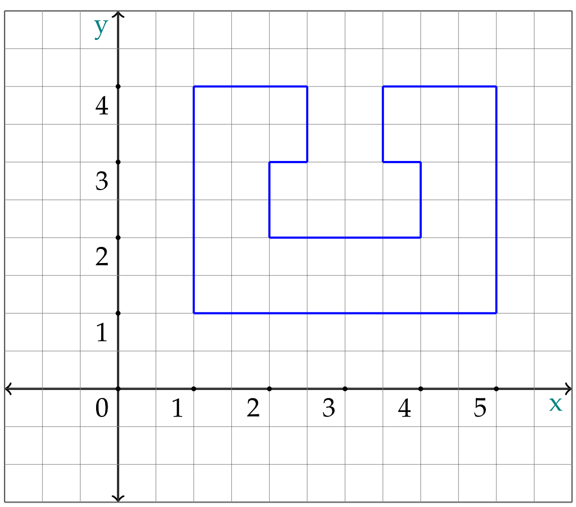
Técnicas de simulación por computadora

Grupo O

3 de junio de 2018

Dada la figura y ecuación diferencial:



$$\nabla(10\nabla T) = -100$$

Mallado de la figura y establecimiento de contornos para condiciones de Dirichlet y Neumann:

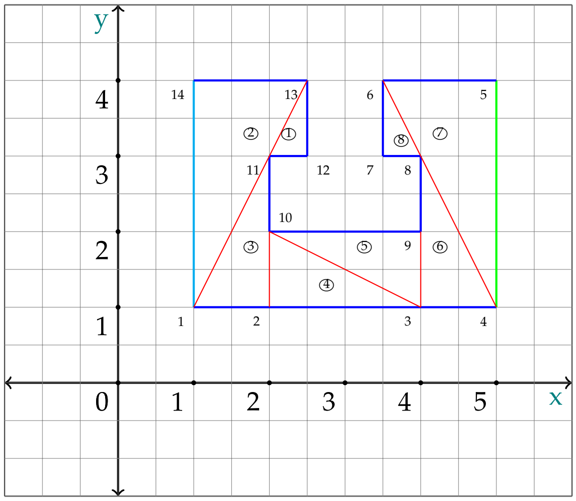


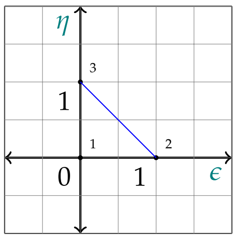
Tabla de conectividad:

elemento	1	2	3
1	11	12	13
2	1	13	14
3	1	2	11
4	2	3	10
5	10	3	9
6	3	4	8
7	6	4	5
8	6	7	8

Condiciones a utilizar:

$$\Gamma_D =$$
$$\Gamma_N =$$

Aproximación en el plano isoparamétrico:



$$
\begin{aligned}
T &\approx f_1T_1 + f_2T_2 + f_3T_3 \\
&\approx N_1T_1 + N_2T_2 + N_3T_3 \\
&\approx \begin{bmatrix} N_1 & N_2 & N_3 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} \\
&\approx \underline{\underline{N}}\vec{T}
\end{aligned}
$$

$$
\begin{aligned}
&\text{con } N_1 = 1 - \epsilon - \eta,\ N_2 = \epsilon,\ N_3 = \eta \\
&x \approx (x_2 - x_1)\epsilon + (x_3 - x_1)\eta + x_1 \\
&y \approx (y_2 - y_1)\epsilon + (y_3 - y_1)\eta + y_1
\end{aligned}
$$

Sustitución de la función por su aproximación:

$$
\begin{aligned}
&\nabla(10\nabla T) = -100 \\
&\nabla(10\nabla \underline{\underline{N}}\vec{T}) \approx -100 \\
&\nabla(10\nabla \underline{\underline{N}}\vec{T}) + 100 \neq 0 \\
&\nabla(10\nabla \underline{\underline{N}}\vec{T}) + 100 = \mathcal{R}
\end{aligned}
$$

W.R.M.:

$$
\begin{aligned}
&\int_\Omega \underline{\underline{W}}\mathcal{R}\, d\Omega = 0 \\
&\int_A \underline{\underline{W}}\mathcal{R}\, dA = 0 \\
&\int_A \underline{\underline{W}}(\nabla(10\nabla \underline{\underline{N}}\vec{T}) + 100)\, dA = 0
\end{aligned}
$$

