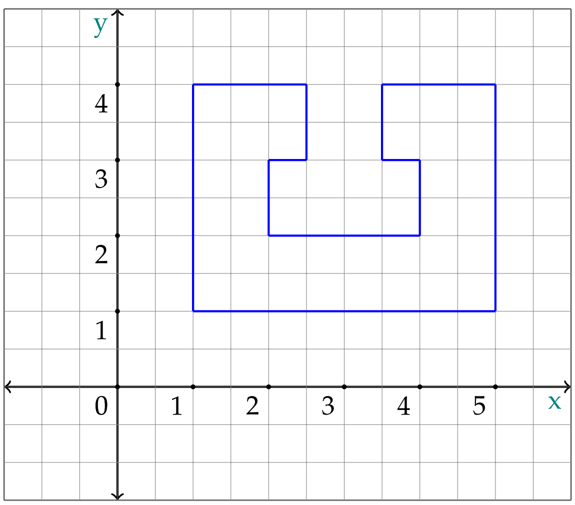
# Técnicas de simulación por computadora

Grupo O

3 de junio de 2018

Dada la figura y ecuación diferencial:




$$\nabla(10\nabla T) = -100$$

Mallado de la figura y establecimiento de contornos para condiciones de Dirichlet y Neumann:

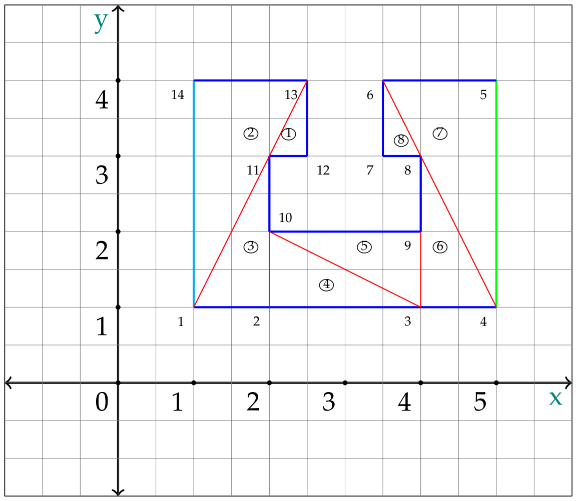




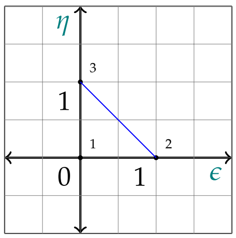
Tabla de conectividad:

| elemento | 1 | 2 | 3 |
|---|---|---|---|
| 1 | 11 | 12 | 13 |
| 2 | 1 | 13 | 14 |
| 3 | 1 | 2 | 11 |
| 4 | 2 | 3 | 10 |
| 5 | 10 | 3 | 9 |
| 6 | 3 | 4 | 8 |
| 7 | 6 | 4 | 5 |
| 8 | 6 | 7 | 8 |

Condiciones a utilizar:

$$\Gamma_D =$$
$$\Gamma_N =$$

Aproximación en el plano isoparamétrico:




$$
\begin{aligned}
T &\approx f_1T_1 + f_2T_2 + f_3T_3 \\
&\approx N_1T_1 + N_2T_2 + N_3T_3 \\
&\approx \begin{bmatrix} N_1 & N_2 & N_3 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} \\
&\approx \underline{\underline{N}}\vec{T}
\end{aligned}
$$

$$
\begin{aligned}
&\text{con } N_1 = 1 - \epsilon - \eta,\, N_2 = \epsilon,\, N_3 = \eta \\
&x \approx (x_2 - x_1)\epsilon + (x_3 - x_1)\eta + x_1 \\
&y \approx (y_2 - y_1)\epsilon + (y_3 - y_1)\eta + y_1
\end{aligned}
$$

Sustitución de la función por su aproximación:

$$
\begin{aligned}
&\nabla(10\nabla T) = -100 \\
&\nabla(10\nabla \underline{\underline{N}}\vec{T}) \approx -100 \\
&\nabla(10\nabla \underline{\underline{N}}\vec{T}) + 100 \neq 0 \\
&\nabla(10\nabla \underline{\underline{N}}\vec{T}) + 100 = \mathcal{R}
\end{aligned}
$$

W.R.M.:

$$
\begin{aligned}
&\int_\Omega \underline{\underline{W}}\mathcal{R}\, d\Omega = 0 \\
&\int_A \underline{\underline{W}}\mathcal{R}\, dA = 0 \\
&\int_A \underline{\underline{W}}(\nabla(10\nabla \underline{\underline{N}}\vec{T}) + 100)\, dA = 0
\end{aligned}
$$

