

# Álgebra Lineal

## Ejercicios Tema 1: Números complejos

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1) Calcula las siguientes operaciones de números complejos.

a)  $(-2 + j) + \left(-\frac{1}{2} - 3j\right)$

$$(-2 + j) + \left(-\frac{1}{2} - 3j\right) = -\frac{5}{2} - 2j$$

b)  $(-2 - 3j) - \left(-3 + \frac{1}{2}j\right)$

$$(-2 - 3j) - \left(-3 + \frac{1}{2}j\right) = 1 - \frac{7}{2}j$$

c)  $-2 \cdot (-1 - j) - 3 \cdot (-2 + j) + 2 \cdot (1 - 2j)$

$$-2 \cdot (-1 - j) - 3 \cdot (-2 + j) + 2 \cdot (1 - 2j) = 2 + 2j + 6 - 3j + 2 - 4j = 10 - 5j$$

d)  $j \cdot (-1 + j)$

$$j \cdot (-1 + j) = -j - 1$$

e)  $(-2 - j) \cdot \left(-3 + \frac{1}{2}j\right)$

$$(-2 - j) \cdot \left(-3 + \frac{1}{2}j\right) = 6 - j + 3j + \frac{1}{2} = \frac{13}{2} + 2j$$

f)  $\frac{1}{-1 - 2j}$

$$\frac{1}{-1 - 2j} = \frac{1}{-1 - 2j} \cdot \frac{-1 + 2j}{-1 + 2j} = \frac{-1 + 2j}{5} = -\frac{1}{5} + \frac{2}{5}j$$

g)  $\frac{-j}{2 - 3j}$

$$\frac{-j}{2 - 3j} = \frac{-j}{2 - 3j} \cdot \frac{2 + 3j}{2 + 3j} = \frac{-j \cdot (2 + 3j)}{13} = \frac{-2j + 3}{13} = \frac{3}{13} - \frac{2}{13}j$$

h)  $\frac{-1 - j}{-2 + j}$

$$\frac{-1 - j}{-2 + j} = \frac{-1 - j}{-2 + j} \cdot \frac{-2 - j}{-2 - j} = \frac{(-1 - j) \cdot (-2 - j)}{5} = \frac{1 + 3j}{5} = \frac{1}{5} + \frac{3}{5}j$$

i)  $\frac{1 - j}{j} - \frac{j}{1 - j}$

$$\frac{1 - j}{j} - \frac{j}{1 - j} = -1 - j + \frac{1}{2} - \frac{1}{2}j = -\frac{1}{2} - \frac{3}{2}j$$

$$\frac{1 - j}{j} = \frac{1 - j}{j} \cdot \frac{-j}{-j} = \frac{(1 - j) \cdot (-j)}{1} = -1 - j$$

$$\frac{j}{1 - j} = \frac{j}{1 - j} \cdot \frac{1 + j}{1 + j} = \frac{j \cdot (1 + j)}{2} = \frac{-1 + j}{2} = -\frac{1}{2} + \frac{1}{2}j$$

2) Obtén las formas polares y trigonométricas de los siguientes números complejos:

a)  $1 + j$

$$\begin{cases} |z| = \sqrt{1^2 + 1^2} = \sqrt{2} \\ \theta = \arctan\left(\frac{1}{1}\right) = \frac{\pi}{4} \end{cases} \longrightarrow \sqrt{2} \cdot e^{\frac{\pi}{4}j}$$

b)  $-j$

$$\begin{cases} |z| = \sqrt{0^2 + (-1)^2} = 1 \\ \theta = \arctan(-j) = -\frac{\pi}{2} \equiv \frac{3\pi}{2} \end{cases} \longrightarrow 1 \cdot e^{\frac{3\pi}{2}j}$$

c)  $-1 + \sqrt{3}j$

$$\begin{cases} |z| = \sqrt{(-1)^2 + (\sqrt{3})^2} = 2 \\ \theta = \arctan\left(\frac{\sqrt{3}}{-1}\right) = \frac{2\pi}{3} \end{cases} \longrightarrow 2e^{\frac{2\pi}{3}j}$$

d)  $2\sqrt{3} - 2j$

$$\begin{cases} |z| = \sqrt{(2\sqrt{3})^2 + (-2)^2} = 4 \\ \theta = \arctan\left(-\frac{2}{2\sqrt{3}}\right) = -\frac{\pi}{6} \equiv \frac{11\pi}{6} \end{cases} \longrightarrow 4e^{\frac{11\pi}{6}j}$$

