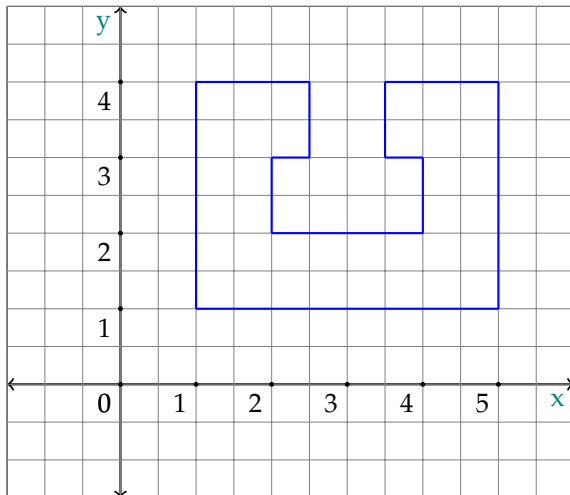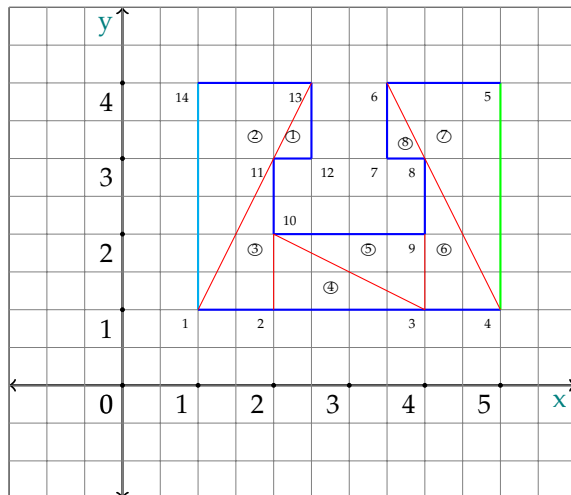# Técnicas de simulación por computadora

Grupo O

2 de junio de 2018

Dada la figura y ecuación diferencial:

$$\nabla(10\nabla T) = -100$$

Mallado de la figura y establecimiento de contornos para condiciones de Dirichlet y Neumann:

O Neumann
O Dirichlet

Tabla de conectividad:

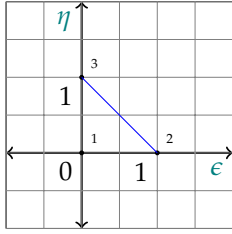| elemento	1	2	3
①	11	12	13
②	1	13	14
③	1	2	11
④	2	3	10
⑤	10	3	9
⑥	3	4	8
⑦	6	4	5
⑧	6	7	8

Condiciones a utilizar:

$$\Gamma_D =$$
$$\Gamma_N =$$

Aproximación en el plano isoparamétrico:

$$T \approx f_1 T_1 + f_2 T_2 + f_3 T_3$$
$$\approx N_1 T_1 + N_2 T_2 + N_3 T_3$$
$$\approx \begin{bmatrix} N_1 & N_2 & N_3 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix}$$
$$\approx \underline{\underline{N}} \vec{T}$$

con $N_1 = 1 - \epsilon - \eta$, $N_2 = \epsilon$, $N_3 = \eta$
$x \approx (x_2 - x_1)\epsilon + (x_3 - x_1)\eta + x_1$
$y \approx (y_2 - y_1)\epsilon + (y_3 - y_1)\eta + y_1$

Sustitución de la función por su aproximación:

$$\nabla(10\nabla T) = -100$$
$$\nabla(10\nabla \underline{\underline{N}}\vec{T}) \approx -100$$
$$\nabla(10\nabla \underline{\underline{N}}\vec{T}) + 100 \neq 0$$
$$\nabla(10\nabla \underline{\underline{N}}\vec{T}) + 100 = \mathcal{R}$$

W.R.M.:

$$\int_{\Omega} \underline{\underline{W}} \mathcal{R} \, d\Omega = 0$$
$$\int_{A} \underline{\underline{W}} \mathcal{R} \, dA = 0$$
$$\int_{A} \underline{\underline{W}} (\nabla(10\nabla \underline{\underline{N}}\vec{T}) + 100) \, dA = 0$$

Galerkin:

$$\underline{\underline{W}} = \underline{N}^T$$
$$\int_{A} \underline{N}^T (\nabla(10\nabla \underline{\underline{N}}\vec{T}) + 100) \, dA = 0$$
$$\int_{A} \underline{N}^T (\nabla(10\nabla \underline{\underline{N}}\vec{T})) \, dA + \int_{A} \underline{N}^T (100) \, dA = 0$$