Galerkin:

$$
\begin{aligned}
&\underline{\underline{W}} = \underline{\underline{N}}^T \\
&\int_A \underline{\underline{N}}^T(\nabla(10\nabla \underline{\underline{N}}\vec{T}) + 100)\, dA = 0 \\
&\int_A \underline{\underline{N}}^T(\nabla(10\nabla \underline{\underline{N}}\vec{T}))\, dA + \int_A \underline{\underline{N}}^T(100)\, dA = 0
\end{aligned}
$$

Integración por partes:

$$
\begin{aligned}
&\text{sea } U = \underline{\underline{N}}^T,\ dU = \nabla \underline{\underline{N}}^T \\
&\text{sea } \ dV = \nabla(10\nabla \underline{\underline{N}}\vec{T}),\, V = 10\nabla(\underline{\underline{N}}\vec{T})
\end{aligned}
$$

$$
\int_A \underline{\underline{N}}^T(\nabla(10\nabla \underline{\underline{N}}\vec{T}))\, dA = [\underline{\underline{N}}^T 10\nabla(\underline{\underline{N}}\vec{T})]_{\Gamma_N} - \int_A \nabla \underline{\underline{N}}^t 10\nabla(\underline{\underline{N}}\vec{T})\, dA
$$

Trabajando en la ecuación:

$$
\begin{aligned}
-\int_A \nabla \underline{\underline{N}}^T 10 \nabla(\underline{\underline{N}}\vec{T})\, dA + \int_A \underline{\underline{N}}^T (100)\, dA &= 0 \\
\int_A \nabla \underline{\underline{N}}^T 10 \nabla(\underline{\underline{N}}\vec{T})\, dA &= \int_A \underline{\underline{N}}^T (100)\, dA \\
10 \int_A \nabla \underline{\underline{N}}^T \nabla(\underline{\underline{N}})\, dA(\vec{T}) &= 100 \int_A \underline{\underline{N}}^T\, dA
\end{aligned}
$$

Lado derecho:

$$
\begin{aligned}
100 \int_A \underline{\underline{N}}^T\, dA &= 100 \int_A \begin{bmatrix} 1-\epsilon-\eta \\ \epsilon \\ \eta \end{bmatrix} dA \\
&= 100 \int_A \begin{bmatrix} 1-\epsilon-\eta \\ \epsilon \\ \eta \end{bmatrix} dx\, dy
\end{aligned}
$$

Llevando la integral al plano isoparamétrico con un jacobiano:

$$
\begin{aligned}
dx\, dy &= D\, d\epsilon\, d\eta \\
J &= \begin{bmatrix} \frac{\delta x}{\delta \epsilon} & \frac{\delta x}{\delta \eta} \\ \frac{\delta y}{\delta \epsilon} & \frac{\delta y}{\delta \eta} \end{bmatrix} \\
&= \begin{bmatrix} (x_2 - x_1) & (x_3 - x_1) \\ (y_2 - y_1) & (y_3 - y_1) \end{bmatrix} \\
D &= |J| \\
&= (x_2 - x_1)(y_3 - y_1) - (x_3 - x_1)(y_2 - y_1)
\end{aligned}
$$

Sustituyendo en la integral:

$$
\begin{aligned}
100 \int_A \begin{bmatrix} 1-\epsilon-\eta \\ \epsilon \\ \eta \end{bmatrix} dA &= 100 \int_0^1 \int_0^1 \begin{bmatrix} 1-\epsilon-\eta \\ \epsilon \\ \eta \end{bmatrix} \begin{vmatrix} (x_2 - x_1) & (x_3 - x_1) \\ (y_2 - y_1) & (y_3 - y_1) \end{vmatrix} d\epsilon\, d\eta \\
&= 100 \begin{vmatrix} (x_2 - x_1) & (x_3 - x_1) \\ (y_2 - y_1) & (y_3 - y_1) \end{vmatrix} \int_0^1 \int_0^1 \begin{bmatrix} 1-\epsilon-\eta \\ \epsilon \\ \eta \end{bmatrix} d\epsilon\, d\eta \\
&= 100 \begin{vmatrix} (x_2 - x_1) & (x_3 - x_1) \\ (y_2 - y_1) & (y_3 - y_1) \end{vmatrix} \begin{bmatrix} \int_0^1 \int_0^1 (1-\epsilon-\eta)\, d\epsilon\, d\eta \\ \\ \int_0^1 \int_0^1 \epsilon\, d\epsilon\, d\eta \\ \\ \int_0^1 \int_0^1 \eta\, d\epsilon\, d\eta \end{bmatrix} \\
&= 100 \begin{vmatrix} (x_2 - x_1) & (x_3 - x_1) \\ (y_2 - y_1) & (y_3 - y_1) \end{vmatrix} \begin{bmatrix} 0 \\ 1/2 \\ 1/2 \end{bmatrix}
\end{aligned}
$$