e)  $-1 - j$

$$\begin{cases} |z| = \sqrt{(-1)^2 + (-1)^2} = \sqrt{2} \\ \theta = \arctan\left(\frac{-1}{-1}\right) = -\frac{3\pi}{4} \equiv \frac{5\pi}{4} \end{cases} \longrightarrow \sqrt{2}e^{\frac{5\pi}{4}j}$$

f)  $-2 + j$

$$\begin{cases} |z| = \sqrt{(-2)^2 + 1^2} = \sqrt{5} \\ \theta = \arctan\left(-\frac{1}{2}\right) \simeq 2.6779 \end{cases} \longrightarrow \sqrt{5}e^{2.6779j}$$

3) Obtén la forma binómica de los siguientes números complejos:

a)  $2\frac{\pi}{3}$

$$2 \cdot \left( \cos\left(\frac{\pi}{3}\right) + j \sin\left(\frac{\pi}{3}\right) \right) = 1 + \sqrt{3}j$$

b)  $1_{\pi}$

$$\cos(\pi) + j \sin(\pi) = -1$$

c)  $3\frac{5\pi}{4}$

$$3 \cdot \left( \cos\left(\frac{5\pi}{4}\right) + j \sin\left(\frac{5\pi}{4}\right) \right) = -\frac{3\sqrt{2}}{2} - \frac{3\sqrt{2}}{2}j$$

4) Calcula las siguientes operaciones de números complejos, expresando el resultado en forma exponencial:

a)  $2\frac{5\pi}{3} \cdot 3\frac{\pi}{2}$

$$2\frac{5\pi}{3} \cdot 3\frac{\pi}{2} = (1 - \sqrt{3}j) \cdot 3j = 3\sqrt{3} + 3j$$

$$2\frac{5\pi}{3} = 2 \cdot \left( \cos\left(\frac{5\pi}{3}\right) + j \sin\left(\frac{5\pi}{3}\right) \right) = 1 - \sqrt{3}j$$

$$3\frac{\pi}{2} = 3 \cdot \left( \cos\left(\frac{\pi}{2}\right) + j \sin\left(\frac{\pi}{2}\right) \right) = 3j$$

b)  $1\frac{\pi}{4} \cdot 2\frac{7\pi}{3}$

$$1\frac{\pi}{4} \cdot 2\frac{7\pi}{3} = \left( \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}j \right) \cdot (1 + \sqrt{3}j) = \frac{\sqrt{6} + \sqrt{2}}{2} + \frac{\sqrt{6} - \sqrt{2}}{2}j$$

$$1_{\frac{7\pi}{4}} = \cos\left(\frac{7\pi}{4}\right) + j \sin\left(\frac{7\pi}{4}\right) = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}j$$

$$2_{\frac{7\pi}{3}} = 2 \cdot \left(\cos\left(\frac{7\pi}{3}\right) + j \sin\left(\frac{7\pi}{3}\right)\right) = 1 + \sqrt{3}j$$

c)  $2\left(\cos \frac{\pi}{2} + j \sin \frac{\pi}{2}\right) \cdot 3_{\frac{11\pi}{6}}$

$$2\left(\cos \frac{\pi}{2} + j \sin \frac{\pi}{2}\right) \cdot 3_{\frac{11\pi}{6}} = 2j \cdot \left(\frac{2\sqrt{3}}{2} - \frac{3}{2}j\right) = 3 + 3\sqrt{3}j$$

$$2\left(\cos \frac{\pi}{2} + j \sin \frac{\pi}{2}\right) = 2j$$

$$3_{\frac{11\pi}{6}} = 3 \cdot \left(\cos \frac{11\pi}{6} + j \sin \frac{11\pi}{6}\right) = \frac{2\sqrt{3}}{2} - \frac{3}{2}j$$

d)  $\frac{4_{\frac{5\pi}{2}}}{2_{\frac{2\pi}{3}}}$

$$\frac{4_{\frac{5\pi}{2}}}{2_{\frac{2\pi}{3}}} = \frac{4j}{-1 + \sqrt{3}j} = \frac{4j}{-1 + \sqrt{3}j} \cdot \frac{-1 - \sqrt{3}j}{-1 - \sqrt{3}j} = \frac{4j \cdot (-1 - \sqrt{3}j)}{4} = \frac{4\sqrt{3} - 4j}{4} = \sqrt{3} - j$$

