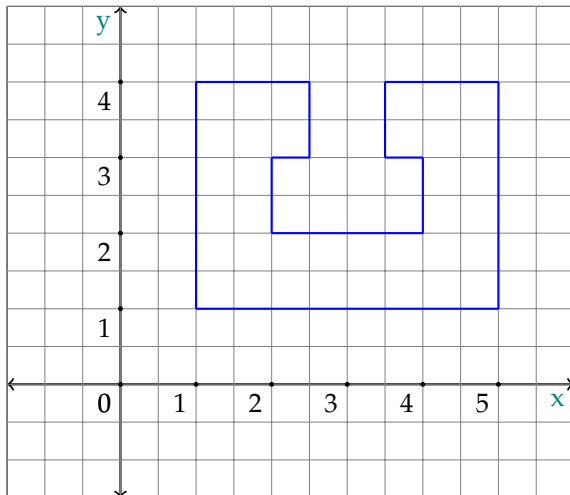# Técnicas de simulación por computadora
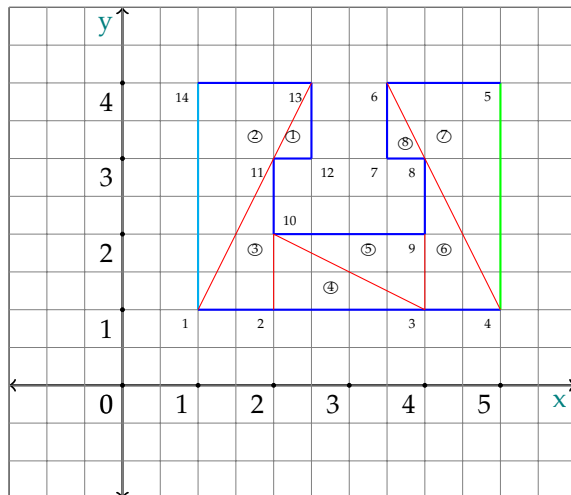
Grupo O

3 de junio de 2018

Dada la figura y ecuación diferencial:

$$\nabla(10\nabla T) = -100$$

Mallado de la figura y establecimiento de contornos para condiciones de Dirichlet y Neumann:

O Neumann
O Dirichlet

Tabla de conectividad:

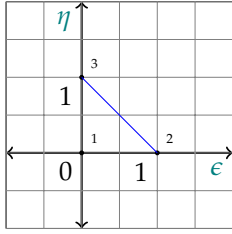| elemento	1	2	3
①	11	12	13
②	1	13	14
③	1	2	11
④	2	3	10
⑤	10	3	9
⑥	3	4	8
⑦	6	4	5
⑧	6	7	8

Condiciones a utilizar:

$$\Gamma_D =$$
$$\Gamma_N =$$

Aproximación en el plano isoparamétrico:

$$T \approx f_1 T_1 + f_2 T_2 + f_3 T_3$$
$$\approx N_1 T_1 + N_2 T_2 + N_3 T_3$$
$$\approx \begin{bmatrix} N_1 & N_2 & N_3 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix}$$
$$\approx \underline{\underline{N}} \vec{T}$$

con $N_1 = 1 - \epsilon - \eta$, $N_2 = \epsilon$, $N_3 = \eta$
$x \approx (x_2 - x_1)\epsilon + (x_3 - x_1)\eta + x_1$
$y \approx (y_2 - y_1)\epsilon + (y_3 - y_1)\eta + y_1$

Sustitución de la función por su aproximación:

$$\nabla(10\nabla T) = -100$$
$$\nabla(10\nabla \underline{\underline{N}}\vec{T}) \approx -100$$
$$\nabla(10\nabla \underline{\underline{N}}\vec{T}) + 100 \neq 0$$
$$\nabla(10\nabla \underline{\underline{N}}\vec{T}) + 100 = \mathcal{R}$$

W.R.M.:

$$\int_\Omega \underline{\underline{W}} \mathcal{R} \, d\Omega = 0$$
$$\int_A \underline{\underline{W}} \mathcal{R} \, dA = 0$$
$$\int_A \underline{\underline{W}} (\nabla(10\nabla \underline{\underline{N}}\vec{T}) + 100) \, dA = 0$$