Trabajando lado izquierdo de la ecuación:

$$\begin{aligned}
10\int_A \nabla \underline{\underline{N}}^T \nabla(\underline{\underline{N}})\, dA(\vec{T}) &= 10\int_c^d \int_a^b \nabla \underline{\underline{N}}^T \nabla(\underline{\underline{N}})\, dx\, dy(\vec{T}) \\
\text{con } \nabla_x \underline{\underline{N}} &= (\nabla_\epsilon \underline{\underline{x}})^{-1} \nabla_\epsilon \underline{\underline{N}} \\
&= \begin{bmatrix} (x_2 - x_1) & (y_2 - y_1) \\ (x_3 - x_1) & (y_3 - y_1) \end{bmatrix}^{-1} \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \\
&= \frac{1}{\begin{vmatrix} (x_2 - x_1) & (y_2 - y_1) \\ (x_3 - x_1) & (y_3 - y_1) \end{vmatrix}} \begin{bmatrix} (y_3 - y_1) & (y_1 - y_2) \\ (x_1 - x_3) & (x_2 - x_1) \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \\
&= \frac{1}{\begin{vmatrix} (x_2 - x_1) & (y_2 - y_1) \\ (x_3 - x_1) & (y_3 - y_1) \end{vmatrix}} \begin{bmatrix} (y_2 - y_3) & (y_3 - y_1) & (y_1 - y_2) \\ (x_3 - x_2) & (x_1 - x_3) & (x_2 - x_1) \end{bmatrix} \\
\nabla \underline{\underline{N}}^T &= \frac{1}{\begin{vmatrix} (x_2 - x_1) & (y_2 - y_1) \\ (x_3 - x_1) & (y_3 - y_1) \end{vmatrix}} \begin{bmatrix} (y_2 - y_3) & (x_3 - x_2) \\ (y_3 - y_1) & (x_1 - x_3) \\ (y_1 - y_2) & (x_2 - x_1) \end{bmatrix}
\end{aligned}$$

Sustituyendo en la integral:

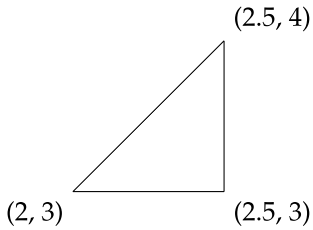
$$\begin{aligned}
&= 10\int_c^d \int_a^b \frac{1}{\begin{vmatrix} (x_2 - x_1) & (y_2 - y_1) \\ (x_3 - x_1) & (y_3 - y_1) \end{vmatrix}^2} \begin{bmatrix} (y_2 - y_3) & (x_3 - x_2) \\ (y_3 - y_1) & (x_1 - x_3) \\ (y_1 - y_2) & (x_2 - x_1) \end{bmatrix} \begin{bmatrix} (y_2 - y_3) & (y_3 - y_1) & (y_1 - y_2) \\ (x_3 - x_2) & (x_1 - x_3) & (x_2 - x_1) \end{bmatrix} dx\, dy(\vec{T}) \\
&= 10 \frac{(x\Big|_a^b)(y\Big|_c^d)}{\begin{vmatrix} (x_2 - x_1) & (y_2 - y_1) \\ (x_3 - x_1) & (y_3 - y_1) \end{vmatrix}^2} \begin{bmatrix} (y_2 - y_3) & (x_3 - x_2) \\ (y_3 - y_1) & (x_1 - x_3) \\ (y_1 - y_2) & (x_2 - x_1) \end{bmatrix} \begin{bmatrix} (y_2 - y_3) & (y_3 - y_1) & (y_1 - y_2) \\ (x_3 - x_2) & (x_1 - x_3) & (x_2 - x_1) \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix}
\end{aligned}$$

Sistema local final:

$$\frac{10(x\Big|_a^b)(y\Big|_c^d)}{\begin{vmatrix} (x_2 - x_1) & (y_2 - y_1) \\ (x_3 - x_1) & (y_3 - y_1) \end{vmatrix}^2} \begin{bmatrix} (y_2 - y_3) & (x_3 - x_2) \\ (y_3 - y_1) & (x_1 - x_3) \\ (y_1 - y_2) & (x_2 - x_1) \end{bmatrix} \begin{bmatrix} (y_2 - y_3) & (y_3 - y_1) & (y_1 - y_2) \\ (x_3 - x_2) & (x_1 - x_3) & (x_2 - x_1) \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = 100 \begin{vmatrix} (x_2 - x_1) & (x_3 - x_1) \\ (y_2 - y_1) & (y_3 - y_1) \end{vmatrix} \begin{bmatrix} 0 \\ 1/2 \\ 1/2 \end{bmatrix}$$

Calculando elementos:

1.

