Final report in

Mechanical Vibrations

System identification of a 3 DOF system

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Chapter 1

Introduction

1.1 System and the experimental setup

For this experiment, we are going to identify the parameters of the Rectilinear Control System (Model 210). The experimental control system is comprised of the three subsystems shown in Figure 1.1. The first of these is the electromechanical plant which consists of the spring/mass mechanism, its actuator and sensors. The design features a brushless DC servomotor, high resolution encoders, adjustable masses, and reconfigurable plant type.
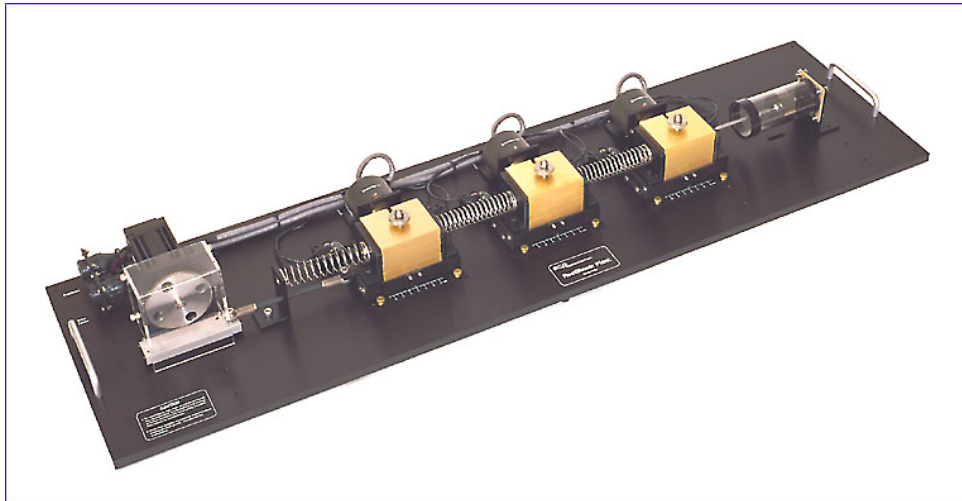

Figure 1.1: Dynamical system

1.1.1 Parameters

The system is configured with three bodies above the mass carriage suspension is an anti-friction ball bearing type with approximately ± 3 [cm] of available travel. The linear

drive is comprised of a gear rack suspended on an anti-friction carriage and pinion (pitch diameter 7.62 [cm]) coupled to the brushless servo-motor shaft. Optical encoders measure the mass carriage positions - also via a rack and pinion with a pinion pitch about 3.18 [cm]. The bodies are connected by known stiffness springs, and a spring connects the third mass to the frame. Instead, the first body is rigidly connected to a pinion gear with a live-powered motor with a PC interface. The position of each body is provided by an encoder. The position zeros are at the equilibrium positions of the springs. For the springs we use the nominal values provided:

- $k_1 = k_2 = 800$ [N m^{-1}] between m_1 and m_2, m_2 and m_3;

- $k_3 = 400$ [N m^{-1}] between m_3 and the ground.

The shifts x_1, x_2, x_3 are provided in encoder counts, where the relationship (1.1) between the measured counts and the displacement was used. Where r_e is the radius of the encoder and $2\pi r_e = 0.0706$ [m]; 16000 is the number of counts per encoder revolution.

$$\Delta x = 2\pi \cdot r_e \cdot \frac{\Delta count}{16000} \tag{1.1}$$

The input data are given by the voltage V. The following relation between the applied voltage and the applied force holds: $f = (k_a \cdot k_t \cdot k_{mp}) \cdot v$.
Where:

k_a is the Servo Amp gain:
$$k_a \approx 2 \quad [\text{A V}^{-1}]$$

k_t is the Servo Motor Torque constant:

$$k_t \approx 0.1 \quad [\text{N m A}^{-1}]$$

k_{mp} is the Motor Pinion pitch radius inverse:

$$k_{mp} = 26.25 \quad [\text{m}^{-1}]$$

1.2 The dynamical model

1.2.1 Assumption

The system described in the previous chapter is modelled as a linear system and for this reason some simplifications are made. It is considered that all the bodies move on the

same axis, assuming therefore that the rack meshed by the pinion plots the force on this axis, so that a straight motion is assumed. In the model there are only viscous frictions. The block containing the engine with the attachment unit and rack is considered rigidly connected to the mass m, according to the equation:

$$\begin{cases} m_1 & = m_{11} + \frac{J_{\text{motor}}}{r^2} \\ c_1 & = c_{11} + \frac{c_{\text{motor}}}{r^2} \end{cases} \tag{1.2}$$

In equation (1.2): r is the radius of the pinion-rack coupling, J_{motor} the inertia of the motor, c_{motor} the rotational damping. While m_{11} and c_{11} are respectively the mass and damping of the first body.