Galerkin:

$$\underline{\underline{W}} = \underline{N}^T$$
$$\int_A \underline{N}^T (\nabla(10\nabla \underline{\underline{N}}\vec{T}) + 100) \, dA = 0$$
$$\int_A \underline{N}^T (\nabla(10\nabla \underline{\underline{N}}\vec{T})) \, dA + \int_A \underline{N}^T (100) \, dA = 0$$

Integración por partes:

sea $U = \underline{N}^T$, $dU = \nabla\underline{N}^T$
sea $dV = \nabla(10\nabla \underline{\underline{N}}\vec{T})$, $V = 10\nabla(\underline{\underline{N}}\vec{T})$

$$\int_A \underline{N}^T (\nabla(10\nabla \underline{\underline{N}}\vec{T})) \, dA = [\underline{N}^T 10\nabla(\underline{\underline{N}}\vec{T})]_{\Gamma_N} - \int_A \nabla\underline{N}^t 10\nabla(\underline{\underline{N}}\vec{T}) \, dA$$

3

Trabajando en la ecuación:

$$-\int_A \nabla \underline{\underline{N}}^T 10 \nabla (\underline{\underline{N}} \vec{T}) \, dA + \int_A \underline{\underline{N}}^T (100) \, dA = 0$$

$$\int_A \nabla \underline{\underline{N}}^T 10 \nabla (\underline{\underline{N}} \vec{T}) \, dA = \int_A \underline{\underline{N}}^T (100) \, dA$$

$$10 \int_A \nabla \underline{\underline{N}}^T \nabla (\underline{\underline{N}}) \, dA (\vec{T}) = 100 \int_A \underline{\underline{N}}^T \, dA$$

Lado derecho:

$$100 \int_A \underline{\underline{N}}^T \, dA = 100 \int_A \begin{bmatrix} 1 - \epsilon - \eta \\ \epsilon \\ \eta \end{bmatrix} dA$$

$$= 100 \int_A \begin{bmatrix} 1 - \epsilon - \eta \\ \epsilon \\ \eta \end{bmatrix} dx \, dy$$

Llevando la integral al plano isoparamétrico con un jacobiano:

$$dx \, dy = D \, d\epsilon \, d\eta$$

$$J = \begin{bmatrix} \frac{\delta x}{\delta \epsilon} & \frac{\delta x}{\delta \eta} \\ \frac{\delta y}{\delta \epsilon} & \frac{\delta y}{\delta \eta} \end{bmatrix}$$

$$= \begin{bmatrix} (x_2 - x_1) & (x_3 - x_1) \\ (y_2 - y_1) & (y_3 - y_1) \end{bmatrix}$$

$$D = |J|$$

$$= (x_2 - x_1)(y_3 - y_1) - (x_3 - x_1)(y_2 - y_1)$$

Sustituyendo en la integral:

$$100 \int_A \begin{bmatrix} 1 - \epsilon - \eta \\ \epsilon \\ \eta \end{bmatrix} dA = 100 \int_0^1 \int_0^1 \begin{bmatrix} 1 - \epsilon - \eta \\ \epsilon \\ \eta \end{bmatrix} \begin{vmatrix} (x_2 - x_1) & (x_3 - x_1) \\ (y_2 - y_1) & (y_3 - y_1) \end{vmatrix} d\epsilon \, d\eta$$

$$= 100 \begin{vmatrix} (x_2 - x_1) & (x_3 - x_1) \\ (y_2 - y_1) & (y_3 - y_1) \end{vmatrix} \int_0^1 \int_0^1 \begin{bmatrix} 1 - \epsilon - \eta \\ \epsilon \\ \eta \end{bmatrix} d\epsilon \, d\eta$$

$$= 100 \begin{vmatrix} (x_2 - x_1) & (x_3 - x_1) \\ (y_2 - y_1) & (y_3 - y_1) \end{vmatrix} \begin{bmatrix} \int_0^1 \int_0^1 (1 - \epsilon - \eta) \, d\epsilon \, d\eta \\ \int_0^1 \int_0^1 \epsilon \, d\epsilon \, d\eta \\ \int_0^1 \int_0^1 \eta \, d\epsilon \, d\eta \end{bmatrix}$$

$$= 100 \begin{vmatrix} (x_2 - x_1) & (x_3 - x_1) \\ (y_2 - y_1) & (y_3 - y_1) \end{vmatrix} \begin{bmatrix} 0 \\ 1/2 \\ 1/2 \end{bmatrix}$$