$$4_{\frac{5\pi}{2}} = 4 \cdot \left(\cos \frac{5\pi}{2} + j \sin \frac{5\pi}{2}\right) = 4j$$

$$2_{\frac{2\pi}{3}} = 2 \cdot \left(\cos \frac{2\pi}{3} + j \sin \frac{2\pi}{3}\right) = -1 + \sqrt{3}j$$

e)  $\frac{1_{\pi}}{2_{\frac{7\pi}{6}}}$

$$\frac{1_{\pi}}{2_{\frac{7\pi}{6}}} = \frac{-1}{-\sqrt{3} - j} = \frac{-1}{-\sqrt{3} - j} \cdot \frac{-\sqrt{3} + j}{-\sqrt{3} + j} = \frac{(-1) \cdot (-\sqrt{3} + j)}{4} = \frac{\sqrt{3} - j}{4} = \frac{\sqrt{3}}{4} - \frac{1}{4}j$$

$$1_{\pi} = \cos(\pi) + j \sin(\pi) = -1$$

$$2_{\frac{7\pi}{6}} = 2 \cdot \left(\cos \frac{7\pi}{6} + j \sin \frac{7\pi}{6}\right) = -\sqrt{3} - j$$

f)  $\frac{12\left(\cos \frac{5\pi}{4} + j \sin \frac{5\pi}{4}\right)}{8_{\frac{5\pi}{4}}}$

$$\frac{12 \cdot \left(\cos \frac{5\pi}{4} + j \sin \frac{5\pi}{4}\right)}{8_{\frac{5\pi}{4}}} = \frac{12 \cdot \left(\cos \frac{5\pi}{4} + j \sin \frac{5\pi}{4}\right)}{8 \cdot \left(\cos \frac{5\pi}{4} + j \sin \frac{5\pi}{4}\right)} = \frac{12}{8} = \frac{3}{2}$$

5) Calcula las siguientes potencias de números complejos:

a)  $(1 - \sqrt{3}j)^6$

$$(1 - \sqrt{3}j)^6 = 64$$

b)  $(1 - j)^8$

$$(1 - j)^8 = 16$$

c)  $(-\sqrt{3} + j)^{10}$

$$(-\sqrt{3} + j)^{10} = 512 + 886.81j$$

d)  $\left(\frac{1-j}{1+j}\right)^5$

$$\left(\frac{1-j}{1+j}\right)^5 = (*) = (-j)^5 = -j$$

$$(*) = \frac{1-j}{1+j} = \frac{1-j}{1+j} \cdot \frac{1-j}{1-j} = \frac{-2j}{2} = -j$$

6) Expresa los siguientes números complejos en forma binómica y en forma exponencial:

a)  $(1+j)^3$

$$(1+j)^3 = -2 + 2j$$

$$\begin{cases} |z| = \sqrt{(-2)^2 + 2^2} = 2\sqrt{2} \\ \theta = \arctan(-2 + 2j) = \frac{3\pi}{4} \end{cases} \longrightarrow 2\sqrt{2} \cdot e^{\frac{3\pi}{4}j}$$

b)  $j^5 + j^{16}$

$$j^5 + j^{16} = 1 + j$$

$$\begin{cases} |z| = \sqrt{1^2 + 1^2} = \sqrt{2} \\ \theta = \arctan(1 + j) = \frac{\pi}{4} \end{cases} \longrightarrow \sqrt{2} \cdot e^{\frac{\pi}{4}j}$$

c)  $1 + 3e^{j\pi}$

$$1 + 3e^{j\pi} = 1 + 3 \cdot (\cos \pi + j \sin \pi) = 1 - 3 = -2$$

$$\begin{cases} |z| = \sqrt{(-2)^2 + 0^2} = 2 \\ \theta = \arctan(-2) = \pi \end{cases} \longrightarrow 2e^{\pi j}$$

d)  $\frac{2+3j}{3-4j}$

$$\frac{2+3j}{3-4j} = \frac{2+3j}{3-4j} \cdot \frac{3+4j}{3+4j} = \frac{-6+17j}{25} = -\frac{6}{25} + \frac{17}{25}j$$

$$\begin{cases} |z| = \sqrt{\left(-\frac{6}{25}\right)^2 + \left(\frac{17}{25}\right)^2} = \frac{\sqrt{13}}{5} \\ \theta = \arctan\left(-\frac{6}{25} + \frac{17}{25}j\right) = 1.91 \end{cases} \longrightarrow \frac{\sqrt{13}}{5} \cdot e^{1.91j}$$

