

# Homework - Kinematics Calibration of a Differential drive wheelchair

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## Abstract

*For a mobile robot, odometry calibration consists of the identification of a set of kinematic parameters that allow reconstructing the vehicle's absolute position and orientation starting from the wheels' encoder measurements.*

## I. INTRODUCTION

The autonomous navigation is understood as the set of techniques by which it is a system able to move with a certain level of autonomy in a certain type of environment (land, air, underwater, space). The problems in the field of autonomous navigation are addressed first and foremost related to the localization of the system with respect to the environment, planning out their duties and motion control. An interesting class of systems covered by the study of autonomous navigation systems is that of the AGV (Autonomous Guided Vehicles). They are now widely used in industries, ports, airports, hospitals, etc, however, to measure the position and the attitude of a vehicle it is still a problem of substantial interest. The sensor fusion is the process that combines information from a number of different sources to provide a complete and robust description (measure) a set of variables of interest. The sensor fusion is of particular utility in any application where many measures have to be combined together, melted and optimized in order to obtain quality information and

integrity suitable for the purpose of the application. The sensor fusion techniques are used in many industrial systems, military, monitoring, civilian surveillance, control processes and systems. The problem of localization of autonomous vehicles, which in almost all cases, the individual transducers are found insufficient in setting up a comprehensive and robust localization system for autonomous navigation, requires the use of sensor fusion techniques for combining measurements from different types of transducers whose characteristics, if fused together, allow us to obtain a more reliable and accurate measure of the state of the system and the environment surrounding it. The sensor fusion techniques have important applications in the field of autonomous navigation, in which it is necessary to obtain a good estimate of the position measurement and alignment (pose) of a mobile robot. Incremental measuring methods or dead-reckoning, using encoders, gyroscopes ultrasuoni etc., have the considerable advantages of being self-contained within the robot, to be relatively simple to use and provide high refresh rate measure. On the other hand, since these measurement systems integrate related increments,

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the errors grow considerably increasing the integration time. The document presented is based on the analysis of a real robot, developed and built at the MIRO - Measurements Instrumentations Robotics Lab at the University of Trento. It is equipped with two rotary incremental encoder keyed to the axes of the two wheels and a chamber pointing upwards for reading specially arranged on the ceiling marker. The two datasets have been collected steering the vehicle clockwise and counterclockwise. The aim of the homework is to:

- Estimate from the encoder and the camera data the kinematics parameters wheels radius  $R_l$  and  $R_r$  and wheel-base  $b$ .
- Estimate the camera position with respect to the wheelchair reference point (the mid point between the wheels).

## II. METHODS

### i. Kinematic Model

For the analysis of differential wheelchair drive parameters using a simplified model in the reference system own odometric placed in the center of the robot indicated by  $RF1$ . Consider that moves in a reference system fixed to the  $RF0$  environment. The robot is subject to pure rolling, then we neglect slippage between wheel and ground. The angular velocities, indicated respectively with  $\omega$ , are applied respectively to the right and left wheels in such a way that the components of the fixed body and the speed of the robot are related to the angular velocity of the wheels according to the equation functional 1. The other two support wheels are considered passive. This schematization is observable in the fig. 1.

$$\begin{bmatrix} v \\ \omega \end{bmatrix} = C \begin{bmatrix} \omega_L \\ \omega_R \end{bmatrix} \quad (1)$$

