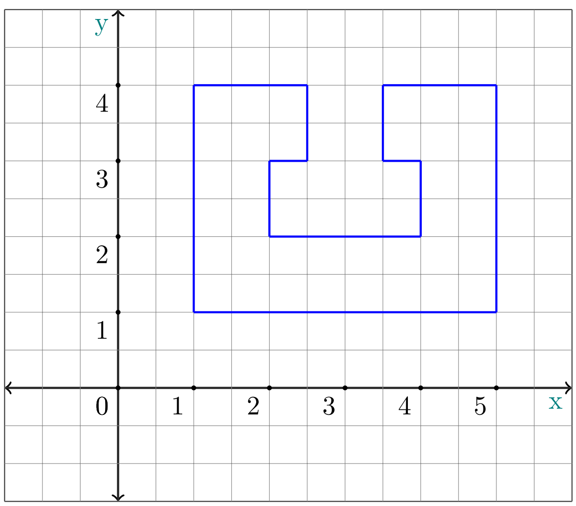
Técnicas de simulación por computadora

Grupo O

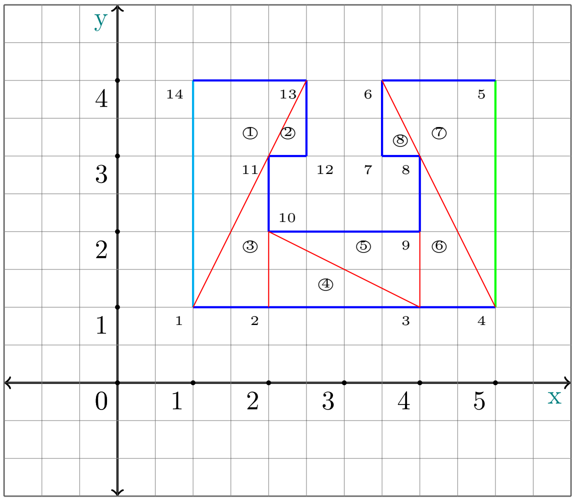
1 de junio de 2018

Dada la figura y ecuación diferencial:



$$\nabla(10\nabla T) = -100$$

Mallado de la figura y establecimiento de contornos para condiciones de Dirichlet y Neumann:



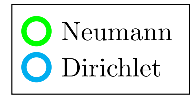


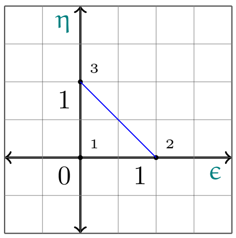
Tabla de conectividad:

elemento	1	2	3
1	1	13	14
2	11	12	13
3	1	2	11
4	2	3	10
5	3	9	10
6	3	4	8
7	4	5	6
8	7	8	6

Condiciones a utilizar:

$$\Gamma_D =$$
$$\Gamma_N =$$

Aproximación en el plano isoparamétrico:



$$
\begin{aligned}
T &\approx f_1T_1 + f_2T_2 + f_3T_3 \\
&\approx N_1T_1 + N_2T_2 + N_3T_3 \\
&\approx \begin{bmatrix} N_1 & N_2 & N_3 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} \\
&\approx \underline{\underline{N}}\vec{T}
\end{aligned}
$$

con $N_1 = 1 - \epsilon - \eta$, $N_2 = \epsilon$, $N_3 = \eta$

$x \approx (x_2 - x_1)\epsilon + (x_3 - x_1)\eta + x_1$

$y \approx (y_2 - y_1)\epsilon + (y_3 - y_1)\eta + y_1$

Sustitución de la función por su aproximación:

$$
\begin{aligned}
&\nabla(10\nabla T) = -100 \\
&\nabla(10\nabla \underline{\underline{N}}\vec{T}) \approx -100 \\
&\nabla(10\nabla \underline{\underline{N}}\vec{T}) + 100 \neq 0 \\
&\nabla(10\nabla \underline{\underline{N}}\vec{T}) + 100 = \mathcal{R}
\end{aligned}
$$

W.R.M.:

$$
\begin{aligned}
&\int_\Omega \underline{\underline{W}}\mathcal{R}\, d\Omega = 0 \\
&\int_A \underline{\underline{W}}\mathcal{R}\, dA = 0 \\
&\int_A \underline{\underline{W}}(\nabla(10\nabla \underline{\underline{N}}\vec{T}) + 100)\, dA = 0
\end{aligned}
$$

