

$$\text{In}[1]:= \text{d}[i_]:= \frac{i}{dx^2} \left(\text{psi}[[i+1]] - 2 \text{psi}[[i]] + \text{psi}[[i-1]] \right) - i \text{v}[[i]] \times \text{psi}[[i]]$$

$$\text{In}[2]:= \text{d}[j]$$

$$\text{Out}[2]= \frac{i \left(\text{psi}[[-1 + j]] - 2 \text{psi}[[j]] + \text{psi}[[1 + j]] \right)}{dx^2} - i \text{psi}[[j]] \text{v}[[j]]$$

$$\text{k1} = \text{d}[i]$$

$$\frac{i \left(\text{psi}[[-1 + i]] - 2 \text{psi}[[i]] + \text{psi}[[1 + i]] \right)}{dx^2} - i \text{psi}[[i]] \text{v}[[i]]$$

$$\text{k1}$$

$$\frac{i \left(\text{psi}[[-1 + i]] - 2 \text{psi}[[i]] + \text{psi}[[1 + i]] \right)}{dx^2} - i \text{psi}[[i]] \text{v}[[i]]$$

$$\text{d}[i] + \frac{dt}{2} \text{k1}$$

$$\frac{i \left(\text{psi}[[-1 + i]] - 2 \text{psi}[[i]] + \text{psi}[[1 + i]] \right)}{dx^2} - i \text{psi}[[i]] \text{v}[[i]] +$$

$$\frac{1}{2} dt \left(\frac{i \left(\text{psi}[[-1 + i]] - 2 \text{psi}[[i]] + \text{psi}[[1 + i]] \right)}{dx^2} - i \text{psi}[[i]] \text{v}[[i]] \right)$$

$$\frac{i \left(\text{psi}[[-1 + i]] - 2 \text{psi}[[i]] + \text{psi}[[1 + i]] \right)}{dx^2} - i \text{psi}[[i]] \text{v}[[i]] + \frac{1}{2} dt \left(\frac{i \left(\text{d}[[-1 + i]] - 2 \text{d}[[i]] + \text{d}[[1 + i]] \right)}{dx^2} - i \text{d}[[i]] \text{v}[[i]] \right)$$

$$\begin{aligned}
k2 = & \frac{i \left(\psi[-1+i] - 2 \psi[i] + \psi[1+i] \right)}{dx^2} - i \psi[i] v[i] + \\
& \frac{1}{2} dt \left(-i v[i] \left(\frac{i \left(\psi[-1+i] - 2 \psi[i] + \psi[1+i] \right)}{dx^2} - i \psi[i] v[i] \right) + \right. \\
& \frac{1}{dx^2} i \left(\frac{i \left(\psi[-2+i] - 2 \psi[-1+i] + \psi[i] \right)}{dx^2} + \right. \\
& \frac{i \left(\psi[i] - 2 \psi[1+i] + \psi[2+i] \right)}{dx^2} - i \psi[-1+i] v[-1+i] - \\
& \left. \left. 2 \left(\frac{i \left(\psi[-1+i] - 2 \psi[i] + \psi[1+i] \right)}{dx^2} - i \psi[i] v[i] \right) - i \psi[1+i] v[1+i] \right) \right) \\
& \frac{i \left(\psi[-1+i] - 2 \psi[i] + \psi[1+i] \right)}{dx^2} - i \psi[i] v[i] + \\
& \frac{1}{2} dt \left(-i v[i] \left(\frac{i \left(\psi[-1+i] - 2 \psi[i] + \psi[1+i] \right)}{dx^2} - i \psi[i] v[i] \right) + \frac{1}{dx^2} \right. \\
& i \left(\frac{i \left(\psi[-2+i] - 2 \psi[-1+i] + \psi[i] \right)}{dx^2} + \frac{i \left(\psi[i] - 2 \psi[1+i] + \psi[2+i] \right)}{dx^2} - i \psi[-1+i] \right. \\
& \left. \left. v[-1+i] - 2 \left(\frac{i \left(\psi[-1+i] - 2 \psi[i] + \psi[1+i] \right)}{dx^2} - i \psi[i] v[i] \right) - i \psi[1+i] v[1+i] \right) \right) \\
& d[i] + \frac{dt}{2} k2
\end{aligned}$$

$$\frac{\hbar \left(\psi[-1+i] - 2 \psi[i] + \psi[1+i] \right)}{dx^2} -$$

$$\hbar \psi[i] v[i] + \frac{1}{2} dt \left(\frac{\hbar \left(d[-1+i] - 2 d[i] + d[1+i] \right)}{dx^2} - \hbar d[i] v[i] + \right.$$

$$\left. \frac{1}{2} dt \left(-\hbar v[i] \left(\frac{\hbar \left(d[-1+i] - 2 d[i] + d[1+i] \right)}{dx^2} - \hbar d[i] v[i] \right) + \right.$$

$$\left. \frac{1}{dx^2} \hbar \left(\frac{\hbar \left(d[-2+i] - 2 d[-1+i] + d[i] \right)}{dx^2} + \frac{\hbar \left(d[i] - 2 d[1+i] + d[2+i] \right)}{dx^2} - \hbar d[-1+i] v[-1+i] - \right.$$

$$\left. \left. 2 \left(\frac{\hbar \left(d[-1+i] - 2 d[i] + d[1+i] \right)}{dx^2} - \hbar d[i] v[i] \right) - \hbar d[1+i] v[1+i] \right) \right)$$

$$k3 = \frac{\hbar \left(\psi[-1+i] - 2 \psi[i] + \psi[1+i] \right)}{dx^2} - \hbar \psi[i] v[i] +$$

$$\frac{1}{2} dt \left(-\hbar v[i] \left(\frac{\hbar \left(\psi[-1+i] - 2 \psi[i] + \psi[1+i] \right)}{dx^2} - \hbar \psi[i] v[i] \right) + \right.$$

$$\left. \frac{1}{dx^2} \hbar \left(\frac{\hbar \left(\psi[-2+i] - 2 \psi[-1+i] + \psi[i] \right)}{dx^2} + \right.$$

$$\left. \frac{\hbar \left(\psi[i] - 2 \psi[1+i] + \psi[2+i] \right)}{dx^2} - \hbar \psi[-1+i] v[-1+i] - \right.$$

$$\left. 2 \left(\frac{\hbar \left(\psi[-1+i] - 2 \psi[i] + \psi[1+i] \right)}{dx^2} - \hbar \psi[i] v[i] \right) - \hbar \psi[1+i] v[1+i] \right) +$$

$$\frac{1}{2} dt \left(-\hbar v[i] \left(-\hbar v[i] \left(\frac{\hbar \left(\psi[-1+i] - 2 \psi[i] + \psi[1+i] \right)}{dx^2} - \hbar \psi[i] v[i] \right) + \right.$$

$$\left. \frac{1}{dx^2} \hbar \left(\frac{\hbar \left(\psi[-2+i] - 2 \psi[-1+i] + \psi[i] \right)}{dx^2} + \right.$$

$$\left. \frac{\hbar \left(\psi[i] - 2 \psi[1+i] + \psi[2+i] \right)}{dx^2} - \hbar \psi[-1+i] v[-1+i] - 2 \right.$$

$$\left. \left(\frac{\hbar \left(\psi[-1+i] - 2 \psi[i] + \psi[1+i] \right)}{dx^2} - \hbar \psi[i] v[i] \right) - \hbar \psi[1+i] v[1+i] \right) \right)$$