Integración por partes:

sea $U = \underline{N}^T$, $dU = \nabla \underline{N}^T$
sea $dV = \nabla(10\nabla \underline{\underline{N}}\vec{T})$, $V = 10\nabla(\underline{\underline{N}}\vec{T})$

$$\int_{A} \underline{N}^T (\nabla(10\nabla \underline{\underline{N}}\vec{T})) \, dA = [\underline{\underline{N}}^T 10\nabla(\underline{\underline{N}}\vec{T})]_{\Gamma_N} - \int_{A} \nabla \underline{N}^t 10\nabla(\underline{\underline{N}}\vec{T}) \, dA$$

Trabajando en la ecuación:

$$-\int_A \nabla \underline{\underline{N}}^T 10 \nabla (\underline{\underline{N}} \vec{T}) \, dA + \int_A \underline{\underline{N}}^T (100) \, dA = 0$$

$$\int_A \nabla \underline{\underline{N}}^T 10 \nabla (\underline{\underline{N}} \vec{T}) \, dA = \int_A \underline{\underline{N}}^T (100) \, dA$$

$$10 \int_A \nabla \underline{\underline{N}}^T \nabla (\underline{\underline{N}}) \, dA (\vec{T}) = 100 \int_A \underline{\underline{N}}^T \, dA$$

Lado derecho:

$$100 \int_A \underline{\underline{N}}^T \, dA = 100 \int_A \begin{bmatrix} 1 - \epsilon - \eta \\ \epsilon \\ \eta \end{bmatrix} dA$$

$$= 100 \int_A \begin{bmatrix} 1 - \epsilon - \eta \\ \epsilon \\ \eta \end{bmatrix} dx \, dy$$

Llevando la integral al plano isoparamétrico con un jacobiano:

$$dx \, dy = D \, d\epsilon \, d\eta$$

$$J = \begin{bmatrix} \frac{\delta x}{\delta \epsilon} & \frac{\delta x}{\delta \eta} \\ \frac{\delta y}{\delta \epsilon} & \frac{\delta y}{\delta \eta} \end{bmatrix}$$

$$= \begin{bmatrix} (x_2 - x_1) & (x_3 - x_1) \\ (y_2 - y_1) & (y_3 - y_1) \end{bmatrix}$$

$$D = |J|$$

$$= (x_2 - x_1)(y_3 - y_1) - (x_3 - x_1)(y_2 - y_1)$$

Sustituyendo en la integral:

$$100 \int_A \begin{bmatrix} 1 - \epsilon - \eta \\ \epsilon \\ \eta \end{bmatrix} dA = 100 \int_0^1 \int_0^1 \begin{bmatrix} 1 - \epsilon - \eta \\ \epsilon \\ \eta \end{bmatrix} \begin{vmatrix} (x_2 - x_1) & (x_3 - x_1) \\ (y_2 - y_1) & (y_3 - y_1) \end{vmatrix} d\epsilon \, d\eta$$

$$= 100 \begin{vmatrix} (x_2 - x_1) & (x_3 - x_1) \\ (y_2 - y_1) & (y_3 - y_1) \end{vmatrix} \int_0^1 \int_0^1 \begin{bmatrix} 1 - \epsilon - \eta \\ \epsilon \\ \eta \end{bmatrix} d\epsilon \, d\eta$$

$$= 100 \begin{vmatrix} (x_2 - x_1) & (x_3 - x_1) \\ (y_2 - y_1) & (y_3 - y_1) \end{vmatrix} \begin{bmatrix} \int_0^1 \int_0^1 (1 - \epsilon - \eta) \, d\epsilon \, d\eta \\ \int_0^1 \int_0^1 \epsilon \, d\epsilon \, d\eta \\ \int_0^1 \int_0^1 \eta \, d\epsilon \, d\eta \end{bmatrix}$$

$$= 100 \begin{vmatrix} (x_2 - x_1) & (x_3 - x_1) \\ (y_2 - y_1) & (y_3 - y_1) \end{vmatrix} \begin{bmatrix} 0 \\ 1/2 \\ 1/2 \end{bmatrix}$$