Galerkin:

$$
\begin{aligned}
&\underline{\underline{W}} = \underline{\underline{N}}^T \\
&\int_A \underline{\underline{N}}^T(\nabla(10\nabla \underline{\underline{N}}\vec{T}) + 100)\, dA = 0 \\
&\int_A \underline{\underline{N}}^T(\nabla(10\nabla \underline{\underline{N}}\vec{T}))\, dA + \int_A \underline{\underline{N}}^T(100)\, dA = 0
\end{aligned}
$$

Integración por partes:

$$
\begin{aligned}
&\text{sea } U = \underline{\underline{N}}^T,\ dU = \nabla \underline{\underline{N}}^T \\
&\text{sea } dV = \nabla(10\nabla \underline{\underline{N}}\vec{T}),\ V = 10\nabla(\underline{\underline{N}}\vec{T})
\end{aligned}
$$

$$
\int_A \underline{\underline{N}}^T(\nabla(10\nabla \underline{\underline{N}}\vec{T}))\, dA = [\underline{\underline{N}}^T 10\nabla(\underline{\underline{N}}\vec{T})]_{\Gamma_N} - \int_A \nabla \underline{\underline{N}}^t 10\nabla(\underline{\underline{N}}\vec{T})\, dA
$$

Trabajando en la ecuación:

$$
\begin{aligned}
-\int_A \nabla \underline{\underline{N}}^T 10 \nabla(\underline{\underline{N}}\vec{T})\, dA + \int_A \underline{\underline{N}}^T (100)\, dA &= 0 \\
\int_A \nabla \underline{\underline{N}}^T 10 \nabla(\underline{\underline{N}}\vec{T})\, dA &= \int_A \underline{\underline{N}}^T (100)\, dA \\
10 \int_A \nabla \underline{\underline{N}}^T \nabla(\underline{\underline{N}})\, dA(\vec{T}) &= 100 \int_A \underline{\underline{N}}^T\, dA
\end{aligned}
$$

Lado derecho:

$$
\begin{aligned}
100 \int_A \underline{\underline{N}}^T\, dA &= 100 \int_A \begin{bmatrix} 1-\epsilon-\eta \\ \epsilon \\ \eta \end{bmatrix} dA \\
&= 100 \int_A \begin{bmatrix} 1-\epsilon-\eta \\ \epsilon \\ \eta \end{bmatrix} dx\, dy
\end{aligned}
$$

Llevando la integral al plano isoparamétrico con un jacobiano:

$$
\begin{aligned}
dx\, dy &= D\, d\epsilon\, d\eta \\
J &= \begin{bmatrix} \frac{\delta x}{\delta \epsilon} & \frac{\delta x}{\delta \eta} \\ \frac{\delta y}{\delta \epsilon} & \frac{\delta y}{\delta \eta} \end{bmatrix} \\
&= \begin{bmatrix} (x_2 - x_1) & (x_3 - x_1) \\ (y_2 - y_1) & (y_3 - y_1) \end{bmatrix} \\
D &= |J| \\
&= (x_2 - x_1)(y_3 - y_1) - (x_3 - x_1)(y_2 - y_1)
\end{aligned}
$$

Sustituyendo en la integral:

$$
\begin{aligned}
100 \int_A \begin{bmatrix} 1-\epsilon-\eta \\ \epsilon \\ \eta \end{bmatrix} dA &= 100 \int_0^1 \int_0^1 \begin{bmatrix} 1-\epsilon-\eta \\ \epsilon \\ \eta \end{bmatrix} \begin{vmatrix} (x_2 - x_1) & (x_3 - x_1) \\ (y_2 - y_1) & (y_3 - y_1) \end{vmatrix} d\epsilon\, d\eta \\
&= 100 \begin{vmatrix} (x_2 - x_1) & (x_3 - x_1) \\ (y_2 - y_1) & (y_3 - y_1) \end{vmatrix} \int_0^1 \int_0^1 \begin{bmatrix} 1-\epsilon-\eta \\ \epsilon \\ \eta \end{bmatrix} d\epsilon\, d\eta \\
&= 100 \begin{vmatrix} (x_2 - x_1) & (x_3 - x_1) \\ (y_2 - y_1) & (y_3 - y_1) \end{vmatrix} \begin{bmatrix} \int_0^1 \int_0^1 (1-\epsilon-\eta)\, d\epsilon\, d\eta \\ \int_0^1 \int_0^1 \epsilon\, d\epsilon\, d\eta \\ \int_0^1 \int_0^1 \eta\, d\epsilon\, d\eta \end{bmatrix} \\
&= 100 \begin{vmatrix} (x_2 - x_1) & (x_3 - x_1) \\ (y_2 - y_1) & (y_3 - y_1) \end{vmatrix} \begin{bmatrix} 0 \\ 1/2 \\ 1/2 \end{bmatrix}
\end{aligned}
$$

Trabajando lado izquierdo de la ecuación:

$$
\begin{aligned}
10\int_A \nabla \underline{\underline{N}}^T \nabla(\underline{\underline{N}})\, dA(\vec{T}) &= 10\int_c^d \int_a^b \nabla \underline{\underline{N}}^T \nabla(\underline{\underline{N}})\, dx\, dy(\vec{T}) \\
\text{con } \nabla_x \underline{\underline{N}} &= (\nabla_\epsilon \underline{\underline{x}})^{-1} \nabla_\epsilon \underline{\underline{N}} \\
&= \begin{bmatrix} (x_2-x_1) & (y_2-y_1) \\ (x_3-x_1) & (y_3-y_1) \end{bmatrix}^{-1} \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \\
&= \frac{1}{\begin{vmatrix} (x_2-x_1) & (y_2-y_1) \\ (x_3-x_1) & (y_3-y_1) \end{vmatrix}} \begin{bmatrix} (y_3-y_1) & (y_1-y_2) \\ (x_1-x_3) & (x_2-x_1) \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \\
&= \frac{1}{\begin{vmatrix} (x_2-x_1) & (y_2-y_1) \\ (x_3-x_1) & (y_3-y_1) \end{vmatrix}} \begin{bmatrix} (y_2-y_3) & (y_3-y_1) & (y_1-y_2) \\ (x_3-x_2) & (x_1-x_3) & (x_2-x_1) \end{bmatrix} \\
\nabla \underline{\underline{N}}^T &= \frac{1}{\begin{vmatrix} (x_2-x_1) & (y_2-y_1) \\ (x_3-x_1) & (y_3-y_1) \end{vmatrix}} \begin{bmatrix} (y_2-y_3) & (x_3-x_2) \\ (y_3-y_1) & (x_1-x_3) \\ (y_1-y_2) & (x_2-x_1) \end{bmatrix}
\end{aligned}
$$

Sustituyendo en la integral:

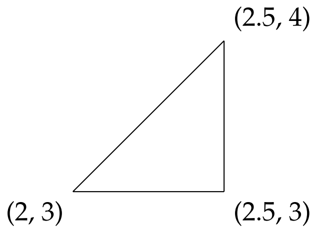
$$
\begin{aligned}
&= 10\int_c^d \int_a^b \frac{1}{\begin{vmatrix} (x_2-x_1) & (y_2-y_1) \\ (x_3-x_1) & (y_3-y_1) \end{vmatrix}^2} \begin{bmatrix} (y_2-y_3) & (x_3-x_2) \\ (y_3-y_1) & (x_1-x_3) \\ (y_1-y_2) & (x_2-x_1) \end{bmatrix} \begin{bmatrix} (y_2-y_3) & (y_3-y_1) & (y_1-y_2) \\ (x_3-x_2) & (x_1-x_3) & (x_2-x_1) \end{bmatrix} dx\, dy(\vec{T}) \\
&= 10 \frac{(x\Big|_a^b)(y\Big|_c^d)}{\begin{vmatrix} (x_2-x_1) & (y_2-y_1) \\ (x_3-x_1) & (y_3-y_1) \end{vmatrix}^2} \begin{bmatrix} (y_2-y_3) & (x_3-x_2) \\ (y_3-y_1) & (x_1-x_3) \\ (y_1-y_2) & (x_2-x_1) \end{bmatrix} \begin{bmatrix} (y_2-y_3) & (y_3-y_1) & (y_1-y_2) \\ (x_3-x_2) & (x_1-x_3) & (x_2-x_1) \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix}
\end{aligned}
$$