Trabajando lado izquierdo de la ecuación:

$$10 \int_A \nabla \underline{\underline{N}}^T \nabla (\underline{N}) \, dA(\vec{T}) = 10 \int_c^d \int_a^b \nabla \underline{\underline{N}}^T \nabla (\underline{N}) \, dx \, dy(\vec{T})$$

$$\text{con } \nabla_x \underline{\underline{N}} = (\nabla_\epsilon \underline{x})^{-1} \nabla_\epsilon \underline{N}$$

$$= \begin{bmatrix} (x_2 - x_1) & (y_2 - y_1) \\ (x_3 - x_1) & (y_3 - y_1) \end{bmatrix}^{-1} \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$$= \frac{1}{\begin{vmatrix} (x_2 - x_1) & (y_2 - y_1) \\ (x_3 - x_1) & (y_3 - y_1) \end{vmatrix}} \begin{bmatrix} (y_3 - y_1) & (y_1 - y_2) \\ (x_1 - x_3) & (x_2 - x_1) \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$$= \frac{1}{\begin{vmatrix} (x_2 - x_1) & (y_2 - y_1) \\ (x_3 - x_1) & (y_3 - y_1) \end{vmatrix}} \begin{bmatrix} (y_2 - y_3) & (y_3 - y_1) & (y_1 - y_2) \\ (x_3 - x_2) & (x_1 - x_3) & (x_2 - x_1) \end{bmatrix}$$

$$\nabla \underline{\underline{N}}^T = \frac{1}{\begin{vmatrix} (x_2 - x_1) & (y_2 - y_1) \\ (x_3 - x_1) & (y_3 - y_1) \end{vmatrix}} \begin{bmatrix} (y_2 - y_3) & (x_3 - x_2) \\ (y_3 - y_1) & (x_1 - x_3) \\ (y_1 - y_2) & (x_2 - x_1) \end{bmatrix}$$

Sustituyendo en la integral:

$$= 10 \int_c^d \int_a^b \frac{1}{\begin{vmatrix} (x_2 - x_1) & (y_2 - y_1) \\ (x_3 - x_1) & (y_3 - y_1) \end{vmatrix}^2} \begin{bmatrix} (y_2 - y_3) & (x_3 - x_2) \\ (y_3 - y_1) & (x_1 - x_3) \\ (y_1 - y_2) & (x_2 - x_1) \end{bmatrix} \begin{bmatrix} (y_2 - y_3) & (y_3 - y_1) & (y_1 - y_2) \\ (x_3 - x_2) & (x_1 - x_3) & (x_2 - x_1) \end{bmatrix} dx \, dy(\vec{T})$$

$$= 10 \frac{(x \big|_a^b)(y \big|_c^d)}{\begin{vmatrix} (x_2 - x_1) & (y_2 - y_1) \\ (x_3 - x_1) & (y_3 - y_1) \end{vmatrix}^2} \begin{bmatrix} (y_2 - y_3) & (x_3 - x_2) \\ (y_3 - y_1) & (x_1 - x_3) \\ (y_1 - y_2) & (x_2 - x_1) \end{bmatrix} \begin{bmatrix} (y_2 - y_3) & (y_3 - y_1) & (y_1 - y_2) \\ (x_3 - x_2) & (x_1 - x_3) & (x_2 - x_1) \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix}$$

Sistema local final:

$$\frac{10(x \big|_a^b)(y \big|_c^d)}{\begin{vmatrix} (x_2 - x_1) & (y_2 - y_1) \\ (x_3 - x_1) & (y_3 - y_1) \end{vmatrix}^2} \begin{bmatrix} (y_2 - y_3) & (x_3 - x_2) \\ (y_3 - y_1) & (x_1 - x_3) \\ (y_1 - y_2) & (x_2 - x_1) \end{bmatrix} \begin{bmatrix} (y_2 - y_3) & (y_3 - y_1) & (y_1 - y_2) \\ (x_3 - x_2) & (x_1 - x_3) & (x_2 - x_1) \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = 100 \begin{vmatrix} (x_2 - x_1) & (x_3 - x_1) \\ (y_2 - y_1) & (y_3 - y_1) \end{vmatrix} \begin{bmatrix} 0 \\ 1/2 \\ 1/2 \end{bmatrix}$$
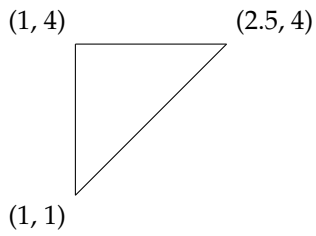
Calculando elementos:

1.