Galerkin:

$$
\begin{aligned}
&\underline{\underline{W}} = \underline{\underline{N}}^T \\
&\int_A \underline{\underline{N}}^T(\nabla(10\nabla \underline{\underline{N}}\vec{T}) + 100)\, dA = 0 \\
&\int_A \underline{\underline{N}}^T(\nabla(10\nabla \underline{\underline{N}}\vec{T}))\, dA + \int_A \underline{\underline{N}}^T(100)\, dA = 0
\end{aligned}
$$

Integración por partes:

$$
\begin{aligned}
&\text{sea } U = \underline{\underline{N}}^T,\ dU = \nabla\underline{\underline{N}}^T \\
&\text{sea } dV = \nabla(10\nabla \underline{\underline{N}}\vec{T}),\ V = 10\nabla(\underline{\underline{N}}\vec{T})
\end{aligned}
$$

$$
\int_A \underline{\underline{N}}^T(\nabla(10\nabla \underline{\underline{N}}\vec{T}))\, dA = [\underline{\underline{N}}^T 10\nabla(\underline{\underline{N}}\vec{T})]_{\Gamma_N} - \int_A \nabla\underline{\underline{N}}^t 10\nabla(\underline{\underline{N}}\vec{T})\, dA
$$

Trabajando en la ecuación:

$$
\begin{aligned}
-\int_A \nabla \underline{\underline{N}}^T 10 \nabla (\underline{\underline{N}}\vec{T})\, dA + \int_A \underline{\underline{N}}^T (100)\, dA = 0 \\
\int_A \nabla \underline{\underline{N}}^T 10 \nabla (\underline{\underline{N}}\vec{T})\, dA = \int_A \underline{\underline{N}}^T (100)\, dA \\
10 \int_A \nabla \underline{\underline{N}}^T \nabla (\underline{\underline{N}})\, dA (\vec{T}) = 100 \int_A \underline{\underline{N}}^T\, dA
\end{aligned}
$$

Lado derecho:

$$
\begin{aligned}
100 \int_A \underline{\underline{N}}^T\, dA &= 100 \int_A \begin{bmatrix} 1-\epsilon-\eta \\ \epsilon \\ \eta \end{bmatrix} dA \\
&= 100 \int_A \begin{bmatrix} 1-\epsilon-\eta \\ \epsilon \\ \eta \end{bmatrix} dx\, dy
\end{aligned}
$$

Llevando la integral al plano isoparamétrico con un jacobiano:

$$
\begin{aligned}
dx\, dy &= D\, d\epsilon\, d\eta \\
J &= \begin{bmatrix} \frac{\delta x}{\delta \epsilon} & \frac{\delta x}{\delta \eta} \\ \frac{\delta y}{\delta \epsilon} & \frac{\delta y}{\delta \eta} \end{bmatrix} \\
&= \begin{bmatrix} (x_2 - x_1) & (x_3 - x_1) \\ (y_2 - y_1) & (y_3 - y_1) \end{bmatrix} \\
D &= |J| \\
&= (x_2 - x_1)(y_3 - y_1) - (x_3 - x_1)(y_2 - y_1)
\end{aligned}
$$

Sustituyendo en la integral:

$$
\begin{aligned}
100 \int_A \begin{bmatrix} 1-\epsilon-\eta \\ \epsilon \\ \eta \end{bmatrix} dA &= 100 \int_0^1 \int_0^1 \begin{bmatrix} 1-\epsilon-\eta \\ \epsilon \\ \eta \end{bmatrix} \begin{vmatrix} (x_2 - x_1) & (x_3 - x_1) \\ (y_2 - y_1) & (y_3 - y_1) \end{vmatrix} d\epsilon\, d\eta \\
&= 100 \begin{vmatrix} (x_2 - x_1) & (x_3 - x_1) \\ (y_2 - y_1) & (y_3 - y_1) \end{vmatrix} \int_0^1 \int_0^1 \begin{bmatrix} 1-\epsilon-\eta \\ \epsilon \\ \eta \end{bmatrix} d\epsilon\, d\eta \\
&= 100 \begin{vmatrix} (x_2 - x_1) & (x_3 - x_1) \\ (y_2 - y_1) & (y_3 - y_1) \end{vmatrix} \begin{bmatrix} \int_0^1 \int_0^1 (1-\epsilon-\eta)\, d\epsilon\, d\eta \\ \int_0^1 \int_0^1 \epsilon\, d\epsilon\, d\eta \\ \int_0^1 \int_0^1 \eta\, d\epsilon\, d\eta \end{bmatrix} \\
&= 100 \begin{vmatrix} (x_2 - x_1) & (x_3 - x_1) \\ (y_2 - y_1) & (y_3 - y_1) \end{vmatrix} \begin{bmatrix} 0 \\ 1/2 \\ 1/2 \end{bmatrix}
\end{aligned}
$$

