} - \int \frac{\text{sen}(ax)}{a} \cdot -s e^{-sx} dx \right)$$

$$e^{-sx} \cdot \frac{-\cos(ax)}{a} - \frac{s}{a} \left(e^{-sx} \cdot \frac{\text{sen}(ax)}{a} + \frac{s}{a} \underbrace{\int \text{sen}(ax) e^{-sx} dx}_{I} \right)$$

$$-\frac{1}{a} e^{-sx} \cdot \cos(ax) - \frac{s}{a^2} e^{-sx} \text{sen}(ax) - \frac{s^2}{a^2} \int \text{sen}(ax) e^{-sx} dx$$

$$I = -\frac{1}{a} e^{-sx} \cdot \cos(ax) - \frac{s}{a^2} e^{-sx} - \frac{s^2}{a^2} I$$

$$I + \frac{s^2}{a^2} I = e^{-sx} \left(-\frac{1}{a} \cos(ax) - \frac{s}{a^2} \text{sen}(ax) \right)$$

$$I \left(1 + \frac{s^2}{a^2} \right) = \left[e^{-sx} \left(-\frac{1}{a} \cos(ax) - \frac{s}{a^2} \text{sen}(ax) \right) \right]$$

$$I \left(\frac{a^2 + s^2}{a^2} \right) = \frac{a^2}{s^2 + a^2} \left[e^{-sx} \left(-\frac{1}{a} \cos(ax) - \frac{s}{a^2} \text{sen}(ax) \right) \right]$$

$$\lim_{t \to +\infty} \int_0^t \text{sen}(ax) e^{-sx} dx = \lim_{t \to +\infty} \left[\frac{a^2}{s^2 + a^2} e^{-sx} \left(-\frac{1}{a} \cos(ax) - \frac{s}{a^2} \text{sen}(ax) \right) \right]_0^t$$

$$\lim_{t\to+\infty} \frac{a^2}{s^2+a^2}\left(e^{-st}\left(-\frac{1}{a}\cos(at)-\frac{s}{a^2}\operatorname{sen}(at)\right)-e^{0}\left(-\frac{1}{a}\cos(0)\right)\right)$$

$$\lim_{t\to+\infty} \frac{a^2}{s^2+a^2}\left(e^{-st}\left(-\frac{1}{a}\cos(at)-\frac{s}{a^2}\operatorname{sen}(at)\right)+\frac{1}{a}\right)$$

$$=\frac{a^2}{s^2+a^2}\left(\underset{\downarrow 0}{e^{-\infty}}\cancel{\left(-\frac{1}{a}\cos(at)-\frac{s}{a^2}\operatorname{sen}(at)\right)}+\frac{1}{a}\right)$$

$$=\boxed{\frac{a}{s^2+a^2}}$$

94 $\int_0^{\infty} x^2 e^{-2x}\,dx$; p. conflictivo; * calculamos su primitiva $\int x^2\cdot e^{-2x} = \left\{\begin{matrix} u=x^2 & du=2x \\ dv=e^{-2x} & v=-\frac{e^{-2x}}{2}\end{matrix}\right\}$

$$-x^2\cdot\frac{e^{-2x}}{2} - \int -\frac{e^{-2x}}{2}\cdot 2x = \frac{-x^2\cdot e^{-2x}}{2}+\int e^{-2x}\cdot x\,dx$$

$$=\left\{\begin{matrix} u=x & du=dx \\ dv=e^{-2x} & v=-\frac{e^{-2x}}{2}\end{matrix}\right\}=\frac{-x^2\cdot e^{-2x}}{2}+\frac{-x\cdot e^{-2x}}{2}-\int\frac{-e^{-2x}}{2}\,dx$$

$$=-\frac{1}{2}x^2\cdot e^{-2x}-\frac{1}{2}x e^{-2x}-\frac{1}{4}e^{-2x} \quad =\frac{1}{2}e^{-2x}\left(-x^2-x-\frac{1}{2}\right)$$

$$\lim_{t\to\infty}\frac{1}{2}e^{-2t}\left(-t^2-t-\frac{1}{2}\right)-\underbrace{\frac{1}{2}\cdot-\frac{1}{2}}_{-\frac{1}{4}} \quad **$$

$$* \lim_{t\to\infty}\int_0^t x^2 e^{-2x}\,dx=\lim_{t\to\infty}\left[\frac{1}{2}e^{-2x}\left(-x^2-x-\frac{1}{2}\right)\right]_0^t$$

$$\lim_{t\to\infty}\frac{1}{2}e^{-2t}\left(-t^2-t-\frac{1}{2}\right)=\frac{-t^2-t-\frac{1}{2}}{e^{2t}}=\frac{\infty}{\infty}\overset{H}{=}\frac{2t-1}{2e^{2t}}=\frac{\infty}{\infty}\overset{H}{=}$$

$$\lim_{t\to\infty}\frac{2}{4e^{2t}}=\frac{-2}{+\infty}=0 \qquad ** \lim_{t\to\infty}=+\frac{1}{4} \quad \text{converge}$$

95 Calcule con una integral

$$\lim_{n\to\infty} \frac{1^5+2^5+\dots+n^5}{n^6} = \lim_{n\to+\infty} \frac{1}{n}\cdot\frac{1^5+2^5+\dots+n^5}{n^5}$$

$$= \lim_{n\to\infty} \frac{1}{5}\left(\left(\frac{1}{n}\right)^5+\left(\frac{2}{n}\right)^5+\dots+\left(\frac{n}{n}\right)^5\right) = \lim_{n\to+\infty} \frac{1}{n}\sum_{j=1}^{n}\left(\frac{j}{n}\right)^5$$

$f(x)=x^5$

$$= \int_0^1 x^5\,dx = \left[\frac{x^6}{6}\right]_0^1 = \frac{1}{6}$$

96 $$\lim_{n\to\infty} \frac{1^3+2^3+\dots+n^3}{n^4} = \lim_{n\to+\infty} \frac{1}{n}\cdot\left(\frac{1^3+2^3+\dots+n^3}{n^3}\right)$$

$$= \lim_{n\to+\infty} \frac{1}{n}\sum_{j=1}^{n}\left(\frac{j}{n}\right)^3 = \int_0^1 x^3\,dx = \left[\frac{x^4}{4}\right]_0^1 = \frac{1}{4}$$

$f(x)=x^3$

97 $$\lim_{n\to+\infty} \frac{\operatorname{sen}\left(\frac{1}{n}\right)+\operatorname{sen}\left(\frac{2}{n}\right)+\dots+\operatorname{sen}\left(\frac{n}{n}\right)}{n}$$

$$= \lim_{n\to\infty} \frac{1}{n}\cdot \operatorname{sen}\left(\frac{1}{n}\right)+\operatorname{sen}\left(\frac{2}{n}\right)+\dots+\operatorname{sen}\left(\frac{n}{n}\right) = \lim_{n\to\infty} \frac{1}{n}\sum_{j=1}^{n}\operatorname{sen}(j/n)$$

$$= \int_0^1 \operatorname{sen}(x)\,dx = \left[-\cos(x)\right]_0^1 = -\cos(1)+\cos(0)$$

