Homework - Kinematics Calibration of a Differential drive wheelchair

FRANCESCO ARGENTIERI*

University of Trento
francesco.argentieri@studenti.unitn.it

Abstract

For a mobile robot, odometry calibration consists of the identification of a set of kinematic parameters that allow reconstructing the vehicle's absolute position and orientation starting from the wheels' encoder measurements.

I. INTRODUCTION

The autonomous navigation is understood as the set of techniques by which it is a system able to move with a certain level of autonomy in a certain type of environment (land, air, underwater, space). The problems in the field of autonomous navigation are addressed first and foremost related to the localization of the system with respect to the environment, planning out their duties and motion control. An interesting class of systems covered by the study of autonomous navigation systems is that of the AGV (Autonomous Guided Vehicles). They are now widely used in industries, ports, airports, hospitals, etc, however, to measure the position and the attitude of a vehicle it is still a problem of substantial interest. The sensor fusion is the process that combines information from a number of different sources to provide a complete and robust description (measure) a set of variables of interest. The sensor fusion is of particular utility in any application where many measures have to be combined together, melted and optimized in order to obtain quality information and integrity suitable for the purpose of the application. The sensor fusion techniques are used in many industrial systems, military, monitoring, civilian surveillance, control processes and systems. The problem of localization of autonomous vehicles, which in almost all cases, the individual transducers are found insufficient in setting up a comprehensive and robust localization system for autonomous navigation, requires the use of sensor fusion techniques for combining measurements from different types of transducers whose characteristics, if fused together, allow us to obtain a more reliable and accurate measure of the state of the system and the environment surrounding it. The sensor fusion techniques have important applications in the field of autonomous navigation, in which it is necessary to obtain a good estimate of the position measurement and alignment (pose) of a mobile robot. Incremental measuring methods or dead-reckoning, using encoders, gyroscopes, ultrasonic etc., have the considerable advantages of being self-contained within the robot, to be relatively simple to use and provide high refresh rate measure. On the other hands, since these measurement systems integrate related increments,

---

*ID: 183892

the errors grow considerably increasing the integration time. The document presented is based on the analysis of a real robot, developed and built at the MIRO - Measurements Instrumentations Robotics Lab at the University of Trento. It is equipped with two rotary incremental encoder keyed to the axes of the two wheels and a chamber pointing upwards for reading specially arranged on the ceiling marker. The two datasets have been collected steering the vehicle clockwise and counterclockwise. The aim of the homework is to:

- Estimate from the encoder and the camera data the kinematics parameters wheels radius Rl and Rr and wheelbase b.

- Estimate the camera position with respect to the wheelchair reference point (the mid point between the wheels).

II. Methods

i. Kinematic Model

For the analysis of wheelchair differential drive parameters is used a simplified model; the reference system, indicated by $RF1$, is placed in the odometric center of the robot. Consider that the robot moves in a reference system fixed to the $RF0$ envoiroment. The robot is subject to pure rolling, then we neglect slippage between wheel and ground. The angular velocities, indicated with ω_r and ω_l, are applied respectively to the right and left wheels in such a way that the components of the fixed body and the speed of the robot are related to the angular velocity of the wheels according to the equation (1). The other two support wheels are considered passive. This schematization is observable in fig. 1.

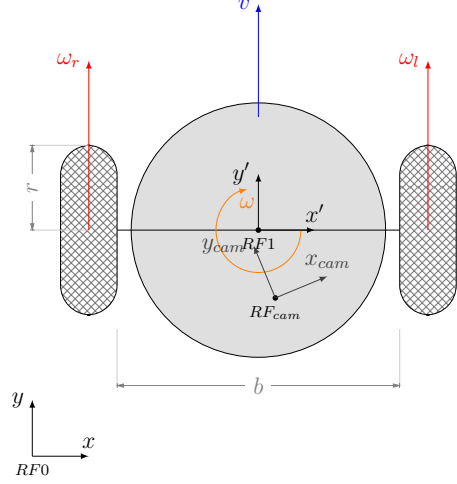$$\begin{bmatrix} v \\ \omega \end{bmatrix} = C \begin{bmatrix} \omega_{\mathrm{L}} \\ \omega_{\mathrm{R}} \end{bmatrix} \qquad (1)$$

