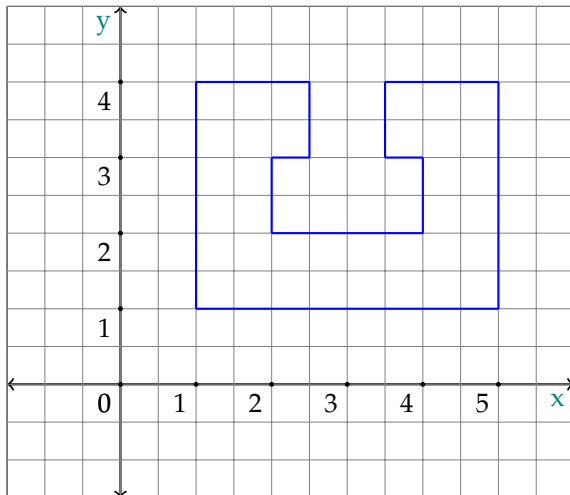


# Técnicas de simulación por computadora

Grupo O

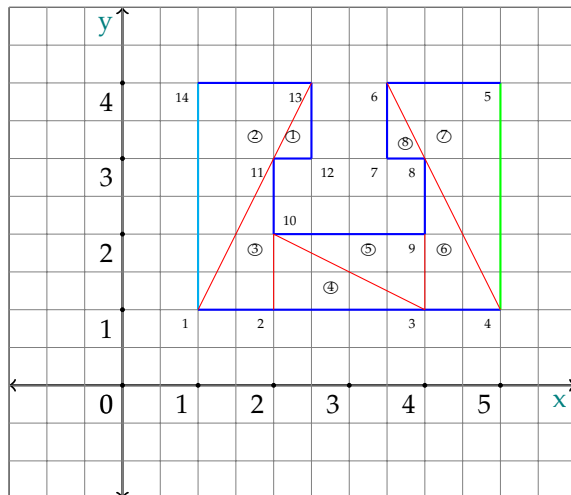
3 de junio de 2018

Dada la figura y ecuación diferencial:



$$\nabla(10\nabla T) = -100$$

Mallado de la figura y establecimiento de contornos para condiciones de Dirichlet y Neumann:



○ Neumann  
○ Dirichlet

Tabla de conectividad:

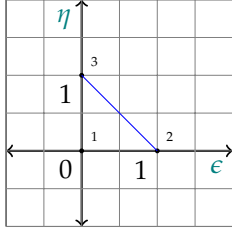
elemento	1	2	3
①	11	12	13
②	1	13	14
③	1	2	11
④	2	3	10
⑤	10	3	9
⑥	3	4	8
⑦	6	4	5
⑧	6	7	8

Condiciones a utilizar:

$$\Gamma_D =$$

$$\Gamma_N =$$

Aproximación en el plano isoparamétrico:



$$\begin{aligned}
 T &\approx f_1 T_1 + f_2 T_2 + f_3 T_3 \\
 &\approx N_1 T_1 + N_2 T_2 + N_3 T_3 \\
 &\approx [N_1 \quad N_2 \quad N_3] \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} \\
 &\approx \underline{\underline{N}}^T
 \end{aligned}$$

$$\begin{aligned}
 \text{con } N_1 &= 1 - \epsilon - \eta, N_2 = \epsilon, N_3 = \eta \\
 x &\approx (x_2 - x_1)\epsilon + (x_3 - x_1)\eta + x_1 \\
 y &\approx (y_2 - y_1)\epsilon + (y_3 - y_1)\eta + y_1
 \end{aligned}$$

Sustitución de la función por su aproximación:

$$\begin{aligned}
 \nabla(10\nabla T) &= -100 \\
 \nabla(10\nabla \underline{\underline{N}}^T) &\approx -100 \\
 \nabla(10\nabla \underline{\underline{N}}^T) + 100 &\neq 0 \\
 \nabla(10\nabla \underline{\underline{N}}^T) + 100 &= \mathcal{R}
 \end{aligned}$$