99 La función gamma se define como:

$$\Gamma(n) = \int_0^{\infty\,\to\text{p.crítico}} t^{n-1}e^{-t}\,dt$$

a) Demuestra que es convergente

$$\lim_{t\to+\infty} \frac{t^{n-1}e^{-t}}{t^{-2}} = \frac{t^{n-1}\cdot t^2}{e^t} = \frac{+\infty}{+\infty} \overset{\hat{H}}{=} \lim_{t\to\infty}\frac{t\cdot 2t}{e^t} = \frac{+\infty}{+\infty} \overset{\hat{H}}{=} \lim_{t\to\infty}\frac{1\cdot 2}{e^t} = \frac{2}{+\infty} = 0$$

Como $t^{n-1}e^{-t} < t^2$ para t muy grande $\Rightarrow \int_0^{+\infty} t^{n-1}\cdot e^{-t}\,dt < \int_1^{+\infty} t^{-2}\,dt$

$$\int_1^{+\infty} t^{-2}\,dt = \lim_{v\to+\infty}\int_1^{v} t^{-2}\,dt = \lim_{v\to+\infty}\left[\frac{t^{-1}}{-1}\right]_1^{v} = 1 \Rightarrow \text{converge}$$ por lo que $\int_0^{+\infty} t^{n-1}\cdot e^{-t}$ también converge ya que $\int t^{-2}$ es mayor

b) Pruebe que $\Gamma(n+1) = n\Gamma(n)$

$$\Gamma(n+1) = \int_0^{\overset{\to \text{p. conflictivo}}{+\infty}} t^n \cdot e^{-t} = \lim_{x\to+\infty}\int_0^x t^n\cdot e^{-t}dt$$; Calculamos la primitiva por partes

$$\left\{\begin{matrix} u = t^n & du = nt^{n-1} \\ dv = e^{-t} & v = -e^{-t}\end{matrix}\right\} \left[-t^n e^{-t}\right]_0^x + \int e^{-t}\cdot nt^{n-1}dt$$

$$= \underbrace{-x^n e^{-x}} + \int_0^x e^{-t}\cdot nt^{n-1}dt = *$$

$$\lim_{x\to\infty} -x^n e^{-x} = \lim_{x\to\infty}\frac{x^n}{e^x} = \left(\frac{+\infty}{\infty}\right) \overset{H}{=} \lim_{x\to+\infty}\frac{nx^{n-1}}{e^x} = \frac{+\infty}{\infty} = \lim_{x\to\infty}\frac{-nx^{n-1}}{e^x} = \left(\frac{+\infty}{\infty}\right) \overset{H}{=}$$
$$= \dots = \lim_{x\to+\infty}\frac{-n!}{e^x} = \frac{-n!}{\infty} = 0$$

$$* \lim_{x\to\infty} n\int_0^x t^{n-1}e^{-t}dt = \underbrace{n\cdot\int_0^{+\infty} t^{n-1}e^{-t}dt}_{n\cdot\Gamma(n)} \; ; \; \Gamma(n+1) = n\Gamma(n)$$

c) Demuestre que $\Gamma(n+1) = n!$ si $n\geq 1$ es un número natural

$$\Gamma(n+1) = n\Gamma(n) = n\cdot(n-1)\cdot\Gamma(n-1) = \underbrace{n\cdot(n-1)\cdot(n-2)\cdots}_{n!}\Gamma(1)$$

$$\Gamma(1) = \int_0^{+\infty} e^{-t}dt = \lim_{u\to\infty}\left[-e^{-t}\right]_0^u = -e^{-u}+1 = +1$$

$$\Gamma(n+1) = n!$$

99

$$T(x) = \int_{\sqrt{x}}^{3\sqrt{x}} \cos(t^2)$$; Recordamos $T'(x)$ de $\int_{h(x)}^{g(x)} f(x) = f(g(x))\cdot g'(x) - f(h(x))\cdot h'(x)$

$$T'(x) = \cos(9x) \cdot \frac{3}{2} \cdot \frac{1}{\sqrt{x}} - \cos(x) \cdot \frac{1}{2\sqrt{x}}$$

$$= \frac{1}{2\sqrt{x}} \left(3\cos(9x) - \cos(x) \right)$$

100 $T(x) = \int_0^{\sqrt{x}} x^2 \cdot \operatorname{sen} t^2 \, dt \quad T(x) = x^2 \cdot \int_0^{\sqrt{x}} \operatorname{sen}(t^2)$

$$T'(x) = 2x \cdot \int_0^{\sqrt{x}} \operatorname{sen}(t^2) + x^2 \cdot \left(\operatorname{sen}(x) \cdot \frac{1}{2\sqrt{x}} \right)$$

$$T'(x) = 2x \cdot \int_0^{\sqrt{x}} \operatorname{sen}(t^2) + x^2 \cdot \operatorname{sen}(x) \cdot \frac{1}{2\sqrt{x}}$$

$$T'(x) = 2x \cdot \int_0^{\sqrt{x}} \operatorname{sen}(t^2) + \frac{1}{2} x^{3/2} \cdot \operatorname{sen}(x)$$

101 $T(x) = \int_x^{x^2} e^t \cdot \operatorname{sen}(t^3) \, dt$

$$T'(x) = e^{x^2} \cdot \operatorname{sen}(x^6) \cdot 2x - e^{x^2} \cdot \operatorname{sen}(x^3)$$

102 $T(x) = \int_0^{x^3} x^2 e^{t^3} dt \; ; \; T(x) \quad x^2 \int_0^{x^3} e^{\tau^3} dt$

$$T'(x) = 2x \int_0^{x^3} e^{\tau^3} dt + x^2 \cdot \left(e^{x^9} \cdot 3x^2 \right)$$

103 Demuestra $y = \frac{1}{a} \int_0^x f(t) \operatorname{sen}(a(x-t)) \, dt$

$$\frac{d^2 y}{d^2 x} + a^2 y = f(x) \; ; \; \frac{dy}{dx}(0) = 0 \; ; \; y(0) = 0$$

Lo primero que debemos hacer, es demostrar que $y(0) = 0$

$$y(0)=\frac{1}{a}\int_0^0 f(t)\,\text{sen}(a(-t))\,dt=0$$ (porque no hay intervalo de integración)

$$\text{sen}(ax-at)=\text{sen}(ax)\cos(at)-\text{sen}(at)\cos(ax)$$

Por tanto:

$$\text{sen}(a(x-t))\cdot\text{sen}(ax)\cdot\cos(at)-\text{sen}(at)\cos(ax)\,dt$$

$$=\frac{1}{a}\left[\text{sen}(ax)\int_0^x f(t)\cos(at)\,dt-\cos(ax)\cdot\int_0^x f(t)\,\text{sen}(at)\right]$$

$$y(x)=\frac{1}{a}\left[\text{sen}(ax)\int_0^x f(t)\cos(at)\,dt-\cos(ax)\cdot\int_0^x f(t)\,\text{sen}(at)\right]$$

Derivamos

$$y'(x)=\frac{1}{a}\Big[a\cos(ax)\int_0^x f(t)\cos(at)\,dt+\text{sen}(ax)\Big(\underbrace{\int_0^x f(t)\,\text{sen}(at)}_{I_1}\Big)'$$

$$+\,a\,\text{sen}(ax)\int_0^x f(t)\,\text{sen}(at)\,dt-\cos(ax)\Big(\underbrace{\int_0^x f(t)\,\text{sen}(at)}_{I_2}\Big)'$$

$$I_1=\left(\int_0^x f(t)\,\text{sen}(at)\right)'=f(x)\cos(ax)-f(0)\cdot\cos(0)\cdot 0=f(x)\cdot\cos(ax)$$

$$I_2=\left(\int_0^x f(t)\,\text{sen}(at)\right)'=f(x)\cdot\text{sen}(ax)-f(0)\cdot\text{sen}(0)\cdot 0=f(x)\cdot\text{sen}(ax)$$

$$y'(x)=\cos(ax)\int_0^x f(t)\cos(at)\,dt+\cancel{\frac{1}{a}\text{sen}(ax)\cdot}\big(f(x)\cos(ax)\big)$$

$$+\,\text{sen}(ax)\int_0^x f(t)\,\text{sen}(at)\,dt-\cancel{\frac{1}{a}\cos(ax)\cdot f(x)}\cdot\text{sen}(ax)$$