$$\begin{aligned}
& \frac{1}{dx^2} i \left(-i v[-1+i] \left(\frac{i (\psi[-2+i] - 2 \psi[-1+i] + \psi[i])}{dx^2} - i \psi[-1+i] v[-1+i] \right) + \right. \\
& \frac{1}{dx^2} i \left(\frac{i (\psi[-3+i] - 2 \psi[-2+i] + \psi[-1+i])}{dx^2} + \right. \\
& \frac{i (\psi[-1+i] - 2 \psi[i] + \psi[1+i])}{dx^2} - i \psi[-2+i] v[-2+i] - \\
& 2 \left(\frac{i (\psi[-2+i] - 2 \psi[-1+i] + \psi[i])}{dx^2} - i \psi[-1+i] v[-1+i] \right) - i \psi[i] \\
& v[i] \left. \right) - i v[1+i] \left(\frac{i (\psi[i] - 2 \psi[1+i] + \psi[2+i])}{dx^2} - i \psi[1+i] v[1+i] \right) - \\
& 2 \left(-i v[i] \left(\frac{i (\psi[-1+i] - 2 \psi[i] + \psi[1+i])}{dx^2} - i \psi[i] v[i] \right) + \frac{1}{dx^2} \right. \\
& i \left(\frac{i (\psi[-2+i] - 2 \psi[-1+i] + \psi[i])}{dx^2} + \right. \\
& \frac{i (\psi[i] - 2 \psi[1+i] + \psi[2+i])}{dx^2} - i \psi[-1+i] v[-1+i] - \\
& 2 \left(\frac{i (\psi[-1+i] - 2 \psi[i] + \psi[1+i])}{dx^2} - i \psi[i] v[i] \right) - \\
& i \psi[1+i] v[1+i] \left. \right) \left. \right) + \frac{1}{dx^2} i \left(\frac{i (\psi[-1+i] - 2 \psi[i] + \psi[1+i])}{dx^2} + \right. \\
& \frac{i (\psi[1+i] - 2 \psi[2+i] + \psi[3+i])}{dx^2} - i \psi[i] v[i] - \\
& 2 \left(\frac{i (\psi[i] - 2 \psi[1+i] + \psi[2+i])}{dx^2} - i \psi[1+i] v[1+i] \right) - \\
& i \psi[2+i] v[2+i] \left. \right) \left. \right) \left. \right) \\
& \frac{i (\psi[-1+i] - 2 \psi[i] + \psi[1+i])}{dx^2} - i \psi[i] v[i] +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{2} \, dt \left(-i \, v[i] \left(\frac{i \left(\psi[-1+i] - 2 \, \psi[i] + \psi[1+i] \right)}{dx^2} - i \, \psi[i] \, v[i] \right) + \frac{1}{dx^2} \right. \\
& \quad i \left(\frac{i \left(\psi[-2+i] - 2 \, \psi[-1+i] + \psi[i] \right)}{dx^2} + \frac{i \left(\psi[i] - 2 \, \psi[1+i] + \psi[2+i] \right)}{dx^2} - i \, \psi[-1+i] \right. \\
& \quad \left. \left. v[-1+i] - 2 \left(\frac{i \left(\psi[-1+i] - 2 \, \psi[i] + \psi[1+i] \right)}{dx^2} - i \, \psi[i] \, v[i] \right) - i \, \psi[1+i] \, v[1+i] \right) \right) + \\
& \quad \frac{1}{2} \, dt \left(-i \, v[i] \left(-i \, v[i] \left(\frac{i \left(\psi[-1+i] - 2 \, \psi[i] + \psi[1+i] \right)}{dx^2} - i \, \psi[i] \, v[i] \right) + \right. \right. \\
& \quad \frac{1}{dx^2} \, i \left(\frac{i \left(\psi[-2+i] - 2 \, \psi[-1+i] + \psi[i] \right)}{dx^2} + \right. \\
& \quad \frac{i \left(\psi[i] - 2 \, \psi[1+i] + \psi[2+i] \right)}{dx^2} - i \, \psi[-1+i] \, v[-1+i] - \\
& \quad \left. \left. 2 \left(\frac{i \left(\psi[-1+i] - 2 \, \psi[i] + \psi[1+i] \right)}{dx^2} - i \, \psi[i] \, v[i] \right) - i \, \psi[1+i] \, v[1+i] \right) \right) \right) + \\
& \quad \frac{1}{dx^2} \, i \left(-i \, v[-1+i] \left(\frac{i \left(\psi[-2+i] - 2 \, \psi[-1+i] + \psi[i] \right)}{dx^2} - i \, \psi[-1+i] \, v[-1+i] \right) + \right. \\
& \quad \frac{1}{dx^2} \, i \left(\frac{i \left(\psi[-3+i] - 2 \, \psi[-2+i] + \psi[-1+i] \right)}{dx^2} + \right. \\
& \quad \frac{i \left(\psi[-1+i] - 2 \, \psi[i] + \psi[1+i] \right)}{dx^2} - i \, \psi[-2+i] \, v[-2+i] - 2 \\
& \quad \left(\frac{i \left(\psi[-2+i] - 2 \, \psi[-1+i] + \psi[i] \right)}{dx^2} - i \, \psi[-1+i] \, v[-1+i] \right) - i \, \psi[i] \, v[i] \right) - \\
& \quad i \, v[1+i] \left(\frac{i \left(\psi[i] - 2 \, \psi[1+i] + \psi[2+i] \right)}{dx^2} - i \, \psi[1+i] \, v[1+i] \right) - \\
& \quad 2 \left(-i \, v[i] \left(\frac{i \left(\psi[-1+i] - 2 \, \psi[i] + \psi[1+i] \right)}{dx^2} - i \, \psi[i] \, v[i] \right) + \frac{1}{dx^2} \, i \right. \\
& \quad \left. \left(\frac{i \left(\psi[-2+i] - 2 \, \psi[-1+i] + \psi[i] \right)}{dx^2} + \frac{i \left(\psi[i] - 2 \, \psi[1+i] + \psi[2+i] \right)}{dx^2} - \right. \right.
\end{aligned}$$

$$\begin{aligned}
& i \psi[-1+i] v[-1+i] - 2 \left(\frac{i (\psi[-1+i] - 2 \psi[i] + \psi[1+i])}{dx^2} - \right. \\
& \left. i \psi[i] v[i] \right) - i \psi[1+i] v[1+i] \Bigg) + \\
& \frac{1}{dx^2} i \left(\frac{i (\psi[-1+i] - 2 \psi[i] + \psi[1+i])}{dx^2} + \frac{i (\psi[1+i] - 2 \psi[2+i] + \psi[3+i])}{dx^2} - \right. \\
& i \psi[i] v[i] - 2 \left(\frac{i (\psi[i] - 2 \psi[1+i] + \psi[2+i])}{dx^2} - \right. \\
& \left. i \psi[1+i] v[1+i] \right) - i \psi[2+i] v[2+i] \Bigg) \Bigg)
\end{aligned}$$