W.R.M.:

$$\begin{aligned}
 \int_{\Omega} \underline{\underline{W}} \mathcal{R} d\Omega &= 0 \\
 \int_A \underline{\underline{W}} \mathcal{R} dA &= 0 \\
 \int_A \underline{\underline{W}} (\nabla(10\nabla \underline{\underline{N}}^T) + 100) dA &= 0
 \end{aligned}$$

Galerkin:

$$\begin{aligned}
 \underline{\underline{W}} &= \underline{\underline{N}}^T \\
 \int_A \underline{\underline{N}}^T (\nabla(10\nabla \underline{\underline{N}}^T) + 100) dA &= 0 \\
 \int_A \underline{\underline{N}}^T (\nabla(10\nabla \underline{\underline{N}}^T)) dA + \int_A \underline{\underline{N}}^T (100) dA &= 0
 \end{aligned}$$

Integración por partes:

$$\begin{aligned}
 \text{sea } U &= \underline{\underline{N}}^T, dU = \nabla \underline{\underline{N}}^T \\
 \text{sea } dV &= \nabla(10\nabla \underline{\underline{N}}^T), V = 10\nabla(\underline{\underline{N}}^T)
 \end{aligned}$$

$$\int_A \underline{\underline{N}}^T (\nabla(10\nabla \underline{\underline{N}}^T)) dA = [\underline{\underline{N}}^T 10\nabla(\underline{\underline{N}}^T)]_{\Gamma_N} - \int_A \nabla \underline{\underline{N}}^T 10\nabla(\underline{\underline{N}}^T) dA$$

Trabajando en la ecuación:

$$\begin{aligned}
& - \int_A \nabla \underline{\underline{N}}^T 10 \nabla (\underline{\underline{N}} \vec{T}) dA + \int_A \underline{\underline{N}}^T (100) dA = 0 \\
& \int_A \nabla \underline{\underline{N}}^T 10 \nabla (\underline{\underline{N}} \vec{T}) dA = \int_A \underline{\underline{N}}^T (100) dA \\
& 10 \int_A \nabla \underline{\underline{N}}^T \nabla (\underline{\underline{N}}) dA (\vec{T}) = 100 \int_A \underline{\underline{N}}^T dA
\end{aligned}$$

Lado derecho:

$$\begin{aligned}
100 \int_A \underline{\underline{N}}^T dA &= 100 \int_A \begin{bmatrix} 1 - \epsilon - \eta \\ \epsilon \\ \eta \end{bmatrix} dA \\
&= 100 \int_A \begin{bmatrix} 1 - \epsilon - \eta \\ \epsilon \\ \eta \end{bmatrix} dx dy
\end{aligned}$$

Llevando la integral al plano isoparamétrico con un jacobiano:

$$\begin{aligned}
dx dy &= D d\epsilon d\eta \\
J &= \begin{bmatrix} \frac{\delta x}{\delta \epsilon} & \frac{\delta x}{\delta \eta} \\ \frac{\delta y}{\delta \epsilon} & \frac{\delta y}{\delta \eta} \end{bmatrix} \\
&= \begin{bmatrix} (x_2 - x_1) & (x_3 - x_1) \\ (y_2 - y_1) & (y_3 - y_1) \end{bmatrix} \\
D &= |J| \\
&= (x_2 - x_1)(y_3 - y_1) - (x_3 - x_1)(y_2 - y_1)
\end{aligned}$$

Sustituyendo en la integral:

$$\begin{aligned}
100 \int_A \begin{bmatrix} 1 - \epsilon - \eta \\ \epsilon \\ \eta \end{bmatrix} dA &= 100 \int_0^1 \int_0^1 \begin{bmatrix} 1 - \epsilon - \eta \\ \epsilon \\ \eta \end{bmatrix} \begin{vmatrix} (x_2 - x_1) & (x_3 - x_1) \\ (y_2 - y_1) & (y_3 - y_1) \end{vmatrix} d\epsilon d\eta \\
&= 100 \begin{vmatrix} (x_2 - x_1) & (x_3 - x_1) \\ (y_2 - y_1) & (y_3 - y_1) \end{vmatrix} \int_0^1 \int_0^1 \begin{bmatrix} 1 - \epsilon - \eta \\ \epsilon \\ \eta \end{bmatrix} d\epsilon d\eta \\
&= 100 \begin{vmatrix} (x_2 - x_1) & (x_3 - x_1) \\ (y_2 - y_1) & (y_3 - y_1) \end{vmatrix} \begin{bmatrix} \int_0^1 \int_0^1 (1 - \epsilon - \eta) d\epsilon d\eta \\ \int_0^1 \int_0^1 \epsilon d\epsilon d\eta \\ \int_0^1 \int_0^1 \eta d\epsilon d\eta \end{bmatrix} \\
&= 100 \begin{vmatrix} (x_2 - x_1) & (x_3 - x_1) \\ (y_2 - y_1) & (y_3 - y_1) \end{vmatrix} \begin{bmatrix} 0 \\ 1/2 \\ 1/2 \end{bmatrix}
\end{aligned}$$