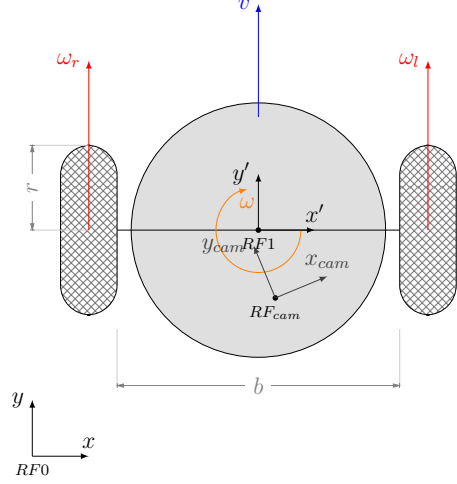


Figure 1: Robot kinematic model

Where the matrix  $C \in \mathbb{R}^{2 \times 2}$  is defined as (2):

$$\begin{bmatrix} \frac{r_R}{2} & \frac{r_L}{2} \\ \frac{r_L}{b} & -\frac{r_L}{b} \end{bmatrix} \quad (2)$$

in which  $r_R$  and  $r_L$  are the radii of the right and left wheel, respectively. The odometry of a vehicle is usually implemented by discrete-time integration, such as (3):

$$\begin{cases} x_{k+1} = x_k + T v_k \cos(\theta_k + T \omega_k/2) \\ y_{k+1} = y_k + T v_k \sin(\theta_k + T \omega_k/2) \\ \theta_{k+1} = \theta_k + T \omega_k \end{cases} \quad (3)$$

Notice that low sampling frequency and high vehicle velocities can be a significant source of odometric error.

### ii. Data Analysis

To achieve the objective of calibration of the geometric parameters of the robot there were provided four datasets where in each of which collected the information from the camera and from the incremental encoder. A first data operation was carried out by a python script to correct the angle sign registered as evaluated with a negative sign. A second operation and change the left encoder incremental measuring as being mounted on reverse respect to the right

its value must be taken with a negative sign. In the file is observed the possibility of the presence of two successive rows with the same timestamp. This means that at the same time have been changed both the encoder ticks and the camera pose or covariance. It may be noted that in one line has changed the camera information and the other the odometer ticks. Thus, at the same timestamp, in the second row is the information are modified. So you chose to eliminate duplicate rows preserve the last where both information camera pose or covariance and ticks has changed.

### iii. Calibration Techinque

To estimate the parameters of the robot expressed in the equation 2, namely, the wheel radius values indicated with " $r_R$ " and " $r_L$ ", and the axle track as indicated " $b$ ", using the method described in the report [1]. Experiments of odometry calibration require measurement of the absolute position and orientation of the mobile robot at suitable locations along the motion trajectories. For instance this calibration technique requires measurement of the starting and final robot configuration for each motion execution. The datasets supplied information related to: the camera position  $x, y, \psi$  and increments of the encoder. Each information is saved in columns:

- column 1: acquisitions time [ms];
- columns 2–3–4: camera pose  $x, y, \theta$  [cm, cm, rad];
- column from 5 to 13: camera covariance ordered by rows [cm<sup>2</sup>] for pose, [rad<sup>2</sup>] for angle and [cm\*rad] for mixed terms;
- column 14–15: odometric Encoder ticks Left and Right;

It has been chosen to perform the odometry calibration whereas in the same calculation all four datasets provided as suggested by the technique used. Then the equations 4 and 5 are rewritten limited to the four paths to obtain:

$$\begin{bmatrix} \hat{c}_{2,1} \\ \hat{c}_{2,2} \end{bmatrix} = (\bar{\Phi}_\theta^T \bar{\Phi}_\theta)^{-1} \bar{\Phi}_\theta^T \begin{bmatrix} \theta_{N,1} - \theta_{N,0} \\ \theta_{N,2} - \theta_{N,0} \\ \theta_{N,3} - \theta_{N,0} \\ \theta_{N,4} - \theta_{N,0} \end{bmatrix} \quad (4)$$

$$\begin{bmatrix} \hat{c}_{1,1} \\ \hat{c}_{1,2} \end{bmatrix} = (\bar{\Phi}_{xy}^T \bar{\Phi}_{xy})^{-1} \bar{\Phi}_{xy}^T \begin{bmatrix} xy_{N,1} - xy_{N,0} \\ xy_{N,2} - xy_{N,0} \\ xy_{N,3} - xy_{N,0} \\ xy_{N,4} - xy_{N,0} \end{bmatrix} \quad (5)$$