Trabajando lado izquierdo de la ecuación:

$$10 \int_A \nabla \underline{\underline{N}}^T \nabla (\underline{N}) \, dA(\vec{T}) = 10 \int_c^d \int_a^b \nabla \underline{\underline{N}}^T \nabla (\underline{N}) \, dx \, dy (\vec{T})$$

$$\text{con } \nabla_x \underline{N} = (\nabla_\epsilon \underline{x})^{-1} \nabla_\epsilon \underline{N}$$

$$= \begin{bmatrix} (x_2 - x_1) & (y_2 - y_1) \\ (x_3 - x_1) & (y_3 - y_1) \end{bmatrix}^{-1} \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$$= \frac{1}{\begin{vmatrix} (x_2 - x_1) & (y_2 - y_1) \\ (x_3 - x_1) & (y_3 - y_1) \end{vmatrix}} \begin{bmatrix} (y_3 - y_1) & (y_1 - y_2) \\ (x_1 - x_3) & (x_2 - x_1) \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$$= \frac{1}{\begin{vmatrix} (x_2 - x_1) & (y_2 - y_1) \\ (x_3 - x_1) & (y_3 - y_1) \end{vmatrix}} \begin{bmatrix} (y_2 - y_3) & (y_3 - y_1) & (y_1 - y_2) \\ (x_3 - x_2) & (x_1 - x_3) & (x_2 - x_1) \end{bmatrix}$$

$$\nabla \underline{\underline{N}}^T = \frac{1}{\begin{vmatrix} (x_2 - x_1) & (y_2 - y_1) \\ (x_3 - x_1) & (y_3 - y_1) \end{vmatrix}} \begin{bmatrix} (y_2 - y_3) & (x_3 - x_2) \\ (y_3 - y_1) & (x_1 - x_3) \\ (y_1 - y_2) & (x_2 - x_1) \end{bmatrix}$$

Sustituyendo en la integral:

$$= 10 \int_c^d \int_a^b \frac{1}{\begin{vmatrix} (x_2 - x_1) & (y_2 - y_1) \\ (x_3 - x_1) & (y_3 - y_1) \end{vmatrix}^2} \begin{bmatrix} (y_2 - y_3) & (x_3 - x_2) \\ (y_3 - y_1) & (x_1 - x_3) \\ (y_1 - y_2) & (x_2 - x_1) \end{bmatrix} \begin{bmatrix} (y_2 - y_3) & (y_3 - y_1) & (y_1 - y_2) \\ (x_3 - x_2) & (x_1 - x_3) & (x_2 - x_1) \end{bmatrix} dx \, dy (\vec{T})$$

$$= 10 \frac{(x \big|_a^b)(y \big|_c^d)}{\begin{vmatrix} (x_2 - x_1) & (y_2 - y_1) \\ (x_3 - x_1) & (y_3 - y_1) \end{vmatrix}^2} \begin{bmatrix} (y_2 - y_3) & (x_3 - x_2) \\ (y_3 - y_1) & (x_1 - x_3) \\ (y_1 - y_2) & (x_2 - x_1) \end{bmatrix} \begin{bmatrix} (y_2 - y_3) & (y_3 - y_1) & (y_1 - y_2) \\ (x_3 - x_2) & (x_1 - x_3) & (x_2 - x_1) \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix}$$

Sistema local final:

$$\frac{10(x \big|_a^b)(y \big|_c^d)}{\begin{vmatrix} (x_2 - x_1) & (y_2 - y_1) \\ (x_3 - x_1) & (y_3 - y_1) \end{vmatrix}^2} \begin{bmatrix} (y_2 - y_3) & (x_3 - x_2) \\ (y_3 - y_1) & (x_1 - x_3) \\ (y_1 - y_2) & (x_2 - x_1) \end{bmatrix} \begin{bmatrix} (y_2 - y_3) & (y_3 - y_1) & (y_1 - y_2) \\ (x_3 - x_2) & (x_1 - x_3) & (x_2 - x_1) \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = 100 \begin{vmatrix} (x_2 - x_1) & (x_3 - x_1) \\ (y_2 - y_1) & (y_3 - y_1) \end{vmatrix} \begin{bmatrix} 0 \\ 1/2 \\ 1/2 \end{bmatrix}$$
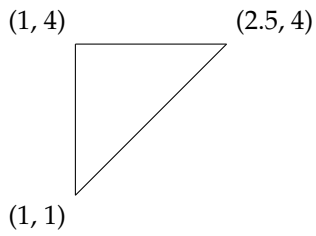
Calculando elementos:

1.