Trabajando lado izquierdo de la ecuación:

$$\begin{aligned}
10 \int_A \nabla \underline{\underline{N}}^T \nabla(\underline{\underline{N}}) dA(\vec{T}) &= 10 \int_c^d \int_a^b \nabla \underline{\underline{N}}^T \nabla(\underline{\underline{N}}) dx dy(\vec{T}) \\
\text{con } \nabla_x \underline{\underline{N}} &= (\nabla_{\epsilon} \underline{\underline{x}})^{-1} \nabla_{\epsilon} \underline{\underline{N}} \\
&= \begin{bmatrix} (x_2 - x_1) & (y_2 - y_1) \\ (x_3 - x_1) & (y_3 - y_1) \end{bmatrix}^{-1} \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \\
&= \frac{1}{\begin{vmatrix} (x_2 - x_1) & (y_2 - y_1) \\ (x_3 - x_1) & (y_3 - y_1) \end{vmatrix}} \begin{bmatrix} (y_3 - y_1) & (y_1 - y_2) \\ (x_1 - x_3) & (x_2 - x_1) \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \\
&= \frac{1}{\begin{vmatrix} (x_2 - x_1) & (y_2 - y_1) \\ (x_3 - x_1) & (y_3 - y_1) \end{vmatrix}} \begin{bmatrix} (y_2 - y_3) & (y_3 - y_1) & (y_1 - y_2) \\ (x_3 - x_2) & (x_1 - x_3) & (x_2 - x_1) \end{bmatrix} \\
\nabla \underline{\underline{N}}^T &= \frac{1}{\begin{vmatrix} (x_2 - x_1) & (y_2 - y_1) \\ (x_3 - x_1) & (y_3 - y_1) \end{vmatrix}} \begin{bmatrix} (y_2 - y_3) & (x_3 - x_2) \\ (y_3 - y_1) & (x_1 - x_3) \\ (y_1 - y_2) & (x_2 - x_1) \end{bmatrix}
\end{aligned}$$

Sustituyendo en la integral:

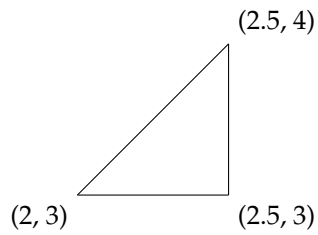
$$\begin{aligned}
&= 10 \int_c^d \int_a^b \frac{1}{\begin{vmatrix} (x_2 - x_1) & (y_2 - y_1) \\ (x_3 - x_1) & (y_3 - y_1) \end{vmatrix}^2} \begin{bmatrix} (y_2 - y_3) & (x_3 - x_2) \\ (y_3 - y_1) & (x_1 - x_3) \\ (y_1 - y_2) & (x_2 - x_1) \end{bmatrix} \begin{bmatrix} (y_2 - y_3) & (y_3 - y_1) & (y_1 - y_2) \\ (x_3 - x_2) & (x_1 - x_3) & (x_2 - x_1) \end{bmatrix} dx dy(\vec{T}) \\
&= 10 \frac{(x|_a^b)(y|_c^d)}{\begin{vmatrix} (x_2 - x_1) & (y_2 - y_1) \\ (x_3 - x_1) & (y_3 - y_1) \end{vmatrix}^2} \begin{bmatrix} (y_2 - y_3) & (x_3 - x_2) \\ (y_3 - y_1) & (x_1 - x_3) \\ (y_1 - y_2) & (x_2 - x_1) \end{bmatrix} \begin{bmatrix} (y_2 - y_3) & (y_3 - y_1) & (y_1 - y_2) \\ (x_3 - x_2) & (x_1 - x_3) & (x_2 - x_1) \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix}
\end{aligned}$$