As a result of simulations shows the values of the matrix C:

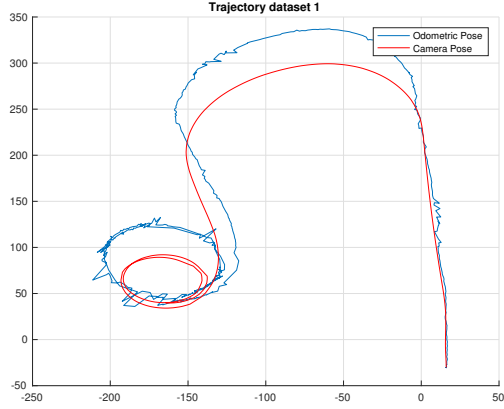
$$\begin{bmatrix} 8.1873 & 6.5229 \\ 0.2823 & -0.2807 \end{bmatrix} \quad (6)$$

It is noted that the parameters  $c_{1,1}$  and  $c_{1,2}$  relative to the spokes of the wheels are different from each other, this is due to the simplifications introduced by the kinematic model. On the other hands, the axle values,  $c_{2,1}$  and  $c_{2,2}$ , without the negative sign, are much more similar because formerly estimated at  $c_{1,j}$  therefore do not contain the error propagation. Subsequently, it calculates the average between the values of the obtained rays and the standard deviation, these are shown in the table (1). Finally,

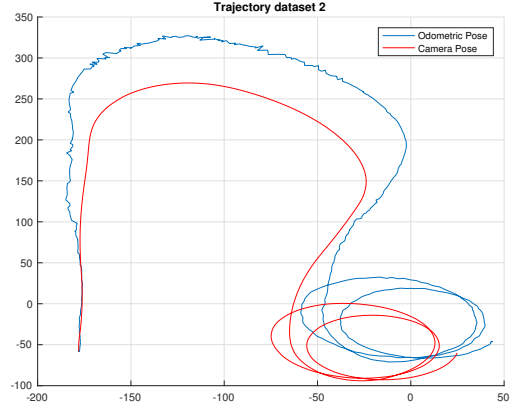
	Radius [mm]	Mean [mm]	standard deviation [mm]
$r_R$	130.457	147.102	$\pm 23.538$
$r_L$	163.746	147.102	$\pm 23.538$
$b$	522.43		

Table 1: estimated value

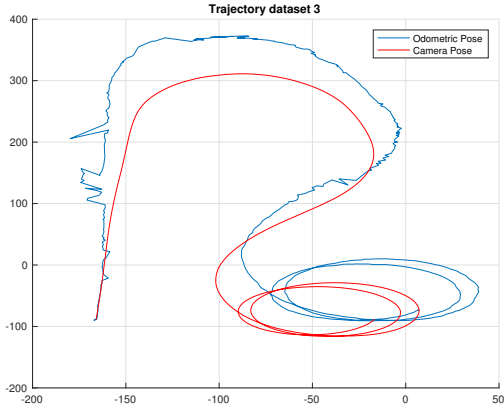
we show in the figures 2, it shows the calculation for each path odometric with parameters previously estimated in comparison with the trajectory recorded by the camera.