e)  $2\frac{3\pi}{2} + j$

$$2\frac{3\pi}{2} + j = 2 \cdot \left( \cos \frac{3\pi}{2} + j \sin \frac{3\pi}{2} \right) + j = -2j + j = -j$$

$$\begin{cases} |z| = \sqrt{0^2 + (-1)^2} = 1 \\ \theta = \arctan(-j) = -\frac{\pi}{2} \equiv \frac{3\pi}{2} \end{cases} \longrightarrow 1 \cdot e^{\frac{3\pi}{2}j}$$

f)  $\frac{1}{j}$

$$\frac{1}{j} = \frac{1}{j} \cdot \frac{-j}{-j} = -j$$

$$\begin{cases} |z| = \sqrt{0^2 + (-1)^2} = 1 \\ \theta = \arctan(-j) = -\frac{\pi}{2} \equiv \frac{3\pi}{2} \end{cases} \longrightarrow 1 \cdot e^{\frac{3\pi}{2}j}$$

7) Calcula el módulo y el argumento principal de los siguientes números complejos:

a)  $2j$

$$\begin{cases} |z| = \sqrt{0^2 + 2^2} = 2 \\ \theta = \arctan(2j) = \frac{\pi}{2} \end{cases}$$

b)  $1-j$

$$\begin{cases} |z| = \sqrt{1^2 + (-1)^2} = \sqrt{2} \\ \theta = \arctan(1-j) = -\frac{\pi}{4} \equiv \frac{7\pi}{4} \end{cases}$$

c)  $-1$

$$\begin{cases} |z| = \sqrt{(-1)^2 + 0^2} = 1 \\ \theta = \arctan(-1) = \pi \end{cases}$$

d)  $\frac{1+j}{1-j}$

$$\frac{1+j}{1-j} = \frac{1+j}{1-j} \cdot \frac{1+j}{1+j} = \frac{2j}{2} = j$$

$$\begin{cases} |z| = \sqrt{0^2 + 1^2} = 1 \\ \theta = \arctan(j) = \frac{\pi}{2} \end{cases}$$

e)  $\frac{1}{j\pi}$

$$\frac{1}{j\pi} = \frac{1}{j\pi} \cdot \frac{-j}{-j} = -\frac{j}{\pi}$$

$$\begin{cases} |z| = \sqrt{0^2 + \left(-\frac{1}{\pi}\right)^2} = \frac{1}{\pi} \\ \theta = \arctan\left(-\frac{j}{\pi}\right) = -\frac{\pi}{2} \equiv \frac{3\pi}{2} \end{cases}$$

f)  $-3 + j\sqrt{3}$

$$\begin{cases} |z| = \sqrt{(-3)^2 + (\sqrt{3})^2} = 2\sqrt{3} \\ \theta = \arctan(-3 + \sqrt{3}j) = \frac{5\pi}{6} \end{cases}$$

8) Calcula las raíces cúbicas de los números  $-8$  y  $1+j$

- $\sqrt[3]{-8} = -2$
- $\sqrt[3]{1+j}$

La raíz cúbica de un número complejo se calcula expresándolo en su **forma polar** y luego aplicando la fórmula de las raíces de un número complejo.

Paso 1: Convertir  $1+j$  a su forma polar

$$\begin{cases} r = |z| = \sqrt{1^2 + 1^2} = \sqrt{2} \\ \theta = \arctan\left(\frac{1}{1}\right) = \frac{\pi}{4} \end{cases}$$

Por lo tanto, la forma polar de  $1+j$  es:

$$1+j = \sqrt{2}e^{j\frac{\pi}{4}}$$

Paso 2: Calcular la raíz cúbica

La fórmula para las  $n$ -ésimas de un número complejo es:

$$z_K = r^{\frac{1}{n}} e^{j\left(\frac{\theta+2k\pi}{n}\right)} \quad k = 0, 1, \dots, n-1$$

Aquí:

- $n = 3$
- $r = \sqrt{2}$
- $\theta = \frac{\pi}{4}$

Por lo tanto:

$$z_k = \left(\sqrt{2}\right)^{1/3} e^{j\left(\frac{\frac{\pi}{4}+2k\pi}{3}\right)}, \quad k = 0, 1, 2$$