104 $\int x(a+bx)^{-3/2}dx = \frac{1}{b}\int x(a+bx)\, b\, dx = \begin{cases} t=(a+bx) \\ dt=b \\ x=\frac{t-a}{b} \end{cases} = \frac{1}{b}\int \frac{t-a}{b}(t)^{-3/2}dt$

$= \frac{1}{b^2}\int t^{-1/2} - a t^{-3/2}\, dt = \frac{1}{b^2}\left(\cdot\frac{t^{1/2}}{1/2} + a\frac{t^{-1/2}}{1/2}\right)$

$= \frac{1}{b^2}\cdot\left(2\sqrt{t} + 2a\frac{1}{\sqrt{t}}\right) = \frac{1}{b^2}\cdot 2\sqrt{a+bx} + \frac{2a}{\sqrt{a+bx}} + C$

105 $\int x\cdot e^{-\frac{x^2}{2a}} = -a\int \frac{-x}{a}\cdot e^{-\frac{x^2}{2a}}dx = \begin{cases} t=-\frac{x^2}{2a} \\ dt=\frac{-x}{a} \end{cases} a\int e^t dt = ae^t = ae^{-\frac{x^2}{2a}} + C$

$\left(-\frac{x^2}{2a}\right)' = \frac{-2x(\cancel{2}a)}{(2a)^{\cancel{2}}} = -\frac{\cancel{2}x}{\cancel{2}a} = \frac{-x}{a}$

106 $\int x^5 \arctan\left(\frac{x^6+4}{5}\right) = \frac{5}{6}\int \frac{6x^5}{25}\arctan\left(\frac{x^6+4}{5}\right) = \begin{cases} t=\left(\frac{x^6+4}{5}\right) \\ dt=\frac{6x^5}{25} \end{cases}$

$\frac{5}{6}\int \arctan(t)\,dt = \begin{cases} u=\arctan(t) & du=\frac{1}{1+t^2} \\ dv=1dt & v=t \end{cases} \frac{5}{6}\left(t\arctan(t) - \int t\cdot\frac{1}{1+t^2}dt\right)$

$\frac{5}{6}\left(t\arctan(t) - \frac{1}{2}\log(1+t^2)\right) + C = \frac{5}{6}\arctan\left(\frac{x^6+4}{5}\right) - \frac{5}{12}\log\left(1+\left(\frac{x^6+4}{5}\right)^2\right) + C$

107 $\int \frac{1}{a^2e^x+b^2e^{-x}} = \int \frac{1}{a^2e^x+\frac{b^2}{e^x}}dx = \int \frac{e^x}{a^2(e^x)^2+b^2}dx = \begin{cases} t=e^x \\ dt=e^x \end{cases} \int \frac{1}{a^2t^2+b^2}dt$

$= b^2\int \frac{1/b^2}{\frac{a^2t^2}{b^2}+1}dt = \frac{1}{b^2}\int \frac{1/b^2}{\left(\frac{at}{b}\right)^2+1} = \frac{1}{ab}\int \frac{a/b}{\left(\frac{at}{b}\right)^2+1} = \frac{1}{ab}\arctan\left(\frac{at}{b}\right) + C$

$g'(x) = \frac{ab}{b^2} = \frac{a}{b}$

$\arctan\left(\frac{at}{b}\right)$

108 $\int \frac{3x^2+2x+4}{x^3+x^2+x+1}dx = \int \frac{3x^2+2x+4}{(x^2+1)\cdot(x+1)}dx$

g numerador < denominador

$x=1 \quad 2A=5 \quad A=\frac{5}{2}$

$Ax^2+A+Bx^2+Bx+Cx+C$

	1	1	1	1
-1		-1	0	-1
	1	0	1	$\boxed{0}$

$(x+1)\cdot(x^2+1)$

$\frac{3x^2+2x+4}{(x^2+1)\cdot(x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1} = \frac{A(x^2+1)+(Bx+C)\cdot(x+1)}{(x+1)\cdot(x^2+1)}$

$x=0 \quad \frac{5}{2}+C=4 \quad C=\frac{3}{2} \quad x=1 \quad B=\frac{1}{2}$

$$\int \frac{5/2}{x+1} + \frac{\frac{1}{2}x+\frac{3}{2}}{x^2+1}\,dx = \frac{5}{2}\int \frac{1}{x+1}\,dx + \frac{1}{2}\int \frac{x}{x^2+1}\,dx + \frac{3}{2}\int \frac{1}{x^2+1}\,dx$$

$$= \frac{5}{2}\log|x+1| + \log|x^2+1| + \frac{3}{2}\arctan(x) + C$$

109 $F(x) = \int_y^{\left(\int_1^x \operatorname{sen}^3(t)\,dt\right)} \frac{1}{1+\operatorname{sen}^6(t)+t^2}\,dt$

$F'(x) = f(g(x))\cdot g'(x) + f(h(x))\cdot h'(x)$

$$\int_1^x \operatorname{sen}^3(t)\,dt =^{*} \left[-\cos(t) + \frac{\cos^3(t)}{3}\right]_1^x = \boxed{-\cos(x) + \frac{\cos^3(x)}{3}}$$

$$\int \operatorname{sen}^3(t)\,dt = \int \operatorname{sen}(t)\cdot\operatorname{sen}^2(t)\,dt = \int \operatorname{sen}(t)\cdot(1-\cos^2(t))\,dt$$

$$= -\int -\operatorname{sen}(t)\cdot(1-\cos^2(t)) = \left\{\begin{matrix} u=\cos(t) \\ du = -\operatorname{sen}(t) \end{matrix}\right\} - \int 1-u^2\,dt = -u + \frac{u^3}{3} =$$

$$= -\cos(t) + \frac{\cos^3(t)}{3}$$

$$F'(x) = \frac{1}{1+\operatorname{sen}^6\left(\int_1^x \operatorname{sen}^3(t)\,dt\right) + \left(\int_1^x \operatorname{sen}^3(t)\,dt\right)^2}\cdot\left(\int_1^x \operatorname{sen}^3(t)\,dt\right)' - \frac{1}{1+\operatorname{sen}^6(y)+y^2}\cdot 0$$

$$F'(x) = \frac{1}{1+\operatorname{sen}^6\left(\int_1^x \operatorname{sen}^3(t)\,dt\right) + \left(\int_1^x \operatorname{sen}^3(t)\,dt\right)^2}\cdot\left(\operatorname{sen}^3(x)\cdot 1 - \cancel{\operatorname{sen}^3(1)}\cdot 0\right)$$

$$F'(x) = \frac{1}{1+\operatorname{sen}^6\left(\int_1^x \operatorname{sen}^3(t)\,dt\right) + \left(\int_1^x \operatorname{sen}^3(t)\,dt\right)^2}\left(\operatorname{sen}^3(x)\right)$$