(2.5, 4)

(2, 3)   (2.5, 3)
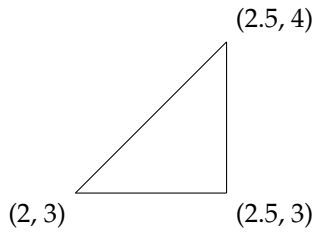
$$x_1 = 2 \quad y_1 = 3 \quad a = 2$$
$$x_2 = 2{,}5 \quad y_2 = 3 \quad b = 2{,}5$$
$$x_3 = 2{,}5 \quad y_3 = 4 \quad c = 3$$
$$d = 4$$

$$\frac{10(0{,}5)(1)}{\begin{vmatrix} 0{,}5 & 0 \\ 0{,}5 & 1 \end{vmatrix}^2} \begin{bmatrix} -1 & 0 \\ 1 & -0{,}5 \\ 0 & 0{,}5 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ 0 & -0{,}5 & 0{,}5 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = 100 \begin{vmatrix} 0{,}5 & 0{,}5 \\ 0 & 1 \end{vmatrix} \begin{bmatrix} 0 \\ 1/2 \\ 1/2 \end{bmatrix}$$

$$\begin{bmatrix} 20 & -20 & 0 \\ -20 & 25 & -5 \\ 0 & -5 & 5 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 25 \\ 25 \end{bmatrix}$$
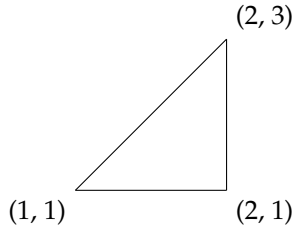
2.

(1, 4)   (2.5, 4)

(1, 1)

$$x_1 = 1 \quad y_1 = 1 \quad a = 1$$
$$x_2 = 2{,}5 \quad y_2 = 4 \quad b = 2{,}5$$
$$x_3 = 1 \quad y_3 = 4 \quad c = 1$$
$$d = 4$$

$$\frac{10(1{,}5)(3)}{\begin{vmatrix} 1{,}5 & 3 \\ 0 & 3 \end{vmatrix}^2} \begin{bmatrix} 0 & -1{,}5 \\ 3 & 0 \\ -3 & 1{,}5 \end{bmatrix} \begin{bmatrix} 0 & 3 & -3 \\ -1{,}5 & 0 & 1{,}5 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = 100 \begin{vmatrix} 1{,}5 & 0 \\ 3 & 3 \end{vmatrix} \begin{bmatrix} 0 \\ 1/2 \\ 1/2 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 0 & -5 \\ 0 & 20 & -20 \\ -5 & -20 & 25 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 255 \\ 255 \end{bmatrix}$$
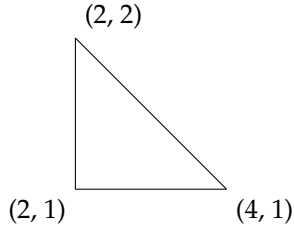
3.

$(2, 3)$

$(1, 1)$ $(2, 1)$

$$x_1 = 1 \quad y_1 = 1 \quad a = 1$$
$$x_2 = 2 \quad y_2 = 1 \quad b = 2$$
$$x_3 = 2 \quad y_3 = 3 \quad c = 1$$
$$d = 3$$

$$\frac{10(1)(2)}{\begin{vmatrix} 1 & 0 \\ 1 & 2 \end{vmatrix}^2} \begin{bmatrix} -2 & 0 \\ 2 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -2 & 2 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = 100 \begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} \begin{bmatrix} 0 \\ 0{,}5 \\ 0{,}5 \end{bmatrix}$$

$$\begin{bmatrix} 20 & -20 & 0 \\ -20 & 25 & -5 \\ 0 & -5 & 5 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 100 \\ 100 \end{bmatrix}$$
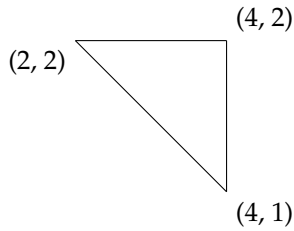
4.

$(2, 2)$

$(2, 1)$ $(4, 1)$

$$x_1 = 2 \quad y_1 = 1 \quad a = 2$$
$$x_2 = 3 \quad y_2 = 1 \quad b = 4$$
$$x_3 = 2 \quad y_3 = 2 \quad c = 1$$
$$d = 2$$

$$\frac{10(2)(1)}{\begin{vmatrix} 2 & 0 \\ 0 & 1 \end{vmatrix}^2} \begin{bmatrix} -1 & -2 \\ 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ -2 & 0 & 2 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = 100 \begin{vmatrix} 2 & 0 \\ 0 & 1 \end{vmatrix} \begin{bmatrix} 0 \\ 0{,}5 \\ 0{,}5 \end{bmatrix}$$

$$\begin{bmatrix} 25 & -5 & -20 \\ -5 & 5 & 0 \\ -20 & 0 & 20 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 100 \\ 100 \end{bmatrix}$$
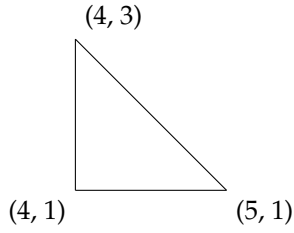
5.