Figure 1: Robot kinematic model

The matrix $\mathbf{C} \in \mathbb{R}^{2x2}$ is defined as (2):

$$\begin{bmatrix} \frac{r_{\mathrm{R}}}{2} & \frac{r_{\mathrm{L}}}{2} \\ \frac{r_{\mathrm{L}}}{b} & -\frac{r_{\mathrm{L}}}{b} \end{bmatrix} \qquad (2)$$

in which r_{R} and r_{L} are the radii of the right and left wheel, respectively. The odometry of a vehicle is usually implemented by discrete-time integration, such as (3):

$$\begin{cases} x_{k+1} = x_k + T\, v_k\, cos(\theta_k + T\,\omega_k/2) \\ y_{k+1} = y_k + T\, v_k\, sin(\theta_k + T\,\omega_k/2) \\ \theta_{k+1} = \theta_k + T\,\omega_k \end{cases}$$
$$(3)$$

Notice that low sampling frequency and high vehicle velocities can be a significant source of odometric error.

ii. Data Analysis

To achieve the objective of calibration of the geometric parameters of the robot four datasets were provided in each of which the information is collected from the camera and from the incremental encoder. A first data operation was carried out by a python script to correct the registered angle sign as evaluated with a negative sign. A second operation was to change the left encoder incremental measuring, since being mounted on reverse respect to the right its

value must be taken with a negative sign. In the file is observed the possibility of the presence of two successive rows with the same timestamp. This means that at the same time both the encoder ticks and the camera pose or covariance changed. It may be noted that in one line the camera information and the other the odometer ticks have changed. Thus, at the same timestamp, in the second row the information is modified. So you chose to eliminate duplicate rows to preserve the last where both information camera pose or covariance and ticks has changed.

iii.   Calibration Techinque

To estimate the parameters of the robot expressed in the equation 2, namely, the wheel radius values indicated with "r_R" and "r_L", and the axle track as indicated "b", using the method described in the report [1]. Experiments of odometry calibration require measurement of the absolute position and orientation of the mobile robot at suitable locations along the motion trajectories. For instance this calibration technique requires measurement of the starting and final robot configuration for each motion execution. The datasets supplied information related to: the camera position x, y, θ and increments of the encoder. Each information is saved in columns:

- column 1: acquisistions time [ms];

- columns 2–3–4:   camera pose x, y, θ [cm, cm, rad];

- column from 5 to 13:   camera covariance ordered by rows [cm^2] for pose, [rad^2] for angle and [cm $*$ rad] for mixed terms;

- colunm 14–15:   odometric encoder [ticks left] and [ticks right];

It has been chosen to perform the odometry calibration whereas in the same calculation all four datasets provided as suggested by the technique used. Then the equations (4) and (5) are rewritten limited to the four paths to obtain:

$$
\begin{bmatrix} \hat{c}_{2,1} \\ \hat{c}_{2,2} \end{bmatrix} = (\bar{\Phi}_\theta^\mathrm{T} \bar{\Phi}_\theta)^{-1} \bar{\Phi}_\theta^\mathrm{T} \begin{bmatrix} \theta_{\mathrm{N},1} - \theta_{0,1} \\ \theta_{\mathrm{N},2} - \theta_{0,2} \\ \theta_{\mathrm{N},3} - \theta_{0,3} \\ \theta_{\mathrm{N},4} - \theta_{0,4} \end{bmatrix} \quad (4)
$$

$$
\begin{bmatrix} \hat{c}_{1,1} \\ \hat{c}_{1,2} \end{bmatrix} = (\bar{\Phi}_{xy}^\mathrm{T} \bar{\Phi}_{xy})^{-1} \bar{\Phi}_{xy}^\mathrm{T} \begin{bmatrix} x_{\mathrm{N},1} - x_{0,1} \\ y_{\mathrm{N},1} - y_{0,1} \\ \vdots \\ x_{\mathrm{N},4} - x_{0,4} \\ y_{\mathrm{N},4} - y_{0,4} \end{bmatrix}
$$
(5)