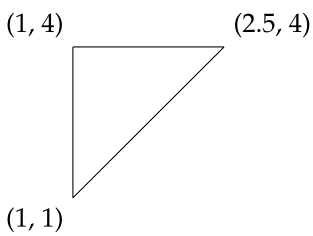



$$\begin{matrix} x_1 = 2 & y_1 = 3 & a = 2 \\ x_2 = 2{,}5 & y_2 = 3 & b = 2{,}5 \\ x_3 = 2{,}5 & y_3 = 4 & c = 3 \\ & & d = 4 \end{matrix}$$

$$\frac{10(0{,}5)(1)}{\begin{vmatrix} 0{,}5 & 0 \\ 0{,}5 & 1 \end{vmatrix}^2} \begin{bmatrix} -1 & 0 \\ 1 & -0{,}5 \\ 0 & 0{,}5 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ 0 & -0{,}5 & 0{,}5 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = 100 \begin{vmatrix} 0{,}5 & 0{,}5 \\ 0 & 1 \end{vmatrix} \begin{bmatrix} 0 \\ 1/2 \\ 1/2 \end{bmatrix}$$

$$\begin{bmatrix} 20 & -20 & 0 \\ -20 & 25 & -5 \\ 0 & -5 & 5 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 25 \\ 25 \end{bmatrix}$$

2.

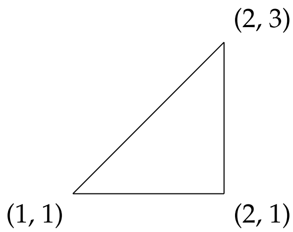



$$\begin{matrix} x_1 = 1 & y_1 = 1 & a = 1 \\ x_2 = 2{,}5 & y_2 = 4 & b = 2{,}5 \\ x_3 = 1 & y_3 = 4 & c = 1 \\ & & d = 4 \end{matrix}$$

$$\frac{10(1{,}5)(3)}{\begin{vmatrix} 1{,}5 & 3 \\ 0 & 3 \end{vmatrix}^2} \begin{bmatrix} 0 & -1{,}5 \\ 3 & 0 \\ -3 & 1{,}5 \end{bmatrix} \begin{bmatrix} 0 & 3 & -3 \\ -1{,}5 & 0 & 1{,}5 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = 100 \begin{vmatrix} 1{,}5 & 0 \\ 3 & 3 \end{vmatrix} \begin{bmatrix} 0 \\ 1/2 \\ 1/2 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 0 & -5 \\ 0 & 20 & -20 \\ -5 & -20 & 25 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 255 \\ 255 \end{bmatrix}$$

3.

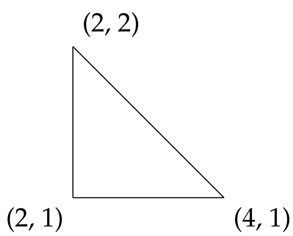



$$\begin{aligned} x_1 &= 1 & y_1 &= 1 & a &= 1 \\ x_2 &= 2 & y_2 &= 1 & b &= 2 \\ x_3 &= 2 & y_3 &= 3 & c &= 1 \\ & & & & d &= 3 \end{aligned}$$

$$\frac{10(1)(2)}{\begin{vmatrix} 1 & 0 \\ 1 & 2 \end{vmatrix}^2} \begin{bmatrix} -2 & 0 \\ 2 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -2 & 2 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = 100 \begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} \begin{bmatrix} 0 \\ 0{,}5 \\ 0{,}5 \end{bmatrix}$$

$$\begin{bmatrix} 20 & -20 & 0 \\ -20 & 25 & -5 \\ 0 & -5 & 5 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 100 \\ 100 \end{bmatrix}$$

4.

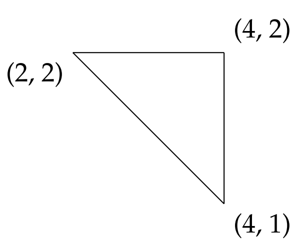



$$\begin{aligned} x_1 &= 2 & y_1 &= 1 & a &= 2 \\ x_2 &= 3 & y_2 &= 1 & b &= 4 \\ x_3 &= 2 & y_3 &= 2 & c &= 1 \\ & & & & d &= 2 \end{aligned}$$

$$\frac{10(2)(1)}{\begin{vmatrix} 2 & 0 \\ 0 & 1 \end{vmatrix}^2} \begin{bmatrix} -1 & -2 \\ 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ -2 & 0 & 2 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = 100 \begin{vmatrix} 2 & 0 \\ 0 & 1 \end{vmatrix} \begin{bmatrix} 0 \\ 0{,}5 \\ 0{,}5 \end{bmatrix}$$

$$\begin{bmatrix} 25 & -5 & -20 \\ -5 & 5 & 0 \\ -20 & 0 & 20 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 100 \\ 100 \end{bmatrix}$$

5.