d[i] + dt k3

$$\begin{aligned}
& \frac{i (\psi[-1+i] - 2 \psi[i] + \psi[1+i])}{dx^2} - i \psi[i] v[i] + dt \left(\frac{i (d[-1+i] - 2 d[i] + d[1+i])}{dx^2} - \right. \\
& i d[i] v[i] + \frac{1}{2} dt \left(-i v[i] \left(\frac{i (d[-1+i] - 2 d[i] + d[1+i])}{dx^2} - i d[i] v[i] \right) + \right. \\
& \frac{1}{dx^2} i \left(\frac{i (d[-2+i] - 2 d[-1+i] + d[i])}{dx^2} + \frac{i (d[i] - 2 d[1+i] + d[2+i])}{dx^2} - i d[-1+i] v[-1+i] - \right. \\
& 2 \left(\frac{i (d[-1+i] - 2 d[i] + d[1+i])}{dx^2} - i d[i] v[i] \right) - i d[1+i] v[1+i] \Bigg) + \\
& \frac{1}{2} dt \left(-i v[i] \left(-i v[i] \left(\frac{i (d[-1+i] - 2 d[i] + d[1+i])}{dx^2} - i d[i] v[i] \right) + \right. \right. \\
& \frac{1}{dx^2} i \left(\frac{i (d[-2+i] - 2 d[-1+i] + d[i])}{dx^2} + \frac{i (d[i] - 2 d[1+i] + d[2+i])}{dx^2} - i d[-1+i] \right. \\
& \left. v[-1+i] - 2 \left(\frac{i (d[-1+i] - 2 d[i] + d[1+i])}{dx^2} - i d[i] v[i] \right) - i d[1+i] v[1+i] \Bigg) \Bigg) + \\
& \frac{1}{dx^2} i \left(-i v[-1+i] \left(\frac{i (d[-2+i] - 2 d[-1+i] + d[i])}{dx^2} - i d[-1+i] v[-1+i] \right) + \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{dx^2} \left(\frac{\hbar \left(\psi[-2+i] - 2 \psi[-1+i] + \psi[i] \right)}{dx^2} + \right. \\
& \quad \frac{\hbar \left(\psi[i] - 2 \psi[1+i] + \psi[2+i] \right)}{dx^2} - \hbar \psi[-1+i] v[-1+i] - 2 \\
& \quad \left. \left(\frac{\hbar \left(\psi[-1+i] - 2 \psi[i] + \psi[1+i] \right)}{dx^2} - \hbar \psi[i] v[i] \right) - \hbar \psi[1+i] v[1+i] \right) + \\
& \frac{1}{dx^2} \left(-\hbar v[-1+i] \left(\frac{\hbar \left(\psi[-2+i] - 2 \psi[-1+i] + \psi[i] \right)}{dx^2} - \hbar \psi[-1+i] v[-1+i] \right) + \right. \\
& \quad \frac{1}{dx^2} \left(\frac{\hbar \left(\psi[-3+i] - 2 \psi[-2+i] + \psi[-1+i] \right)}{dx^2} + \right. \\
& \quad \frac{\hbar \left(\psi[-1+i] - 2 \psi[i] + \psi[1+i] \right)}{dx^2} - \hbar \psi[-2+i] v[-2+i] - \\
& \quad 2 \left(\frac{\hbar \left(\psi[-2+i] - 2 \psi[-1+i] + \psi[i] \right)}{dx^2} - \hbar \psi[-1+i] v[-1+i] \right) - \hbar \psi[i] \\
& \quad \left. v[i] \right) - \hbar v[1+i] \left(\frac{\hbar \left(\psi[i] - 2 \psi[1+i] + \psi[2+i] \right)}{dx^2} - \hbar \psi[1+i] v[1+i] \right) - \\
& \quad 2 \left(-\hbar v[i] \left(\frac{\hbar \left(\psi[-1+i] - 2 \psi[i] + \psi[1+i] \right)}{dx^2} - \hbar \psi[i] v[i] \right) + \frac{1}{dx^2} \right. \\
& \quad \left(\frac{\hbar \left(\psi[-2+i] - 2 \psi[-1+i] + \psi[i] \right)}{dx^2} + \right. \\
& \quad \frac{\hbar \left(\psi[i] - 2 \psi[1+i] + \psi[2+i] \right)}{dx^2} - \hbar \psi[-1+i] v[-1+i] - \\
& \quad 2 \left(\frac{\hbar \left(\psi[-1+i] - 2 \psi[i] + \psi[1+i] \right)}{dx^2} - \hbar \psi[i] v[i] \right) - \\
& \quad \left. \hbar \psi[1+i] v[1+i] \right) + \frac{1}{dx^2} \left(\frac{\hbar \left(\psi[-1+i] - 2 \psi[i] + \psi[1+i] \right)}{dx^2} + \right. \\
& \quad \frac{\hbar \left(\psi[1+i] - 2 \psi[2+i] + \psi[3+i] \right)}{dx^2} - \hbar \psi[i] v[i] -
\end{aligned}$$

$$\begin{aligned}
& 2 \left(\frac{\hbar (\psi[i] - 2 \psi[1+i] + \psi[2+i])}{dx^2} - \hbar \psi[1+i] v[1+i] \right) - \\
& \hbar \psi[2+i] v[2+i] \Big) \Big) + \\
& \frac{1}{2} \, dt \left(-\hbar v[i] \left(-\hbar v[i] \left(-\hbar v[i] \left(\frac{\hbar (\psi[-1+i] - 2 \psi[i] + \psi[1+i])}{dx^2} - \hbar \psi[i] v[i] \right) + \right. \right. \right. \\
& \frac{1}{dx^2} \hbar \left(\frac{\hbar (\psi[-2+i] - 2 \psi[-1+i] + \psi[i])}{dx^2} + \right. \\
& \frac{\hbar (\psi[i] - 2 \psi[1+i] + \psi[2+i])}{dx^2} - \hbar \psi[-1+i] v[-1+i] - \\
& 2 \left(\frac{\hbar (\psi[-1+i] - 2 \psi[i] + \psi[1+i])}{dx^2} - \hbar \psi[i] v[i] \right) - \\
& \hbar \psi[1+i] v[1+i] \Big) \Big) + \frac{1}{dx^2} \hbar \left(-\hbar v[-1+i] \right. \\
& \left. \left(\frac{\hbar (\psi[-2+i] - 2 \psi[-1+i] + \psi[i])}{dx^2} - \hbar \psi[-1+i] v[-1+i] \right) + \right. \\
& \frac{1}{dx^2} \hbar \left(\frac{\hbar (\psi[-3+i] - 2 \psi[-2+i] + \psi[-1+i])}{dx^2} + \right. \\
& \frac{\hbar (\psi[-1+i] - 2 \psi[i] + \psi[1+i])}{dx^2} - \hbar \psi[-2+i] v[-2+i] - \\
& 2 \left(\frac{\hbar (\psi[-2+i] - 2 \psi[-1+i] + \psi[i])}{dx^2} - \hbar \psi[-1+i] v[-1+i] \right) - \\
& \hbar \psi[i] v[i] \Big) - \hbar v[1+i] \left(\frac{\hbar (\psi[i] - 2 \psi[1+i] + \psi[2+i])}{dx^2} - \right. \\
& \hbar \psi[1+i] v[1+i] \Big) - 2 \left(-\hbar v[i] \left(\frac{\hbar (\psi[-1+i] - 2 \psi[i] + \psi[1+i])}{dx^2} - \right. \right. \\
& \hbar \psi[i] v[i] \Big) + \frac{1}{dx^2} \hbar \left(\frac{\hbar (\psi[-2+i] - 2 \psi[-1+i] + \psi[i])}{dx^2} + \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{i \left(\psi[i] - 2 \psi[1+i] + \psi[2+i] \right)}{dx^2} - i \psi[-1+i] v[-1+i] - \\
& 2 \left(\frac{i \left(\psi[-1+i] - 2 \psi[i] + \psi[1+i] \right)}{dx^2} - i \psi[i] v[i] \right) - i \psi[1+i] v[1+i] \Bigg) + \frac{1}{dx^2} i \left(\frac{i \left(\psi[-1+i] - 2 \psi[i] + \psi[1+i] \right)}{dx^2} + \right. \\
& \frac{i \left(\psi[1+i] - 2 \psi[2+i] + \psi[3+i] \right)}{dx^2} - i \psi[i] v[i] - \\
& 2 \left(\frac{i \left(\psi[i] - 2 \psi[1+i] + \psi[2+i] \right)}{dx^2} - i \psi[1+i] v[1+i] \right) - i \psi[2+i] v[2+i] \Bigg) \Bigg) + \\
& \frac{1}{dx^2} i \left(-i v[-1+i] \left(-i v[-1+i] \left(\frac{i \left(\psi[-2+i] - 2 \psi[-1+i] + \psi[i] \right)}{dx^2} - i \psi[-1+i] v[-1+i] \right) + \frac{1}{dx^2} i \left(\frac{i \left(\psi[-3+i] - 2 \psi[-2+i] + \psi[-1+i] \right)}{dx^2} + \right. \right. \right. \\
& \frac{i \left(\psi[-1+i] - 2 \psi[i] + \psi[1+i] \right)}{dx^2} - i \psi[-2+i] v[-2+i] - \\
& 2 \left(\frac{i \left(\psi[-2+i] - 2 \psi[-1+i] + \psi[i] \right)}{dx^2} - i \psi[-1+i] v[-1+i] \right) - \\
& i \psi[i] v[i] \Bigg) \Bigg) + \frac{1}{dx^2} i \left(-i v[-2+i] \left(\frac{i \left(\psi[-3+i] - 2 \psi[-2+i] + \psi[-1+i] \right)}{dx^2} - i \psi[-2+i] v[-2+i] \right) + \right. \\
& \frac{1}{dx^2} i \left(\frac{i \left(\psi[-4+i] - 2 \psi[-3+i] + \psi[-2+i] \right)}{dx^2} + \right. \\
& \frac{i \left(\psi[-2+i] - 2 \psi[-1+i] + \psi[i] \right)}{dx^2} - i \psi[-3+i] v[-3+i] -
\end{aligned}$$