Paso 3: Expresar las raíces

El módulo de las raíces es:

$$r^{\frac{1}{3}} = \left(\sqrt{2}\right)^{\frac{1}{3}}$$

El argumento de las raíces es:

$$\theta_k = \frac{\pi}{4} + \frac{2k\pi}{3}, \quad k = 0, 1, 2$$

Por lo tanto, las raíces son:

$$z_k = \left(\sqrt{2}\right)^{\frac{1}{3}} \left[ \cos\left(\frac{\pi}{4} + \frac{2k\pi}{3}\right) + j \sin\left(\frac{\pi}{4} + \frac{2k\pi}{3}\right) \right], \quad k = 0, 1, 2$$

1) Para  $k = 0$ :

$$z_0 = \left(\sqrt{2}\right)^{\frac{1}{3}} \left[ \cos\left(\frac{\pi}{4}\right) + j \sin\left(\frac{\pi}{4}\right) \right] = \left(\sqrt{2}\right)^{\frac{1}{3}} \cdot \left( \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}j \right) = \frac{j+1}{\sqrt[3]{2}}$$

2) Para  $k = 1$ :

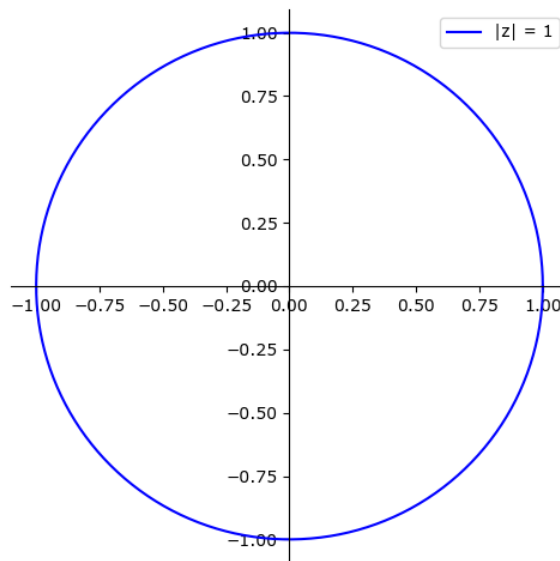
$$z_1 = \left(\sqrt{2}\right)^{\frac{1}{3}} \left[ \cos\left(\frac{\pi}{4} + \frac{2\pi}{3}\right) + j \sin\left(\frac{\pi}{4} + \frac{2\pi}{3}\right) \right] = \left(\sqrt{2}\right)^{\frac{1}{3}} \cdot \left( -\frac{\sqrt{6} + \sqrt{2}}{4} + \frac{\sqrt{6} - \sqrt{2}}{4}j \right)$$

3) Para  $k = 2$ :

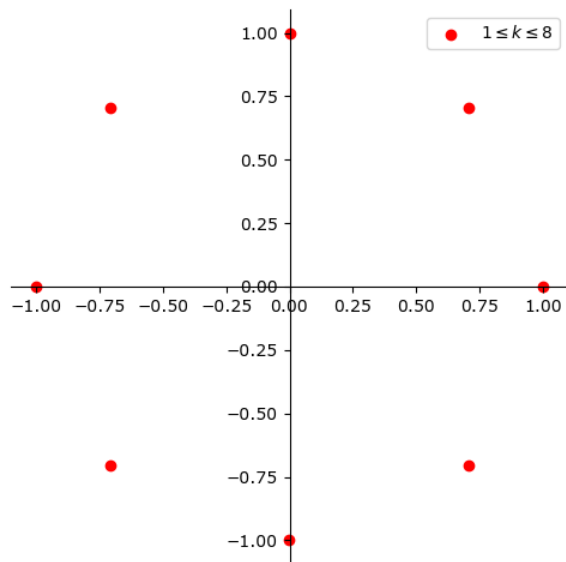
$$z_2 = \left(\sqrt{2}\right)^{\frac{1}{3}} \left[ \cos\left(\frac{\pi}{4} + \frac{4\pi}{3}\right) + j \sin\left(\frac{\pi}{4} + \frac{4\pi}{3}\right) \right] = \left(\sqrt{2}\right)^{\frac{1}{3}} \cdot \left( \frac{\sqrt{6} - \sqrt{2}}{4} - \frac{\sqrt{6} + \sqrt{2}}{4}j \right)$$

9) Representa gráficamente los siguientes subconjuntos del plano complejo:

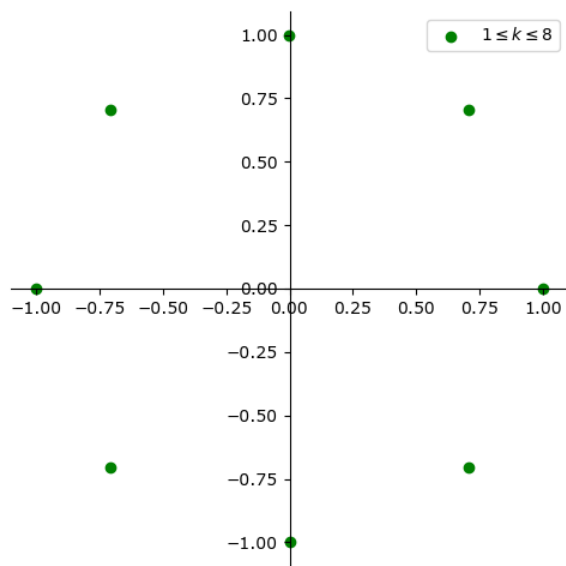
a) Números complejos cuyo módulo es igual a 1.



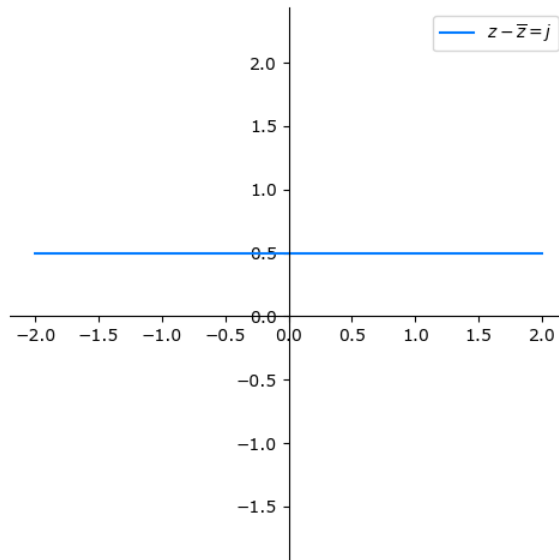
b)  $\{z^k \in C : z = e^{\frac{2\pi j}{8}}, 1 \leq k \leq 8\}$



c)  $\{z^k \in C : z = e^{-\frac{2\pi j}{8}}, 1 \leq k \leq 8\}$



d)  $\{z \in C : z - \bar{z} = j\}$



10) Resuelve las siguientes ecuaciones algebraicas y expresa el resultado en forma binómica:

a)  $2 + 4j + 10e^{\frac{\pi}{3}j} = ze^{\frac{\pi}{3}j}$

$$2 + 4j + 10e^{\frac{\pi}{3}j} = ze^{\frac{\pi}{3}j} \longrightarrow 2 + 4j + 5 + 5\sqrt{3}j = z \cdot \left( \frac{1}{2} + \frac{\sqrt{3}}{2}j \right) \longrightarrow z = \frac{7 + (4 + 5\sqrt{3})j}{\frac{1}{2} + \frac{\sqrt{3}}{2}j} = 11 + 2\sqrt{3} + (2 - \sqrt{3})j$$

$$10e^{\frac{\pi}{3}j} = 10 \cdot \left( \cos \frac{\pi}{3} + j \sin \frac{\pi}{3} \right) = 5 + 5\sqrt{3}j$$

$$ze^{\frac{\pi}{3}j} = z \cdot \left( \cos \frac{\pi}{3} + j \sin \frac{\pi}{3} \right) = z \cdot \left( \frac{1}{2} + \frac{\sqrt{3}}{2}j \right)$$

$$\frac{7 + (4 + 5\sqrt{3})j}{\frac{1}{2} + \frac{\sqrt{3}}{2}j} = \frac{7 + (4 + 5\sqrt{3})j}{\frac{1}{2} + \frac{\sqrt{3}}{2}j} \cdot \frac{\frac{1}{2} - \frac{\sqrt{3}}{2}j}{\frac{1}{2} - \frac{\sqrt{3}}{2}j} = \frac{11 + 2\sqrt{3} + (2 - \sqrt{3})j}{1}$$