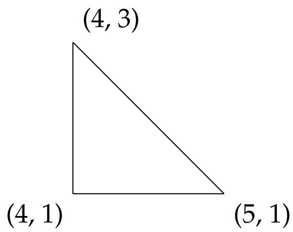



$$\begin{aligned} x_1 &= 2 & y_1 &= 2 & a &= 2 \\ x_2 &= 4 & y_2 &= 1 & b &= 4 \\ x_3 &= 4 & y_3 &= 2 & c &= 1 \\ & & & & d &= 2 \end{aligned}$$

$$\frac{10(2)(1)}{\begin{vmatrix} 2 & -1 \\ 2 & 0 \end{vmatrix}^2} \begin{bmatrix} -1 & 0 \\ 0 & -2 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} -1 & 0 & 1 \\ 0 & -2 & 2 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = 100 \begin{vmatrix} 2 & 2 \\ -1 & 0 \end{vmatrix} \begin{bmatrix} 0 \\ 0{,}5 \\ 0{,}5 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 0 & -5 \\ 0 & 20 & -20 \\ -5 & -20 & 25 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 100 \\ 100 \end{bmatrix}$$

6.

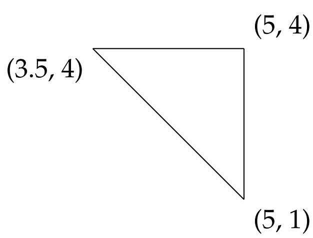



$$\begin{array}{lll} x_1 = 4 & y_1 = 1 & a = 4 \\ x_2 = 5 & y_2 = 1 & b = 5 \\ x_3 = 4 & y_3 = 3 & c = 1 \\ & & d = 3 \end{array}$$

$$\frac{10(1)(2)}{\begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix}^2} \begin{bmatrix} -2 & 1 \\ 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -2 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = 100 \begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix} \begin{bmatrix} 0 \\ 0{,}5 \\ 0{,}5 \end{bmatrix}$$

$$\begin{bmatrix} 25 & -20 & -5 \\ -20 & 20 & 0 \\ -5 & 0 & 5 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 100 \\ 100 \end{bmatrix}$$

7.

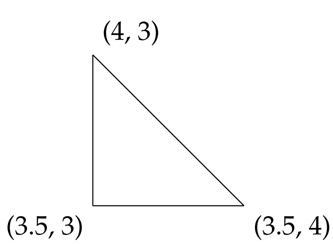



$$\begin{array}{lll} x_1 = 3{,}5 & y_1 = 4 & a = 3{,}5 \\ x_2 = 5 & y_2 = 1 & b = 5 \\ x_3 = 5 & y_3 = 4 & c = 1 \\ & & d = 4 \end{array}$$

$$\frac{10(1{,}5)(3)}{\begin{vmatrix} 1{,}5 & -3 \\ 1{,}5 & 0 \end{vmatrix}^2} \begin{bmatrix} -3 & 0 \\ 0 & -1{,}5 \\ -3 & 1{,}5 \end{bmatrix} \begin{bmatrix} -3 & 0 & 3 \\ 0 & -1{,}5 & 1{,}5 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = 100 \begin{vmatrix} 1{,}5 & 1{,}5 \\ -3 & 0 \end{vmatrix} \begin{bmatrix} 0 \\ 0{,}5 \\ 0{,}5 \end{bmatrix}$$

$$\begin{bmatrix} 20 & 0 & -20 \\ 0 & 5 & -5 \\ -20 & -5 & 25 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 225 \\ 225 \end{bmatrix}$$

8.




$$\begin{array}{lll} x_1 = 3{,}5 & y_1 = 4 & a = 3{,}5 \\ x_2 = 3{,}5 & y_2 = 3 & b = 4 \\ x_3 = 4 & y_3 = 3 & c = 3 \\ & & d = 4 \end{array}$$

$$\frac{10(0{,}5)(1)}{\begin{vmatrix} 0 & -1 \\ 0{,}5 & -1 \end{vmatrix}^2} \begin{bmatrix} 0 & 0{,}5 \\ -1 & -0{,}5 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 & 1 \\ 0-5 & -0{,}5 & 0 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = 100 \begin{vmatrix} 0 & 0{,}5 \\ -1 & -1 \end{vmatrix} \begin{bmatrix} 0 \\ 0{,}5 \\ 0{,}5 \end{bmatrix}$$

$$\begin{bmatrix} 5 & -5 & 0 \\ -5 & 25 & -20 \\ 0 & -20 & 20 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 25 \\ 25 \end{bmatrix}$$