$$\begin{aligned}
& 2 \left(\frac{\hbar \left(\psi[-3+i] - 2 \psi[-2+i] + \psi[-1+i] \right)}{dx^2} - \right. \\
& \quad \left. \hbar \psi[-2+i] v[-2+i] \right) - \hbar \psi[-1+i] v[-1+i] \Big) - \\
& \hbar v[i] \left(\frac{\hbar \left(\psi[-1+i] - 2 \psi[i] + \psi[1+i] \right)}{dx^2} - \hbar \psi[i] v[i] \right) - \\
& 2 \left(-\hbar v[-1+i] \left(\frac{\hbar \left(\psi[-2+i] - 2 \psi[-1+i] + \psi[i] \right)}{dx^2} - \hbar \psi[-1+i] \right. \right. \\
& \quad \left. \left. v[-1+i] \right) + \frac{1}{dx^2} \hbar \left(\frac{\hbar \left(\psi[-3+i] - 2 \psi[-2+i] + \psi[-1+i] \right)}{dx^2} + \right. \right. \\
& \quad \left. \left. \frac{\hbar \left(\psi[-1+i] - 2 \psi[i] + \psi[1+i] \right)}{dx^2} - \hbar \psi[-2+i] v[-2+i] - \right. \right. \\
& \quad \left. \left. 2 \left(\frac{\hbar \left(\psi[-2+i] - 2 \psi[-1+i] + \psi[i] \right)}{dx^2} - \right. \right. \right. \\
& \quad \left. \left. \left. \hbar \psi[-1+i] v[-1+i] \right) - \hbar \psi[i] v[i] \right) \right) \Big) + \\
& \frac{1}{dx^2} \hbar \left(\frac{\hbar \left(\psi[-2+i] - 2 \psi[-1+i] + \psi[i] \right)}{dx^2} + \right. \\
& \quad \left. \frac{\hbar \left(\psi[i] - 2 \psi[1+i] + \psi[2+i] \right)}{dx^2} - \hbar \psi[-1+i] v[-1+i] - \right. \\
& \quad \left. 2 \left(\frac{\hbar \left(\psi[-1+i] - 2 \psi[i] + \psi[1+i] \right)}{dx^2} - \hbar \psi[i] v[i] \right) - \right. \\
& \quad \left. \left. \hbar \psi[1+i] v[1+i] \right) \right) - \hbar v[1+i] \\
& \left(-\hbar v[1+i] \left(\frac{\hbar \left(\psi[i] - 2 \psi[1+i] + \psi[2+i] \right)}{dx^2} - \hbar \psi[1+i] v[1+i] \right) + \right. \\
& \quad \left. \frac{1}{dx^2} \hbar \left(\frac{\hbar \left(\psi[-1+i] - 2 \psi[i] + \psi[1+i] \right)}{dx^2} + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{\hbar \left(\psi[1+i] - 2 \psi[2+i] + \psi[3+i] \right)}{dx^2} - \hbar \psi[i] v[i] - \\
& 2 \left(\frac{\hbar \left(\psi[i] - 2 \psi[1+i] + \psi[2+i] \right)}{dx^2} - \hbar \psi[1+i] v[1+i] \right) - \\
& \hbar \psi[2+i] v[2+i] \Big) \Big) - 2 \\
& \left(-\hbar v[i] \left(-\hbar v[i] \left(\frac{\hbar \left(\psi[-1+i] - 2 \psi[i] + \psi[1+i] \right)}{dx^2} - \hbar \psi[i] v[i] \right) + \right. \right. \\
& \frac{1}{dx^2} \hbar \left(\frac{\hbar \left(\psi[-2+i] - 2 \psi[-1+i] + \psi[i] \right)}{dx^2} + \right. \\
& \frac{\hbar \left(\psi[i] - 2 \psi[1+i] + \psi[2+i] \right)}{dx^2} - \hbar \psi[-1+i] v[-1+i] - \\
& 2 \left(\frac{\hbar \left(\psi[-1+i] - 2 \psi[i] + \psi[1+i] \right)}{dx^2} - \hbar \psi[i] v[i] \right) - \\
& \hbar \psi[1+i] v[1+i] \Big) \Big) + \frac{1}{dx^2} \hbar \left(-\hbar v[-1+i] \right. \\
& \left. \left(\frac{\hbar \left(\psi[-2+i] - 2 \psi[-1+i] + \psi[i] \right)}{dx^2} - \hbar \psi[-1+i] v[-1+i] \right) + \right. \\
& \frac{1}{dx^2} \hbar \left(\frac{\hbar \left(\psi[-3+i] - 2 \psi[-2+i] + \psi[-1+i] \right)}{dx^2} + \right. \\
& \frac{\hbar \left(\psi[-1+i] - 2 \psi[i] + \psi[1+i] \right)}{dx^2} - \hbar \psi[-2+i] v[-2+i] - \\
& 2 \left(\frac{\hbar \left(\psi[-2+i] - 2 \psi[-1+i] + \psi[i] \right)}{dx^2} - \right. \\
& \left. \hbar \psi[-1+i] v[-1+i] \right) - \hbar \psi[i] v[i] \Big) - \hbar v[1+i] \\
& \left(\frac{\hbar \left(\psi[i] - 2 \psi[1+i] + \psi[2+i] \right)}{dx^2} - \hbar \psi[1+i] v[1+i] \right) -
\end{aligned}$$