As a result of simulations shows the values of the matrix C:

$$
\begin{bmatrix} 8.1873 & 6.5229 \\ 0.2823 & -0.2807 \end{bmatrix} \quad (6)
$$

It is noted that the parameters $c_{1,1}$ and $c_{1,2}$ relative to the spokes of the wheels are different from each other, this is due to the simplifications introduced by the kinematic model. On the other hands, the axle values, $c_{2,1}$ and $c_{2,2}$, without the negative sign, are much more similar because formerly estimated at $c_{1,j}$ therefore do not contain the error propagation. Subsequently, it calculates the average between the values of the obtained rays and the standard deviation, these are shown in table (1).

	Radius [mm]	Mean [mm]	standard deviation [mm]
r_R	130.457	147.102	± 23.538
r_L	163.746	147.102	± 23.538
b	522.43		

Table 1: estimated value

Finally, figures 2, it shows the calculation for each path odometric with parameters previously estimated in comparison with the trajectory recorded by the camera.
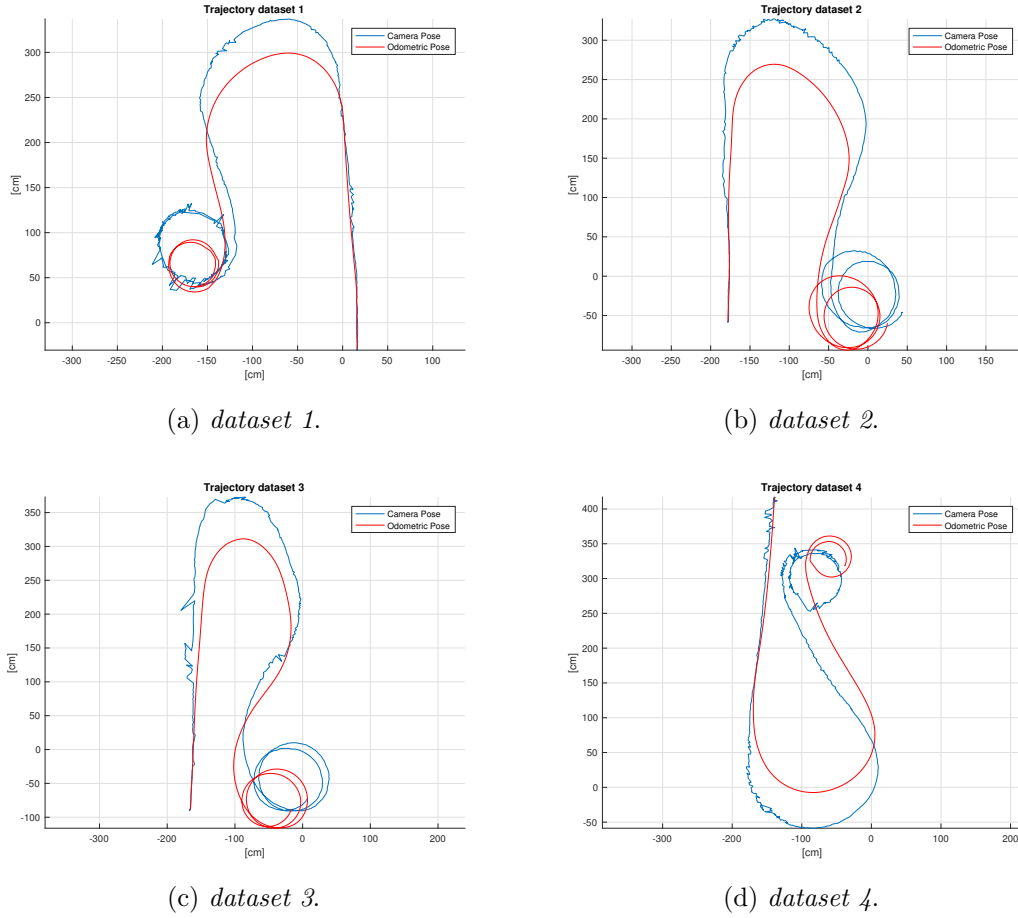
(a) *dataset 1.*