110 $\int \frac{1}{(x-1)(x^2+2)}\,dx = \frac{A}{(x-1)} + \frac{Bx+C}{(x^2+2)} = \frac{A(x^2+2)+(Bx+C)(x-1)}{(x-1)(x^2+2)} = \frac{1}{(x-1)(x^2+2)}$

$x=1 \quad 3A=1 \quad A=\frac{1}{3} \qquad x=0 \quad \frac{2}{3}-C=1 \quad C=\frac{2}{3}-1=-\frac{1}{3} \qquad x=-1 \quad 1-(B+\frac{1}{3})(-2)$

$B=-\frac{1}{3}$

$$\int \frac{1/3}{(x-1)} + \frac{(-1/3)x-(1/3)}{(x^2+2)}\,dx = \frac{1}{3}\int\frac{1}{(x-1)}dx - \frac{1}{3}\int\frac{x}{x^2+2} - \frac{1}{3}\int\frac{1}{\left(\frac{x}{\sqrt{2}}\right)^2+\frac{2}{2}}dx \qquad \frac{x}{\sqrt{2}}=\frac{\sqrt{2}}{(\sqrt{2})^2}=\frac{1}{\sqrt{2}}$$

$$=\frac{1}{3}\log|x-1| - \frac{1}{6}\log(x^2+2) - \frac{\sqrt{2}}{6}\arctan\left(\frac{x}{\sqrt{2}}\right)$$

111 $\int sen^2(x)\cdot\cos^2(x)\,dx = \left\{\begin{matrix} sen(2x)=2sen(x)\cos(x) \\ sen^2(2x)=4sen^2(x)\cos^2(x)\end{matrix}\right\}$

$$\frac{1}{4}\int sen^2(2x)\,dx = \left\{sen^2(x)=\frac{1-\cos^2(x)}{2}\right\}\frac{1}{4}\int\frac{1-\cos^2(4x)}{2}dx = \frac{1}{8}\left(x-\frac{1}{4}sen(4x)\right)+C$$

112 $\int x^2 sen(3x) = \left\{\begin{matrix} u=x^2 & du=2x \\ dv=sen(3x) & v=-\frac{\cos(3x)}{3}\end{matrix}\right\} -\frac{x^2\cdot\cos(3x)}{3} + \int\frac{\cos(3x)}{3}\cdot 2x = ^*$

polinomio * f.tris

$$\left.\begin{matrix} u=2x & du=2 \\ dv=\frac{\cos(3x)}{3} & v=\frac{sen(3x)}{9}\end{matrix}\right\} 2x\cdot\frac{sen(3x)}{9} - \frac{1}{9}\int sen(3x)\cdot 2$$

$$= 2x\cdot\frac{sen(3x)}{9} - \frac{2}{27}\int 3sen(3x)$$

$$-\frac{x^2\cdot\cos(3x)}{3} + 2x\cdot\frac{sen(3x)}{9} + \frac{2}{27}\cdot\cos(3x)$$

113 $\int\sqrt{4-x^2} = \left\{\begin{matrix} x=2sen(t) \\ dx=2\cos(t)dt\end{matrix}\right\}\int\sqrt{4-(2sen(t))^2}\cdot 2\cos(t)dt$

$\sqrt{a^2-x^2}$

$x=a\,sen(t)$

$$=\int\sqrt{4(1-sen^2(t))}\cdot 2\cos(t)dt = \int 2\cdot\sqrt{\cos^2(t)}\cdot 2\cos(t)dt$$

$$=4\int\cos^2(t) = 4\int\frac{1+\cos(2t)}{2} = 2\int 1+\cos(2t)$$

$$2\left(t+\frac{1}{2}sen(2t)\right)+C = 2t+2sen(t)\cos(t)+C = \left\{\begin{matrix} sen(t)=\frac{x}{2} \\ t=arcsen\left(\frac{x}{2}\right)\end{matrix}\right.$$

$$=2arcosen\left(\frac{x}{2}\right) + \cancel{2}\cdot\frac{x}{\cancel{2}}\cdot\sqrt{1-sen^2(t)} = 2arcosen\left(\frac{x}{2}\right)+\frac{1}{2}x\sqrt{4-x^2}$$

114 $\displaystyle\int \frac{1}{(x+2)\sqrt{x+1}}\,dx = \left\{\begin{matrix} x+1=t^2 \\ dx=2t \\ x=t^2-1 \end{matrix}\right\} \int \frac{1}{(t^2-1+2)\sqrt{t^2}}\,2t\,dt = \int \frac{1}{(t^2+1)\cancel{t}}\,2\cancel{t}$

$\displaystyle 2\int \frac{1}{(t^2+1)}\,dt = 2\arctan(t) = 2\cdot \arctan(\sqrt{x+1}) + C$

115 $\displaystyle\int_1^{64} \frac{1}{\sqrt{x}+\sqrt[3]{x}}\,dx \overset{*}{=} \left[2\sqrt{x} - 3\sqrt[3]{x} + 6\sqrt[6]{x} - 6\log|\sqrt[6]{x}+1|\right]_1^{64}$

$= 2\cdot 8 - 3\cdot 4 + 6\cdot 2 - 6\log|3| - (2-3+6-\log(2) = 11 - 6\log(3) + 6\log(2)$

$\displaystyle \overset{*}{=} \int \frac{1}{\sqrt{x}+\sqrt[3]{x}} \left\{\begin{matrix} x=t^6 \\ dx=6t^5 \end{matrix}\right\} = \int \frac{1}{\sqrt[2]{t^6}+\sqrt[3]{t^6}}\,6t^5\,dt = \int \frac{1}{t^3+t^2}\,6t^5$

$\displaystyle = \int \frac{6t^{\cancel{5}\,3}}{\cancel{t^2}(t+1)}\,dt = \int \frac{6t^3}{t+1}\,dt \overset{**}{=} \int 6t^2 - 6t + 6 - \frac{6}{t+1}$

g.num >= denominador

$= 2t^3 - 3t^2 + 6t - 6\log|t+1|$

$= 2\sqrt[6]{x^3} - 3\sqrt[6]{x^2} + 6\sqrt[6]{x} - 6\log|\sqrt[6]{x}+1|$

$= 2\sqrt{x} - 3\sqrt[3]{x} + 6\sqrt[6]{x} - 6\log|\sqrt[6]{x}+1|$

$$\begin{array}{r|l} 6t^3 & t+1 \\ \hline -6t^3 - 6t^2 & 6t^2 - 6t + 6 \\ +6t^2 + 6t & \\ -6t - 6 & \end{array}$$

116 $\displaystyle\int_1^{\infty} \frac{1}{x^{1+a}}\,dx \quad (a>0)$ p-conflictivo

La función que queremos integrar nos da problemas en $x=0$ pero este punto no está en el intervalo de integración.