$$\begin{aligned}
& 2 \left(-i v[i] \left(\frac{i (\psi[-1+i] - 2 \psi[i] + \psi[1+i])}{dx^2} - i \psi[i] v[i] \right) + \right. \\
& \quad \frac{1}{dx^2} i \left(\frac{i (\psi[-2+i] - 2 \psi[-1+i] + \psi[i])}{dx^2} + \right. \\
& \quad \quad \frac{i (\psi[i] - 2 \psi[1+i] + \psi[2+i])}{dx^2} - i \psi[-1+i] \\
& \quad \quad \quad v[-1+i] - 2 \left(\frac{i (\psi[-1+i] - 2 \psi[i] + \psi[1+i])}{dx^2} - \right. \\
& \quad \quad \quad \quad i \psi[i] v[i] \left. \right) - i \psi[1+i] v[1+i] \left. \right) \left. \right) + \\
& \quad \frac{1}{dx^2} i \left(\frac{i (\psi[-1+i] - 2 \psi[i] + \psi[1+i])}{dx^2} + \right. \\
& \quad \quad \frac{i (\psi[1+i] - 2 \psi[2+i] + \psi[3+i])}{dx^2} - i \psi[i] v[i] - \\
& \quad \quad \quad 2 \left(\frac{i (\psi[i] - 2 \psi[1+i] + \psi[2+i])}{dx^2} - i \psi[1+i] v[1+i] \right) - \\
& \quad \quad \quad \quad i \psi[2+i] v[2+i] \left. \right) \left. \right) + \frac{1}{dx^2} \\
& i \left(-i v[i] \left(\frac{i (\psi[-1+i] - 2 \psi[i] + \psi[1+i])}{dx^2} - i \psi[i] v[i] \right) + \right. \\
& \quad \frac{1}{dx^2} i \left(\frac{i (\psi[-2+i] - 2 \psi[-1+i] + \psi[i])}{dx^2} + \right. \\
& \quad \quad \frac{i (\psi[i] - 2 \psi[1+i] + \psi[2+i])}{dx^2} - i \psi[-1+i] v[-1+i] - \\
& \quad \quad \quad 2 \left(\frac{i (\psi[-1+i] - 2 \psi[i] + \psi[1+i])}{dx^2} - i \psi[i] v[i] \right) - \\
& \quad \quad \quad \quad i \psi[1+i] v[1+i] \left. \right) - i v[2+i]
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{i \left(\psi[1+i] - 2 \psi[2+i] + \psi[3+i] \right)}{dx^2} - i \psi[2+i] v[2+i] \right) - \\
& 2 \left(-i v[1+i] \left(\frac{i \left(\psi[i] - 2 \psi[1+i] + \psi[2+i] \right)}{dx^2} - i \psi[1+i] v[1+i] \right) + \right. \\
& \quad \frac{1}{dx^2} i \left(\frac{i \left(\psi[-1+i] - 2 \psi[i] + \psi[1+i] \right)}{dx^2} + \right. \\
& \quad \quad \frac{i \left(\psi[1+i] - 2 \psi[2+i] + \psi[3+i] \right)}{dx^2} - \\
& \quad \quad i \psi[i] v[i] - 2 \left(\frac{i \left(\psi[i] - 2 \psi[1+i] + \psi[2+i] \right)}{dx^2} - \right. \\
& \quad \quad \quad i \psi[1+i] v[1+i] \left. \right) - i \psi[2+i] \\
& \quad \quad \quad v[2+i] \left. \right) + \frac{1}{dx^2} i \left(\frac{i \left(\psi[i] - 2 \psi[1+i] + \psi[2+i] \right)}{dx^2} + \right. \\
& \quad \quad \frac{i \left(\psi[2+i] - 2 \psi[3+i] + \psi[4+i] \right)}{dx^2} - i \psi[1+i] \\
& \quad \quad v[1+i] - 2 \left(\frac{i \left(\psi[1+i] - 2 \psi[2+i] + \psi[3+i] \right)}{dx^2} - \right. \\
& \quad \quad \quad i \psi[2+i] v[2+i] \left. \right) - i \psi[3+i] v[3+i] \left. \right) \left. \right) \left. \right) \left. \right) \left. \right) \\
& \frac{i \left(\psi[-1+i] - 2 \psi[i] + \psi[1+i] \right)}{dx^2} - i \psi[i] v[i] + \\
& dt \left(-i v[i] \left(\frac{i \left(\psi[-1+i] - 2 \psi[i] + \psi[1+i] \right)}{dx^2} - i \psi[i] v[i] \right) + \frac{1}{dx^2} \right. \\
& \quad i \left(\frac{i \left(\psi[-2+i] - 2 \psi[-1+i] + \psi[i] \right)}{dx^2} + \frac{i \left(\psi[i] - 2 \psi[1+i] + \psi[2+i] \right)}{dx^2} - i \psi[-1+i] \right. \\
& \quad \quad v[-1+i] - 2 \left(\frac{i \left(\psi[-1+i] - 2 \psi[i] + \psi[1+i] \right)}{dx^2} - i \psi[i] v[i] \right) - i \psi[1+i] v[1+i] \left. \right) +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{2} \, dt \left(-i \, v[[i]] \left(-i \, v[[i]] \left(\frac{i \left(\psi[-1+i] - 2 \, \psi[[i]] + \psi[1+i] \right)}{dx^2} - i \, \psi[[i]] \, v[[i]] \right) + \right. \right. \\
& \quad \frac{1}{dx^2} \, i \left(\frac{i \left(\psi[-2+i] - 2 \, \psi[-1+i] + \psi[[i]] \right)}{dx^2} + \right. \\
& \quad \left. \frac{i \left(\psi[[i]] - 2 \, \psi[1+i] + \psi[2+i] \right)}{dx^2} - i \, \psi[-1+i] \, v[-1+i] - \right. \\
& \quad \left. \left. 2 \left(\frac{i \left(\psi[-1+i] - 2 \, \psi[[i]] + \psi[1+i] \right)}{dx^2} - i \, \psi[[i]] \, v[[i]] \right) - i \, \psi[1+i] \, v[1+i] \right) \right) + \\
& \quad \frac{1}{dx^2} \, i \left(-i \, v[-1+i] \left(\frac{i \left(\psi[-2+i] - 2 \, \psi[-1+i] + \psi[[i]] \right)}{dx^2} - i \, \psi[-1+i] \, v[-1+i] \right) + \right. \\
& \quad \frac{1}{dx^2} \, i \left(\frac{i \left(\psi[-3+i] - 2 \, \psi[-2+i] + \psi[-1+i] \right)}{dx^2} + \right. \\
& \quad \frac{i \left(\psi[-1+i] - 2 \, \psi[[i]] + \psi[1+i] \right)}{dx^2} - i \, \psi[-2+i] \, v[-2+i] - 2 \\
& \quad \left(\frac{i \left(\psi[-2+i] - 2 \, \psi[-1+i] + \psi[[i]] \right)}{dx^2} - i \, \psi[-1+i] \, v[-1+i] \right) - i \, \psi[[i]] \, v[[i]] \Big) - \\
& \quad i \, v[1+i] \left(\frac{i \left(\psi[[i]] - 2 \, \psi[1+i] + \psi[2+i] \right)}{dx^2} - i \, \psi[1+i] \, v[1+i] \right) - \\
& \quad 2 \left(-i \, v[[i]] \left(\frac{i \left(\psi[-1+i] - 2 \, \psi[[i]] + \psi[1+i] \right)}{dx^2} - i \, \psi[[i]] \, v[[i]] \right) + \frac{1}{dx^2} \, i \right. \\
& \quad \left(\frac{i \left(\psi[-2+i] - 2 \, \psi[-1+i] + \psi[[i]] \right)}{dx^2} + \frac{i \left(\psi[[i]] - 2 \, \psi[1+i] + \psi[2+i] \right)}{dx^2} - \right. \\
& \quad \left. i \, \psi[-1+i] \, v[-1+i] - 2 \left(\frac{i \left(\psi[-1+i] - 2 \, \psi[[i]] + \psi[1+i] \right)}{dx^2} - \right. \\
& \quad \left. \left. i \, \psi[[i]] \, v[[i]] \right) - i \, \psi[1+i] \, v[1+i] \right) \Big) + \\
& \quad \frac{1}{dx^2} \, i \left(\frac{i \left(\psi[-1+i] - 2 \, \psi[[i]] + \psi[1+i] \right)}{dx^2} + \frac{i \left(\psi[1+i] - 2 \, \psi[2+i] + \psi[3+i] \right)}{dx^2} - \right.
\end{aligned}$$

