Interpolation

21/02/12

Linear Interpolation

Recall that the **line segment** between two points $P_1 = (x_1, y_1, z_1)$ and $P_2 = (x_2, y_2, z_2)$ parameterized by t where $0 \le t \le 1$ is given by:

$$P(t) = (1-t)P_1 + tP_2$$

$$P(0) = P_1$$

$$P(1) = P_2$$

The coordinates are:

$$x = (1-t)x_1 + tx_2 \qquad y = (1-t)y_1 + ty_2 \qquad z = (1-t)z_1 + tz_2$$

More generally, to interpolate linearly between $P_1 = (x_1, y_1, z_1)$ and $P_2 = (x_2, y_2, z_2)$ and parameterize the line segment using $t \in [t_1, t_2]$ we can write

$$P(t) = \left(\frac{t-t_2}{t_1-t_2}\right)P_1 + \left(\frac{t-t_1}{t_2-t_1}\right)P_2 \qquad ★★★$$

then $P(t_1) = P_1$ and $P(t_2) = P_2$

Example

Parameterize the line segment between $P_1 = (0, 5, -4)$ and $P_2 = (10, -5, 8)$ such that at $t_1 = 2\,s$ an object is at P_1 and at $t_2 = 30\,s$ the object is at P_2

Solution

$$P(t) = \left(\frac{t-t_2}{t_1-t_2}\right)P_1 + \left(\frac{t-t_1}{t_2-t_1}\right)P_2$$

$$= \left(\frac{t-30}{2-30}\right)(0,5,-4) + \left(\frac{t-2}{30-2}\right)(10,-5,8)$$

$$= \left(\frac{t-30}{2-30}\right)(0,5,-4) + \left(\frac{t-2}{30-2}\right)(10,-5,8)$$

$$= \boxed{\left(\frac{5t-10}{14}\ ,\ \frac{80-5t}{14}\ ,\ \frac{3t-34}{7}\right)}$$

Piecewise Linear Interpolation
Given n points $P_1 = (x_1, y_{1,} z_1)$, $P_2 = (x_2, y_2, z_2)$ ... $P_n = (x_n, y_n, z_n)$ we then have $n-1$ line segments L_1 , L_2 ... L_{n-1} where we can interpolate on each L_i using
$L_i(t) = (1-t)P_i + tP_{i+1}$ for i = 1, 2, ... $n-1$.
The resulting "curve" will be a piecewise line: a series of connected line segments. In general it will not be differentiable at the endpoints. So this may not be suitable for animation of an object.

Polynomial Interpolation: Lagrange Polynomials
Just as a first degree polynomial (a line) can be used to interpolate two given points, a quadratic can be found that will interpolate three given points. Given three points in the plane $P_1 = (x_1, y_1)$, $P_2 = (x_2, y_2)$, $P_n = (x_n, y_n)$ we form the three quadratic functions

$$l_1(x) = \left(\frac{x-x_2}{x_1-x_2}\right)\left(\frac{x-x_3}{x_1-x_3}\right) \qquad ★★★$$

$$l_2(x) = \left(\frac{x-x_1}{x_2-x_1}\right)\left(\frac{x-x_3}{x_2-x_3}\right) \qquad ★★★$$

$$l_3(x) = \left(\frac{x-x_1}{x_3-x_1}\right)\left(\frac{x-x_2}{x_3-x_2}\right) \qquad ★★★$$

Then the unique quadratic that interpolates the three desired points P_1, P_2, P_3 is given by the **Lagrange polynomial**

$$L(x) = y_1 l_1(x) + y_2 l_2(x) + y_3 l_3(x) \qquad ★★★$$

Example
Find the quadratic function that interpolates $(1,2)$, $(3,5)$, and $(6,3)$
Solution

$$l_1(x) = \left(\frac{x-x_2}{x_1-x_2}\right)\left(\frac{x-x_3}{x_1-x_3}\right) = \left(\frac{x-3}{1-3}\right)\left(\frac{x-6}{1-6}\right) = \frac{(x-3)(x-6)}{10}$$

$$l_2(x) = \left(\frac{x-x_1}{x_2-x_1}\right)\left(\frac{x-x_3}{x_2-x_3}\right) = \left(\frac{x-1}{3-1}\right)\left(\frac{x-6}{3-6}\right) = \frac{(x-1)(x-6)}{-6}$$

$$l_3(x) = \left(\frac{x-x_1}{x_3-x_1}\right)\left(\frac{x-x_2}{x_3-x_2}\right) = \left(\frac{x-1}{6-1}\right)\left(\frac{x-3}{6-3}\right) = \frac{(x-1)(x-3)}{15}$$