Sistema local final:

$$
\frac{10(x\Big|_a^b)(y\Big|_c^d)}{\begin{vmatrix} (x_2-x_1) & (y_2-y_1) \\ (x_3-x_1) & (y_3-y_1) \end{vmatrix}^2} \begin{bmatrix} (y_2-y_3) & (x_3-x_2) \\ (y_3-y_1) & (x_1-x_3) \\ (y_1-y_2) & (x_2-x_1) \end{bmatrix} \begin{bmatrix} (y_2-y_3) & (y_3-y_1) & (y_1-y_2) \\ (x_3-x_2) & (x_1-x_3) & (x_2-x_1) \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = 100 \begin{vmatrix} (x_2-x_1) & (x_3-x_1) \\ (y_2-y_1) & (y_3-y_1) \end{vmatrix} \begin{bmatrix} 0 \\ 1/2 \\ 1/2 \end{bmatrix}
$$

Calculando elementos:

1.

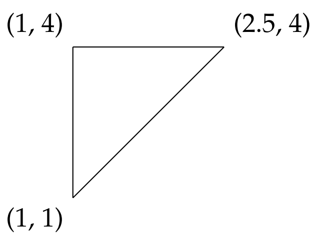


$$\begin{array}{lll} x_1 = 2 & y_1 = 3 & a = 2 \\ x_2 = 2{,}5 & y_2 = 3 & b = 2{,}5 \\ x_3 = 2{,}5 & y_3 = 4 & c = 3 \\ & & d = 4 \end{array}$$

$$\frac{10(0{,}5)(1)}{\begin{vmatrix} 0{,}5 & 0 \\ 0{,}5 & 1 \end{vmatrix}^2} \begin{bmatrix} -1 & 0 \\ 1 & -0{,}5 \\ 0 & 0{,}5 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ 0 & -0{,}5 & 0{,}5 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = 100 \begin{vmatrix} 0{,}5 & 0{,}5 \\ 0 & 1 \end{vmatrix} \begin{bmatrix} 0 \\ 1/2 \\ 1/2 \end{bmatrix}$$

$$\begin{bmatrix} 20 & -20 & 0 \\ -20 & 25 & -5 \\ 0 & -5 & 5 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 25 \\ 25 \end{bmatrix}$$

2.

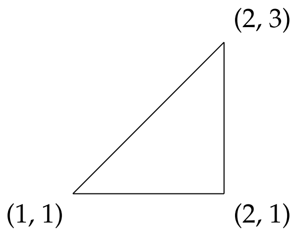


$$\begin{array}{lll} x_1 = 1 & y_1 = 1 & a = 1 \\ x_2 = 2{,}5 & y_2 = 4 & b = 2{,}5 \\ x_3 = 1 & y_3 = 4 & c = 1 \\ & & d = 4 \end{array}$$

$$\frac{10(1{,}5)(3)}{\begin{vmatrix} 1{,}5 & 3 \\ 0 & 3 \end{vmatrix}^2} \begin{bmatrix} 0 & -1{,}5 \\ 3 & 0 \\ -3 & 1{,}5 \end{bmatrix} \begin{bmatrix} 0 & 3 & -3 \\ -1{,}5 & 0 & 1{,}5 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = 100 \begin{vmatrix} 1{,}5 & 0 \\ 3 & 3 \end{vmatrix} \begin{bmatrix} 0 \\ 1/2 \\ 1/2 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 0 & -5 \\ 0 & 20 & -20 \\ -5 & -20 & 25 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 255 \\ 255 \end{bmatrix}$$

3.