$$\begin{aligned}
& i \psi[i] v[i] - 2 \left(\frac{i (\psi[i] - 2 \psi[1+i] + \psi[2+i])}{dx^2} - \right. \\
& \quad \left. i \psi[1+i] v[1+i] - i \psi[2+i] v[2+i] \right) \Bigg) + \\
& \frac{1}{2} dt \left(-i v[i] \left(-i v[i] \left(-i v[i] \left(\frac{i (\psi[-1+i] - 2 \psi[i] + \psi[1+i])}{dx^2} - i \psi[i] v[i] \right) + \frac{1}{dx^2} \right. \right. \right. \\
& \quad \left. \left. i \left(\frac{i (\psi[-2+i] - 2 \psi[-1+i] + \psi[i])}{dx^2} + \frac{i (\psi[i] - 2 \psi[1+i] + \psi[2+i])}{dx^2} - \right. \right. \right. \\
& \quad \left. \left. i \psi[-1+i] v[-1+i] - 2 \left(\frac{i (\psi[-1+i] - 2 \psi[i] + \psi[1+i])}{dx^2} - \right. \right. \right. \\
& \quad \left. \left. i \psi[i] v[i] \right) - i \psi[1+i] v[1+i] \right) \Bigg) + \\
& \frac{1}{dx^2} i \left(-i v[-1+i] \left(\frac{i (\psi[-2+i] - 2 \psi[-1+i] + \psi[i])}{dx^2} - i \psi[-1+i] \right. \right. \\
& \quad \left. \left. v[-1+i] \right) + \frac{1}{dx^2} i \left(\frac{i (\psi[-3+i] - 2 \psi[-2+i] + \psi[-1+i])}{dx^2} + \right. \right. \\
& \quad \left. \left. \frac{i (\psi[-1+i] - 2 \psi[i] + \psi[1+i])}{dx^2} - i \psi[-2+i] v[-2+i] - \right. \right. \\
& \quad \left. \left. 2 \left(\frac{i (\psi[-2+i] - 2 \psi[-1+i] + \psi[i])}{dx^2} - i \psi[-1+i] v[-1+i] \right) - \right. \right. \\
& \quad \left. \left. i \psi[i] v[i] \right) - i v[1+i] \left(\frac{i (\psi[i] - 2 \psi[1+i] + \psi[2+i])}{dx^2} - \right. \right. \\
& \quad \left. \left. i \psi[1+i] v[1+i] \right) - 2 \left(-i v[i] \left(\frac{i (\psi[-1+i] - 2 \psi[i] + \psi[1+i])}{dx^2} - \right. \right. \\
& \quad \left. \left. i \psi[i] v[i] \right) + \frac{1}{dx^2} i \left(\frac{i (\psi[-2+i] - 2 \psi[-1+i] + \psi[i])}{dx^2} + \right. \right. \\
& \quad \left. \left. \frac{i (\psi[i] - 2 \psi[1+i] + \psi[2+i])}{dx^2} - i \psi[-1+i] v[-1+i] - \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 2 \left(\frac{i (\psi[-1+i] - 2 \psi[i] + \psi[1+i])}{dx^2} - i \psi[i] v[i] \right) - \\
& i \psi[1+i] v[1+i] \Bigg) + \frac{1}{dx^2} i \left(\frac{i (\psi[-1+i] - 2 \psi[i] + \psi[1+i])}{dx^2} + \right. \\
& \frac{i (\psi[1+i] - 2 \psi[2+i] + \psi[3+i])}{dx^2} - i \psi[i] v[i] - \\
& 2 \left(\frac{i (\psi[i] - 2 \psi[1+i] + \psi[2+i])}{dx^2} - i \psi[1+i] v[1+i] \right) - \\
& i \psi[2+i] v[2+i] \Bigg) \Bigg) + \\
& \frac{1}{dx^2} i \left(-i v[-1+i] \left(-i v[-1+i] \left(\frac{i (\psi[-2+i] - 2 \psi[-1+i] + \psi[i])}{dx^2} - i \psi[-1+i] v[-1+i] \right) + \right. \right. \\
& \frac{1}{dx^2} i \left(\frac{i (\psi[-3+i] - 2 \psi[-2+i] + \psi[-1+i])}{dx^2} + \right. \\
& \frac{i (\psi[-1+i] - 2 \psi[i] + \psi[1+i])}{dx^2} - i \psi[-2+i] v[-2+i] - \\
& 2 \left(\frac{i (\psi[-2+i] - 2 \psi[-1+i] + \psi[i])}{dx^2} - i \psi[-1+i] v[-1+i] \right) - \\
& i \psi[i] v[i] \Bigg) + \frac{1}{dx^2} i \left(-i v[-2+i] \right. \\
& \left. \left(\frac{i (\psi[-3+i] - 2 \psi[-2+i] + \psi[-1+i])}{dx^2} - i \psi[-2+i] v[-2+i] \right) + \right. \\
& \frac{1}{dx^2} i \left(\frac{i (\psi[-4+i] - 2 \psi[-3+i] + \psi[-2+i])}{dx^2} + \right. \\
& \frac{i (\psi[-2+i] - 2 \psi[-1+i] + \psi[i])}{dx^2} - i \psi[-3+i] v[-3+i] - \\
& 2 \left(\frac{i (\psi[-3+i] - 2 \psi[-2+i] + \psi[-1+i])}{dx^2} - \right.
\end{aligned}$$

$$\begin{aligned}
& i \psi[-2+i] v[-2+i] \Bigg) - i \psi[-1+i] v[-1+i] \Bigg) - \\
& i v[i] \left(\frac{i (\psi[-1+i] - 2 \psi[i] + \psi[1+i])}{dx^2} - i \psi[i] v[i] \right) - \\
& 2 \left(-i v[-1+i] \left(\frac{i (\psi[-2+i] - 2 \psi[-1+i] + \psi[i])}{dx^2} - i \psi[-1+i] \right. \right. \\
& \left. \left. v[-1+i] \right) + \frac{1}{dx^2} i \left(\frac{i (\psi[-3+i] - 2 \psi[-2+i] + \psi[-1+i])}{dx^2} + \right. \right. \\
& \left. \left. \frac{i (\psi[-1+i] - 2 \psi[i] + \psi[1+i])}{dx^2} - i \psi[-2+i] v[-2+i] - \right. \right. \\
& \left. \left. 2 \left(\frac{i (\psi[-2+i] - 2 \psi[-1+i] + \psi[i])}{dx^2} - i \psi[-1+i] v[-1+i] \right) - \right. \right. \\
& \left. \left. i \psi[i] v[i] \right) \right) + \frac{1}{dx^2} i \left(\frac{i (\psi[-2+i] - 2 \psi[-1+i] + \psi[i])}{dx^2} + \right. \\
& \left. \frac{i (\psi[i] - 2 \psi[1+i] + \psi[2+i])}{dx^2} - i \psi[-1+i] v[-1+i] - \right. \\
& \left. 2 \left(\frac{i (\psi[-1+i] - 2 \psi[i] + \psi[1+i])}{dx^2} - i \psi[i] v[i] \right) - i \psi[1+i] \right. \\
& \left. v[1+i] \right) \Bigg) - i v[1+i] \left(-i v[1+i] \left(\frac{i (\psi[i] - 2 \psi[1+i] + \psi[2+i])}{dx^2} - \right. \right. \\
& \left. \left. i \psi[1+i] v[1+i] \right) + \frac{1}{dx^2} i \left(\frac{i (\psi[-1+i] - 2 \psi[i] + \psi[1+i])}{dx^2} + \right. \right. \\
& \left. \left. \frac{i (\psi[1+i] - 2 \psi[2+i] + \psi[3+i])}{dx^2} - i \psi[i] v[i] - \right. \right. \\
& \left. \left. 2 \left(\frac{i (\psi[i] - 2 \psi[1+i] + \psi[2+i])}{dx^2} - i \psi[1+i] v[1+i] \right) - \right. \right. \\
& \left. \left. i \psi[2+i] v[2+i] \right) \right) \Bigg) -
\end{aligned}$$