Ensamblaje y sistema final dadas las matrices $\underline{\underline{K}}$, $\underline{T}$ y $\underline{b}$ :

$$\underline{\underline{K}}\underline{T} = \underline{b}$$

$$\begin{bmatrix}
k_{1,1}^{(2)}+k_{1,1}^{(3)} & k_{1,2}^{(3)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & k_{1,3}^{(3)} & 0 & k_{1,2}^{(2)} & k_{1,3}^{(2)} \\
k_{2,1}^{(3)} & k_{2,2}^{(3)}+k_{1,1}^{(4)} & k_{1,2}^{(4)} & 0 & 0 & 0 & 0 & 0 & 0 & k_{1,3}^{(4)} & k_{2,3}^{(3)} & 0 & 0 & 0 \\
0 & k_{2,1}^{(4)} & k_{2,2}^{(4)}+k_{2,2}^{(5)}+k_{1,1}^{(6)} & k_{1,2}^{(6)} & 0 & 0 & 0 & k_{1,3}^{(6)} & k_{2,3}^{(5)} & k_{2,3}^{(4)}+k_{2,1}^{(5)} & 0 & 0 & 0 & 0 \\
0 & 0 & k_{2,1}^{(6)} & k_{2,2}^{(6)}+k_{2,2}^{(7)} & k_{2,3}^{(7)} & k_{2,1}^{(7)} & 0 & k_{2,3}^{(6)} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & k_{3,2}^{(7)} & k_{3,3}^{(7)} & k_{3,1}^{(7)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & k_{1,2}^{(7)} & k_{1,3}^{(7)} & k_{1,1}^{(7)}+k_{1,1}^{(8)} & k_{1,2}^{(8)} & k_{1,3}^{(8)} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & k_{2,1}^{(8)} & k_{2,2}^{(8)} & k_{2,3}^{(8)} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & k_{3,1}^{(6)} & k_{3,2}^{(6)} & 0 & k_{3,1}^{(8)} & k_{3,2}^{(8)} & k_{3,3}^{(6)}+k_{3,3}^{(8)} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & k_{3,2}^{(5)} & 0 & 0 & 0 & 0 & 0 & k_{3,3}^{(5)} & k_{3,1}^{(5)} & 0 & 0 & 0 & 0 \\
0 & k_{3,1}^{(4)} & k_{3,2}^{(4)}+k_{1,2}^{(5)} & 0 & 0 & 0 & 0 & 0 & k_{1,3}^{(5)} & k_{3,3}^{(4)}+k_{1,1}^{(5)} & 0 & 0 & 0 & 0 \\
k_{3,1}^{(3)} & k_{3,2}^{(3)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & k_{1,1}^{(1)}+k_{3,3}^{(3)} & k_{1,2}^{(1)} & k_{1,3}^{(1)} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & k_{2,1}^{(1)} & k_{2,2}^{(1)} & k_{2,3}^{(1)} & 0 \\
k_{2,1}^{(2)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & k_{3,1}^{(1)} & k_{3,2}^{(1)} & k_{3,3}^{(1)}+k_{2,2}^{(2)} & k_{2,3}^{(2)} \\
k_{3,1}^{(2)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & k_{3,2}^{(2)} & k_{3,3}^{(2)}
\end{bmatrix}
\begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \\ T_8 \\ T_9 \\ T_{10} \\ T_{11} \\ T_{12} \\ T_{13} \\ T_{14} \end{bmatrix}
=
\begin{bmatrix}
b_1^{(2)}+b_1^{(3)} \\
b_2^{(3)}+b_1^{(4)} \\
b_2^{(4)}+b_2^{(5)}+b_1^{(6)} \\
b_2^{(6)}+b_2^{(7)} \\
b_3^{(7)} \\
b_1^{(7)}+b_1^{(8)} \\
b_2^{(8)} \\
b_3^{(6)}+b_3^{(8)} \\
b_3^{(5)} \\
b_3^{(4)}+b_1^{(5)} \\
b_1^{(1)}+b_3^{(3)} \\
b_2^{(1)} \\
b_3^{(1)}+b_2^{(2)} \\
b_3^{(2)}
\end{bmatrix}$$

