MFSURP Notes

Contents

1. Chooser options under the Blachelier Model

We consider the Blachelier model with where the stock prices $\{S_t\}_{t\geq 0}$ evolves according to

$$S_t = S_0 + \kappa W_t \tag{1.1}$$

where $S_0 > 0$ and $\{W_t\}_{t\geq 0}$ is a Brownian motion under the risk neutral measure $\tilde{\mathbb{P}}$.

We consider a contract with maturity T and strike price K, that allows an agent to decide on a choosing date $\tau < T$ to choose the underlying derivative. Many results are known when an agent is allowed to choose between a European call and a European put. In this section we consider a contract that allows an agent to decide between two securities that pays

$$C_T = (A_T - K)^+,\ P_T = (K - A_T)^+,\ A_T = \frac{1}{T}\int_0^T S_t\, dt. \tag{1.2}$$

(fill in derivation that gives V_τ, the value of the contract at time τ). Here, we assume the agent chooses optimally with no outside information. At time τ, the agent will choose the option of higher value between the put and call. The value of contract V at time τ is

$$V_\tau = \max(C_\tau, P_\tau). \tag{1.3}$$

We subtract C_τ out of the max function and get

$$V_\tau = C_\tau + \max(0, P_\tau - C_\tau) \tag{1.4}$$

which we want to evaluate through Put-Call Parity.

Recall that Put-Call Parity tells us

$$P_T - C_T = K - A_T. \tag{1.5}$$

We want to discount this entire equation back to time τ, but note that we cannot bring A_T back to time τ as A_τ (why?). So, we define w_τ to be the price at time τ of a contract to receive A_T at time T. We can now use w_τ to bring the equation back to time τ

$$P_\tau - C_\tau = Ke^{r(\tau - T)} - w_\tau \tag{1.6}$$

So in conclusion, we find that

$$V_\tau = C_\tau + \left(K - S_0 + \frac{\kappa(T-\tau)}{T}W_\tau - \frac{\kappa}{T}\int_0^\tau W_t\, dt\right)^+. \tag{1.7}$$

Then by the risk-neutral pricing formula, the time-zero price of this contract is given by

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