Then the desired quadratic is

$$L(x) = y_1 l_1(x) + y_2 l_2(x) + y_3 l_3(x)$$

$$= 2\left(\frac{(x-3)(x-6)}{10}\right) + 5\left(\frac{(x-1)(x-6)}{-6}\right) + 3\left(\frac{(x-1)(x-3)}{15}\right)$$

$$= \boxed{\frac{(x-3)(x-6)}{5} - \frac{5(x-1)(x-6)}{6} + \frac{(x-1)(x-3)}{5}}$$

We don't bother to simplify algebraically since it will be easy to verify in this form.
Now check that $L(1) = 2$, $L(2) = 5$ and $L(6) = 3$.

Recall that 3 points in 3D determine a plane. A parabola lies in a plane. So we can use the same approach to generate an interpolating quadratic in 3D from any three points. It is natural to parameterize a point using t. The unique quadratic that interpolates three points $P(t_1) = P_1 = (x_1, y_1, z_1)$, $P(t_2) = P_2 = (x_2, y_2, z_2)$ and $P(t_3) = P_3 = (x_3, y_3, z_3)$ in 3D is the **Lagrange polynomial**

$$P(t) = \frac{(t-t_2)(t-t_3)}{(t_1-t_2)(t_1-t_3)}P_1 + \frac{(t-t_1)(t-t_3)}{(t_2-t_1)(t_2-t_3)}P_2 + \frac{(t-t_1)(t-t_2)}{(t_3-t_1)(t_3-t_2)}P_3 \qquad \bigstar\bigstar\bigstar$$

Example

Construct the Lagrange polynomial $P(t)$ that satisfies $P(0) = (2,-3,4)$ $P(5) = (10,3,-1)$ and $P(8) = (-4,0,2)$

Solution

$$P(t) = \frac{(t-t_2)(t-t_3)}{(t_1-t_2)(t_1-t_3)}P_1 + \frac{(t-t_1)(t-t_3)}{(t_2-t_1)(t_2-t_3)}P_2 + \frac{(t-t_1)(t-t_2)}{(t_3-t_1)(t_3-t_2)}P_3$$

$$= \frac{(t-5)(t-8)}{(0-5)(0-8)}(2,-3,4) + \frac{(t-0)(t-8)}{(5-0)(5-8)}(10,3,-1) + \frac{(t-0)(t-5)}{(8-0)(8-5)}(-4,0,2)$$

$$= \boxed{\frac{(t-5)(t-8)}{40}(2,-3,4) - \frac{t(t-8)}{15}(10,3,-1) + \frac{t(t-5)}{24}(-4,0,2)}$$

(Cubic) Polynomial Curves

We have seen that a quadratic can always be found that interpolates three given points. So what is wrong with quadratic interpolation? Draw two points in the plane. The shape of a quadratic that interpolates those two points has limited degrees of freedom. In between the two points it can change direction at most once. This may not be suitable to animate an object. We need the flexibility of a cubic. The parametric representation of an arbitrary cubic curve in 3D is

$$\vec{P}(t) = \vec{a} + \vec{b}\,t + \vec{c}\,t^2 + \vec{d}\,t^3 \qquad ☆☆☆$$

Where $\vec{a} = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}$, $\vec{b} = \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix}$, $\vec{c} = \begin{bmatrix} c_x \\ c_y \\ c_z \end{bmatrix}$, and $\vec{d} = \begin{bmatrix} d_x \\ d_y \\ d_z \end{bmatrix}$ are constant vectors.

Writing $\vec{P}(t)$ in terms of its components:

$$P_x(t) = a_x + b_x t + c_x t^2 + d_x t^3 \qquad ☆☆☆$$

$$P_y(t) = a_y + b_y t + c_y t^2 + d_y t^3 \qquad ☆☆☆$$

$$P_z(t) = a_z + b_z t + c_z t^2 + d_z t^3 \qquad ☆☆☆$$

This can be written in matrix form:

$$\vec{P}(t) = \begin{bmatrix} a_x & b_x & c_x & d_x \\ a_y & b_y & c_y & d_y \\ a_z & b_z & c_z & d_z \end{bmatrix} \begin{bmatrix} 1 \\ t \\ t^2 \\ t^3 \end{bmatrix} \qquad \text{or equivalently} \quad \vec{P}(t) = C\vec{T}(t)$$

where $\vec{T}(t) = \begin{bmatrix} 1 \\ t \\ t^2 \\ t^3 \end{bmatrix}$ and C is the 3×4 coefficient matrix : $C = \begin{bmatrix} a_x & b_x & c_x & d_x \\ a_y & b_y & c_y & d_y \\ a_z & b_z & c_z & d_z \end{bmatrix}$

The coefficient matrix will be determined by the desired geometric properties of the curve $\vec{P}(t)$.