b)  $5_{-\frac{\pi}{6}} + z + 4 + \sqrt{2}j = 6e^{-\frac{\pi}{4}j}z$

$$\begin{aligned} 5_{-\frac{\pi}{6}} + z + 4 + \sqrt{2}j &= 6e^{-\frac{\pi}{4}j}z \longrightarrow \frac{5\sqrt{3}}{2} - \frac{5}{2}j + z + 4 + \sqrt{2}j = z \cdot (3\sqrt{2} - 3\sqrt{2}j) \\ &\longrightarrow \frac{8 + 5\sqrt{3}}{2} + \frac{-5 + 2\sqrt{2}}{2}j = z \cdot (-1 + 3\sqrt{2} - 3\sqrt{2}j) \\ &\longrightarrow z = \frac{\frac{8+5\sqrt{3}}{2} + \frac{-5+2\sqrt{2}}{2}j}{-1 + 3\sqrt{2} - 3\sqrt{2}j} \longrightarrow z = 1.10883824361717 + 1.11594720455811j \end{aligned}$$

$$5_{-\frac{\pi}{6}} = 5 \cdot \left( \cos \left( -\frac{\pi}{6} \right) + j \sin \left( -\frac{\pi}{6} \right) \right) = \frac{5\sqrt{3}}{2} - \frac{5}{2}j$$

$$6e^{-\frac{\pi}{4}j} = 6 \cdot \left( \cos \left( -\frac{\pi}{4} \right) + j \sin \left( -\frac{\pi}{4} \right) \right) = 3\sqrt{2} - 3\sqrt{2}j$$

c)  $2_{\frac{\pi}{3}} + j + ze^{-\frac{\pi}{3}j} = 0$

$$2_{\frac{\pi}{3}} + j + ze^{-\frac{\pi}{3}j} = 0 \longrightarrow 1 + \sqrt{3} + j + z \cdot \left( \frac{1 - \sqrt{3}}{2} \right) = 0 \longrightarrow z = - \left( \frac{1 + \sqrt{3} + j}{\frac{1 - \sqrt{3}}{2}} \right) = 2\sqrt{3} + 4 + j + \sqrt{3}j$$

$$2_{\frac{\pi}{3}} = 2 \cdot \left( \cos \frac{\pi}{3} + j \sin \frac{\pi}{3} \right) = 1 + \sqrt{3}$$

$$ze^{-\frac{\pi}{3}j} = z \cdot \left( \cos \left( -\frac{\pi}{3} \right) + j \sin \left( -\frac{\pi}{3} \right) \right) = z \cdot \left( \frac{1 - \sqrt{3}}{2} \right)$$

d)  $z + 4jz = 1$

$$z + 4jz = 1 \longrightarrow z \cdot (1 + 4j) = 1 \longrightarrow z = \frac{1}{1 + 4j} = \frac{1}{1 + 4j} \cdot \frac{1 - 4j}{1 - 4j} = \frac{1}{17} - \frac{4}{17}j$$



- 11) Dados los números complejos  $z_1 = -1 - j$ ,  $z_2 = 2\frac{\pi}{3}$ ,  $z_3 = 3e^{100\pi j}$ , representa gráficamente los números  $z_1, z_2, z_3, z_1 + z_2 + z_3, z_1 \cdot z_2, z_1 \cdot z_2 \cdot z_3$ .

