

① Historical Volatility

$$\text{Let } R_n = \frac{S_n - S_{n-1}}{S_{n-1}} \quad \text{Daily Return}$$

S_n Stock Price on day n

$$\text{Let } \gamma_n = \ln(1 + R_n)$$

$$\text{Notice } 1 + R_n = \frac{S_n}{S_{n-1}}$$

$$\text{Notice } \ln(1 + R_n) = \ln\left(\frac{S_n}{S_{n-1}}\right)$$

$\approx R_n$

(Since $\ln(1+x) \approx x$)

(2)

$$\hat{\mu}_d = \frac{1}{N} \sum_{n=1}^N \gamma_n \quad \text{Sample Mean}$$

$$\hat{\sigma}_d^2 = \frac{1}{N-1} \sum_{n=1}^N [\gamma_n - \hat{\mu}_d]^2 \quad \text{Sample Variance}$$

$$\sigma_H = \sqrt{252} \hat{\sigma}_d$$

historical
Volatility

252 Trading Days
Per year

Mathematicians Trading Year

256 days