Señales y Sistemas

Problemas Unidad 3

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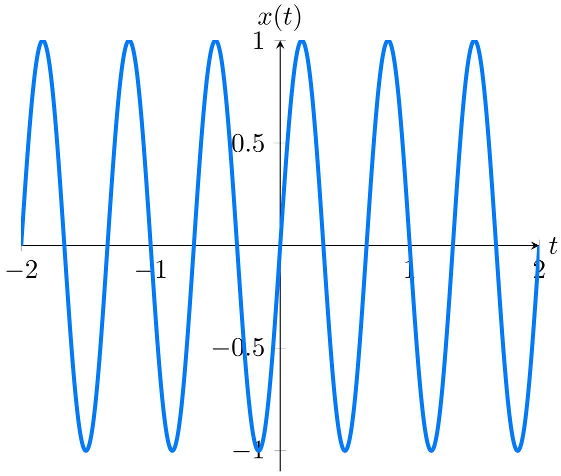
1) Expresar la señal $x(t)=\sin(3\pi t)$ como una combinación lineal de exponenciales complejas.

$$x(t)=\sin(3\pi t)=\begin{cases}\omega_0=3\pi\\ T=\dfrac{2\pi}{\omega_0}=\dfrac{2\pi}{3\pi}=\dfrac{2}{3}\end{cases}$$

$$\sin(3\pi x)=\frac{e^{j3\pi t}-e^{-j3\pi t}}{2j}=\underbrace{\frac{1}{2j}}_{a_1}e^{j3\pi t}-\underbrace{\frac{1}{2j}}_{a_{-1}})e^{-j3\pi t}=\begin{cases}a_1=\dfrac{1}{2j}\\ a_{-1}=-\dfrac{1}{2j}\end{cases}$$



2) Expresar la señal $x(t)=\displaystyle\sum_{n=-\infty}^{\infty}\prod\left(\frac{t-nT}{\tau}\right)$ como una combinación lineal de exponenciales complejas.

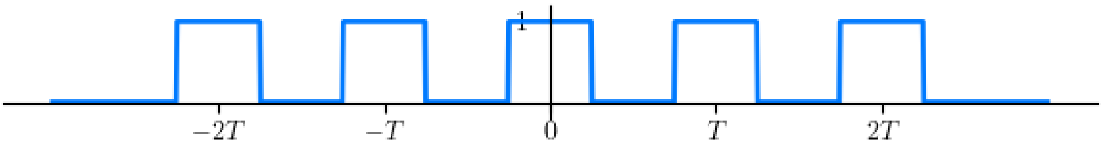
$$\omega_0=\frac{2\pi}{T}$$

$$a_k=\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}\prod\left(\frac{t-nT}{\tau}\right)e^{-jk\omega_0 t}\,\mathrm{dt}=\frac{1}{T}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}}e^{-jk\omega_0 t}\,\mathrm{dt}=\frac{1}{T}\cdot\left[\frac{e^{-jk\omega_0 t}}{-jk\omega_0}\right]_{-\frac{\tau}{2}}^{\frac{\tau}{2}}$$

$$=\frac{1}{T}\cdot\left[\frac{e^{-jk\frac{2\pi}{T}t}}{-j\frac{2\pi}{T}}\right]_{-\frac{\tau}{2}}^{\frac{\tau}{2}}=\frac{1}{T}\cdot\left[\frac{e^{-jk\frac{\pi\tau}{T}}}{-jk\frac{2\pi}{T}}-\frac{e^{jk\frac{\pi\tau}{T}}}{-jk\frac{2\pi}{T}}\right]=\frac{e^{jk\frac{\pi\tau}{T}}}{jk2\pi}-\frac{e^{-jk\frac{\pi\tau}{T}}}{jk2\pi}$$

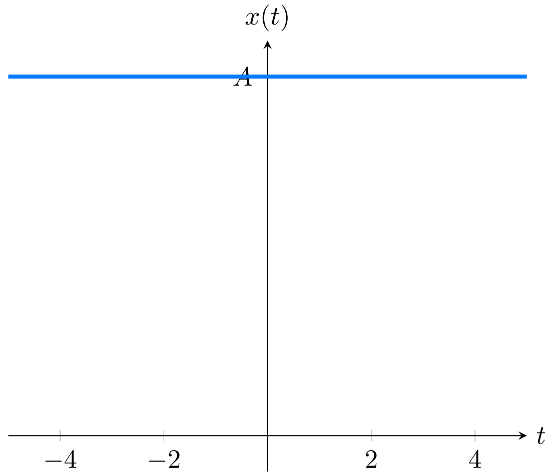
$$=\frac{\sin\left(k\frac{\pi\tau}{T}\right)}{\pi k}=\frac{\tau}{T}\cdot\frac{\sin\left(k\frac{\pi\tau}{T}\right)}{k\frac{\pi\tau}{T}}=\frac{\tau}{T}\cdot\mathrm{sinc}\left(\frac{k\tau}{T}\right)$$

$$x(t)=\sum_k a_k\cdot e^{jk\frac{2\pi}{T}t}=\sum_k\frac{\tau}{T}\cdot\mathrm{sinc}\left(\frac{k\tau}{T}\right)\cdot e^{jk\frac{2\pi}{T}t}$$