1.2.2 Equation of motion

We describe the equations of motion for each body according to the embodiments reported in (1.3), the schematic representation is observable in the figure 1.2

$$\begin{aligned} m_1 \ddot{x}_1 &= k_1(x_2 - x_1) - c_1 \dot{x}_1 + g_{\text{v}} \cdot v \\ m_2 \ddot{x}_2 &= k_1(x_1 - x_2) + k_2(x_3 - x_2) - c_2 \dot{x}_2 \\ m_3 \ddot{x}_3 &= k_2(x_2 - x_3) - c_3 \dot{x}_3 - k_3 x_3 \end{aligned} \tag{1.3}$$

The equation in matrix form is shown below (1.4):

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \ddot{x} + \begin{bmatrix} c_1 & 0 & 0 \\ 0 & c_2 & 0 \\ 0 & 0 & c_3 \end{bmatrix} \dot{x} + \begin{bmatrix} k_1 & -k_1 & 0 \\ -k_1 & k_1 + k_2 & -k_2 \\ 0 & -k_2 & k_2 + k_3 \end{bmatrix} x = \begin{bmatrix} g_{\text{v}} \\ 0 \\ 0 \end{bmatrix} \cdot v \tag{1.4}$$
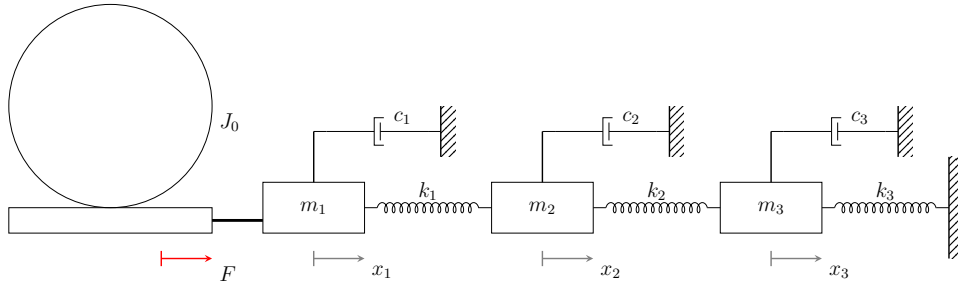

Figure 1.2: model system

Chapter 2

Parameters identification

2.1 Steady state analysis

Using the data of the file *data_steps.mat*, we analysed the step response of the system by going to check the relationships between the stiffness of the springs. And estimating a new coefficient for the voltage-to-force. The estimate of a new voltage-to-force coefficient is carried out using the equation (1.4):

$$[K]x = \begin{bmatrix} g_{\mathrm{v}} \\ 0 \\ 0 \end{bmatrix} \cdot v \tag{2.1}$$

and therefore from (2.1) it is possible to derive:

$$g_{\mathrm{v}} = [K]^{-1} \cdot b \tag{2.2}$$

Using the steady state value of the three output and the equation (2.2) it is possible to evaluate the ratio of the stiffness of the springs with respect to the nominal value of the third spring indicated with k_3, so it is possible to rewrite:

$$k_3 \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = v \cdot \begin{bmatrix} g_{\mathrm{v}} + g_{\mathrm{v}} \cdot \frac{k_3}{k_1} + g_{\mathrm{v}} \cdot \frac{k_3}{k_2} \\ g_{\mathrm{v}} + g_{\mathrm{v}} \cdot \frac{k_3}{k_2} \\ g_{\mathrm{v}} \end{bmatrix} \tag{2.3}$$