Notes

- The sum or difference of two cubic polynomials is a cubic polynomial.
- Addition & subtraction of cubic polynomials are commutative & associative.
- Multiplying a cubic polynomial by a scalar (real number) yields a cubic polynomial.
- Cubic polynomials behave like vectors!

Recall that any vector in 3D can be written as a linear combination

$$\vec{v} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = v_x \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + v_y \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + v_z \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \text{ of the } \textbf{basis vectors} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

We are“blending” the three basis vectors to produce $\vec{v}$. The basis vectors are not unique: any three non-coplanar vectors can be used. The same structure applies to cubic polynomials. The polynomials $1,\ t,\ t^2,\ t^3$ behave like a basis. We will construct a linear combination of them to form the desired cubic. It coefficients will be determined by the geometric properties we want the animation curve to have.

Splines: Piecewise Polynomial Curves

A **spline** is a piecewise polynomial. Just as we used a sequence of line segments L_1 , L_2 ... L_n to construct a piecewise (linear) curve, we will use a sequence of cubic polynomials $P_1(t)$, $P_2(t)$ ... $P_n(t)$ to construct a more general curve in 3D. If the curve is to be used to realistically animate a rigid body, then we want control over the way the end points "match up" at each successive pair $P_{i-1}(t)$ and $P_i(t)$.

Hermite Polynomials

A **Hermite Polynomial** is a cubic polynomial defined on an interval, whose values at the endpoints, and whose tangent vectors at the endpoints, are determined. Hermite polynomials are combined piecewise and are used to construct a path that interpolate a given set of points. If the path of an object is to appear realistic, the curve must be continuous, and pairs of outgoing and incoming tangent vectors must "agree" in some acceptable way (discuss). Suppose $\vec{P}_1$ and $\vec{P}_2$ are the endpoints of a specific Hermite curve, and suppose $\vec{T}_1$ and $\vec{T}_2$ are the associated incoming and outgoing tangent vectors. Then that Hermite polynomial can be expressed as

$$\vec{H}(t) = \begin{bmatrix}\vec{P}_1 & \vec{P}_2 & \vec{T}_1 & \vec{T}_2\end{bmatrix} M_H \begin{bmatrix}1\\ t\\ t^2\\ t^3\end{bmatrix} \qquad \text{for some } 4\times 4 \text{ coefficient matrix } M_H \ .$$

Now $\begin{bmatrix}\vec{P}_1 & \vec{P}_2 & \vec{T}_1 & \vec{T}_2\end{bmatrix}$ and M_H are just constants, so differentiating $\vec{H}(t)$ yields

$$\vec{H}'(t) = \begin{bmatrix}\vec{P}_1 & \vec{P}_2 & \vec{T}_1 & \vec{T}_2\end{bmatrix} M_H \begin{bmatrix}0\\ 1\\ 2t\\ 3t^2\end{bmatrix}$$

The four conditions on the endpoints and tangent vectors can be written as

$$\vec{H}(0) = \vec{P}_1 \qquad \vec{H}(1) = \vec{P}_2 \qquad \vec{H}'(0) = \vec{T}_1 \qquad \vec{H}'(1) = \vec{T}_2$$

which can then be written as a single equation:

$$\begin{bmatrix}\vec{P}_1 & \vec{P}_2 & \vec{T}_1 & \vec{T}_2\end{bmatrix} M_H \begin{bmatrix}1 & 1 & 0 & 0\\ 0 & 1 & 1 & 1\\ 0 & 1 & 0 & 2\\ 0 & 1 & 0 & 3\end{bmatrix} = \begin{bmatrix}\vec{P}_1 & \vec{P}_2 & \vec{T}_1 & \vec{T}_2\end{bmatrix}$$

Therefore

$$M_H = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 1 & 0 & 3 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 0 & 3 & -2 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

We substitute this back into the definition for $\vec{H}(t)$ and summarize as follows:

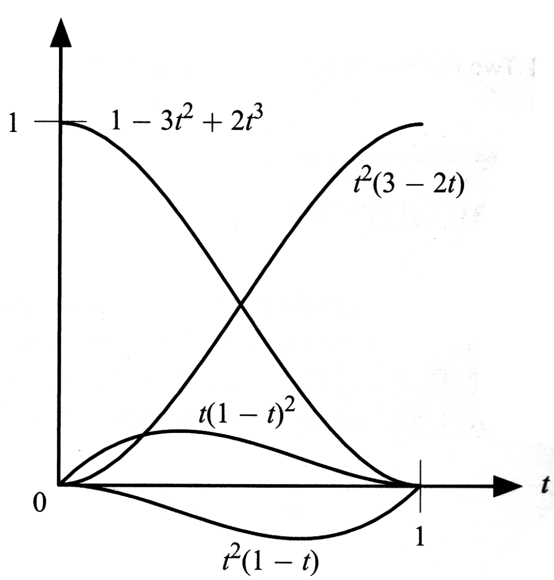
Cubic Hermite Interpolation

Given two points in 3D: $\vec{P}_1$ and $\vec{P}_2$ and two tangent vectors: $\vec{T}_1$ and $\vec{T}_2$ the cubic Hermite polynomial interpolating $\vec{P}_1$ and $\vec{P}_2$ and having tangent vectors $\vec{T}_1$ at $\vec{P}_1$ and $\vec{T}_2$ at $\vec{P}_2$ respectively is

$$\vec{H}(t) = \begin{bmatrix} \vec{P}_1 & \vec{P}_2 & \vec{T}_1 & \vec{T}_2 \end{bmatrix} \begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 0 & 3 & -2 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ t \\ t^2 \\ t^3 \end{bmatrix} \qquad \bigstar\bigstar\bigstar$$

Equivalently

$$\vec{H}(t) = (1-3t^2+2t^3)\vec{P}_1 + t^2(3-2t)\vec{P}_2 + t(t-1)^2\vec{T}_1 + t^2(t-1)\vec{T}_2 \qquad \bigstar\bigstar\bigstar$$



Blending Hermite Polynomials

Example
Construct the Hermite cubic polynomial $\vec{H}(t)$ satisfying

$$\vec{H}(0) = \vec{P}_1 = [0,0,0] \qquad \vec{H}(1) = \vec{P}_2 = [10,60,20]$$

$$\vec{H}'(0) = \vec{T}_1 = [1,3,2] \qquad \vec{H}'(1) = \vec{T}_2 = [-2.0,3]$$

Solution
In matrix form

$$\vec{H}(t) = \begin{bmatrix} \vec{P}_1 & \vec{P}_2 & \vec{T}_1 & \vec{T}_2 \end{bmatrix} \begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 0 & 3 & -2 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ t \\ t^2 \\ t^3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 10 & 1 & -2 \\ 0 & 60 & 3 & 0 \\ 0 & 20 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 0 & 3 & -2 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ t \\ t^2 \\ t^3 \end{bmatrix}$$

Equivalently

$$\vec{H}(t) = (1-3t^2+2t^3)\vec{P}_1 + t^2(3-2t)\vec{P}_2 + t(t-1)^2\vec{T}_1 + t^2(t-1)\vec{T}_2$$

$$= (1-3t^2+2t^3)\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + t^2(3-2t)\begin{bmatrix} 10 \\ 60 \\ 20 \end{bmatrix} + t(t-1)^2\begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} + t^2(t-1)\begin{bmatrix} -2 \\ 0 \\ 3 \end{bmatrix}$$

= etc. Check that these two form are equal, and that the desired conditions apply:

$$\vec{H}(0) = \vec{P}_1 \qquad \vec{H}(1) = \vec{P}_2 \qquad \vec{H}'(0) = \vec{T}_1 \qquad \vec{H}'(1) = \vec{T}_2$$