Sistema local final:

$$\frac{10(x|_a^b)(y|_c^d)}{\begin{vmatrix} (x_2 - x_1) & (y_2 - y_1) \\ (x_3 - x_1) & (y_3 - y_1) \end{vmatrix}^2} \begin{bmatrix} (y_2 - y_3) & (x_3 - x_2) \\ (y_3 - y_1) & (x_1 - x_3) \\ (y_1 - y_2) & (x_2 - x_1) \end{bmatrix} \begin{bmatrix} (y_2 - y_3) & (y_3 - y_1) & (y_1 - y_2) \\ (x_3 - x_2) & (x_1 - x_3) & (x_2 - x_1) \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = 100 \begin{vmatrix} (x_2 - x_1) & (x_3 - x_1) \\ (y_2 - y_1) & (y_3 - y_1) \end{vmatrix} \begin{bmatrix} 0 \\ 1/2 \\ 1/2 \end{bmatrix}$$

Calculando elementos:

1.

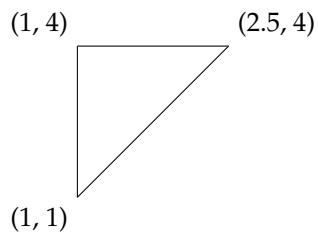


$$\begin{array}{lll} x_1 = 2 & y_1 = 3 & a = 2 \\ x_2 = 2,5 & y_2 = 3 & b = 2,5 \\ x_3 = 2,5 & y_3 = 4 & c = 3 \\ & & d = 4 \end{array}$$

$$\frac{10(0,5)(1)}{\begin{vmatrix} 0,5 & 0 \\ 0,5 & 1 \end{vmatrix}^2} \begin{bmatrix} -1 & 0 \\ 1 & -0,5 \\ 0 & 0,5 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ 0 & -0,5 & 0,5 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = 100 \begin{vmatrix} 0,5 & 0,5 \\ 0 & 1 \end{vmatrix} \begin{bmatrix} 0 \\ 1/2 \\ 1/2 \end{bmatrix}$$

$$\begin{bmatrix} 20 & -20 & 0 \\ -20 & 25 & -5 \\ 0 & -5 & 5 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 25 \\ 25 \end{bmatrix}$$

2.

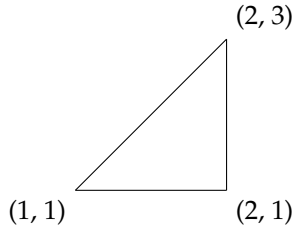


$$\begin{array}{lll} x_1 = 1 & y_1 = 1 & a = 1 \\ x_2 = 2,5 & y_2 = 4 & b = 2,5 \\ x_3 = 1 & y_3 = 4 & c = 1 \\ & & d = 4 \end{array}$$

$$\frac{10(1,5)(3)}{\begin{vmatrix} 1,5 & 3 \\ 0 & 3 \end{vmatrix}^2} \begin{bmatrix} 0 & -1,5 \\ 3 & 0 \\ -3 & 1,5 \end{bmatrix} \begin{bmatrix} 0 & 3 & -3 \\ -1,5 & 0 & 1,5 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = 100 \begin{vmatrix} 1,5 & 0 \\ 3 & 3 \end{vmatrix} \begin{bmatrix} 0 \\ 1/2 \\ 1/2 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 0 & -5 \\ 0 & 20 & -20 \\ -5 & -20 & 25 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 255 \\ 255 \end{bmatrix}$$

3.