(2.5, 4)

(2, 3)   (2.5, 3)
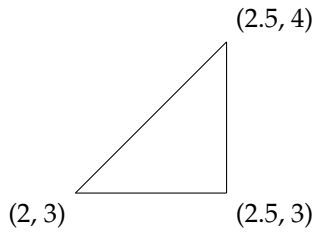
$$x_1 = 2 \quad y_1 = 3 \quad a = 2$$
$$x_2 = 2{,}5 \quad y_2 = 3 \quad b = 2{,}5$$
$$x_3 = 2{,}5 \quad y_3 = 4 \quad c = 3$$
$$d = 4$$

$$\frac{10(0{,}5)(1)}{\begin{vmatrix} 0{,}5 & 0 \\ 0{,}5 & 1 \end{vmatrix}^2} \begin{bmatrix} -1 & 0 \\ 1 & -0{,}5 \\ 0 & 0{,}5 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ 0 & -0{,}5 & 0{,}5 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = 100 \begin{vmatrix} 0{,}5 & 0{,}5 \\ 0 & 1 \end{vmatrix} \begin{bmatrix} 0 \\ 1/2 \\ 1/2 \end{bmatrix}$$

$$\begin{bmatrix} 20 & -20 & 0 \\ -20 & 25 & -5 \\ 0 & -5 & 5 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 25 \\ 25 \end{bmatrix}$$
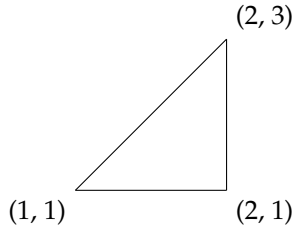
2.

(1, 4)   (2.5, 4)

(1, 1)

$$x_1 = 1 \quad y_1 = 1 \quad a = 1$$
$$x_2 = 2{,}5 \quad y_2 = 4 \quad b = 2{,}5$$
$$x_3 = 1 \quad y_3 = 4 \quad c = 1$$
$$d = 4$$

$$\frac{10(1{,}5)(3)}{\begin{vmatrix} 1{,}5 & 3 \\ 0 & 3 \end{vmatrix}^2} \begin{bmatrix} 0 & -1{,}5 \\ 3 & 0 \\ -3 & 1{,}5 \end{bmatrix} \begin{bmatrix} 0 & 3 & -3 \\ -1{,}5 & 0 & 1{,}5 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = 100 \begin{vmatrix} 1{,}5 & 0 \\ 3 & 3 \end{vmatrix} \begin{bmatrix} 0 \\ 1/2 \\ 1/2 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 0 & -5 \\ 0 & 20 & -20 \\ -5 & -20 & 25 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 255 \\ 255 \end{bmatrix}$$
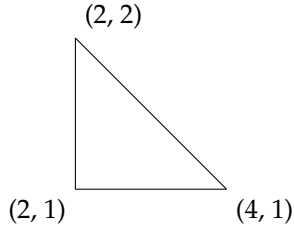
3.

(2, 3)

(1, 1)   (2, 1)

$$x_1 = 1 \quad y_1 = 1 \quad a = 1$$
$$x_2 = 2 \quad y_2 = 1 \quad b = 2$$
$$x_3 = 2 \quad y_3 = 3 \quad c = 1$$
$$d = 3$$

$$\frac{10(1)(2)}{\begin{vmatrix} 1 & 0 \\ 1 & 2 \end{vmatrix}^2} \begin{bmatrix} -2 & 0 \\ 2 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -2 & 2 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = 100 \begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} \begin{bmatrix} 0 \\ 0,5 \\ 0,5 \end{bmatrix}$$

$$\begin{bmatrix} 20 & -20 & 0 \\ -20 & 25 & -5 \\ 0 & -5 & 5 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 100 \\ 100 \end{bmatrix}$$
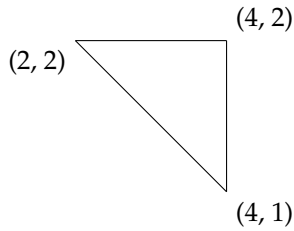
4.