$\displaystyle = \lim_{t\to\infty} \int_1^t \frac{1}{x^{1+a}} \overset{*}{=} \left[\frac{1}{ax^a}\right]_1^t = \frac{1}{at^a} + \frac{1}{a} = \boxed{+\frac{1}{a}}$ La integral converge

$\displaystyle \int \frac{1}{x^{1+a}} = \int x^{-(1+a)} = \frac{x^{-1-a+1}}{a} = -\frac{x^{-a}}{a} = -\frac{1}{ax^a}$

$$\int \frac{1}{\operatorname{sen}(x)\cos(x)}\,dx = \left\{1 = \operatorname{sen}^2(x)+\cos^2(x)\right\} = \int \frac{\operatorname{sen}^2(x)+\cos^2(x)}{\operatorname{sen}(x)\cdot\cos(x)}\,dx$$

$$= \int \frac{\operatorname{sen}^{\cancel{2}}(x)}{\cancel{\operatorname{sen}(x)}\cdot\cos(x)} + \frac{\cos^{\cancel{2}}(x)}{\operatorname{sen}(x)\cancel{\cos(x)}} = \int \frac{\operatorname{sen}(x)}{\cos(x)}\,dx + \int \frac{\cos(x)}{\operatorname{sen}(x)}\,dx$$

$$= \left\{\begin{array}{ll} \cos(x)=t & u=\operatorname{sen}(t) \\ -\operatorname{sen}(x)dx = dt & du=\cos(t)dt \end{array}\right\} = -\int \frac{1\,dt}{t} + \int \frac{1}{u}\,du$$

$$-\log|t| + \log|u| = -\log|\cos(x)| + \log|\operatorname{sen}(x)|$$

$$\log\left|\frac{\operatorname{sen}(x)}{\cos(x)}\right| = \log|tg(x)| + C$$

117 $$\int \sqrt{9-x^2}\,dx = \left\{\begin{array}{l} x=3\operatorname{sen}(t) \\ \operatorname{sen}(t)=\frac{x}{3} \\ t=\operatorname{arcsen}\frac{x}{3} \\ dx=3\cos(t)dt \end{array}\right\} \int \sqrt{9-(3\operatorname{sen}(t))^2}\;3\cos(t)\,dt$$

$$= \int \sqrt{9(1-\operatorname{sen}^2(t))}\cdot 3\cos(t)\,dt = 9\int \sqrt{\cos^2(t)}\cdot\cos(t)\,dt$$

$$= 9\int \cos^2(t) = 9\int \frac{1+\cos(2t)}{2} = 9\left(\frac{1}{2}\int 1\,dt + \frac{1}{2}\cdot\frac{1}{2}\int \cos(2t)\cdot 2\,dt\right.$$

$$= \frac{9}{2}t + \frac{9}{4}\operatorname{sen}(2t) + C = \frac{9}{2}t + \frac{9}{\cancel{4}\,2}\cdot\cancel{2}\operatorname{sen}(t)\cos(t)$$

$$= \frac{9}{2}t + \frac{9}{2}\operatorname{sen}(t)\cos(t)\,dt + C$$

$$= \frac{9}{2}\operatorname{arcsen}\left(\frac{x}{3}\right) + \frac{\cancel{9}}{2}\cdot\frac{x}{\cancel{3}}\cdot\frac{\sqrt{9-x^2}}{\cancel{3}} + C$$

$$\cos(t) = \sqrt{1-\operatorname{sen}^2(t)} = \sqrt{1-\left(\frac{x}{3}\right)^2} = \frac{\sqrt{9-x^2}}{3}$$

118 $\displaystyle\int \sqrt{4-9x^2}\,dx = \left\{\begin{array}{l} 3x = 2\,sen(t) \\ t = arcsen\left(\frac{3}{2}x\right) \\ sen(t) = \frac{3x}{2} \\ x = \frac{2}{3}\,sen(t) \\ dx = \frac{2}{3}\cos(t)\,dt \end{array}\right\} \int \sqrt{4-(2\,sen(t))^2}\cdot\frac{2}{3}\cos(t)\,dt$

$\sqrt{a^2-x^2}$

$x = a\,sen(t)$

$$\int\sqrt{4(1-sen^2(t))}\,\frac{2}{3}\cos(t)\,dt = \frac{4}{3}\int\sqrt{\cos^2(t)}\cdot\cos(t)\,dt$$

$$=\frac{4}{3}\int\cos^2(t) = \frac{4}{3}\int\frac{1+\cos(2t)}{2} = \frac{4}{3}\left(\frac{1}{2}\int 1\,dt + \frac{1}{2}\cdot\frac{1}{2}\int\cos(2t)\cdot 2\,dt\right.$$

$$\frac{\cancel{4}}{3}\cdot\frac{1}{\cancel{2}}\cdot t + \frac{\cancel{4}}{3}\cdot\frac{1}{\cancel{2}}\cdot\frac{1}{\cancel{2}}\,sen(2t) = \frac{2}{3}t + \frac{1}{3}\cdot 2\,sen(t)\cos(t)$$

$$=\frac{2}{3}\,arcsen\left(\frac{3}{2}x\right) + \frac{\cancel{2}}{\cancel{3}}\cdot\frac{\cancel{3}x}{\cancel{2}}\cdot\frac{\sqrt{4-9x^2}}{2} + C$$

$$\cos(t) = \sqrt{1-sen(t)^2} = \sqrt{1-\left(\frac{3x}{2}\right)^2} = \frac{\sqrt{4-9x^2}}{2}$$

119 $\displaystyle\int\log^3(x)\,dx = \left\{\begin{array}{ll} u=\log^3(x) & du = 3\log^2(x)\cdot\frac{1}{x} \\ dv = 1 & v = x\end{array}\right\}\log^3(x)\cdot x - \int\cancel{x}\cdot 3\log^2(x)\cdot\frac{1}{\cancel{x}}$

Sabemos la derivada pero no la integral integramos por partes

$$= x\log^3(x) - 3\int\log^2(x)\,dx = x\log^3(x) - 3\left(x\log^2(x) - 2\left(x\log(x) - x\right)\right)+C$$

$$= x\log^3(x) - 3x\log^2(x) + 6x\log(x) - 6x + C$$

$$\int\log^2(x)\,dx = \left\{\begin{array}{ll} u=\log^2(x) & du = 2\log(x)\cdot\frac{1}{x} \\ dv = 1 & v = x\end{array}\right\} x\log^2(x) - \int\cancel{x}\cdot 2\log(x)\cdot\frac{1}{\cancel{x}}$$

$$= x\log^2(x) - 2\cdot\int\log(x)$$

$$\int\log(x) = \left\{\begin{array}{ll} u=\log(x) & du = \frac{1}{x} \\ dv = 1 & v = x\end{array}\right\} x\log(x) - \int\frac{1}{x}\cdot x\,dx = x\log(x) - x$$

120 Dada una función continua en $[0,1]$, calcular el siguiente límite

$$\lim_{t\to 0}\frac{\int_0^t x^n F(x)\,dx}{t^{n+1}} = \frac{0}{0} \overset{\hat{H}}{=} \lim_{t\to 0}\frac{\left(\int_0^t x^n F(x)\,dx\right)'}{(n+1)\ t^n}$$

$$= \lim_{t\to 0}\frac{\cancel{t^n}\cdot F(t) - \cancel{0}\cdot \cancel{F(0)}\cdot \cancel{0}}{(n+1)\cdot \cancel{t^n}} = \lim_{t\to 0}\frac{F(0)}{n+1} = \frac{K}{n+1}$$

120 $$\lim_{x\to 0}\frac{\int_0^{x^2} x^2 e^{-t^2}\,dt}{\log(1+x^4)} \underset{\text{equivalencia}}{\overset{\text{aplicamos}}{=}} \lim_{x\to 0}\frac{\left(\cancel{x^2}\int_0^{x^2} e^{-t^2}\,dt\right)}{x^{\cancel{4}}} \overset{\hat{H}}{=}$$

$$= \lim_{x\to 0}\frac{e^{-x^4}\cdot 2x - e^0\cdot 0}{2x} = \frac{e^{-x^4}}{1} = e^0 = 1$$