$$\begin{aligned}
& 2 \left(-i v[i] \left(-i v[i] \left(\frac{i (\psi[-1+i] - 2 \psi[i] + \psi[1+i])}{dx^2} - i \psi[i] v[i] \right) + \right. \right. \\
& \quad \frac{1}{dx^2} i \left(\frac{i (\psi[-2+i] - 2 \psi[-1+i] + \psi[i])}{dx^2} + \right. \\
& \quad \quad \frac{i (\psi[i] - 2 \psi[1+i] + \psi[2+i])}{dx^2} - i \psi[-1+i] v[-1+i] - \\
& \quad \quad \quad 2 \left(\frac{i (\psi[-1+i] - 2 \psi[i] + \psi[1+i])}{dx^2} - i \psi[i] v[i] \right) - \\
& \quad \quad \quad \left. \left. i \psi[1+i] v[1+i] \right) \right) + \frac{1}{dx^2} i \left(-i v[-1+i] \right. \\
& \quad \quad \left. \left(\frac{i (\psi[-2+i] - 2 \psi[-1+i] + \psi[i])}{dx^2} - i \psi[-1+i] v[-1+i] \right) + \right. \\
& \quad \quad \frac{1}{dx^2} i \left(\frac{i (\psi[-3+i] - 2 \psi[-2+i] + \psi[-1+i])}{dx^2} + \right. \\
& \quad \quad \quad \frac{i (\psi[-1+i] - 2 \psi[i] + \psi[1+i])}{dx^2} - i \psi[-2+i] v[-2+i] - \\
& \quad \quad \quad 2 \left(\frac{i (\psi[-2+i] - 2 \psi[-1+i] + \psi[i])}{dx^2} - i \psi[-1+i] v[-1+i] \right) - \\
& \quad \quad \quad \left. i \psi[i] v[i] \right) - i v[1+i] \left(\frac{i (\psi[i] - 2 \psi[1+i] + \psi[2+i])}{dx^2} - i \psi[1+i] v[1+i] \right) - \\
& \quad \quad \quad 2 \left(-i v[i] \left(\frac{i (\psi[-1+i] - 2 \psi[i] + \psi[1+i])}{dx^2} - \right. \right. \\
& \quad \quad \quad \left. \left. i \psi[i] v[i] \right) + \frac{1}{dx^2} i \left(\frac{i (\psi[-2+i] - 2 \psi[-1+i] + \psi[i])}{dx^2} + \right. \right. \\
& \quad \quad \quad \left. \left. \frac{i (\psi[i] - 2 \psi[1+i] + \psi[2+i])}{dx^2} - i \psi[-1+i] v[-1+i] - \right. \right. \\
& \quad \quad \quad \left. \left. 2 \left(\frac{i (\psi[-1+i] - 2 \psi[i] + \psi[1+i])}{dx^2} - \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left. \left. \left. \left. i \psi[i] v[i] \right) - i \psi[1+i] v[1+i] \right) \right) \right) + \\
& \frac{1}{dx^2} i \left(\frac{i (\psi[-1+i] - 2 \psi[i] + \psi[1+i])}{dx^2} + \right. \\
& \quad \left. \frac{i (\psi[1+i] - 2 \psi[2+i] + \psi[3+i])}{dx^2} - \right. \\
& \quad \left. i \psi[i] v[i] - 2 \left(\frac{i (\psi[i] - 2 \psi[1+i] + \psi[2+i])}{dx^2} - \right. \right. \\
& \quad \left. \left. i \psi[1+i] v[1+i] \right) - i \psi[2+i] v[2+i] \right) \right) \right) + \\
& \frac{1}{dx^2} i \left(-i v[i] \left(\frac{i (\psi[-1+i] - 2 \psi[i] + \psi[1+i])}{dx^2} - i \psi[i] v[i] \right) + \right. \\
& \quad \frac{1}{dx^2} i \left(\frac{i (\psi[-2+i] - 2 \psi[-1+i] + \psi[i])}{dx^2} + \right. \\
& \quad \left. \frac{i (\psi[i] - 2 \psi[1+i] + \psi[2+i])}{dx^2} - i \psi[-1+i] v[-1+i] - \right. \\
& \quad \left. 2 \left(\frac{i (\psi[-1+i] - 2 \psi[i] + \psi[1+i])}{dx^2} - i \psi[i] v[i] \right) - \right. \\
& \quad \left. i \psi[1+i] v[1+i] \right) - i v[2+i] \\
& \quad \left(\frac{i (\psi[1+i] - 2 \psi[2+i] + \psi[3+i])}{dx^2} - i \psi[2+i] v[2+i] \right) - \\
& \quad \left. 2 \left(-i v[1+i] \left(\frac{i (\psi[i] - 2 \psi[1+i] + \psi[2+i])}{dx^2} - i \psi[1+i] v[1+i] \right) + \right. \right. \\
& \quad \left. \frac{1}{dx^2} i \left(\frac{i (\psi[-1+i] - 2 \psi[i] + \psi[1+i])}{dx^2} + \right. \right. \\
& \quad \left. \left. \frac{i (\psi[1+i] - 2 \psi[2+i] + \psi[3+i])}{dx^2} - i \psi[i] v[i] - \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 2 \left(\frac{i (\psi[i] - 2 \psi[1+i] + \psi[2+i])}{dx^2} - i \psi[1+i] v[1+i] \right) - \\
& i \psi[2+i] v[2+i] \Bigg) + \frac{1}{dx^2} i \left(\frac{i (\psi[i] - 2 \psi[1+i] + \psi[2+i])}{dx^2} + \right. \\
& \frac{i (\psi[2+i] - 2 \psi[3+i] + \psi[4+i])}{dx^2} - i \psi[1+i] v[1+i] - \\
& 2 \left(\frac{i (\psi[1+i] - 2 \psi[2+i] + \psi[3+i])}{dx^2} - i \psi[2+i] v[2+i] \right) - \\
& \left. i \psi[3+i] v[3+i] \right) \Bigg) \Bigg) \Bigg) \Bigg) \Bigg) \Bigg)
\end{aligned}$$

k1

$$\frac{i (\psi[-1+i] - 2 \psi[i] + \psi[1+i])}{dx^2} - i \psi[i] v[i]$$

Simplify[k1]

$$(I (\psi[-1+i] + \psi[1+i] - \psi[i] (2 + dx^2 v[i]))) / dx^2$$

Simplify[k2]

$$\begin{aligned}
& -((1/(2 dx^4)) (dt \psi[-2+i] + 6 dt \psi[i] + 4 I dx^2 \psi[i] - \\
& 4 dt \psi[1+i] - 2 I dx^2 \psi[1+i] + dt \psi[2+i] + \\
& 4 dt dx^2 \psi[i] v[i] + 2 I dx^4 \psi[i] v[i] - \\
& dt dx^2 \psi[1+i] v[i] + dt dx^4 \psi[i] v[i]^2 - \\
& \psi[-1+i] (4 dt + 2 I dx^2 + dt dx^2 v[-1+i] + dt dx^2 v[i]) - \\
& dt dx^2 \psi[1+i] v[1+i]))
\end{aligned}$$

Simplify[k3]

$$\begin{aligned}
& (1/(4dx^6))(I*(-dt^2)*psi[-3 + i] - 15*dt^2*psi[-1 + i] - \\
& \quad 8*I*dt*dx^2*psi[-1 + i] + 4*dx^4*psi[-1 + i] + 20*dt^2*psi[i] + \\
& \quad 12*I*dt*dx^2*psi[i] - 8*dx^4*psi[i] - 15*dt^2*psi[1 + i] - \\
& \quad 8*I*dt*dx^2*psi[1 + i] + 4*dx^4*psi[1 + i] + 6*dt^2*psi[2 + i] + \\
& \quad 2*I*dt*dx^2*psi[2 + i] - dt^2*psi[3 + i] - 6*dt^2*dx^2*psi[-1 + i])* \\
& \quad v[-1 + i] - 2*I*dt*dx^4*psi[-1 + i]*v[-1 + i] + \\
& \quad dt^2*dx^2*psi[i]*v[-1 + i] - dt^2*dx^4*psi[-1 + i]*v[-1 + i]^2 - \\
& \quad 6*dt^2*dx^2*psi[-1 + i]*v[i] - 2*I*dt*dx^4*psi[-1 + i]*v[i] + \\
& \quad 16*dt^2*dx^2*psi[i]*v[i] + 8*I*dt*dx^4*psi[i]*v[i] - \\
& \quad 4*dx^6*psi[i]*v[i] - 6*dt^2*dx^2*psi[1 + i]*v[i] - \\
& \quad 2*I*dt*dx^4*psi[1 + i]*v[i] + dt^2*dx^2*psi[2 + i]*v[i] - \\
& \quad dt^2*dx^4*psi[-1 + i]*v[-1 + i]*v[i] - dt^2*dx^4*psi[-1 + i]*v[i]^2 + \\
& \quad 6*dt^2*dx^4*psi[i]*v[i]^2 + 2*I*dt*dx^6*psi[i]*v[i]^2 - \\
& \quad dt^2*dx^4*psi[1 + i]*v[i]^2 + dt^2*dx^6*psi[i]*v[i]^3 + \\
& \quad dt*psi[-2 + i]*(6*dt + 2*I*dx^2 + dt*dx^2*v[-2 + i] + dt*dx^2*v[-1 + i] + \\
& \quad \quad dt*dx^2*v[i]) + dt^2*dx^2*psi[i]*v[1 + i] - \\
& \quad 6*dt^2*dx^2*psi[1 + i]*v[1 + i] - 2*I*dt*dx^4*psi[1 + i]*v[1 + i] + \\
& \quad dt^2*dx^2*psi[2 + i]*v[1 + i] - dt^2*dx^4*psi[1 + i]*v[i]*v[1 + i] - \\
& \quad dt^2*dx^4*psi[1 + i]*v[1 + i]^2 + dt^2*dx^2*psi[2 + i]*v[2 + i])
\end{aligned}$$
Simplify[k4]