From the above equation (2.3) we obtain the results: nominal and estimated ratios between the stiffness of the springs as well as the coefficient voltage-to-force. The table 2.1 shows values and it is possible to appreciate in the last column the error made on the

estimate of these values, according to the formulation:

$$\mathbf{err} = 100 \cdot \frac{\text{computed}}{\text{nominal}} \tag{2.4}$$

	nominal	computed	error [%]
ratio k_3/k_1	0.50000	0.48935	2.17651
ratio k_3/k_2	0.50000	0.52086	4.00560
gain ratio g_v	5.25000	6.04866	15.21248

Table 2.1: Stiffnesses ratios and voltage-to-force coefficients

2.2 System identification

It is necessary to estimate the parameters of three DOF system at simple identification, so that the impulse response is used to suppose that a model is based on a set of parameters, identifying the inputs as the initial force and conditions and the output of the system. thus comparing the system response measured with that predicted by the model. Defined a model for the system: depends on a set of parameters θ. Identify the input (strength and i.c.) and output $(x(t))$ of the system. By giving the same input to the model and comparing the measured response $x(t)$ to that expected by the model $(x(t, \theta))$.
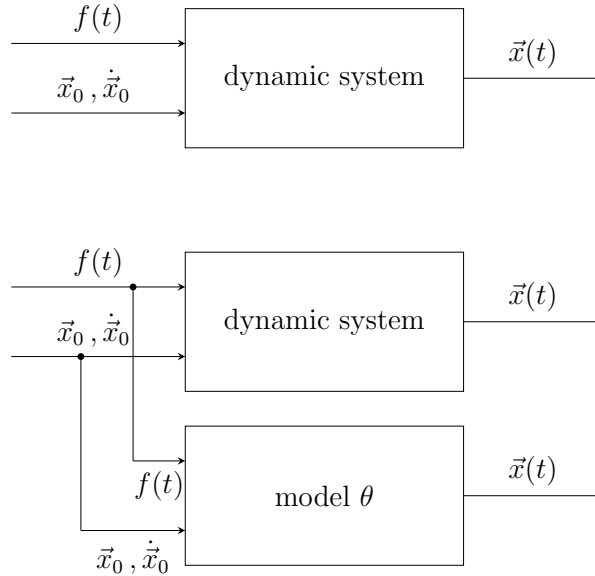

Figure 2.1: System identification: real system and model

We define the residue as the difference between the two signals as in the equation (2.5).

$$\varepsilon(t, \theta) = x(t) - x(t, \theta) \tag{2.5}$$

The best choice for the set of parameter θ is the one that minimizes the residue integer (2.6) of the residue.

$$\theta_{optim} = \operatorname{argmin}\left(\int \varepsilon(t, \theta)^2 \, dt\right) \tag{2.6}$$

In this case, as the discrete signals with N samples, the optimal value N calculated as (2.7), this correspond to solving a least square problem.

$$\theta_{optim} = \operatorname{argmin}\left(\sum_{n=1}^{N} \varepsilon(t, \theta)^2\right) \tag{2.7}$$

A linear model is adopted as described above, refer 1.2.1, assuming the stiffness matrix $[K]$ given. Then quantifying the $[M]$ and $[C]$ parameters are used to minimize the problem. Using the *fmincon* algorithm, the problem is resolved. In addition, the selected algorithm is required to specify the lower and upper search limits; as well as the first guess conditons from which the result is strictly dependent.

2.2.1 Free damping case

Using the data of the file *data_impulse.mat* identify the seven parameters previously identified in the model and described by the equation (1.4). In this case, since the response to impulse force, two to approximation of the force estimation, will be analysed, the voltage-to-force coefficient will be studied again. The results of this optimization are shown in table 2.2.

m_1	m_2	m_3	c_1	c_2	c_3	g_v
	[kg]			[N s m^{-1}]		[V]
1.5712	1.5020	1.2018	2.7956	1.9978	2.1950	1.2096