Problem

Construct the Hermite cubic polynomial $\vec{H}(t)$ satisfying

$$\vec{H}(1) = \vec{P}_1 = [0,0,0]$$

$$\vec{H}(5) = \vec{P}_2 = [10,60,20]$$

$$\vec{H}'(1) = \vec{T}_1 = [1,3,2]$$

$$\vec{H}'(5) = \vec{T}_2 = [-2.0,3]$$

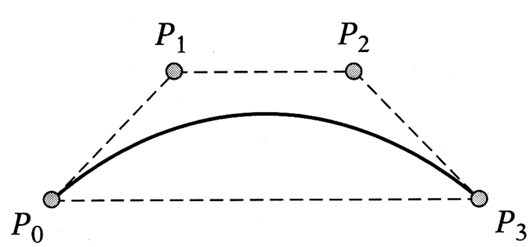
Solution

Here $t_1 = 1$ and $t_2 = 5$. The Hermite polynomial $\vec{H}(t)$ from the previous example expects a parameter $0 \le t \le 1$. Observe that the parameter $s = \dfrac{t-1}{4}$ varies between 0 and 1 as t varies from 1 to 5. So the desired Hermite cubic will be

$$\vec{J}(t) = \vec{H}(s) = \vec{H}\left(\frac{t-1}{4}\right)$$

(Cubic) Bezier Curves

The idea behind Bezier curves is similar to Hermite curves. However instead of using tangent vectors, a Bezier curve $\vec{B}(t)$ uses four control points $\vec{P}_0$, $\vec{P}_1$, $\vec{P}_2$, $\vec{P}_3$: The curve interpolate $\vec{P}_0$ and $\vec{P}_3$ and approximates $\vec{P}_1$ and $\vec{P}_2$. Thus the direction vectors from $\vec{P}_0$ to $\vec{P}_1$ and from $\vec{P}_2$ to $\vec{P}_3$ act as tangent vectors.



Recall that the equation of the cubic Hermite curve was constructed by using the conditions imposed on the end points (as well as the conditions imposed on the tangents at the end points) to compute the coefficient matrix M_H :

$$\vec{H}(t) = \begin{bmatrix}\vec{P}_1 & \vec{P}_2 & \vec{T}_1 & \vec{T}_2\end{bmatrix} M_H \begin{bmatrix}1\\ t\\ t^2\\ t^3\end{bmatrix} \quad \text{where} \quad M_H = \begin{bmatrix}1 & 0 & -3 & 2\\ 0 & 0 & 3 & -2\\ 0 & 1 & -2 & 1\\ 0 & 0 & -1 & 1\end{bmatrix}$$

or writing $\vec{H}(t)$ algebraically:

$$\vec{H}(t) = (1-3t^2+2t^3)\vec{P}_1 + t^2(3-2t)\vec{P}_2 + t(t-1)^2\vec{T}_1 + t^2(t-1)\vec{T}_2$$

In the case of Bezier curves we begin instead with the basis functions:

$$B_{3,0}(t) = (1-t)^3 \quad B_{3,1}(t) = 3t(1-t)^2 \quad B_{3,2}(t) = 3t^2(1-t) \quad B_{3,3}(t) = t^3 \qquad ★★★$$

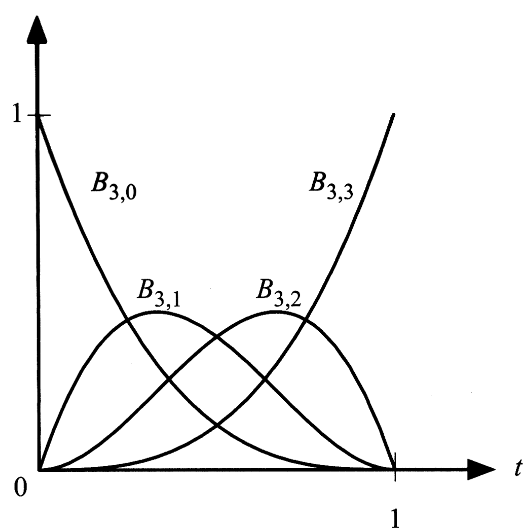
So

$$\vec{B}(t) = (1-t)^3\vec{P}_0 + 3t(1-t)^2\vec{P}_1 + 3t^2(1-t)\vec{P}_2 + t^3\vec{P}_3 \qquad ★★★$$

Writing $\vec{B}(t)$ in matrix form in terms of the four control points $\vec{P}_0$, $\vec{P}_1$, $\vec{P}_2$, $\vec{P}_3$ yields

$$\vec{B}(t) = \begin{bmatrix}\vec{P}_0 & \vec{P}_1 & \vec{P}_2 & \vec{P}_3\end{bmatrix} M_B \begin{bmatrix}1\\ t\\ t^2\\ t^3\end{bmatrix} \quad \text{which gives} \quad M_B = \begin{bmatrix}1 & -3 & 3 & -1\\ 0 & 3 & -6 & 3\\ 0 & 0 & 3 & -3\\ 0 & 0 & 0 & 1\end{bmatrix}$$

The four polynomials $B_{3,0}(t) = (1-t)^3$ $B_{3,1}(t) = 3t(1-t)^2$ $B_{3,2}(t) = 3t^2(1-t)$ and $B_{3,3}(t) = t^3$ are the degree three **Bernstein Polynomials**.