121 Demuestra que la integral impropia converge

$$\int_{1\to \text{p.crítico}}^{+\infty \to \text{p.crítico}} e^{-x}\cos(x)\,dx = \lim_{t\to\infty}\int_1^t e^{-x}\cos(x)\,dx \overset{*}{=}$$

$$\lim_{t\to\infty}\left[\frac{1}{2}e^{-x}\cdot(\operatorname{sen}(x)-\cos(x))\right]_1^t = \lim_{t\to+\infty}\left(\cancel{\tfrac{1}{2}e^{-t}(\operatorname{sen}(t)-\cos(t))}\right)+\boxed{\frac{1}{2}e^{-1}(\operatorname{sen}(1)-\cos(1))}$$

$\to e^{-\infty}=0$ converge

$$\int \underbrace{e^{-x}\cos(x)\,dx}_{I} = \left\{\begin{matrix} u=e^{-x} & du=-e^{-x}dx \\ dv=\cos(x) & v=\operatorname{sen}(x)\end{matrix}\right\} e^{-x}\operatorname{sen}(x)+\int \operatorname{sen}(x)\,e^{-x}$$

$$= \left\{\begin{matrix} u=e^{-x} & du=-e^{-x} \\ dv=\operatorname{sen}(x) & v=-\cos(x)\end{matrix}\right\} -e^{-x}\cdot\cos(x) - \int \underbrace{\cos(x)\cdot e^{-x}dx}_{I}$$

$$e^{-x}\operatorname{sen}(x) - e^{-x}\cos(x) - \int e^{-x}\cos(x)\,dx$$

$$I = e^{-x}\cdot sen(x) - e^{-x}\cos(x) - I$$

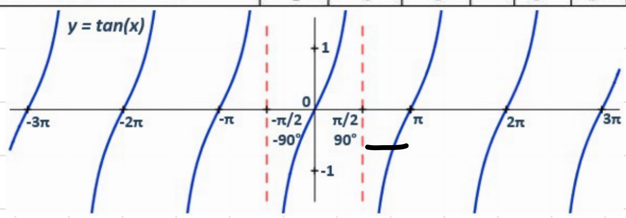
$$2I = e^{-x}\cdot sen(x) - e^{-x}\cos(x) \qquad I = \frac{e^{-x}(sen(x) - \cos(x))}{2}$$

122 $$\int_0^{+\infty \to \text{p. conflictivo}} \frac{x}{x^4+1}\,dx = \lim_{t\to+\infty}\int_0^t \underbrace{\frac{x}{x^4+1}}_{\text{g. num < g. denom}}dx = \left[\frac{1}{2}\arctan(x^2)\right]_0^t$$

$$= \lim_{t\to\infty} \frac{1}{2}\arctan(t) - \cancel{\frac{1}{2}\arctan(0)\cdot 0} = \lim_{t\to\infty}\frac{1}{2}\underbrace{\arctan(\infty)}_{\frac{\pi}{2}} = \frac{1}{2}\cdot\frac{\pi}{2} = \frac{\pi}{4}$$

$$\int\frac{x}{x^4+1}\,dx = \int\frac{x}{(x^2)^2+1}\,dx = \frac{1}{2}\int\frac{2x}{(x^2)^2+1} = \frac{1}{2}\arctan(x^2)$$

x	$-\infty$	$-\sqrt{3}$	$\frac{-\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$	$+\infty$
$\arctan x$ (grados sexagesimales)	-90º	-60º	-30º	0º	30º	60º	90º
$\arctan x$ (radianes)	$-\frac{1}{2}\pi$	$-\frac{1}{3}\pi$	$-\frac{1}{6}\pi$	0	$\frac{1}{6}\pi$	$\frac{1}{3}\pi$	$\frac{1}{2}\pi$



123 $$\int_{0^+ \to \text{p. conflictivo}}^1 \log(x)\,dx = \lim_{t\to0^+}\int_t^1 \log(x)\,dx \overset{*}{=} \Big[x(\log(x)-1)\Big]_t^1 =$$

$$\lim_{t\to0^+} 1\cdot\log(1) - 1 - (t(\log(t)-1)) = -1 - \lim_{x\to0^+} t\log(t)$$

$$\overset{**}{=} -1 \Rightarrow \text{converge}$$

$$\int^{*}\log(x) = \left\{\begin{matrix} u=\log(x) & du=\frac{1}{x} \\ dv=1 & v=x \end{matrix}\right\}\ x\log(x) - \int\cancel{x}\cdot\frac{1}{\cancel{x}}\,dx = x\log(x) - x$$

$$** \lim_{x\to0^+} t\log(t) = (0\cdot\infty) = \lim_{t\to+\infty}\frac{\log(t)}{\frac{1}{t}} = \left(\frac{\infty}{\infty}\right) \overset{\hat{H}}{=} \lim_{t\to0^+}\frac{\frac{1}{t}}{-\frac{1}{t^2}} = \lim_{t\to0^+}\frac{-t^2}{t} = -t = 0$$

124 $\int_1^{+\infty} \frac{1}{x(1+\ln(x))^p}\,dx \quad p>0 = \left\{ \begin{array}{l} t=(1+\ln(x)) \\ dt=\frac{1}{x}dx \\ x=1 \to t=1 \\ x=+\infty \to t=+\infty \end{array} \right\} \int_1^{+\infty}\frac{1}{t^p}\,dt$

Diverge $0<p\le 1$

Converge $p>1$ p.crítico

125 $\int_0^{+\infty}\frac{1}{a^2+x^2}\,dx = \lim_{t\to\infty}\int_0^t \frac{1}{a^2+x^2}\,dx \overset{*}{=} \frac{1}{a}\lim_{t\to\infty}\left[\arctan\left(\frac{x}{a}\right)\right]_0^t$

$= \text{arcotang}(t) - \text{arctang}(0)\cdot 0 = \frac{1}{a}\lim_{t\to\infty} \text{arctg}(t) = \frac{1}{a}\cdot\frac{\pi}{2}$

$\frac{1}{a^2}\int \frac{1}{\left(\frac{x}{a}\right)^2+1} = \frac{1}{a^2}\int\frac{1/a}{\left(\frac{x}{a}\right)^2+1} = \frac{1}{a}\cdot\text{arctang}\left(\frac{x}{a}\right) \qquad \left(\frac{x}{a}\right)' = \frac{a}{a^2}=\frac{1}{a}$

126 Hallar el área acotada por las curvas

$y_1 = 2x^3 - x^2 - 5x \qquad y_2 = -x^2+3x$

Para calcular el área entre dos curvas, realizaremos:

1 Igualaremos las funciones y obtendremos los p. de corte

$2x^3 - \cancel{x^2} - 5x = -\cancel{x^2} + 3x;\ 2x^3 - 8x = x(2x^2-8)\begin{cases} x_1 = 0 \\ x_2 = x = \pm\sqrt{4} = \pm 2 \end{cases}$