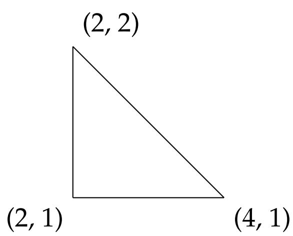


$$
\begin{array}{lll}
x_1 = 1 & y_1 = 1 & a = 1 \\
x_2 = 2 & y_2 = 1 & b = 2 \\
x_3 = 2 & y_3 = 3 & c = 1 \\
 & & d = 3
\end{array}
$$

$$
\frac{10(1)(2)}{\begin{vmatrix} 1 & 0 \\ 1 & 2 \end{vmatrix}^2} \begin{bmatrix} -2 & 0 \\ 2 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -2 & 2 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = 100 \begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} \begin{bmatrix} 0 \\ 0{,}5 \\ 0{,}5 \end{bmatrix}
$$

$$
\begin{bmatrix} 20 & -20 & 0 \\ -20 & 25 & -5 \\ 0 & -5 & 5 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 100 \\ 100 \end{bmatrix}
$$

4.

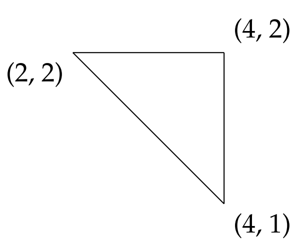


$$
\begin{array}{lll}
x_1 = 2 & y_1 = 1 & a = 2 \\
x_2 = 3 & y_2 = 1 & b = 4 \\
x_3 = 2 & y_3 = 2 & c = 1 \\
 & & d = 2
\end{array}
$$

$$
\frac{10(2)(1)}{\begin{vmatrix} 2 & 0 \\ 0 & 1 \end{vmatrix}^2} \begin{bmatrix} -1 & -2 \\ 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ -2 & 0 & 2 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = 100 \begin{vmatrix} 2 & 0 \\ 0 & 1 \end{vmatrix} \begin{bmatrix} 0 \\ 0{,}5 \\ 0{,}5 \end{bmatrix}
$$

$$
\begin{bmatrix} 25 & -5 & -20 \\ -5 & 5 & 0 \\ -20 & 0 & 20 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 100 \\ 100 \end{bmatrix}
$$

5.

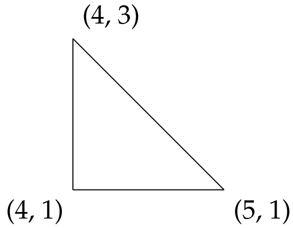


$$
\begin{array}{lll}
x_1 = 2 & y_1 = 2 & a = 2 \\
x_2 = 4 & y_2 = 1 & b = 4 \\
x_3 = 4 & y_3 = 2 & c = 1 \\
 & & d = 2
\end{array}
$$

$$
\frac{10(2)(1)}{\begin{vmatrix} 2 & -1 \\ 2 & 0 \end{vmatrix}^2} \begin{bmatrix} -1 & 0 \\ 0 & -2 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} -1 & 0 & 1 \\ 0 & -2 & 2 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = 100 \begin{vmatrix} 2 & 2 \\ -1 & 0 \end{vmatrix} \begin{bmatrix} 0 \\ 0{,}5 \\ 0{,}5 \end{bmatrix}
$$

$$
\begin{bmatrix} 5 & 0 & -5 \\ 0 & 20 & -20 \\ -5 & -20 & 25 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 100 \\ 100 \end{bmatrix}
$$

6.

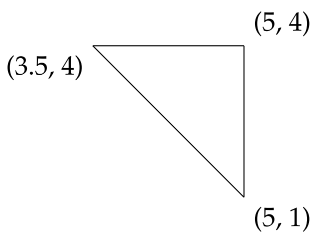


$$
\begin{array}{lll}
x_1 = 4 & y_1 = 1 & a = 4 \\
x_2 = 5 & y_2 = 1 & b = 5 \\
x_3 = 4 & y_3 = 3 & c = 1 \\
 & & d = 3
\end{array}
$$

$$
\frac{10(1)(2)}{\begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix}^2} \begin{bmatrix} -2 & 1 \\ 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -2 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = 100 \begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix} \begin{bmatrix} 0 \\ 0{,}5 \\ 0{,}5 \end{bmatrix}
$$

$$
\begin{bmatrix} 25 & -20 & -5 \\ -20 & 20 & 0 \\ -5 & 0 & 5 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 100 \\ 100 \end{bmatrix}
$$

7.