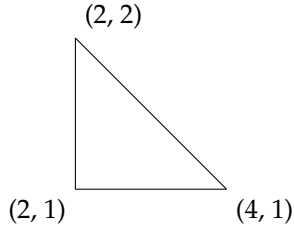


$$\begin{array}{lll} x_1 = 1 & y_1 = 1 & a = 1 \\ x_2 = 2 & y_2 = 1 & b = 2 \\ x_3 = 2 & y_3 = 3 & c = 1 \\ & & d = 3 \end{array}$$

$$\frac{10(1)(2)}{\begin{vmatrix} 1 & 0 \\ 1 & 2 \end{vmatrix}}^2 \begin{bmatrix} -2 & 0 \\ 2 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -2 & 2 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = 100 \begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} \begin{bmatrix} 0 \\ 0,5 \\ 0,5 \end{bmatrix}$$

$$\begin{bmatrix} 20 & -20 & 0 \\ -20 & 25 & -5 \\ 0 & -5 & 5 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 100 \\ 100 \end{bmatrix}$$

4.

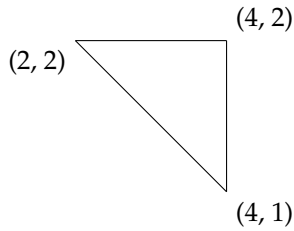


$$\begin{array}{lll} x_1 = 2 & y_1 = 1 & a = 2 \\ x_2 = 3 & y_2 = 1 & b = 4 \\ x_3 = 2 & y_3 = 2 & c = 1 \\ & & d = 2 \end{array}$$

$$\frac{10(2)(1)}{\begin{vmatrix} 2 & 0 \\ 0 & 1 \end{vmatrix}}^2 \begin{bmatrix} -1 & -2 \\ 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ -2 & 0 & 2 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = 100 \begin{vmatrix} 2 & 0 \\ 0 & 1 \end{vmatrix} \begin{bmatrix} 0 \\ 0,5 \\ 0,5 \end{bmatrix}$$

$$\begin{bmatrix} 25 & -5 & -20 \\ -5 & 5 & 0 \\ -20 & 0 & 20 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 100 \\ 100 \end{bmatrix}$$

5.

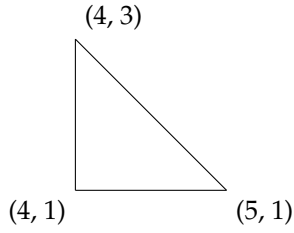


$$\begin{array}{lll} x_1 = 2 & y_1 = 2 & a = 2 \\ x_2 = 4 & y_2 = 1 & b = 4 \\ x_3 = 4 & y_3 = 2 & c = 1 \\ & & d = 2 \end{array}$$

$$\frac{10(2)(1)}{\begin{vmatrix} 2 & -1 \\ 2 & 0 \end{vmatrix}}^2 \begin{bmatrix} -1 & 0 \\ 0 & -2 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} -1 & 0 & 1 \\ 0 & -2 & 2 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = 100 \begin{vmatrix} 2 & 2 \\ -1 & 0 \end{vmatrix} \begin{bmatrix} 0 \\ 0,5 \\ 0,5 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 0 & -5 \\ 0 & 20 & -20 \\ -5 & -20 & 25 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 100 \\ 100 \end{bmatrix}$$

6.

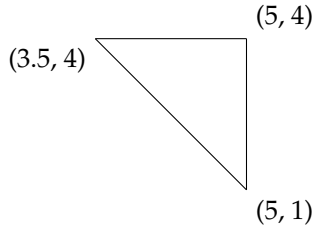


$$\begin{array}{lll} x_1 = 4 & y_1 = 1 & a = 4 \\ x_2 = 5 & y_2 = 1 & b = 5 \\ x_3 = 4 & y_3 = 3 & c = 1 \\ & & d = 3 \end{array}$$

$$\frac{10(1)(2)}{\begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix}}^2 \begin{bmatrix} -2 & 1 \\ 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -2 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = 100 \begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix} \begin{bmatrix} 0 \\ 0,5 \\ 0,5 \end{bmatrix}$$

$$\begin{bmatrix} 25 & -20 & -5 \\ -20 & 20 & 0 \\ -5 & 0 & 5 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 100 \\ 100 \end{bmatrix}$$

7.