(b) *dataset 2.*

(c) *dataset 3.*

(d) *dataset 4.*

Figure 2: Odometry reconstruction

iv. Optimization

To achieve the goal of determining the offset of the camera mounted on the robot, it is resorted to multivariable. Using the parameters obtained previously, summarized in the table 1, they were used as the boundary conditions limits. The algorithm used is part of the family of "*genetic algorithms* [2]" used for finding optimal solution. In a genetic algorithm, a population of candidate solutions (called individuals, creatures, or phenotypes) to an optimization problem is evolved toward better solutions. Each candidate solution has a set of properties which can be mutated and altered; traditionally, solutions are represented in binary as strings of $0s$ and $1s$, but other encodings are also possible [3]. The evolution usually starts from a population of randomly generated individuals, and is an iterative process, with the population in each iteration called a generation. In each generation, the fitness of every individual in the population is evaluated; the fitness is usually the value of the objective function in the optimization problem being solved. The more fit individuals are stochastically selected from the current population, and each individual's genome is modified (recombined and possibly randomly mutated) to form a new generation. The new generation of candidate solutions is then used in the next iteration of the algorithm. Commonly, the algorithm terminates when either a maximum number of generations has been produced, or a satisfactory fitness level has been reached for
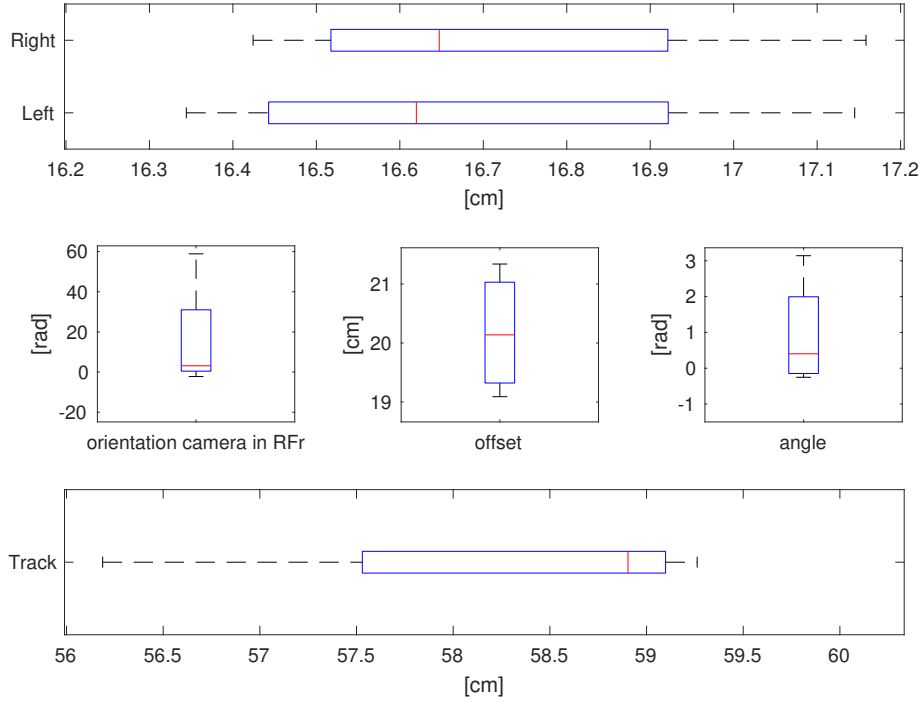
4

Figure 3: optimization among all dataset

the population. A typical genetic algorithm requires:

- a genetic representation of the solution domain,

- a fitness function to evaluate the solution domain.