(2, 2)

(2, 1)   (4, 1)

$$x_1 = 2 \quad y_1 = 1 \quad a = 2$$
$$x_2 = 3 \quad y_2 = 1 \quad b = 4$$
$$x_3 = 2 \quad y_3 = 2 \quad c = 1$$
$$d = 2$$

$$\frac{10(2)(1)}{\begin{vmatrix} 2 & 0 \\ 0 & 1 \end{vmatrix}^2} \begin{bmatrix} -1 & -2 \\ 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ -2 & 0 & 2 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = 100 \begin{vmatrix} 2 & 0 \\ 0 & 1 \end{vmatrix} \begin{bmatrix} 0 \\ 0,5 \\ 0,5 \end{bmatrix}$$

$$\begin{bmatrix} 25 & -5 & -20 \\ -5 & 5 & 0 \\ -20 & 0 & 20 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 100 \\ 100 \end{bmatrix}$$
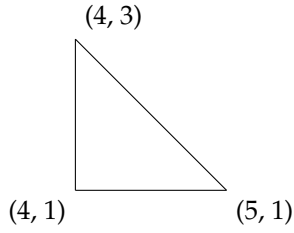
5.

(2, 2)   (4, 2)

(4, 1)

$$x_1 = 2 \quad y_1 = 2 \quad a = 2$$
$$x_2 = 4 \quad y_2 = 1 \quad b = 4$$
$$x_3 = 4 \quad y_3 = 2 \quad c = 1$$
$$d = 2$$

$$\frac{10(2)(1)}{\begin{vmatrix} 2 & -1 \\ 2 & 0 \end{vmatrix}^2} \begin{bmatrix} -1 & 0 \\ 0 & -2 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} -1 & 0 & 1 \\ 0 & -2 & 2 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = 100 \begin{vmatrix} 2 & 2 \\ -1 & 0 \end{vmatrix} \begin{bmatrix} 0 \\ 0,5 \\ 0,5 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 0 & -5 \\ 0 & 20 & -20 \\ -5 & -20 & 25 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 100 \\ 100 \end{bmatrix}$$

6.

$(4, 3)$

$(4, 1)$ $(5, 1)$

$$
\begin{aligned}
x_1 &= 4 & y_1 &= 1 & a &= 4 \\
x_2 &= 5 & y_2 &= 1 & b &= 5 \\
x_3 &= 4 & y_3 &= 3 & c &= 1 \\
 & & & & d &= 3
\end{aligned}
$$

$$
\frac{10(1)(2)}{\begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix}^2}
\begin{bmatrix} -2 & 1 \\ 2 & 0 \\ 0 & 1 \end{bmatrix}
\begin{bmatrix} -2 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}
\begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix}
= 100 \begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix}
\begin{bmatrix} 0 \\ 0{,}5 \\ 0{,}5 \end{bmatrix}
$$

$$
\begin{bmatrix} 25 & -20 & -5 \\ -20 & 20 & 0 \\ -5 & 0 & 5 \end{bmatrix}
\begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix}
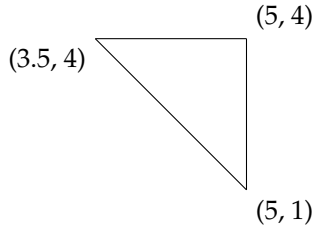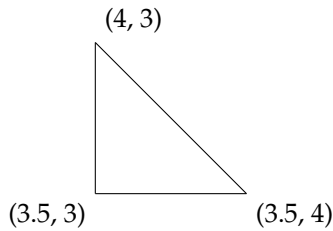= \begin{bmatrix} 0 \\ 100 \\ 100 \end{bmatrix}
$$

7.