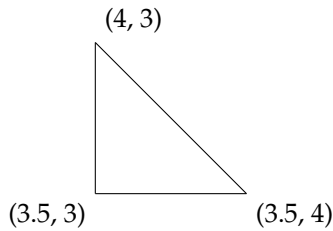


$$\begin{array}{lll} x_1 = 3,5 & y_1 = 4 & a = 3,5 \\ x_2 = 5 & y_2 = 1 & b = 5 \\ x_3 = 5 & y_3 = 4 & c = 1 \\ & & d = 4 \end{array}$$

$$\frac{10(1,5)(3)}{\begin{vmatrix} 1,5 & -3 \\ 1,5 & 0 \end{vmatrix}}^2 \begin{bmatrix} -3 & 0 \\ 0 & -1,5 \\ -3 & 1,5 \end{bmatrix} \begin{bmatrix} -3 & 0 & 3 \\ 0 & -1,5 & 1,5 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = 100 \begin{vmatrix} 1,5 & 1,5 \\ -3 & 0 \end{vmatrix} \begin{bmatrix} 0 \\ 0,5 \\ 0,5 \end{bmatrix}$$

$$\begin{bmatrix} 20 & 0 & -20 \\ 0 & 5 & -5 \\ -20 & -5 & 25 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 225 \\ 225 \end{bmatrix}$$

8.



$$\begin{array}{lll} x_1 = 3,5 & y_1 = 4 & a = 3,5 \\ x_2 = 3,5 & y_2 = 3 & b = 4 \\ x_3 = 4 & y_3 = 3 & c = 3 \\ & & d = 4 \end{array}$$

$$\frac{10(0,5)(1)}{\begin{vmatrix} 0 & -1 \\ 0,5 & -1 \end{vmatrix}}^2 \begin{bmatrix} 0 & 0,5 \\ -1 & -0,5 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 & 1 \\ 0 & -5 & -0,5 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = 100 \begin{vmatrix} 0 & 0,5 \\ -1 & -1 \end{vmatrix} \begin{bmatrix} 0 \\ 0,5 \\ 0,5 \end{bmatrix}$$

$$\begin{bmatrix} 5 & -5 & 0 \\ -5 & 25 & 20 \\ 0 & -20 & 20 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 25 \\ 25 \end{bmatrix}$$



Ensamblaje y sistema final dadas las matrices  $\underline{K}$ ,  $\underline{T}$  y  $\underline{b}$  :