A standard representation of each candidate solution is as an array of bits[3]. Arrays of other types and structures can be used in essentially the same way. The main property that makes these genetic representations convenient is that their parts are easily aligned due to their fixed size, which facilitates simple crossover operations. Variable length representations may also be used, but crossover implementation is more complex in this case. Tree-like representations are explored in genetic programming and graph-form representations are explored in evolutionary programming; a mix of both linear chromosomes and trees is explored in gene expression programming. Once the genetic representation and the fitness function are defined, a GA proceeds to initialize a population of solutions and then to improve it through repetitive application of the mutation, crossover, inversion and selection operators. The population size depends on the nature of the problem, but typically contains several hundreds or thousands of possible solutions. Often, the initial population is generated randomly, allowing the entire range of possible solutions (the search space). Occasionally, the solutions may be "seeded" in areas where optimal solutions are likely to be found. During each successive generation, a portion of the existing population is selected to breed a new generation. Individual solutions are selected through a fitness-based process, where fitter solutions (as measured by a fitness function) are typically more likely

5

to be selected. Certain selection methods rate the fitness of each solution and preferentially select the best solutions. Other methods rate only a random sample of the population, as the former process may be very time-consuming. The fitness function is defined over the genetic representation and measures the quality of the represented solution. The fitness function is always problem dependent. For instance, in the knapsack problem one wants to maximize the total value of objects that can be put in a knapsack of some fixed capacity. A representation of a solution might be an array of bits, where each bit represents a different object, and the value of the bit (0 or 1) represents whether or not the object is in the knapsack. Not every such representation is valid, as the size of objects may exceed the capacity of the knapsack. The fitness of the solution is the sum of values of all objects in the knapsack if the representation is valid, or 0 otherwise. In some problems, it is hard or even impossible to define the fitness expression; in these cases, a simulation may be used to determine the fitness function value of a phenotype, or even interactive genetic algorithms are used. The next step is to generate a second generation population of solutions from those selected through a combination of genetic operators: crossover, and mutation. For each new solution to be produced, a pair of "parent" solutions is selected for breeding from the pool selected previously. By producing a "child" solution using the above methods of crossover and mutation, a new solution is created which typically shares many of the characteristics of its "parents". New parents are selected for each new child, and the process continues until a new population of solutions of appropriate size is generated. Although reproduction methods that are based on the use of two parents are more "biology inspired", some research suggests that more than two

"parents" generate higher quality chromosomes. These processes ultimately result in the next generation population of chromosomes that is different from the initial generation. Generally the average fitness will have increased by this procedure for the population, since only the best organisms from the first generation are selected for breeding, along with a small proportion of less fit solutions. These less fit solutions ensure genetic diversity within the genetic pool of the parents and therefore ensure the genetic diversity of the subsequent generation of children. Although crossover and mutation are known as the main genetic operators, it is possible to use other operators such as regrouping, colonization-extinction, or migration in genetic algorithms. It is worth tuning parameters such as the mutation probability, crossover probability and population size to find reasonable settings for the problem class being worked on. A very small mutation rate may lead to genetic drift (which is non-ergodic in nature). A recombination rate that is too high may lead to premature convergence of the genetic algorithm. A mutation rate that is too high may lead to loss of good solutions, unless elitist selection is employed. This generational process is repeated until a termination condition has been reached. Common terminating conditions are:

- A solution is found that satisfies minimum criteria

- Fixed number of generations reached

- Allocated budget (computation time/money) reached

- The highest ranking solution's fitness is reaching or has reached a plateau such that successive iterations no longer produce better results

- Manual inspection

- Combinations of the above

It is made of a fitness function that evaluates the difference of position and orientation between the path registered by the robot's camera with the odometry reconstruction carried out starting from the data, by obtaining the minimum for the function as the best possible combination, thanks to the peculiarities of genetic recombination algorithms. The optimization is performed on all the datasets, thus returning four best combinations for the path observable in boxplot, fig. 3. It is possible to observe the behavior of the four best samples:

- extreme values, represented black dashed lines;

- median values, represented by the red segment;