$(5, 4)$

$(3.5, 4)$

$(5, 1)$

$$
\begin{aligned}
x_1 &= 3{,}5 & y_1 &= 4 & a &= 3{,}5 \\
x_2 &= 5 & y_2 &= 1 & b &= 5 \\
x_3 &= 5 & y_3 &= 4 & c &= 1 \\
 & & & & d &= 4
\end{aligned}
$$

$$
\frac{10(1{,}5)(3)}{\begin{vmatrix} 1{,}5 & -3 \\ 1{,}5 & 0 \end{vmatrix}^2}
\begin{bmatrix} -3 & 0 \\ 0 & -1{,}5 \\ -3 & 1{,}5 \end{bmatrix}
\begin{bmatrix} -3 & 0 & 3 \\ 0 & -1{,}5 & 1{,}5 \end{bmatrix}
\begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix}
= 100 \begin{vmatrix} 1{,}5 & 1{,}5 \\ -3 & 0 \end{vmatrix}
\begin{bmatrix} 0 \\ 0{,}5 \\ 0{,}5 \end{bmatrix}
$$

$$
\begin{bmatrix} 20 & 0 & -20 \\ 0 & 5 & -5 \\ -20 & -5 & 25 \end{bmatrix}
\begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix}
= \begin{bmatrix} 0 \\ 225 \\ 225 \end{bmatrix}
$$

8.

$(4, 3)$

$(3.5, 3)$ $(3.5, 4)$

$$
\begin{aligned}
x_1 &= 3{,}5 & y_1 &= 4 & a &= 3{,}5 \\
x_2 &= 3{,}5 & y_2 &= 3 & b &= 4 \\
x_3 &= 4 & y_3 &= 3 & c &= 3 \\
 & & & & d &= 4
\end{aligned}
$$

$$
\frac{10(0{,}5)(1)}{\begin{vmatrix} 0 & -1 \\ 0{,}5 & -1 \end{vmatrix}^2}
\begin{bmatrix} 0 & 0{,}5 \\ -1 & -0{,}5 \\ 1 & 0 \end{bmatrix}
\begin{bmatrix} 0 & -1 & 1 \\ 0-5 & -0{,}5 & 0 \end{bmatrix}
\begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix}
= 100 \begin{vmatrix} 0 & 0{,}5 \\ -1 & -1 \end{vmatrix}
\begin{bmatrix} 0 \\ 0{,}5 \\ 0{,}5 \end{bmatrix}
$$

$$
\begin{bmatrix} 5 & -5 & 0 \\ -5 & 25 & -20 \\ 0 & -20 & 20 \end{bmatrix}
\begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix}
= \begin{bmatrix} 0 \\ 25 \\ 25 \end{bmatrix}
$$

8

Ensamblaje y sistema final dadas las matrices $\underline{\underline{K}}$, $\underline{T}$ y $\underline{b}$:

$$\underline{\underline{K}}\,\underline{T} = \underline{b}$$

$$
\begin{bmatrix}
k_{1,1}^{(2)}+k_{1,1}^{(3)} & k_{1,2}^{(3)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & k_{1,3}^{(3)} & 0 & k_{1,2}^{(2)} & k_{1,3}^{(2)} \\
k_{2,1}^{(3)} & k_{2,2}^{(3)}+k_{1,1}^{(4)} & k_{1,2}^{(4)} & 0 & 0 & 0 & 0 & 0 & 0 & k_{1,3}^{(4)} & k_{2,3}^{(3)} & 0 & 0 & 0 \\
0 & k_{2,1}^{(4)} & k_{2,2}^{(4)}+k_{2,2}^{(5)}+k_{1,1}^{(6)} & k_{1,2}^{(6)} & 0 & 0 & 0 & k_{1,3}^{(6)} & k_{2,3}^{(5)} & k_{2,3}^{(4)}+k_{2,1}^{(5)} & 0 & 0 & 0 & 0 \\
0 & 0 & k_{2,1}^{(6)} & k_{2,2}^{(6)}+k_{2,2}^{(7)} & k_{2,3}^{(7)} & k_{2,1}^{(7)} & 0 & k_{2,3}^{(6)} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & k_{3,2}^{(7)} & k_{3,3}^{(7)} & k_{3,1}^{(7)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & k_{1,2}^{(7)} & k_{1,3}^{(7)} & k_{1,1}^{(7)}+k_{1,1}^{(8)} & k_{1,2}^{(8)} & k_{1,3}^{(8)} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & k_{2,1}^{(8)} & k_{2,2}^{(8)} & k_{2,3}^{(8)} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & k_{3,1}^{(6)} & k_{3,2}^{(6)} & 0 & k_{3,1}^{(8)} & k_{3,2}^{(8)} & k_{3,3}^{(6)}+k_{3,3}^{(8)} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & k_{3,2}^{(5)} & 0 & 0 & 0 & 0 & 0 & k_{3,3}^{(5)} & k_{3,1}^{(5)} & 0 & 0 & 0 & 0 \\
0 & k_{3,1}^{(4)} & k_{3,2}^{(4)}+k_{1,2}^{(5)} & 0 & 0 & 0 & 0 & 0 & k_{1,3}^{(5)} & k_{3,3}^{(4)}+k_{1,1}^{(5)} & 0 & 0 & 0 & 0 \\
k_{3,1}^{(3)} & k_{3,2}^{(3)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & k_{1,1}^{(1)}+k_{3,3}^{(3)} & k_{1,2}^{(1)} & k_{1,3}^{(1)} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & k_{2,1}^{(1)} & k_{2,2}^{(1)} & k_{2,3}^{(1)} & 0 \\
k_{2,1}^{(2)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & k_{3,1}^{(1)} & k_{3,2}^{(1)} & k_{3,3}^{(1)}+k_{2,2}^{(2)} & k_{2,3}^{(2)} \\
k_{3,1}^{(2)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & k_{3,2}^{(2)} & k_{3,3}^{(2)}
\end{bmatrix}
\begin{bmatrix}
T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \\ T_8 \\ T_9 \\ T_{10} \\ T_{11} \\ T_{12} \\ T_{13} \\ T_{14}
\end{bmatrix}
=
\begin{bmatrix}
b_1^{(2)}+b_1^{(3)} \\
b_2^{(3)}+b_1^{(4)} \\
b_2^{(4)}+b_2^{(5)}+b_1^{(6)} \\
b_2^{(6)}+b_2^{(7)} \\
b_3^{(7)} \\
b_1^{(7)}+b_1^{(8)} \\
b_2^{(8)} \\
b_3^{(6)}+b_3^{(8)} \\
b_3^{(5)} \\
b_3^{(4)}+b_1^{(5)} \\
b_1^{(1)}+b_3^{(3)} \\
b_2^{(1)} \\
b_3^{(1)}+b_2^{(2)} \\
b_3^{(2)}
\end{bmatrix}
$$

$$
\begin{bmatrix}
25,0 & -20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -5,0 \\
-20 & 50 & -5 & 0 & 0 & 0 & 0 & 0 & 0 & -20 & -5 & 0 & 0 & 0 \\
0 & -5 & 50 & -20 & 0 & 0 & 0 & -5 & -20 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -20 & 25,0 & -5,0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -5,0 & 25,0 & -20,0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -20,0 & 25,0 & -5,0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -5,0 & 25,0 & -20,0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -5 & 0 & 0 & 0 & -20,0 & 25,0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -20 & 0 & 0 & 0 & 0 & 0 & 25 & -5 & 0 & 0 & 0 & 0 \\
0 & -20 & 0 & 0 & 0 & 0 & 0 & 0 & -5 & 25 & 0 & 0 & 0 & 0 \\
0 & -5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 25,0 & -20,0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -20,0 & 25,0 & -5,0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -5,0 & 25,0 & -20,0 \\
-5,0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -20,0 & 25,0
\end{bmatrix}
\begin{bmatrix}
T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \\ T_8 \\ T_9 \\ T_{10} \\ T_{11} \\ T_{12} \\ T_{13} \\ T_{14}
\end{bmatrix}
=
\begin{bmatrix}
0 \\ 100,0 \\ 200,0 \\ 325,0 \\ 225,0 \\ 0 \\ 25,0 \\ 125,0 \\ 100,0 \\ 100,0 \\ 100,0 \\ 25,0 \\ 250,0 \\ 225,0
\end{bmatrix}
$$