$$\begin{bmatrix}
 k_{1,1}^{(2)} + k_{1,1}^{(3)} & k_{1,2}^{(3)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & k_{1,3}^{(3)} & 0 & k_{1,2}^{(2)} & k_{1,3}^{(2)} \\
 k_{2,1}^{(3)} & k_{2,2}^{(3)} + k_{1,1}^{(4)} & k_{1,2}^{(4)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & k_{2,3}^{(3)} & 0 & 0 & 0 \\
 0 & k_{2,1}^{(4)} & k_{2,2}^{(4)} + k_{2,2}^{(5)} + k_{1,1}^{(6)} & k_{1,2}^{(6)} & 0 & 0 & 0 & k_{1,3}^{(6)} & k_{2,3}^{(5)} & k_{2,3}^{(4)} + k_{2,1}^{(5)} & 0 & 0 & 0 & 0 \\
 0 & 0 & k_{2,1}^{(6)} & k_{2,2}^{(6)} + k_{2,2}^{(7)} & k_{2,3}^{(7)} & k_{2,1}^{(7)} & 0 & k_{2,3}^{(6)} & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & k_{3,2}^{(7)} & k_{3,3}^{(7)} & k_{3,1}^{(7)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & k_{1,2}^{(7)} & k_{1,3}^{(7)} & k_{1,1}^{(7)} + k_{1,1}^{(8)} & k_{1,2}^{(8)} & k_{1,3}^{(8)} & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & k_{2,1}^{(8)} & k_{2,2}^{(8)} & k_{2,3}^{(8)} & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & k_{3,1}^{(6)} & k_{3,2}^{(6)} & 0 & k_{3,1}^{(8)} & k_{3,2}^{(8)} & k_{3,3}^{(8)} + k_{3,3}^{(8)} & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & k_{3,2}^{(5)} & 0 & 0 & 0 & 0 & 0 & k_{3,3}^{(5)} & k_{3,1}^{(5)} & 0 & 0 & 0 & 0 \\
 0 & k_{3,1}^{(4)} & k_{3,2}^{(4)} + k_{1,2}^{(5)} & 0 & 0 & 0 & 0 & 0 & k_{1,3}^{(5)} & k_{3,3}^{(4)} + k_{1,1}^{(5)} & 0 & 0 & 0 & 0 \\
 k_{3,1}^{(3)} & k_{3,2}^{(3)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & k_{1,1}^{(1)} + k_{3,3}^{(3)} & k_{1,2}^{(1)} & k_{1,3}^{(1)} & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & k_{2,1}^{(1)} & k_{2,2}^{(1)} & k_{2,3}^{(1)} & 0 \\
 k_{2,1}^{(2)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & k_{3,1}^{(1)} & k_{3,2}^{(1)} & k_{3,3}^{(1)} + k_{2,2}^{(2)} & k_{2,3}^{(2)} \\
 k_{3,1}^{(2)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & k_{3,2}^{(2)} & k_{3,3}^{(2)} & k_{3,3}^{(2)}
 \end{bmatrix}
 \begin{bmatrix}
 T_1 \\
 T_2 \\
 T_3 \\
 T_4 \\
 T_5 \\
 T_6 \\
 T_7 \\
 T_8 \\
 T_9 \\
 T_{10} \\
 T_{11} \\
 T_{12} \\
 T_{13} \\
 T_{14}
 \end{bmatrix}
 =
 \begin{bmatrix}
 b_2^{(2)} + b_1^{(3)} \\
 b_2^{(4)} + b_2^{(5)} + b_1^{(6)} \\
 b_2^{(6)} + b_2^{(7)} \\
 b_3^{(7)} \\
 b_1^{(7)} + b_1^{(8)} \\
 b_2^{(8)} \\
 b_3^{(6)} + b_3^{(8)} \\
 b_3^{(5)} \\
 b_3^{(4)} + b_1^{(5)} \\
 b_1^{(1)} + b_3^{(3)} \\
 b_2^{(1)} \\
 b_3^{(1)} + b_2^{(2)} \\
 b_3^{(2)}
 \end{bmatrix}$$

$$\begin{bmatrix}
 40,0 & -20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 -20 & 50 & -5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -20 & -5 & 0 & 0 \\
 0 & -5 & 35 & -20 & 0 & 0 & 0 & 0 & -5 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & -20 & 40,0 & 0 & -20,0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 5,0 & -5,0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & -20,0 & -5,0 & 45,0 & 40,0 & -60,0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 40,0 & 85,0 & -125,0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & -5 & 0 & 0 & -60,0 & -125,0 & 190,0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 20 & -20 & 0 & 0 & 0 & 0 \\
 0 & -20 & -5 & 0 & 0 & 0 & 0 & 0 & -20 & 45 & 0 & 0 & 0 & 0 \\
 0 & -5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10,0 & 0 & -5,0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 20,0 & -20,0 & 0 \\
 -20,0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -5,0 & -20,0 & 50,0 & -5,0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -5,0 & 5,0 & 0
 \end{bmatrix}
 \begin{bmatrix}
 T_1 \\
 T_2 \\
 T_3 \\
 T_4 \\
 T_5 \\
 T_6 \\
 T_7 \\
 T_8 \\
 T_9 \\
 T_{10} \\
 T_{11} \\
 T_{12} \\
 T_{13} \\
 T_{14}
 \end{bmatrix}
 =
 \begin{bmatrix}
 0 \\
 100,0 \\
 100,0 \\
 100,0 \\
 225,0 \\
 250,0 \\
 0 \\
 125,0 \\
 100,0 \\
 200,0 \\
 100,0 \\
 225,0 \\
 250,0 \\
 25,0
 \end{bmatrix}$$