$$\begin{bmatrix}
25{,}0 & -20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -5{,}0 \\
-20 & 50 & -5 & 0 & 0 & 0 & 0 & 0 & 0 & -20 & -5 & 0 & 0 & 0 \\
0 & -5 & 50 & -20 & 0 & 0 & 0 & -5 & -20 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -20 & 25{,}0 & -5{,}0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -5{,}0 & 25{,}0 & -20{,}0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -20{,}0 & 25{,}0 & -5{,}0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -5{,}0 & 25{,}0 & -20{,}0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -5 & 0 & 0 & 0 & -20{,}0 & 25{,}0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -20 & 0 & 0 & 0 & 0 & 0 & 25 & -5 & 0 & 0 & 0 & 0 \\
0 & -20 & 0 & 0 & 0 & 0 & 0 & 0 & -5 & 25 & 0 & 0 & 0 & 0 \\
0 & -5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 25{,}0 & -20{,}0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -20{,}0 & 25{,}0 & -5{,}0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -5{,}0 & 25{,}0 & -20{,}0 \\
-5{,}0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -20{,}0 & 25{,}0
\end{bmatrix}
\begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \\ T_8 \\ T_9 \\ T_{10} \\ T_{11} \\ T_{12} \\ T_{13} \\ T_{14} \end{bmatrix}
=
\begin{bmatrix} 0 \\ 100{,}0 \\ 200{,}0 \\ 325{,}0 \\ 225{,}0 \\ 0 \\ 25{,}0 \\ 125{,}0 \\ 100{,}0 \\ 100{,}0 \\ 100{,}0 \\ 25{,}0 \\ 250{,}0 \\ 225{,}0 \end{bmatrix}$$

Aplicaciones de condiciones de contorno

$$\Gamma_D = \{T_1, T_{14}\}$$
$$\Gamma_N = \left\{ \frac{\mathrm{d}T_4}{\mathrm{d}\hat{n}}, \frac{\mathrm{d}T_5}{\mathrm{d}\hat{n}} \right\}$$

$$\begin{bmatrix}
k_{2,2}^{(3)}+k_{1,1}^{(4)} & k_{1,2}^{(4)} & 0 & 0 & 0 & 0 & 0 & 0 & k_{1,3}^{(4)} & k_{2,3}^{(3)} & 0 & 0 \\
k_{2,1}^{(4)} & k_{2,2}^{(4)}+k_{2,2}^{(5)}+k_{1,1}^{(6)} & k_{1,2}^{(6)} & 0 & 0 & 0 & k_{1,3}^{(6)} & k_{2,3}^{(5)} & k_{2,3}^{(4)}+k_{2,1}^{(5)} & 0 & 0 & 0 \\
0 & k_{2,1}^{(6)} & k_{2,2}^{(6)}+k_{2,2}^{(7)} & k_{2,3}^{(7)} & k_{2,1}^{(7)} & 0 & k_{2,3}^{(6)} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & k_{3,2}^{(7)} & k_{3,3}^{(7)} & k_{3,1}^{(7)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & k_{1,2}^{(7)} & k_{1,3}^{(7)} & k_{1,1}^{(7)}+k_{1,1}^{(8)} & k_{1,2}^{(8)} & k_{1,3}^{(8)} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & k_{2,1}^{(8)} & k_{2,2}^{(8)} & k_{2,3}^{(8)} & 0 & 0 & 0 & 0 & 0 \\
0 & k_{3,1}^{(6)} & k_{3,2}^{(6)} & 0 & k_{3,1}^{(8)} & k_{3,2}^{(8)} & k_{3,3}^{(6)}+k_{3,3}^{(8)} & 0 & 0 & 0 & 0 & 0 \\
0 & k_{3,2}^{(5)} & 0 & 0 & 0 & 0 & 0 & k_{3,3}^{(5)} & k_{3,1}^{(5)} & 0 & 0 & 0 \\
k_{3,1}^{(4)} & k_{3,2}^{(4)}+k_{1,2}^{(5)} & 0 & 0 & 0 & 0 & 0 & k_{1,3}^{(5)} & k_{3,3}^{(4)}+k_{1,1}^{(5)} & 0 & 0 & 0 \\
k_{3,2}^{(3)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & k_{1,1}^{(1)}+k_{3,3}^{(3)} & k_{1,2}^{(1)} & k_{1,3}^{(1)} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & k_{2,1}^{(1)} & k_{2,2}^{(1)} & k_{2,3}^{(1)} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & k_{3,1}^{(1)} & k_{3,2}^{(1)} & k_{3,3}^{(1)}+k_{2,2}^{(2)}
\end{bmatrix}
\begin{bmatrix} T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \\ T_8 \\ T_9 \\ T_{10} \\ T_{11} \\ T_{12} \\ T_{13} \end{bmatrix}
=
\begin{bmatrix}
b_2^{(3)}+b_1^{(4)} \\
b_2^{(4)}+b_2^{(5)}+b_1^{(6)} \\
b_2^{(6)}+b_2^{(7)} \\
b_3^{(7)} \\
b_1^{(7)}+b_1^{(8)} \\
b_2^{(8)} \\
b_3^{(6)}+b_3^{(8)} \\
b_3^{(5)} \\
b_3^{(4)}+b_1^{(5)} \\
b_1^{(1)}+b_3^{(3)} \\
b_2^{(1)} \\
b_3^{(1)}+b_2^{(2)}
\end{bmatrix}
+
\begin{bmatrix}
-T_1 k_{2,1}^{(3)} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -T_1 k_{3,1}^{(3)} \\ 0 \\ -T_{14} k_{2,3}^{(2)} - T_1 k_{2,1}^{(2)}
\end{bmatrix}
+
\begin{bmatrix}
0 \\ 0 \\ \frac{\mathrm{d}T_4}{\mathrm{d}\hat{n}} \\ \frac{\mathrm{d}T_5}{\mathrm{d}\hat{n}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0
\end{bmatrix}$$