3) Calcular los coeficientes del desarrollo en series de Fourier:

$$x(t) = A$$

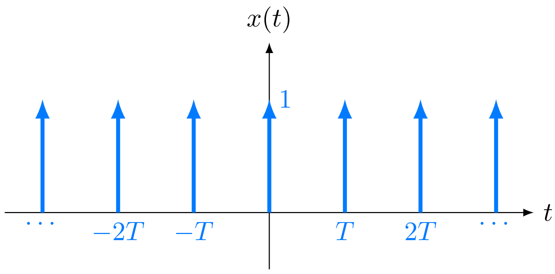


$$\omega_0 = \frac{2\pi}{T}$$

$$a_k = \frac{1}{T}\cdot\int_{-\frac{T}{2}}^{\frac{T}{2}} A\cdot e^{-jk\omega_0 t}\mathrm{d}t = \frac{A}{T}\cdot\left[\frac{e^{-jk\frac{2\pi}{T}t}}{-jk\frac{1\pi}{T}}\right]_{-\frac{T}{2}}^{\frac{T}{2}} = \frac{A}{T}\cdot\left[\frac{e^{-jk\pi}}{-jk\frac{2\pi}{T}} - \frac{e^{jk\pi}}{jk\frac{2\pi}{T}}\right] = A\cdot\left(\frac{e^{jk\pi}}{jk2\pi} - \frac{e^{-jk\pi}}{jk2\pi}\right) = A\cdot\frac{\sin(k\pi)}{k\pi} = A\cdot\mathrm{sinc}(k)$$

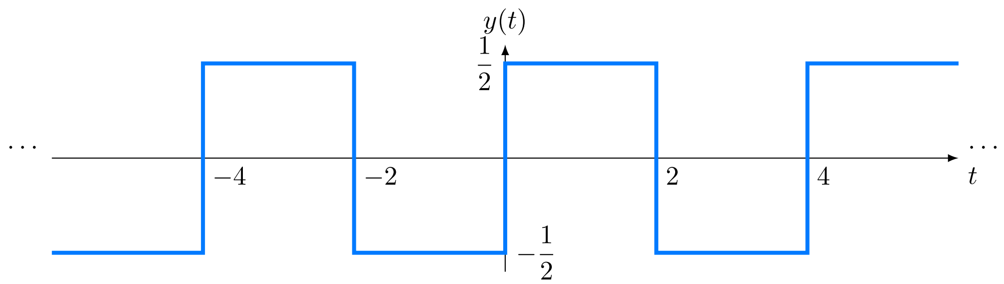
4) Calcular los coeficientes del desarrollo en series de Fourier:

$$x(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$



$$a_k = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \delta(t)\cdot e^{-jk\frac{2\pi}{T}t}\,\mathrm{d}t = \left\{\delta(t) \to t_0 = 0 \to e^{-jk\frac{2\pi}{T}\cdot 0} = 1\right\} = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \delta(t)\,\mathrm{d}t = \frac{1}{T}$$

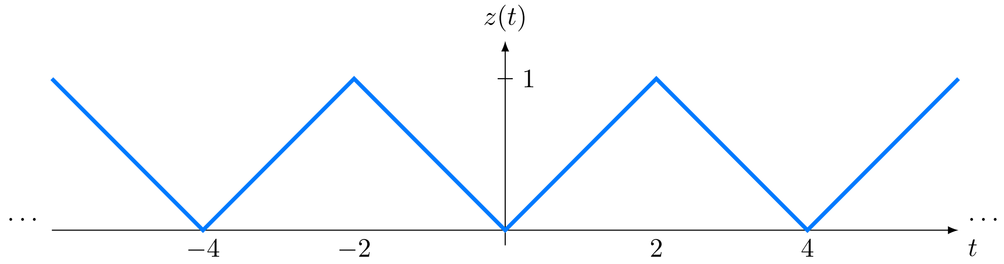
5) Calcular los coeficientes del desarrollo en series de Fourier:



$$a_k = \frac{1}{4}\cdot\left(\int_{-2}^{0} -\frac{1}{2}\cdot e^{-jk\frac{\pi}{2}t}\,\mathrm{d}t + \int_{0}^{2} \frac{1}{2}\cdot e^{-jk\frac{\pi}{2}t}\,\mathrm{d}t\right) = \frac{1}{4}\cdot\left(\left[-\frac{1}{2}\cdot\left(\frac{e^0}{jk\frac{\pi}{2}} - \frac{e^{jk\pi}}{-jk\frac{\pi}{2}}\right)\right] + \left[\frac{1}{2}\cdot\left(\frac{e^{-jk\pi}}{-jk\frac{\pi}{2}} - \frac{e^0}{-jk\frac{\pi}{2}}\right)\right]\right)$$

$$= \frac{1}{8}\cdot\left(\frac{e^0}{jk\frac{\pi}{2}} - \frac{e^{jk\pi}}{jk\frac{\pi}{2}} - \frac{e^{-jk\pi}}{jk\frac{\pi}{2}} + \frac{e^0}{jk\frac{\pi}{2}}\right) = \frac{1}{8}\cdot\left(\frac{2 - 2\cdot(-1)^k}{jk\frac{\pi}{2}}\right) = \frac{1}{4}\cdot\frac{1-(-1)^k}{jk\frac{\pi}{2}} = \frac{1-(-1)^k}{jk2\pi}$$

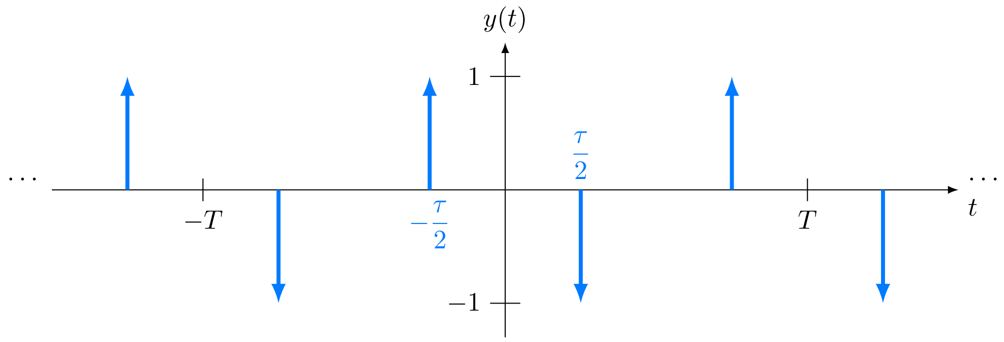
6) Calcular los coeficientes del desarrollo en series de Fourier:



$$\frac{\partial}{\partial t}y(t) = x(t) \implies y(t) = \int_{-\infty}^{t} x(\tau)\mathrm{d}\tau$$

$$y(t) \xrightarrow{DSF} \frac{a_k}{jk\omega_0} = \frac{1-(-1)^k}{\frac{jk2\pi}{jk\frac{\pi}{2}}} = \frac{1-(-1)^k}{\frac{jk2\pi}{\frac{jk\pi}{2}}} = \frac{2-2(-1)^k}{-k^2\pi^2 2} = \frac{1-(-1)^k}{-k^2\pi^2}$$

7) Calcular los coeficientes del desarrollo en series de Fourier:



$$\sum_{n=-\infty}^{\infty} \delta\left(t - nT + \frac{\tau}{2}\right) - \delta\left(t - nT - \frac{\tau}{2}\right) \xrightarrow{DSF} \frac{1}{T}\cdot e^{jk\frac{2\pi}{T}\cdot\frac{\tau}{2}} - \frac{1}{T}\cdot e^{-jk\frac{2\pi}{T}\cdot\frac{\tau}{2}} = \frac{e^{jk\frac{\pi\tau}{T}} - e^{-jk\frac{\pi\tau}{T}}}{T} = \frac{1}{T}\cdot 2j \cdot \frac{e^{jk\frac{\pi\tau}{T}} - e^{-jk\frac{\pi\tau}{T}}}{2j}$$

$$= \frac{2j}{T}\cdot \sin\left(k\frac{\pi}{T}\tau\right)$$

8) Calcular la potencia total de la señal: $x(t) = \sin(3\pi t)$

$$P_\infty = P_T = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} |x(t)|^2 \,\mathrm{d}t = \frac{3}{2}\int_{-\frac{1}{3}}^{\frac{1}{3}} |\sin(3\pi t)|^2 \,\mathrm{d}t \equiv \sum_{k=-\infty}^{\infty} |a_k|^2 = \left|\frac{1}{2j}\right|^2 + \left|-\frac{1}{2j}\right|^2 = \frac{1}{2}W$$