2 Ordenamos los puntos de corte de menor a mayor creando subintervalos dos a dos

$-2 \quad 0 \quad 2 \qquad \begin{cases} I_1\,[-2,0] \\ I_2\,[0,2] \end{cases}$

3 Sobre cada subintervalo, tomamos un punto interior y lo evaluamos en ambas curvas para saber cual está por encima y quién por debajo:

$$I_1[-2,0]\left\{\begin{array}{l} y_1(-1)=2 \\ y_2(-1)=-4 \end{array}\right\}$$ En I_1, y_1 está por encima de y_2

$$A_1=\int_{-2}^{0} (2x^3-x^2-5x)-(-x^2+3x)\,dx=\int_{-2}^{0} 2x^3-8x\,dx$$

$$A_1=\left[\frac{1}{2}x^4-4x^2\right]_{-2}^{0}=-(8-16)=+8$$

$$I_2[0,2]\left\{\begin{array}{l} y_1(1)=-4 \\ y_2(1)=2 \end{array}\right\}$$ En I_2, y_2 está por encima de y_1

$$A_2=\int_{0}^{2} (-x^2+3x)-(2x^3-x^2-5x)\,dx=\int_{0}^{2} -2x^3+8x\,dx$$

$$=\left[-\frac{1}{2}x^4+4x^2\right]_{0}^{2}=-8+16=8$$

4 El área total, será la suma de las áreas encerradas en cada subintervalo

$$A_T=A_1+A_2=8+8=16u^2$$

127 Hallar el área limitada por las curvas

$$\left.\begin{array}{l} x_1=4-4y^2 \\ x_2=1-y^4 \end{array}\right\} x\geq 0 \left\{\begin{array}{l} 4-4y^2\geq 0 \Rightarrow y^2\leq 1 \Rightarrow -1\leq y\leq 1 \\ 1-y^4\geq 0 \Rightarrow y^4\leq 1 \Rightarrow -1\leq y\leq 1 \end{array}\right.$$

1. Igualamos las funciones y obtenemos mas p. de corte

$4-4y^2 = 1-y^4 \qquad y^4-4y^2+3=0$

$w=y^2 \Rightarrow w^2-4w+3=0 \quad w=\dfrac{+4\pm\sqrt{16-4\cdot 3}}{2} \begin{cases} w_1=3 \\ w_2=1 \end{cases}$

$y=\pm\sqrt{1}=\pm 1$

$y=\pm\sqrt{3}>1 \Rightarrow$ No sirve porque está fuera de nuestro estudio

2. Ordenamos los p. corte de menor a mayor y los tomaremos dos a dos creando así subintervalos

$\{ I_1=[-1,1]$

3. Sobre cada subintervalo, tomaremos un punto intermedio y lo evaluaremos en ambas funciones para saber cual está por encima de la otra

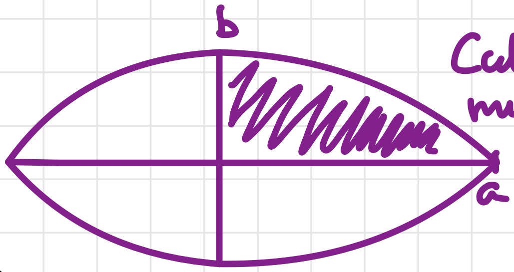
$\left.\begin{array}{l} x_1(0)=4 \\ x_2(0)=1 \end{array}\right\}$ En I_1, x_1 esta por encima de x_2

$$A_1=\int_{-1}^{1}(4-4y^2)-(1-y^4)\,dx=\int_{-1}^{1}y^4-4y^2+3\,dx$$

$$=\left[\frac{y^5}{5}-\frac{4y^3}{3}+3y\right]_{-1}^{1}=\frac{1}{5}-\frac{4}{3}+3-\left(-\frac{1}{5}+\frac{4}{3}-3\right)=\frac{56}{15}$$

128 Calcular el área de una elipse de semiejes a y b

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$



Calculamos $\frac{1}{4}$ y multiplicamos por 4

$$y^2 = b^2\left(\frac{a^2}{a^2}-\frac{x^2}{a^2}\right) = \frac{b^2}{a^2}\cdot a^2-x^2$$

$$y = \frac{b}{a}\sqrt{a^2-x^2}$$

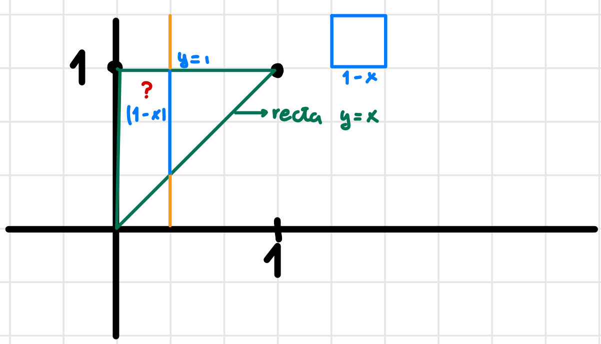
$$4\int_0^a \frac{b}{a}\sqrt{a^2-x^2}\,dx = \frac{4b}{a}\int_0^a \sqrt{a^2-x^2} = \begin{cases} x = a\,sen(t) \\ t = arcsen\left(\frac{x}{a}\right) \\ dx = a\cos(t)\,dt \\ x=0 \quad t=0 \\ x=a \quad t=\frac{\pi}{2} \end{cases}$$

$$\frac{4b}{a}\int_0^{\frac{\pi}{2}} \sqrt{a^2-a^2sen^2(t)}\cdot a\cos(t)\,dt = \frac{4b}{a}\int_0^{\frac{\pi}{2}} \sqrt{a^2(1-sen^2(t)}\cdot a\cos(t)\,dt$$

$$4ab\int_0^{\frac{\pi}{2}} \cos^2(t)\,dt = 4ab\int_0^{\pi/2} \frac{1+\cos(2t)}{2} = 2ab\left[t+\frac{sen\,2t}{2}\right]_0^{\frac{\pi}{2}}$$

$$= \pi ab$$

129 La base de un sólido es el triángulo de vértices (0,0), (0,1) y (1,1) en el plano XOY. Sus secciones por planos perpendiculares al eje OX son cuadrados. Calcular el volumen del sólido

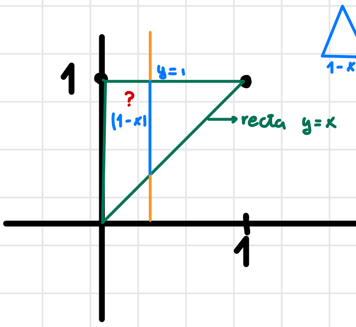


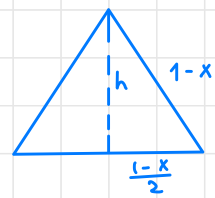
$$V=\int_0^1 A_x\,dx = \int_0^1 (1-x)^2\,dx$$

$$A_x = (1-x)^2$$

$$= \left[-\frac{(1-x^3)}{3}\right] = \frac{1}{3}$$

130 La base de un sólido es el triángulo de vértices $(0,0)$, $(0,1)$ y $(1,1)$ en el plano XOY. Sus secciones por planos perpendiculares al eje OX son triángulos equiláteros. Calcular el volumen del sólido





$$(1-x)^2 = h^2 + \left(\frac{1-x}{2}\right)^2$$

$$h^2 = (1-x)^2 - \frac{(1-x)^2}{4} = \frac{3}{4}(1-x)^2$$

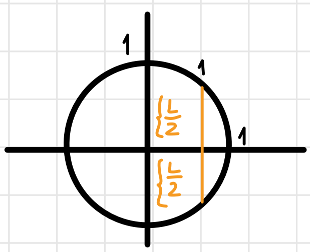
$$h = \frac{\sqrt{3}}{2}(1-x)$$

$$A_x = \frac{b \cdot h}{2} = \frac{(1-x)\cdot\frac{\sqrt{3}}{2}(1-x)}{2} = \frac{\sqrt{3}\,(1-x)^2}{4}$$

$$V = \int_0^1 \frac{\sqrt{3}}{4}(1-x)^2dx = \frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{4}\int_0^1 (1-x)^2dx = \frac{\sqrt{3}}{4}\left[\frac{-(1-x)^3}{3}\right]_0^1$$

$$= \frac{\sqrt{3}}{4}\cdot\frac{1}{3} = \frac{\sqrt{3}}{12}$$

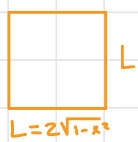
131 La base de un sólido de centro$(0,0)$ y radio 1 en el plano XOY. Sus secciones perpendiculares al eje OX son cuadrados. Calcular el volumen del sólido