$$\begin{bmatrix}
50 & -5 & 0 & 0 & 0 & 0 & 0 & 0 & -20 & -5 & 0 & 0 \\
-5 & 50 & -20 & 0 & 0 & 0 & -5 & -20 & 0 & 0 & 0 & 0 \\
0 & -20 & 25{,}0 & -5{,}0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -5{,}0 & 25{,}0 & -20{,}0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -20{,}0 & 25{,}0 & -5{,}0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -5{,}0 & 25{,}0 & -20{,}0 & 0 & 0 & 0 & 0 & 0 \\
0 & -5 & 0 & 0 & 0 & -20{,}0 & 25{,}0 & 0 & 0 & 0 & 0 & 0 \\
0 & -20 & 0 & 0 & 0 & 0 & 0 & 25 & -5 & 0 & 0 & 0 \\
-20 & 0 & 0 & 0 & 0 & 0 & 0 & -5 & 25 & 0 & 0 & 0 \\
-5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 25{,}0 & -20{,}0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -20{,}0 & 25{,}0 & -5{,}0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -5{,}0 & 25{,}0
\end{bmatrix}
\begin{bmatrix} T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \\ T_8 \\ T_9 \\ T_{10} \\ T_{11} \\ T_{12} \\ T_{13} \end{bmatrix}
=
\begin{bmatrix} 100{,}0 \\ 200{,}0 \\ 325{,}0 \\ 225{,}0 \\ 0 \\ 25{,}0 \\ 125{,}0 \\ 100{,}0 \\ 100{,}0 \\ 100{,}0 \\ 25{,}0 \\ 250{,}0 \end{bmatrix}
+
\begin{bmatrix} 34{,}8 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 34{,}8 \end{bmatrix}
+
\begin{bmatrix} 0 \\ 0 \\ 0{,}87 \\ 0{,}87 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix}
50 & -5 & 0 & 0 & 0 & 0 & 0 & 0 & -20 & -5 & 0 & 0 \\
-5 & 50 & -20 & 0 & 0 & 0 & -5 & -20 & 0 & 0 & 0 & 0 \\
0 & -20 & 25{,}0 & -5{,}0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -5{,}0 & 25{,}0 & -20{,}0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -20{,}0 & 25{,}0 & -5{,}0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -5{,}0 & 25{,}0 & -20{,}0 & 0 & 0 & 0 & 0 & 0 \\
0 & -5 & 0 & 0 & 0 & -20{,}0 & 25{,}0 & 0 & 0 & 0 & 0 & 0 \\
0 & -20 & 0 & 0 & 0 & 0 & 0 & 25 & -5 & 0 & 0 & 0 \\
-20 & 0 & 0 & 0 & 0 & 0 & 0 & -5 & 25 & 0 & 0 & 0 \\
-5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 25{,}0 & -20{,}0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -20{,}0 & 25{,}0 & -5{,}0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -5{,}0 & 25{,}0
\end{bmatrix}
\begin{bmatrix} T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \\ T_8 \\ T_9 \\ T_{10} \\ T_{11} \\ T_{12} \\ T_{13} \end{bmatrix}
=
\begin{bmatrix} 134{,}8 \\ 200{,}0 \\ 325{,}87 \\ 225{,}87 \\ 0 \\ 25{,}0 \\ 125{,}0 \\ 100{,}0 \\ 100{,}0 \\ 100{,}0 \\ 25{,}0 \\ 284{,}8 \end{bmatrix}$$

Solución del sistema de ecuaciones lineales:

$$\underline{\underline{T}} = \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \\ T_8 \\ T_9 \\ T_{10} \\ T_{11} \\ T_{12} \\ T_{13} \\ T_{14} \end{bmatrix} := \begin{bmatrix} 1{,}74 \\ 60{,}7963636363636 \\ 181{,}005163636364 \\ 205{,}824763636364 \\ 239{,}929163636363 \\ 237{,}161763636363 \\ 226{,}092163636363 \\ 222{,}074763636363 \\ 165{,}970363636364 \\ 85{,}8311636363636 \\ 56{,}6738181818182 \\ 50{,}6431818181818 \\ 21{,}5206363636364 \\ 1{,}74 \end{bmatrix}$$