$(4, 2)$

$(2, 2)$

$(4, 1)$

$$x_1 = 2 \quad y_1 = 2 \quad a = 2$$
$$x_2 = 4 \quad y_2 = 1 \quad b = 4$$
$$x_3 = 4 \quad y_3 = 2 \quad c = 1$$
$$d = 2$$

$$\frac{10(2)(1)}{\begin{vmatrix} 2 & -1 \\ 2 & 0 \end{vmatrix}^2} \begin{bmatrix} -1 & 0 \\ 0 & -2 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} -1 & 0 & 1 \\ 0 & -2 & 2 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = 100 \begin{vmatrix} 2 & 2 \\ -1 & 0 \end{vmatrix} \begin{bmatrix} 0 \\ 0{,}5 \\ 0{,}5 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 0 & -5 \\ 0 & 20 & -20 \\ -5 & -20 & 25 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 100 \\ 100 \end{bmatrix}$$
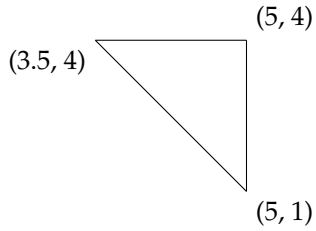
6.

(4, 3)

(4, 1)   (5, 1)

$$x_1 = 4 \quad y_1 = 1 \quad a = 4$$
$$x_2 = 5 \quad y_2 = 1 \quad b = 5$$
$$x_3 = 4 \quad y_3 = 3 \quad c = 1$$
$$d = 3$$

$$\frac{10(1)(2)}{\begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix}^2} \begin{bmatrix} -2 & 1 \\ 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -2 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = 100 \begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix} \begin{bmatrix} 0 \\ 0,5 \\ 0,5 \end{bmatrix}$$

$$\begin{bmatrix} 25 & -20 & -5 \\ -20 & 20 & 0 \\ -5 & 0 & 5 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 100 \\ 100 \end{bmatrix}$$
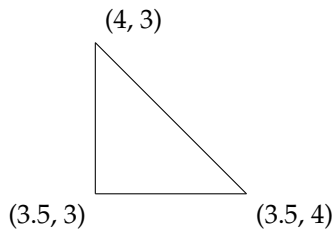
7.

(3.5, 4)   (5, 4)

(5, 1)

$$x_1 = 3,5 \quad y_1 = 4 \quad a = 3,5$$
$$x_2 = 5 \quad y_2 = 1 \quad b = 5$$
$$x_3 = 5 \quad y_3 = 4 \quad c = 1$$
$$d = 4$$

$$\frac{10(1,5)(3)}{\begin{vmatrix} 1,5 & -3 \\ 1,5 & 0 \end{vmatrix}^2} \begin{bmatrix} -3 & 0 \\ 0 & -1,5 \\ -3 & 1,5 \end{bmatrix} \begin{bmatrix} -3 & 0 & 3 \\ 0 & -1,5 & 1,5 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = 100 \begin{vmatrix} 1,5 & 1,5 \\ -3 & 0 \end{vmatrix} \begin{bmatrix} 0 \\ 0,5 \\ 0,5 \end{bmatrix}$$

$$\begin{bmatrix} 20 & 0 & -20 \\ 0 & 5 & -5 \\ -20 & -5 & 25 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 225 \\ 225 \end{bmatrix}$$

8.

(4, 3)

(3.5, 3)   (3.5, 4)

$$x_1 = 3,5 \quad y_1 = 4 \quad a = 3,5$$
$$x_2 = 3,5 \quad y_2 = 3 \quad b = 4$$
$$x_3 = 4 \quad y_3 = 3 \quad c = 3$$
$$d = 4$$

$$\frac{10(0,5)(1)}{\begin{vmatrix} 0 & -1 \\ 0,5 & -1 \end{vmatrix}^2} \begin{bmatrix} 0 & 0,5 \\ -1 & -0,5 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 & 1 \\ 0-5 & -0,5 & 0 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = 100 \begin{vmatrix} 0 & 0,5 \\ -1 & -1 \end{vmatrix} \begin{bmatrix} 0 \\ 0,5 \\ 0,5 \end{bmatrix}$$

$$\begin{bmatrix} 5 & -5 & 0 \\ -5 & 25 & 20 \\ 0 & -20 & 20 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 25 \\ 25 \end{bmatrix}$$

8