$r = 1$

$x^2+y^2=1$

$y = \sqrt{1-x^2}$



$$A_x = L^2 = 4(1-x^2)$$

$$V = 2\int_0^1 4(1-x^2)dx = 8\int_0^1 (1-x^2)dx$$

$$= 8\cdot\left[x - \frac{x^3}{3}\right]_0^1 = 8\cdot\left(1-\frac{1}{3}\right) = \frac{16}{3}$$

132 Sabiendo que $I=\int_{-\infty}^{\infty} e^{-x^2}dx=\sqrt{\pi}$ calcula:

a) $$\int_0^{\infty} e^{-x^2}dx = \lim_{t\to\infty}\int_0^t x^2 e^{-x^2}dx \overset{*}{=} \lim_{t\to\infty}\left[-\frac{1}{2}xe^{-x^2}\right]_0^t + \frac{1}{2}\int_0^t e^{-x^2}dx$$

$$= \underbrace{\lim_{t\to\infty} -\frac{1}{2}te^{-t^2}}_{I_1} + \frac{1}{2}\underbrace{\int_0^{+\infty} e^{-x^2}dx}_{I_2} = \frac{\sqrt{\pi}}{4}$$

hacemos esto para poder aplicar I

$$I_1=\lim_{t\to\infty} -\frac{1}{2}\cdot\frac{t}{e^{t^2}} = \left(\frac{-\infty}{\infty}\right) \overset{H}{=} -\frac{1}{2}\lim_{t\to\infty}\frac{-1}{4t\cdot e^{t^2}} = \frac{1}{\infty}=0$$

$$I_2=\underbrace{\int_0^{+\infty}}_{\text{Esta es la mitad de } \int_{-\infty}^{+\infty}} e^{-x^2}dx = \frac{1}{2}\int_{-\infty}^{+\infty} e^{-x^2}dx = \frac{\sqrt{\pi}}{2}$$

hacemos esto, ya que la derivada del exp $=2x$

$$\int x^2\cdot e^{-2x} = \int x\, xe^{-x^2}dx = \left\{\begin{matrix} u=x & du=dx \\ dv=xe^{-x} & v=-\frac{1}{2}e^{-x}\end{matrix}\right\} -x\frac{1}{2}e^{-x^2} + \int\frac{1}{2}e^{-x^2}dx$$

$$= -x\frac{1}{2}e^{-x^2} + \frac{1}{2}\int e^{-x^2}dx$$

calculo la derivada para luego cambiar de variable

b) $$\int_{-\infty}^{+\infty} e^{-\frac{(x-\mu)^2}{2\sigma^2}}dx = \int_{-\infty}^{+\infty} e^{-\left[\frac{x-\mu}{\sqrt{2}\sigma}\right]^2}dx = \sqrt{2}\cdot\sigma\cdot\int_{-\infty}^{+\infty}\frac{1}{\sqrt{2}\sigma}e^{-\left[\frac{x-\mu}{\sqrt{2}\sigma}\right]^2}$$

$$= \left\{\begin{matrix} \frac{x-\mu}{2\sigma}=t \\ \frac{1}{\sqrt{2}\sigma}=dt\end{matrix}\right\} \sqrt{2}\cdot\sigma\int_{-\infty}^{+\infty} e^{-t^2}dt = \sqrt{2}\sigma\cdot\sqrt{\pi} = \sqrt{2\pi}\cdot\sigma$$

c) $$\int_0^{+\infty}\frac{e^{-x}}{\sqrt{x}}dx = \left\{\begin{matrix} x=t^2 \\ dx=2t \\ x=0 \quad t=0 \\ x=+\infty \quad t=+\infty\end{matrix}\right\} \int_0^{+\infty}\frac{e^{-t^2}}{\cancel{t}}2\cancel{t} = 2\int_0^{+\infty} e^{-t^2} = \int_{-\infty}^{+\infty} e^{-t^2}$$

$$= \sqrt{\pi}$$

133 $\Gamma(\alpha)=\int_0^{\infty} e^{-x}\, x^{\alpha-1}\,dx$ si $\alpha\geq 0$

a) Justifica porque la integral converge para $\alpha\geq 0$

1° caso $0<\alpha<1$

$$\int_0^{+\infty}\frac{e^{-x}}{x^{1-\alpha}}=\int_0^1\frac{e^{-x}}{x^{1-\alpha}}+\int_1^{+\infty}\frac{e^{-x}}{x^{1-\alpha}}dx$$

$$\int_0^1\frac{e^{-x}}{x^{1-\alpha}}dx \qquad \lim_{t\to 0^+}\frac{\frac{e^{-x}}{\cancel{x^{1-\alpha}}}}{\frac{1}{\cancel{x^{1-\alpha}}}}=\lim_{t\to 0^+} e^{-x}=1=cte\neq 0\Rightarrow \text{mismo carácter}$$

Th. comparación

$\Rightarrow$ converge cuando $1-\alpha<1$

$$\int_1^{+\infty}\frac{e^{-x}}{x^{1-\alpha}} \qquad \lim_{x\to+\infty}\frac{\frac{e^{-x}}{\cancel{x^{1-\alpha}}}}{\frac{1}{\cancel{x^{1-\alpha}}}}=0\rightarrow\int_0^{+\infty}\frac{e^{-x}}{x^{1-\alpha}}dx<\int_0^{+\infty}e^{-x}$$

$$=\left[-e^{-x}\right]_0^{+\infty}=1 \text{ convergente}$$

Como $\int_1^{+\infty}e^{-x}dx$ es convergente $\rightarrow\int_0^{+\infty}\frac{e^{-x}}{x^{1-\alpha}}dx$ es converge

2° caso $\alpha=1$ $\int_0^{+\infty}e^{-x}dx=\left[-e^{-x}\right]_0^{+\infty}=1$ converge

3° caso $\alpha>1$ $\int_0^{+\infty}e^{-x}\cdot x^{\alpha-1}dx=$ convergente por integración por partes hasta que el exponente de $x^{\alpha-1}$ está entre 0 y 1

b) $\Gamma(\alpha+1) = \int_0^{+\infty} e^{-x} \cdot x^{\alpha}\, dx = \left\{ \begin{matrix} u = x^{\alpha} & du = \alpha x^{\alpha-1} \\ dv = e^{-x} dx & v = -e^{-x} \end{matrix} \right\} -x^{\alpha} e^{-x} + \int_0^{+\infty} e^{-x} \cdot \alpha x^{\alpha-1}$

$$= \underbrace{\left[-x^{\alpha} \cdot e^{-x} \right]_0^{+\infty}}_{0} + \alpha \underbrace{\int_0^{+\infty} e^{-x} \cdot x^{\alpha-1}\, dx}_{\Gamma(\alpha)} = \alpha\, \Gamma(\alpha)$$

$$\Gamma(\alpha+1) = \alpha\, \Gamma(\alpha)$$