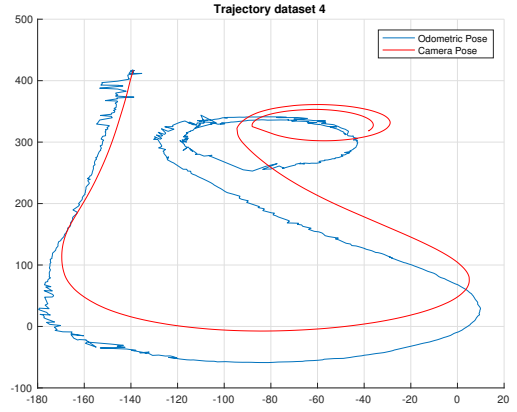
(a) *dataset 1.*



(b) *dataset 2.*



(c) *dataset 3.*



(d) *dataset 4.*

Figure 2: Odometry reconstruction

#### iv. Optimization

To achieve the goal of determining the offset of the camera mounted on the robot, recourse was made to a six-variable optimization. Following the parameters obtained previously, summarized in the table, were used as the maximum and minimum limits of the search space along with other parameters to determine: the distance from the robot center, the angle of rotation with respect to the reference system and the camera's orientation. It is made of a matrix containing a large number of samples to cover a large number of combinations of these parameters generated randomly. Were an-

alyzed by a function that minimizes the error between ritch recorded by the camera and what rebuilt with odometry, thus they are obtained the values sought. The optimization is performed for all the datasets thus returning four best combinations for each path, shows the values in a box plot in fig. 3, where it is possible to observe:

- extreme values, represented by blacks hatches;
- the median values, represented by the red segment;

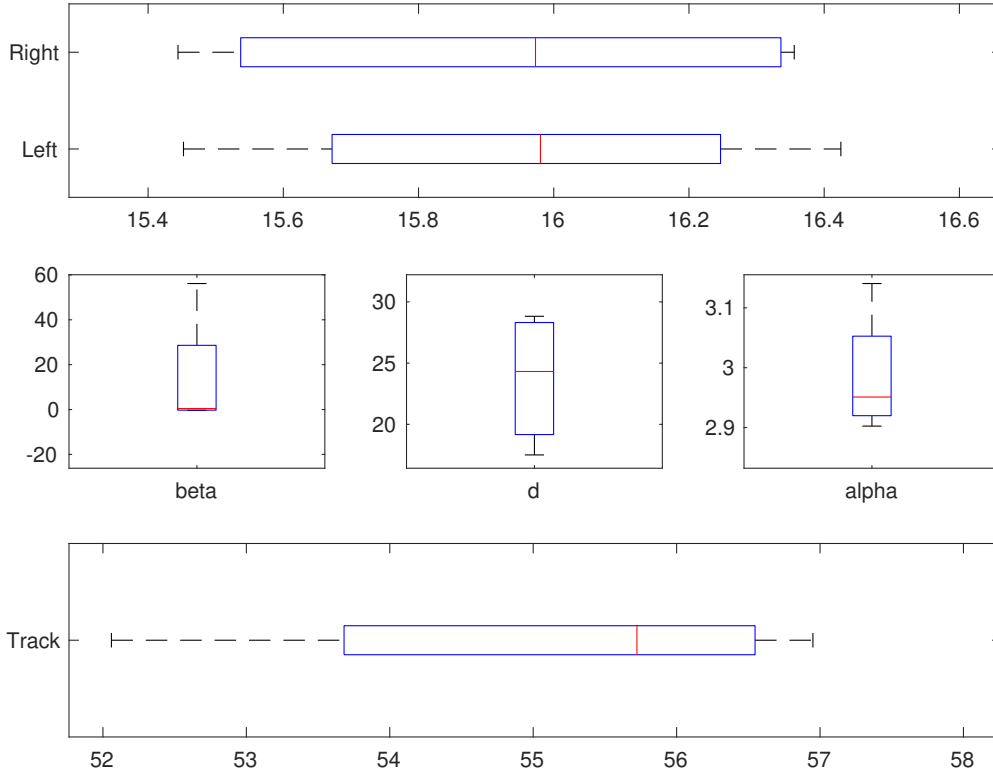
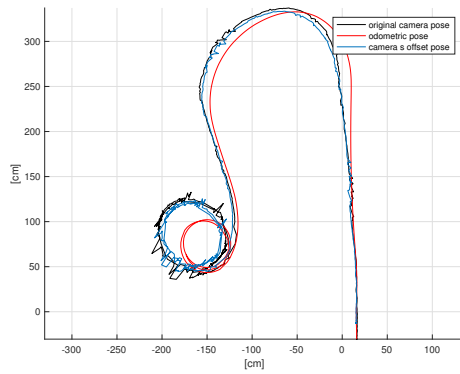