In the first box at the top it shows the distribution of values for the radii's wheels left and right evaluated for the four data series. Mid boxes show the orientation values, camera distance from robot's center and realtive angle orienatation. Finally, the last box shows the track value. After performing the optimization it is impossible to determine with greater accuracy the searched data and the results for all four datasets are visible in figures 4–5–6–7. It is observed that the odometry reconstruction,in red, deviates from the original with odometric center in robot's middle axle, in black; on the other side, the optimal position of the calculated position camera, in blue, is much closer to the original though it takes into account the displacement from the midpoint previously considered.
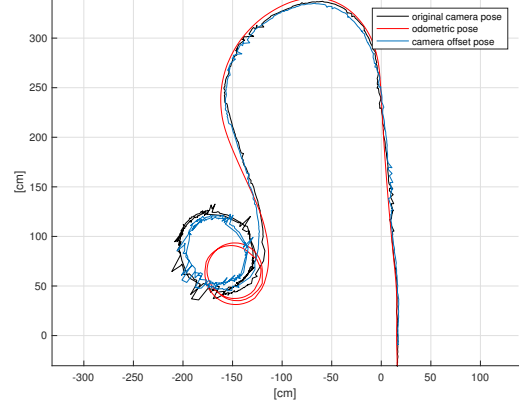

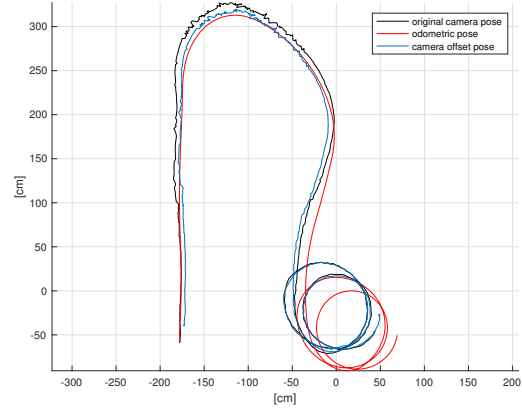Figure 4: Optimal odometric and camera position, dataset: 1

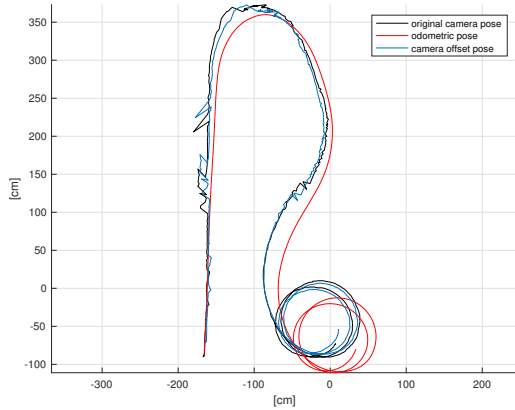Figure 5: Optimal odometric and camera position, dataset: 2

7

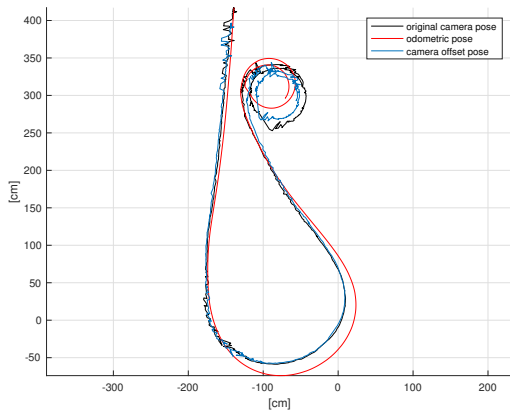Figure 6: Optimal odometric and camera position, dataset: 3

Figure 7: Optimal odometric and camera position, dataset: 4

III. RESULTS

Using the method proposed in scientific article [1] allows to obtain the values in the table (1); it is observed that the distance between the robot's actual parameters and the estimates are introduced by the assumed model simplifications and errors. In the first analysis, the odometric center is placed in mid's robot axle as well as in the model used, but this method does not take into account that the camera's position may be not aligned with the point previously analyzed. Errors also depend on assumptions regarding the non-deformability of the wheels, the

tire above keyed and misalignments.