$$z_1 = -1 - j$$

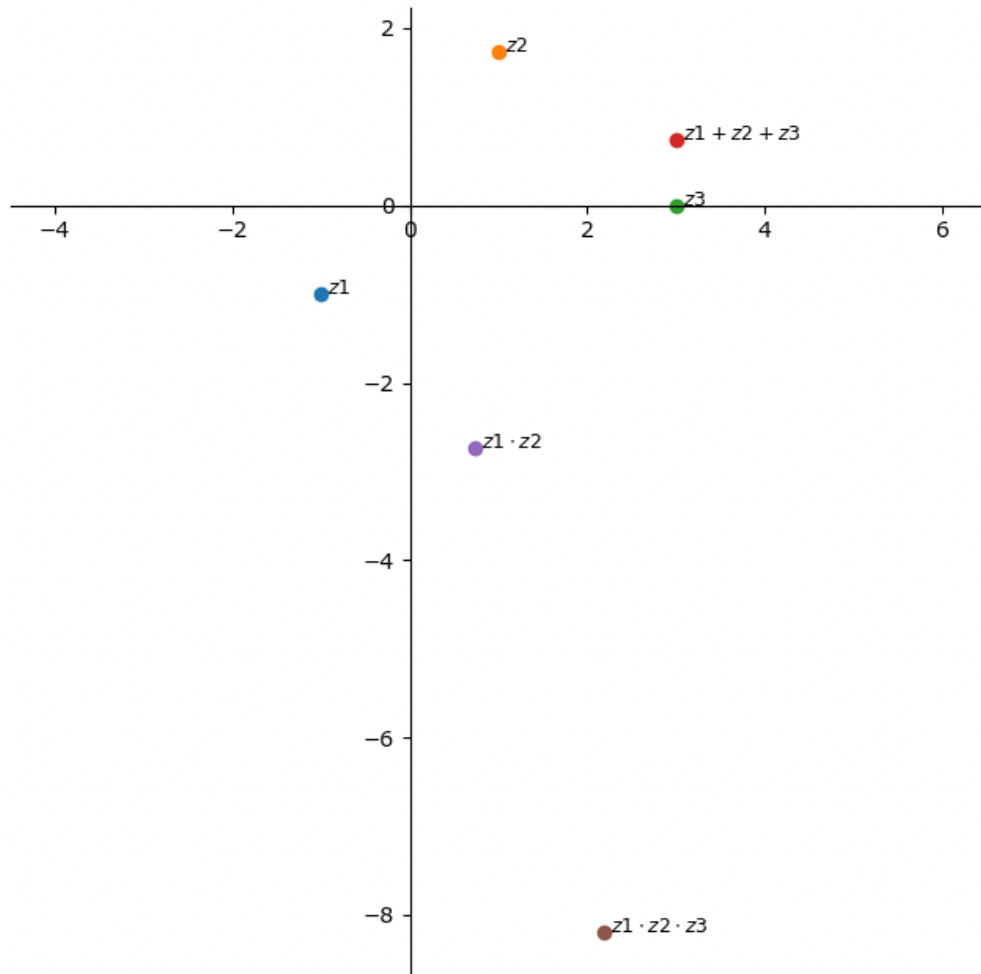
$$z_2 = 2\frac{\pi}{3} = 2 \cdot \left( \cos \frac{\pi}{3} + j \sin \frac{\pi}{3} \right) = 1 + \sqrt{3}j$$

$$z_3 = 3e^{100\pi j} = 3 \cdot (\cos(100\pi) + j \sin(100\pi)) = 3$$

$$z_1 + z_2 + z_3 = -1 - j + 1 + \sqrt{3}j + 3 = 3 + (-1 + \sqrt{3})j$$

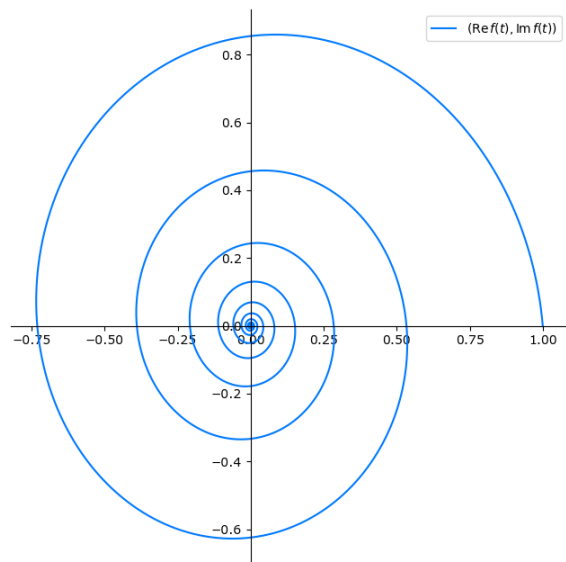
$$z_1 \cdot z_2 = (-1 - j) \cdot (1 + \sqrt{3}j) = -1 + \sqrt{3} - (1 + \sqrt{3})j$$

$$z_1 \cdot z_2 \cdot z_3 = (-1 - j) \cdot (1 + \sqrt{3}j) \cdot 3 = -3 + 3\sqrt{3} - (3 + 3\sqrt{3})j$$



- 12) Consideremos la función  $f(t) = e^{-t+j_{10}t}$ ,  $0 \leq t \leq 2\pi$ . Se pide:

a) Dibuja de manera aproximada el conjunto  $\{(\text{Re}f(t), \text{Im}f(t)), 0 \leq t \leq 2\pi\}$



b) Dibuja de manera aproximada las funciones  $\text{Re} f(t)$ ,  $\text{Im} f(t)$ , y  $|f(t)|$