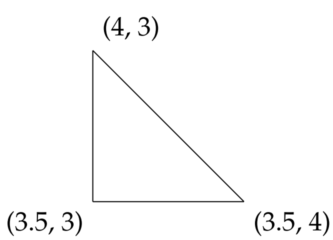


$$
\begin{array}{lll}
x_1 = 3{,}5 & y_1 = 4 & a = 3{,}5 \\
x_2 = 5 & y_2 = 1 & b = 5 \\
x_3 = 5 & y_3 = 4 & c = 1 \\
 & & d = 4
\end{array}
$$

$$
\frac{10(1{,}5)(3)}{\begin{vmatrix} 1{,}5 & -3 \\ 1{,}5 & 0 \end{vmatrix}^2} \begin{bmatrix} -3 & 0 \\ 0 & -1{,}5 \\ -3 & 1{,}5 \end{bmatrix} \begin{bmatrix} -3 & 0 & 3 \\ 0 & -1{,}5 & 1{,}5 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = 100 \begin{vmatrix} 1{,}5 & 1{,}5 \\ -3 & 0 \end{vmatrix} \begin{bmatrix} 0 \\ 0{,}5 \\ 0{,}5 \end{bmatrix}
$$

$$
\begin{bmatrix} 20 & 0 & -20 \\ 0 & 5 & -5 \\ -20 & -5 & 25 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 225 \\ 225 \end{bmatrix}
$$

8.



$$
\begin{array}{lll}
x_1 = 3{,}5 & y_1 = 4 & a = 3{,}5 \\
x_2 = 3{,}5 & y_2 = 3 & b = 4 \\
x_3 = 4 & y_3 = 3 & c = 3 \\
 & & d = 4
\end{array}
$$

$$
\frac{10(0{,}5)(1)}{\begin{vmatrix} 0 & -1 \\ 0{,}5 & -1 \end{vmatrix}^2} \begin{bmatrix} 0 & 0{,}5 \\ -1 & -0{,}5 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 & 1 \\ 0-5 & -0{,}5 & 0 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = 100 \begin{vmatrix} 0 & 0{,}5 \\ -1 & -1 \end{vmatrix} \begin{bmatrix} 0 \\ 0{,}5 \\ 0{,}5 \end{bmatrix}
$$

$$
\begin{bmatrix} 5 & -5 & 0 \\ -5 & 25 & 20 \\ 0 & -20 & 20 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 25 \\ 25 \end{bmatrix}
$$

Ensamblaje y sistema final dadas las matrices $\underline{\underline{K}}$, $\underline{\underline{T}}$ y $\underline{\underline{b}}$:

$$\underline{\underline{KT}} = \underline{\underline{b}}$$

$$\begin{bmatrix}
k^{(2)}_{1,1}+k^{(3)}_{1,1} & k^{(3)}_{1,2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & k^{(3)}_{1,3} & 0 & k^{(2)}_{1,2} & k^{(2)}_{1,3} \\
k^{(3)}_{2,1} & k^{(3)}_{2,2}+k^{(4)}_{1,1} & k^{(4)}_{1,2} & 0 & 0 & 0 & 0 & 0 & 0 & k^{(4)}_{1,3} & k^{(3)}_{2,3} & 0 & 0 & 0 \\
0 & k^{(4)}_{2,1} & k^{(4)}_{2,2}+k^{(5)}_{2,2}+k^{(6)}_{1,1} & k^{(6)}_{1,2} & 0 & 0 & 0 & k^{(6)}_{1,3} & k^{(5)}_{2,3} & k^{(4)}_{2,3}+k^{(5)}_{2,1} & 0 & 0 & 0 & 0 \\
0 & 0 & k^{(6)}_{2,1} & k^{(6)}_{2,2}+k^{(7)}_{2,2} & k^{(7)}_{2,3} & k^{(7)}_{2,1} & 0 & k^{(6)}_{2,3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & k^{(7)}_{3,2} & k^{(7)}_{3,3} & k^{(7)}_{3,1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & k^{(7)}_{1,2} & k^{(7)}_{1,3} & k^{(7)}_{1,1}+k^{(8)}_{1,1} & k^{(8)}_{1,2} & k^{(8)}_{1,3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & k^{(8)}_{2,1} & k^{(8)}_{2,2} & k^{(8)}_{2,3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & k^{(6)}_{3,1} & k^{(6)}_{3,2} & 0 & k^{(8)}_{3,1} & k^{(8)}_{3,2} & k^{(6)}_{3,3}+k^{(8)}_{3,3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & k^{(5)}_{3,2} & 0 & 0 & 0 & 0 & 0 & k^{(5)}_{3,3} & k^{(5)}_{3,1} & 0 & 0 & 0 & 0 \\
0 & k^{(4)}_{3,1} & k^{(4)}_{3,2}+k^{(5)}_{1,2} & 0 & 0 & 0 & 0 & 0 & k^{(5)}_{1,3} & k^{(4)}_{3,3}+k^{(5)}_{1,1} & 0 & 0 & 0 & 0 \\
k^{(3)}_{3,1} & k^{(3)}_{3,2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & k^{(1)}_{1,1}+k^{(3)}_{3,3} & k^{(1)}_{1,2} & k^{(1)}_{1,3} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & k^{(1)}_{2,1} & k^{(1)}_{2,2} & k^{(1)}_{2,3} & 0 \\
k^{(2)}_{2,1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & k^{(1)}_{3,1} & k^{(1)}_{3,2} & k^{(1)}_{3,3}+k^{(2)}_{2,2} & k^{(2)}_{2,3} \\
k^{(2)}_{3,1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & k^{(2)}_{3,2} & k^{(2)}_{3,3}
\end{bmatrix}
\begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \\ T_8 \\ T_9 \\ T_{10} \\ T_{11} \\ T_{12} \\ T_{13} \\ T_{14} \end{bmatrix}
=
\begin{bmatrix}
b^{(2)}_1+b^{(3)}_1 \\
b^{(3)}_2+b^{(4)}_1 \\
b^{(4)}_2+b^{(5)}_2+b^{(6)}_1 \\
b^{(6)}_2+b^{(7)}_2 \\
b^{(7)}_3 \\
b^{(7)}_1+b^{(8)}_1 \\
b^{(8)}_2 \\
b^{(6)}_3+b^{(8)}_3 \\
b^{(5)}_3 \\
b^{(4)}_3+b^{(5)}_1 \\
b^{(1)}_1+b^{(3)}_3 \\
b^{(1)}_2 \\
b^{(1)}_3+b^{(2)}_2 \\
b^{(2)}_3
\end{bmatrix}$$

$$\begin{bmatrix}
40{,}0 & -20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -20{,}0 & 0 \\
-20 & 50 & -5 & 0 & 0 & 0 & 0 & 0 & 0 & -20 & -5 & 0 & 0 & 0 \\
0 & -5 & 35 & -20 & 0 & 0 & 0 & -5 & 0 & -5 & 0 & 0 & 0 & 0 \\
0 & 0 & -20 & 40{,}0 & 0 & -20{,}0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 5{,}0 & -5{,}0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -20{,}0 & -5{,}0 & 45{,}0 & 40{,}0 & -60{,}0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 40{,}0 & 85{,}0 & -125{,}0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -5 & 0 & 0 & -60{,}0 & -125{,}0 & 190{,}0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 20 & -20 & 0 & 0 & 0 & 0 \\
0 & -20 & -5 & 0 & 0 & 0 & 0 & 0 & -20 & 45 & 0 & 0 & 0 & 0 \\
0 & -5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10{,}0 & 0 & -5{,}0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 20{,}0 & -20{,}0 & 0 \\
-20{,}0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -5{,}0 & -20{,}0 & 50{,}0 & -5{,}0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -5{,}0 & 5{,}0
\end{bmatrix}
\begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \\ T_8 \\ T_9 \\ T_{10} \\ T_{11} \\ T_{12} \\ T_{13} \\ T_{14} \end{bmatrix}
=
\begin{bmatrix} 0 \\ 100{,}0 \\ 100{,}0 \\ 100{,}0 \\ 225{,}0 \\ 250{,}0 \\ 0 \\ 125{,}0 \\ 100{,}0 \\ 200{,}0 \\ 100{,}0 \\ 225{,}0 \\ 250{,}0 \\ 25{,}0 \end{bmatrix}$$