$$\begin{aligned}
& (1/(4dx^8))(dt^3*psi[-4 + i] + 28*dt^3*psi[-2 + i] + \\
& \quad 12*I*dt^2*dx^2*psi[-2 + i] - 4*dt*dx^4*psi[-2 + i] - 56*dt^3*psi[-1 + i] - \\
& \quad 30*I*dt^2*dx^2*psi[-1 + i] + 16*dt*dx^4*psi[-1 + i] + \\
& \quad 4*I*dx^6*psi[-1 + i] + 70*dt^3*psi[i] + 40*I*dt^2*dx^2*psi[i] - \\
& \quad 24*dt*dx^4*psi[i] - 8*I*dx^6*psi[i] - 56*dt^3*psi[1 + i] - \\
& \quad 30*I*dt^2*dx^2*psi[1 + i] + 16*dt*dx^4*psi[1 + i] + 4*I*dx^6*psi[1 + i] + \\
& \quad 28*dt^3*psi[2 + i] + 12*I*dt^2*dx^2*psi[2 + i] - 4*dt*dx^4*psi[2 + i] - \\
& \quad 8*dt^3*psi[3 + i] - 2*I*dt^2*dx^2*psi[3 + i] + dt^3*psi[4 + i] + \\
& \quad 8*dt^3*dx^2*psi[-2 + i]*v[-2 + i] + 2*I*dt^2*dx^4*psi[-2 + i]*v[-2 + i] - \\
& \quad dt^3*dx^2*psi[-1 + i]*v[-2 + i] + dt^3*dx^4*psi[-2 + i]*v[-2 + i]^2 + \\
& \quad 8*dt^3*dx^2*psi[-2 + i]*v[-1 + i] + 2*I*dt^2*dx^4*psi[-2 + i]*v[-1 + i] - \\
& \quad 29*dt^3*dx^2*psi[-1 + i]*v[-1 + i] - 12*I*dt^2*dx^4*psi[-1 + i]* \\
& \quad \quad v[-1 + i] + 4*dt*dx^6*psi[-1 + i]*v[-1 + i] + \\
& \quad 8*dt^3*dx^2*psi[i]*v[-1 + i] + 2*I*dt^2*dx^4*psi[i]*v[-1 + i] - \\
& \quad dt^3*dx^2*psi[1 + i]*v[-1 + i] + dt^3*dx^4*psi[-2 + i]*v[-2 + i]* \\
& \quad \quad v[-1 + i] + dt^3*dx^4*psi[-2 + i]*v[-1 + i]^2 - \\
& \quad 8*dt^3*dx^4*psi[-1 + i]*v[-1 + i]^2 - 2*I*dt^2*dx^6*psi[-1 + i]* \\
& \quad \quad v[-1 + i]^2 + dt^3*dx^4*psi[i]*v[-1 + i]^2 - \\
& \quad dt^3*dx^6*psi[-1 + i]*v[-1 + i]^3 + 8*dt^3*dx^2*psi[-2 + i]*v[i] + \\
& \quad 2*I*dt^2*dx^4*psi[-2 + i]*v[i] - 29*dt^3*dx^2*psi[-1 + i]*v[i] - \\
& \quad 12*I*dt^2*dx^4*psi[-1 + i]*v[i] + 4*dt*dx^6*psi[-1 + i]*v[i] + \\
& \quad 64*dt^3*dx^2*psi[i]*v[i] + 32*I*dt^2*dx^4*psi[i]*v[i] - \\
& \quad 16*dt*dx^6*psi[i]*v[i] - 4*I*dx^8*psi[i]*v[i] - \\
& \quad 29*dt^3*dx^2*psi[1 + i]*v[i] - 12*I*dt^2*dx^4*psi[1 + i]*v[i] + \\
& \quad 4*dt*dx^6*psi[1 + i]*v[i] + 8*dt^3*dx^2*psi[2 + i]*v[i] + \\
& \quad 2*I*dt^2*dx^4*psi[2 + i]*v[i] - dt^3*dx^2*psi[3 + i]*v[i] + \\
& \quad dt^3*dx^4*psi[-2 + i]*v[-2 + i]*v[i] + dt^3*dx^4*psi[-2 + i]*v[-1 + i]*
\end{aligned}$$

```

v[[i]] - 8*dt^3*dx^4*psi[[-1 + i]]*v[[-1 + i]]*v[[i]] -
2*I*dt^2*dx^6*psi[[-1 + i]]*v[[-1 + i]]*v[[i]] + 2*dt^3*dx^4*psi[[i]]*v[[-1 + i]]*
v[[i]] - dt^3*dx^6*psi[[-1 + i]]*v[[-1 + i]]^2*v[[i]] +
dt^3*dx^4*psi[[-2 + i]]*v[[i]]^2 - 8*dt^3*dx^4*psi[[-1 + i]]*v[[i]]^2 -
2*I*dt^2*dx^6*psi[[-1 + i]]*v[[i]]^2 + 30*dt^3*dx^4*psi[[i]]*v[[i]]^2 +
12*I*dt^2*dx^6*psi[[i]]*v[[i]]^2 - 4*dt*dx^8*psi[[i]]*v[[i]]^2 -
8*dt^3*dx^4*psi[[1 + i]]*v[[i]]^2 - 2*I*dt^2*dx^6*psi[[1 + i]]*v[[i]]^2 +
dt^3*dx^4*psi[[2 + i]]*v[[i]]^2 - dt^3*dx^6*psi[[-1 + i]]*v[[-1 + i]]*v[[i]]^2 -
dt^3*dx^6*psi[[-1 + i]]*v[[i]]^3 + 8*dt^3*dx^6*psi[[i]]*v[[i]]^3 +
2*I*dt^2*dx^8*psi[[i]]*v[[i]]^3 - dt^3*dx^6*psi[[1 + i]]*v[[i]]^3 +
dt^3*dx^8*psi[[i]]*v[[i]]^4 - dt^2*psi[[-3 + i]]*
(8*dt + 2*I*dx^2 + dt*dx^2*v[[-3 + i]] + dt*dx^2*v[[-2 + i]] +
dt*dx^2*v[[-1 + i]] + dt*dx^2*v[[i]]) - dt^3*dx^2*psi[[-1 + i]]*v[[1 + i]] +
8*dt^3*dx^2*psi[[i]]*v[[1 + i]] + 2*I*dt^2*dx^4*psi[[i]]*v[[1 + i]] -
29*dt^3*dx^2*psi[[1 + i]]*v[[1 + i]] - 12*I*dt^2*dx^4*psi[[1 + i]]*v[[1 + i]] +
4*dt*dx^6*psi[[1 + i]]*v[[1 + i]] + 8*dt^3*dx^2*psi[[2 + i]]*v[[1 + i]] +
2*I*dt^2*dx^4*psi[[2 + i]]*v[[1 + i]] - dt^3*dx^2*psi[[3 + i]]*v[[1 + i]] +
2*dt^3*dx^4*psi[[i]]*v[[i]]*v[[1 + i]] - 8*dt^3*dx^4*psi[[1 + i]]*v[[i]]*
v[[1 + i]] - 2*I*dt^2*dx^6*psi[[1 + i]]*v[[i]]*v[[1 + i]] +
dt^3*dx^4*psi[[2 + i]]*v[[i]]*v[[1 + i]] - dt^3*dx^6*psi[[1 + i]]*v[[i]]^2*
v[[1 + i]] + dt^3*dx^4*psi[[i]]*v[[1 + i]]^2 - 8*dt^3*dx^4*psi[[1 + i]]*
v[[1 + i]]^2 - 2*I*dt^2*dx^6*psi[[1 + i]]*v[[1 + i]]^2 +
dt^3*dx^4*psi[[2 + i]]*v[[1 + i]]^2 - dt^3*dx^6*psi[[1 + i]]*v[[i]]*
v[[1 + i]]^2 - dt^3*dx^6*psi[[1 + i]]*v[[1 + i]]^3 -
dt^3*dx^2*psi[[1 + i]]*v[[2 + i]] + 8*dt^3*dx^2*psi[[2 + i]]*v[[2 + i]] +
2*I*dt^2*dx^4*psi[[2 + i]]*v[[2 + i]] - dt^3*dx^2*psi[[3 + i]]*v[[2 + i]] +
dt^3*dx^4*psi[[2 + i]]*v[[i]]*v[[2 + i]] + dt^3*dx^4*psi[[2 + i]]*v[[1 + i]]*
v[[2 + i]] + dt^3*dx^4*psi[[2 + i]]*v[[2 + i]]^2 -
dt^3*dx^2*psi[[3 + i]]*v[[3 + i]])

```