Aplicaciones de condiciones de contorno

$$\Gamma_D = \{T_1, T_{14}\}$$

$$\Gamma_N = \left\{ \frac{\mathrm{d}T_4}{\mathrm{d}\hat{n}}, \frac{\mathrm{d}T_5}{\mathrm{d}\hat{n}} \right\}$$

$$
\begin{bmatrix}
k_{2,2}^{(3)}+k_{1,1}^{(4)} & k_{1,2}^{(4)} & 0 & 0 & 0 & 0 & 0 & 0 & k_{1,3}^{(4)} & k_{2,3}^{(3)} & 0 & 0 \\
k_{2,1}^{(4)} & k_{2,2}^{(4)}+k_{2,2}^{(5)}+k_{1,1}^{(6)} & k_{1,2}^{(6)} & 0 & 0 & 0 & k_{1,3}^{(6)} & k_{2,3}^{(5)} & k_{2,3}^{(4)}+k_{2,1}^{(5)} & 0 & 0 & 0 \\
0 & k_{2,1}^{(6)} & k_{2,2}^{(6)}+k_{2,2}^{(7)} & k_{2,3}^{(7)} & k_{2,1}^{(7)} & 0 & k_{2,3}^{(6)} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & k_{3,2}^{(7)} & k_{3,3}^{(7)} & k_{3,1}^{(7)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & k_{1,2}^{(7)} & k_{1,3}^{(7)} & k_{1,1}^{(7)}+k_{1,1}^{(8)} & k_{1,2}^{(8)} & k_{1,3}^{(8)} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & k_{2,1}^{(8)} & k_{2,2}^{(8)} & k_{2,3}^{(8)} & 0 & 0 & 0 & 0 & 0 \\
0 & k_{3,1}^{(6)} & k_{3,2}^{(6)} & 0 & k_{3,1}^{(8)} & k_{3,2}^{(8)} & k_{3,3}^{(6)}+k_{3,3}^{(8)} & 0 & 0 & 0 & 0 & 0 \\
0 & k_{3,2}^{(5)} & 0 & 0 & 0 & 0 & 0 & k_{3,3}^{(5)} & k_{3,1}^{(5)} & 0 & 0 & 0 \\
k_{3,1}^{(4)} & k_{3,2}^{(4)}+k_{1,2}^{(5)} & 0 & 0 & 0 & 0 & 0 & k_{1,3}^{(5)} & k_{3,3}^{(4)}+k_{1,1}^{(5)} & 0 & 0 & 0 \\
k_{3,2}^{(3)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & k_{1,1}^{(1)}+k_{3,3}^{(3)} & k_{1,2}^{(1)} & k_{1,3}^{(1)} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & k_{2,1}^{(1)} & k_{2,2}^{(1)} & k_{2,3}^{(1)} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & k_{3,1}^{(1)} & k_{3,2}^{(1)} & k_{3,3}^{(1)}+k_{2,2}^{(2)}
\end{bmatrix}
\begin{bmatrix}
T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \\ T_8 \\ T_9 \\ T_{10} \\ T_{11} \\ T_{12} \\ T_{13}
\end{bmatrix}
=
\begin{bmatrix}
b_2^{(3)}+b_1^{(4)} \\
b_2^{(4)}+b_2^{(5)}+b_1^{(6)} \\
b_2^{(6)}+b_2^{(7)} \\
b_3^{(7)} \\
b_1^{(7)}+b_1^{(8)} \\
b_2^{(8)} \\
b_3^{(6)}+b_3^{(8)} \\
b_3^{(5)} \\
b_3^{(4)}+b_1^{(5)} \\
b_1^{(1)}+b_3^{(3)} \\
b_2^{(1)} \\
b_3^{(1)}+b_2^{(2)}
\end{bmatrix}
+
\begin{bmatrix}
-T_1 k_{2,1}^{(3)} \\
0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\
-T_1 k_{3,1}^{(3)} \\
0 \\
0 \\
-T_{14}k_{2,3}^{(2)}-T_1 k_{2,1}^{(2)}
\end{bmatrix}
+
\begin{bmatrix}
0 \\ 0 \\ \frac{\mathrm{d}T_4}{\mathrm{d}\hat{n}} \\ \frac{\mathrm{d}T_5}{\mathrm{d}\hat{n}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0
\end{bmatrix}
$$