Trabajando lado izquierdo de la ecuación:

$$
\begin{aligned}
10\int_A \nabla\underline{\underline{N}}^T\nabla(\underline{\underline{N}})\,dA(\vec{T}) &= 10\int_c^d\int_a^b \nabla\underline{\underline{N}}^T\nabla(\underline{\underline{N}})\,dx\,dy(\vec{T}) \\
\text{con } \nabla_x\underline{\underline{N}} &= (\nabla_\epsilon\underline{\underline{x}})^{-1}\nabla_\epsilon\underline{\underline{N}} \\
&= \begin{bmatrix}(x_2-x_1) & (y_2-y_1)\\ (x_3-x_1) & (y_3-y_1)\end{bmatrix}^{-1}\begin{bmatrix}-1 & 1 & 1\\ -1 & 0 & 1\end{bmatrix} \\
&= \frac{1}{\begin{vmatrix}(x_2-x_1) & (y_2-y_1)\\ (x_3-x_1) & (y_3-y_1)\end{vmatrix}}\begin{bmatrix}(y_3-y_1) & (y_1-y_2)\\ (x_1-x_3) & (x_2-x_1)\end{bmatrix}\begin{bmatrix}-1 & 1 & 1\\ -1 & 0 & 1\end{bmatrix} \\
&= \frac{1}{\begin{vmatrix}(x_2-x_1) & (y_2-y_1)\\ (x_3-x_1) & (y_3-y_1)\end{vmatrix}}\begin{bmatrix}(y_2-y_3) & (y_3-y_1) & (y_1-y_2)\\ (x_3-x_2) & (x_1-x_3) & (x_2-x_1)\end{bmatrix} \\
\nabla\underline{\underline{N}}^T &= \frac{1}{\begin{vmatrix}(x_2-x_1) & (y_2-y_1)\\ (x_3-x_1) & (y_3-y_1)\end{vmatrix}}\begin{bmatrix}(y_2-y_3) & (x_3-x_2)\\ (y_3-y_1) & (x_1-x_3)\\ (y_1-y_2) & (x_2-x_1)\end{bmatrix}
\end{aligned}
$$

Sustituyendo en la integral:

$$
= 10\int_c^d\int_a^b \frac{1}{\begin{vmatrix}(x_2-x_1) & (y_2-y_1)\\ (x_3-x_1) & (y_3-y_1)\end{vmatrix}^2}\begin{bmatrix}(y_2-y_3) & (x_3-x_2)\\ (y_3-y_1) & (x_1-x_3)\\ (y_1-y_2) & (x_2-x_1)\end{bmatrix}\begin{bmatrix}(y_2-y_3) & (y_3-y_1) & (y_1-y_2)\\ (x_3-x_2) & (x_1-x_3) & (x_2-x_1)\end{bmatrix}dx\,dy(\vec{T})
$$

$$
= 10\frac{(x\Big|_a^b)(y\Big|_c^d)}{\begin{vmatrix}(x_2-x_1) & (y_2-y_1)\\ (x_3-x_1) & (y_3-y_1)\end{vmatrix}^2}\begin{bmatrix}(y_2-y_3) & (x_3-x_2)\\ (y_3-y_1) & (x_1-x_3)\\ (y_1-y_2) & (x_2-x_1)\end{bmatrix}\begin{bmatrix}(y_2-y_3) & (y_3-y_1) & (y_1-y_2)\\ (x_3-x_2) & (x_1-x_3) & (x_2-x_1)\end{bmatrix}\begin{bmatrix}T_1\\ T_2\\ T_3\end{bmatrix}
$$

Sistema local final:

$$
\frac{10(x\Big|_a^b)(y\Big|_c^d)}{\begin{vmatrix}(x_2-x_1) & (y_2-y_1)\\ (x_3-x_1) & (y_3-y_1)\end{vmatrix}^2}\begin{bmatrix}(y_2-y_3) & (x_3-x_2)\\ (y_3-y_1) & (x_1-x_3)\\ (y_1-y_2) & (x_2-x_1)\end{bmatrix}\begin{bmatrix}(y_2-y_3) & (y_3-y_1) & (y_1-y_2)\\ (x_3-x_2) & (x_1-x_3) & (x_2-x_1)\end{bmatrix}\begin{bmatrix}T_1\\ T_2\\ T_3\end{bmatrix} = 100\begin{vmatrix}(x_2-x_1) & (x_3-x_1)\\ (y_2-y_1) & (y_3-y_1)\end{vmatrix}\begin{bmatrix}0\\ 1/2\\ 1/2\end{bmatrix}
$$