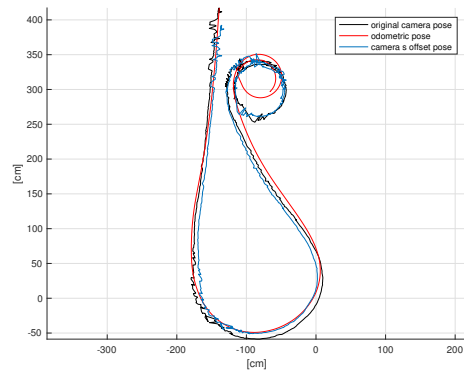
Figure 3: optimization among all dataset

In the first box at the top is show the distribution of values for the radii of the left and right wheels evaluated for the four datasets. In the box in the middle shows the orientation values and camera distance from the center of robot minimized for the data. Finally, the same for the last box shows the track value dell'axle. After performing the optimization is unable to determine with greater accuracy the searched data and the results for all four datasets are visible in figures 4. It is observed that the odometry reconstruction, represented in red, deviates from the original, in black, as odometric center of the middle of the axle track point robot, the other side of the optimal position of the calculated camera, in blue, is much closer to the original though it takes account of the displacement from the mid-

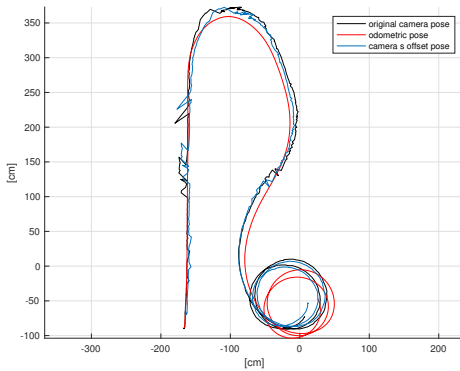
point.



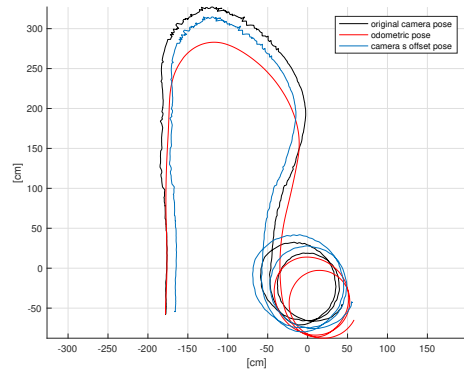
(a) dataset 1.



(b) dataset 2.



(c) dataset 3.



(d) dataset 4.

Figure 4: Optimal odometric and camera position

### III. RESULTS

Using the scientific article by proposed method allows to obtain the values obtained in the table, it is observed that the distance between the robot's actual parameters and those estimates are introduced by the assumed model simplifications and errors. In the first analysis, the method proposed, treats that the odometric center placed in mid's robot axle as well as in the model used, but does not take account that the camera's position may be not aligned with point previously analyzed. Errors also depend on assumptions regarding the non-deformability of the wheels and the tire above keyed, any misalignments. The motion is achieved by considering a regu-

lar and uniform surface, and then ignoring bumps, obstacles, unevenness, etc. Also recommend to use predetermined paths, possibly, whereas those in a straight line, with counterclockwise and clockwise rotations in order to minimize errors and to allow independent calibration of the parameters.

### REFERENCES

- [1] G. Antonelli, S. Chiaverini, and G. Fusco, "A calibration method for odometry of mobile robots based on the least-squares technique: theory and experimental validation," *IEEE Transactions on Robotics*, vol. 21, no. 5, pp. 994–1004, Oct 2005.