Aplicaciones de condiciones de contorno

$$\Gamma_D = \{T_1, T_{14}\}$$

$$\Gamma_N = \left\{ \frac{dT_4}{dt}, \frac{dT_5}{dt} \right\}$$

$$\begin{bmatrix}
 k_{2,1}^{(3)} & k_{2,2}^{(3)} + k_{1,1}^{(4)} & k_{1,2}^{(4)} & 0 & 0 & 0 & 0 & 0 & 0 & k_{1,3}^{(4)} & k_{2,3}^{(3)} & 0 & 0 & 0 \\
 0 & k_{2,1}^{(4)} & k_{2,2}^{(4)} + k_{2,2}^{(5)} + k_{1,1}^{(6)} & k_{1,2}^{(6)} & 0 & 0 & 0 & k_{1,3}^{(6)} & k_{2,3}^{(5)} & k_{2,3}^{(4)} + k_{2,1}^{(5)} & 0 & 0 & 0 & 0 \\
 0 & 0 & k_{2,1}^{(6)} & k_{2,2}^{(6)} + k_{2,2}^{(7)} & k_{2,3}^{(7)} & k_{2,1}^{(7)} & 0 & k_{2,3}^{(6)} & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & k_{3,2}^{(7)} & k_{3,3}^{(7)} & k_{3,1}^{(7)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & k_{1,2}^{(7)} & k_{1,3}^{(7)} & k_{1,1}^{(7)} + k_{1,1}^{(8)} & k_{1,2}^{(8)} & k_{1,3}^{(8)} & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & k_{2,1}^{(8)} & k_{2,2}^{(8)} & k_{2,3}^{(8)} & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & k_{3,1}^{(6)} & k_{3,2}^{(6)} & 0 & k_{3,1}^{(8)} & k_{3,2}^{(8)} & k_{3,3}^{(8)} + k_{3,3}^{(8)} & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & k_{3,2}^{(5)} & 0 & 0 & 0 & 0 & 0 & k_{3,3}^{(5)} & k_{3,1}^{(5)} & 0 & 0 & 0 & 0 \\
 0 & k_{3,1}^{(4)} & k_{3,2}^{(4)} + k_{1,2}^{(5)} & 0 & 0 & 0 & 0 & 0 & k_{1,3}^{(5)} & k_{3,3}^{(4)} + k_{1,1}^{(5)} & 0 & 0 & 0 & 0 \\
 k_{3,1}^{(3)} & k_{3,2}^{(3)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & k_{1,1}^{(1)} + k_{3,3}^{(3)} & k_{1,2}^{(1)} & k_{1,3}^{(1)} & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & k_{2,1}^{(1)} & k_{2,2}^{(1)} & k_{2,3}^{(1)} & 0 \\
 k_{2,1}^{(2)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & k_{3,1}^{(1)} & k_{3,2}^{(1)} & k_{3,3}^{(1)} + k_{2,2}^{(2)} & k_{2,3}^{(2)}
 \end{bmatrix}
 \begin{bmatrix}
 T_2 \\
 T_3 \\
 T_4 \\
 T_5 \\
 T_6 \\
 T_7 \\
 T_8 \\
 T_9 \\
 T_{10} \\
 T_{11} \\
 T_{12} \\
 T_{13}
 \end{bmatrix}
 =
 \begin{bmatrix}
 b_2^{(3)} + b_1^{(4)} \\
 b_2^{(4)} + b_2^{(5)} + b_1^{(6)} \\
 b_2^{(6)} + b_2^{(7)} \\
 b_3^{(7)} \\
 b_1^{(7)} + b_1^{(8)} \\
 b_2^{(8)} \\
 b_3^{(6)} + b_3^{(8)} \\
 b_3^{(5)} \\
 b_3^{(4)} + b_1^{(5)} \\
 b_1^{(1)} + b_3^{(3)} \\
 b_2^{(1)} \\
 b_3^{(1)} + b_2^{(2)}
 \end{bmatrix}
 +
 \begin{bmatrix}
 -T_1 k_{2,1}^{(3)} \\
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 -T_1 k_{3,1}^{(3)} \\
 0 \\
 -T_{14} k_{2,3}^{(2)} - T_1 k_{2,1}^{(2)}
 \end{bmatrix}
 +
 \begin{bmatrix}
 0 \\
 0 \\
 \frac{dT_4}{dt} \\
 \frac{dT_5}{dt} \\
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0
 \end{bmatrix}$$