In general, the i-th Bernstein polynomial of degree n is

$$B_{n,i}(t) = \binom{n}{i} t^i (1-t)^{n-i} \qquad \text{where} \qquad \binom{n}{i} = \frac{n!}{i!(n-i)!}$$

Note

- The notation for the Bernstein polynomials is standard and convenient. However, using it results in a different ordering of the control points than was used for Hermite curves. The two interpolated points on the Bezier curve have been labelled $\vec{P}_0$ and $\vec{P}_3$ rather than $\vec{P}_1$ and $\vec{P}_2$ as was done in Hermite interpolation.

Example

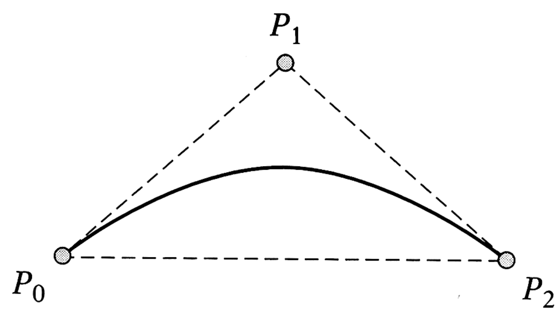
A Bezier curve of degree 1 reduces to a straight line segment: $\vec{B}(t) = t\,\vec{P}_0 + (1-t)\vec{P}_1$

It has two interpolated points and no approximated points.

Example

A degree two Bezier curve, namely a quadratic Bezier curve, uses three control points. The curve interpolates two, and approximates the third. It takes the form

$\vec{B}(t) = (1-t)^2\vec{P}_0 + 2t(1-t)\vec{P}_1 + t^2\vec{P}_2$



Bezier vs Hermite

Recall that two points determine a direction vector, and conversely having a point and a vector determines all other points along that vector. So cubic Hermite and Bezier curves are equivalent. Start with the Bezier curve:

$$\vec{B}(t) = (1-t)^3\vec{P}_0 + 3t(1-t)^2\vec{P}_1 + 3t^2(1-t)\vec{P}_2 + t^3\vec{P}_3$$

Differentiating gives

$$\vec{B}'(t) = -3(1-t)^2\vec{P}_0 + (3(1-t)^2-6t(1-t))\vec{P}_1 + (6t(1-t)-3t^2)\vec{P}_2 + 3t^2\vec{P}_3$$

$$= -3(1-t)^2\vec{P}_0 + (9t^2-12t+3)\vec{P}_1 + (6t-9t^2)\vec{P}_2 + 3t^2\vec{P}_3$$

Therefore

$$\vec{B}'(0) = -3\vec{P}_0 + 3\vec{P}_1 \quad \text{and} \quad \vec{B}'(1) = -3\vec{P}_2 + 3\vec{P}_3$$

Equating these with the tangent vectors of the Hermite curve yields

$$\vec{T}_1 = 3(\vec{P}_1 - \vec{P}_0) \quad \text{and} \quad \vec{T}_2 = 3(\vec{P}_3 - \vec{P}_2) \qquad ☆☆☆$$

where $\vec{T}_1$ is the tangent vector at $\vec{P}_0$ of $\vec{B}(t)$ and $\vec{T}_2$ is the tangent vector at $\vec{P}_3$ of $\vec{B}(t)$. To construct the corresponding Hermite curve we plug $\vec{P}_0$, $\vec{P}_3$ and these tangent vectors $\vec{T}_1$ and $\vec{T}_2$ into the definition of the Hermite curve. (using the Bezier labels for the points). We have

$$\vec{H}(t) = (1-3t^2+2t^3)\vec{P}_0 + t^2(3-2t)\vec{P}_3 + t(t-1)^2 3(\vec{P}_1-\vec{P}_0) + t^2(t-1)3(\vec{P}_3-\vec{P}_2)$$

A little algebra shows this is equal to the Bezier curve

$$\vec{B}(t) = (1-t)^3\vec{P}_0 + 3t(1-t)^2\vec{P}_1 + 3t^2(1-t)\vec{P}_2 + t^3\vec{P}_3$$

Problem

Go the other way around. Begin with a Hermite curve and construct an Associated Bezier formulation.