The motion is achieved by considering a regular and uniform surface, ignoring bumps, obstacles, unevenness, etc. In this scientific article, as in other scientific works [4]–[5]–[6], is recommended to use predetermined paths, possibly in a straight line, with counterclockwise and clockwise rotations in order to minimize errors and to allow independent calibration of the parameters. On the other hand for the results obtained from optimization we must take into account that there are limitations of the use of a genetic algorithm compared to alternative optimization algorithms. Repeated fitness function evaluation for complex problems is often the most prohibitive and limiting segment of artificial evolutionary algorithms. Finding the optimal solution to complex high-dimensional, multimodal problems often requires very expensive fitness function evaluations. In real world problems such as structural optimisation problems, a single function evaluation may require several hours to several days of complete simulation. Typical optimisation methods can not deal with such types of problem. In this case, it may be necessary to forgo an exact evaluation and use an approximated fitness that is computationally efficient. It is apparent that amalgamation of approximate models may be one of the most promising approaches to convincingly use GA to solve complex real-life problems. The "better" solution is only in comparison to other solutions. As a result, the stop criterion is not clear in every problem. In many problems, GAs may have a tendency to converge towards local optima or even arbitrary points rather than the global optimum of the problem. This means that it does not "know how" to sacrifice short-term fitness to gain longer-term fitness. The likelihood of this occurring depends on the shape of the fitness landscape: certain problems may provide an easy ascent towards a

global optimum, others may make it easier for the function to find the local optima. This problem may be alleviated by using a different fitness function, increasing the rate of mutation, or by using selection techniques that maintain a diverse population of solutions, although the "*No Free Lunch theorem*" proves that there is no general solution to this problem. A common technique to maintain diversity is to impose a "niche penalty", wherein, any group of individuals of sufficient similarity (niche radius) have a penalty added, which will reduce the representation of that group in subsequent generations, permitting other (less similar) individuals to be maintained in the population. This trick, however, may not be effective, depending on the landscape of the problem. Another possible technique would be to simply replace part of the population with randomly generated individuals, when most of the population is too similar to each other. Diversity is important in genetic algorithms (and genetic programming) because crossing over a homogeneous population does not yield new solutions. In evolution strategies and evolutionary programming, diversity is not essential because of a greater reliance on mutation.

References

[1] G. Antonelli, S. Chiaverini, and G. Fusco, "A calibration method for odometry of mobile robots based on the least-squares technique: theory and experimental validation," *IEEE Transactions on Robotics*, vol. 21, no. 5, pp. 994–1004, Oct 2005.

[2] Wikipedia, "Genetic algorithm — wikipedia, the free encyclopedia," 2017. [Online]. Available: https://en.wikipedia.org/w/index.php?title=Genetic_algorithm&oldid=774791868

[3] D. Whitley, "A genetic algorithm tutorial," *Statistics and Computing*, vol. 4, no. 2, pp. 65–85, 1994.

[4] A. Censi, A. Franchi, L. Marchionni, and G. Oriolo, "Simultaneous calibration of odometry and sensor parameters for mobile robots," *IEEE Transactions on Robotics*, vol. 29, no. 2, pp. 475–492, April 2013. [Online]. Available: http://purl.org/censi/2012/joint_calibration

[5] D. Jung, J. Seong, C.-b. Moon, J. Jin, and W. Chung, "Accurate calibration of systematic errors for car-like mobile robots using experimental orientation errors," *International Journal of Precision Engineering and Manufacturing*, vol. 17, no. 9, pp. 1113–1119, 2016.

[6] K. S. Chong and L. Kleeman, "Feature-based mapping in real, large scale environments using an ultrasonic array," *I. J. Robotics Res.*, vol. 18, no. 1, pp. 3–19, 1999.

[7] Y. Davidor, H. Schwefel, and R. Männer, *Parallel Problem Solving from Nature - PPSN III: International Conference on Evolutionary Computation. The Third Conference on Parallel Problem Solving from Nature, Jerusalem, Israel, October 9 - 14, 1994. Proceedings*, ser. Lecture Notes in Computer Science. Springer Berlin Heidelberg, 1994.

[8] M. Capcarrere, A. Freitas, P. Bentley, C. Johnson, and J. Timmis, *Advances in Artificial Life: 8th European Conference, ECAL 2005, Canterbury, UK, September 5-9, 2005, Proceedings*, ser. Lecture Notes in Computer Science. Springer Berlin Heidelberg, 2005.

[9] M. Mitchell, *An Introduction to Genetic Algorithms*. Cambridge, MA, USA: MIT Press, 1998.