Aplicaciones de condiciones de contorno

$$\Gamma_D = \{T_1, T_{14}\}$$
$$\Gamma_N = \left\{ \frac{\mathrm{d}T_4}{\mathrm{d}t}, \frac{\mathrm{d}T_5}{\mathrm{d}t} \right\}$$

$$\begin{bmatrix}
k^{(3)}_{2,1} & k^{(3)}_{2,2}+k^{(4)}_{1,1} & k^{(4)}_{1,2} & 0 & 0 & 0 & 0 & 0 & 0 & k^{(4)}_{1,3} & k^{(3)}_{2,3} & 0 & 0 & 0 \\
0 & k^{(4)}_{2,1} & k^{(4)}_{2,2}+k^{(5)}_{2,2}+k^{(6)}_{1,1} & k^{(6)}_{1,2} & 0 & 0 & 0 & k^{(6)}_{1,3} & k^{(5)}_{2,3} & k^{(4)}_{2,3}+k^{(5)}_{2,1} & 0 & 0 & 0 & 0 \\
0 & 0 & k^{(6)}_{2,1} & k^{(6)}_{2,2}+k^{(7)}_{2,2} & k^{(7)}_{2,3} & k^{(7)}_{2,1} & 0 & k^{(6)}_{2,3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & k^{(7)}_{3,2} & k^{(7)}_{3,3} & k^{(7)}_{3,1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & k^{(7)}_{1,2} & k^{(7)}_{1,3} & k^{(7)}_{1,1}+k^{(8)}_{1,1} & k^{(8)}_{1,2} & k^{(8)}_{1,3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & k^{(8)}_{2,1} & k^{(8)}_{2,2} & k^{(8)}_{2,3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & k^{(6)}_{3,1} & k^{(6)}_{3,2} & 0 & k^{(8)}_{3,1} & k^{(8)}_{3,2} & k^{(6)}_{3,3}+k^{(8)}_{3,3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & k^{(5)}_{3,2} & 0 & 0 & 0 & 0 & 0 & k^{(5)}_{3,3} & k^{(5)}_{3,1} & 0 & 0 & 0 & 0 \\
0 & k^{(4)}_{3,1} & k^{(4)}_{3,2}+k^{(5)}_{1,2} & 0 & 0 & 0 & 0 & 0 & k^{(5)}_{1,3} & k^{(4)}_{3,3}+k^{(5)}_{1,1} & 0 & 0 & 0 & 0 \\
k^{(3)}_{3,1} & k^{(3)}_{3,2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & k^{(1)}_{1,1}+k^{(3)}_{3,3} & k^{(1)}_{1,2} & k^{(1)}_{1,3} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & k^{(1)}_{2,1} & k^{(1)}_{2,2} & k^{(1)}_{2,3} & 0 \\
k^{(2)}_{2,1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & k^{(1)}_{3,1} & k^{(1)}_{3,2} & k^{(1)}_{3,3}+k^{(2)}_{2,2} & k^{(2)}_{2,3}
\end{bmatrix}
\begin{bmatrix} T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \\ T_8 \\ T_9 \\ T_{10} \\ T_{11} \\ T_{12} \\ T_{13} \end{bmatrix}
=
\begin{bmatrix} b^{(3)}_2+b^{(4)}_1 \\ b^{(4)}_2+b^{(5)}_2+b^{(6)}_1 \\ b^{(6)}_2+b^{(7)}_2 \\ b^{(7)}_3 \\ b^{(7)}_1+b^{(8)}_1 \\ b^{(8)}_2 \\ b^{(6)}_3+b^{(8)}_3 \\ b^{(5)}_3 \\ b^{(4)}_3+b^{(5)}_1 \\ b^{(1)}_1+b^{(3)}_3 \\ b^{(1)}_2 \\ b^{(1)}_3+b^{(2)}_2 \end{bmatrix}
+
\begin{bmatrix} -T_1k^{(3)}_{2,1} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -T_1k^{(3)}_{3,1} \\ 0 \\ -T_{14}k^{(2)}_{2,3}-T_1k^{(2)}_{2,1} \end{bmatrix}
+
\begin{bmatrix} 0 \\ 0 \\ \frac{\mathrm{d}T_4}{\mathrm{d}t} \\ \frac{\mathrm{d}T_5}{\mathrm{d}t} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$