Table 2.2: Optimizations results in free damping case

(a) *residual.*

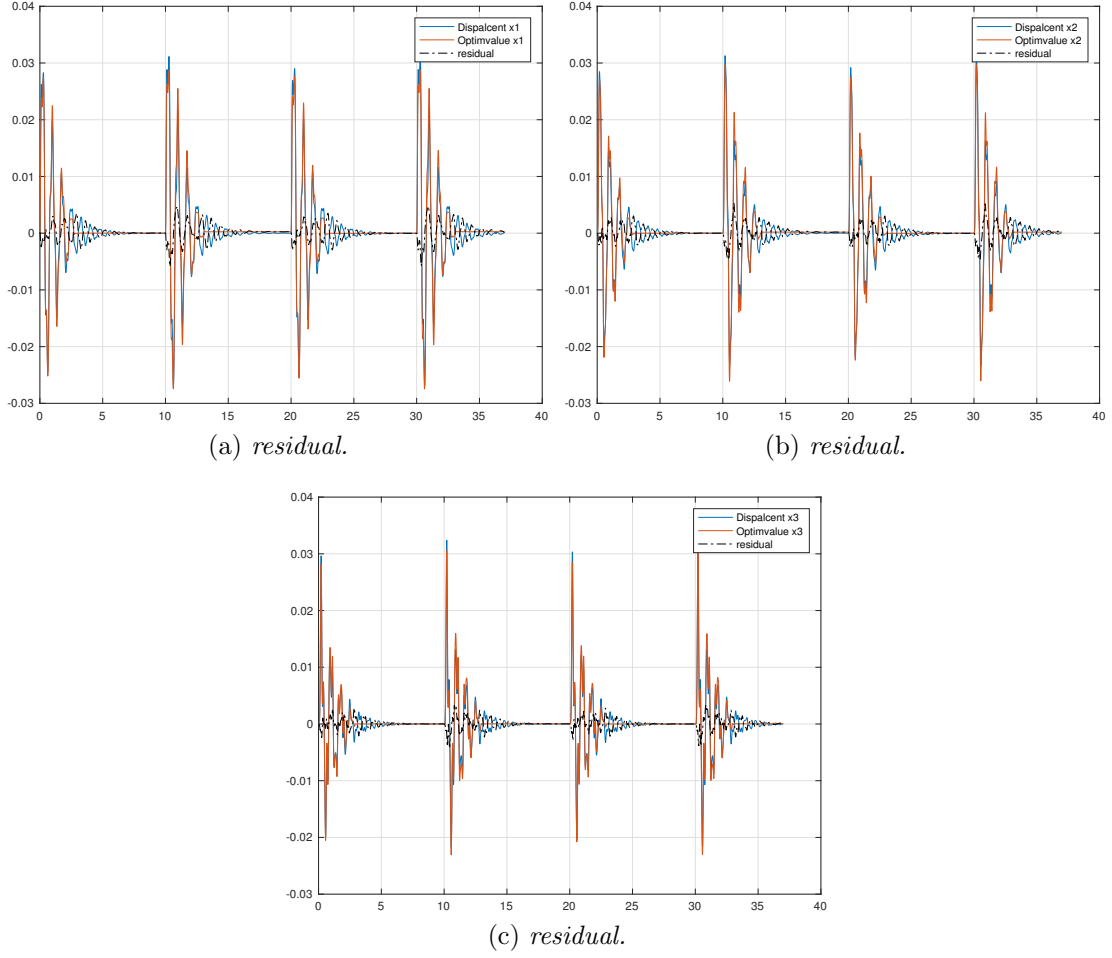(b) *residual.*

(c) *residual.*

Figure 2.2: Comparison between the response of the model and the response of the system free damping case

2.2.2 Proportional damping case

In this case, the procedure used to solve the problem raised in the case of free damping to obtain the parameters is used; changing some conditions. We define damping through the equation (2.8) where it is possible to observe that this depends on the linear combination of the mass matrix multiply by constant and the stiffness matrix multiply by the another constant. Then the search limits and first guess will be changed to search the mass values and the unknown constants. The interesting property of this representation is the ability to perform modal decomposition on the system.

$$[C] = \alpha \cdot [M] + \beta \cdot [K] \tag{2.8}$$

The optimization results are available in the table 2.3.

m_1	m_2	m_3	g_v	α	β
	[kg]		[V]	[N s m^{-1}]	
1.5761	1.4970	1.1996	1.2094	1.6209	0.0001

Table 2.3: Optimizations results in proportional damping case

$$[C] = \begin{bmatrix} 2.64026 & -0.08550 & 0.00000 \\ -0.08550 & 2.59748 & -0.08550 \\ 0.00000 & -0.08550 & 2.07270 \end{bmatrix} \tag{2.9}$$
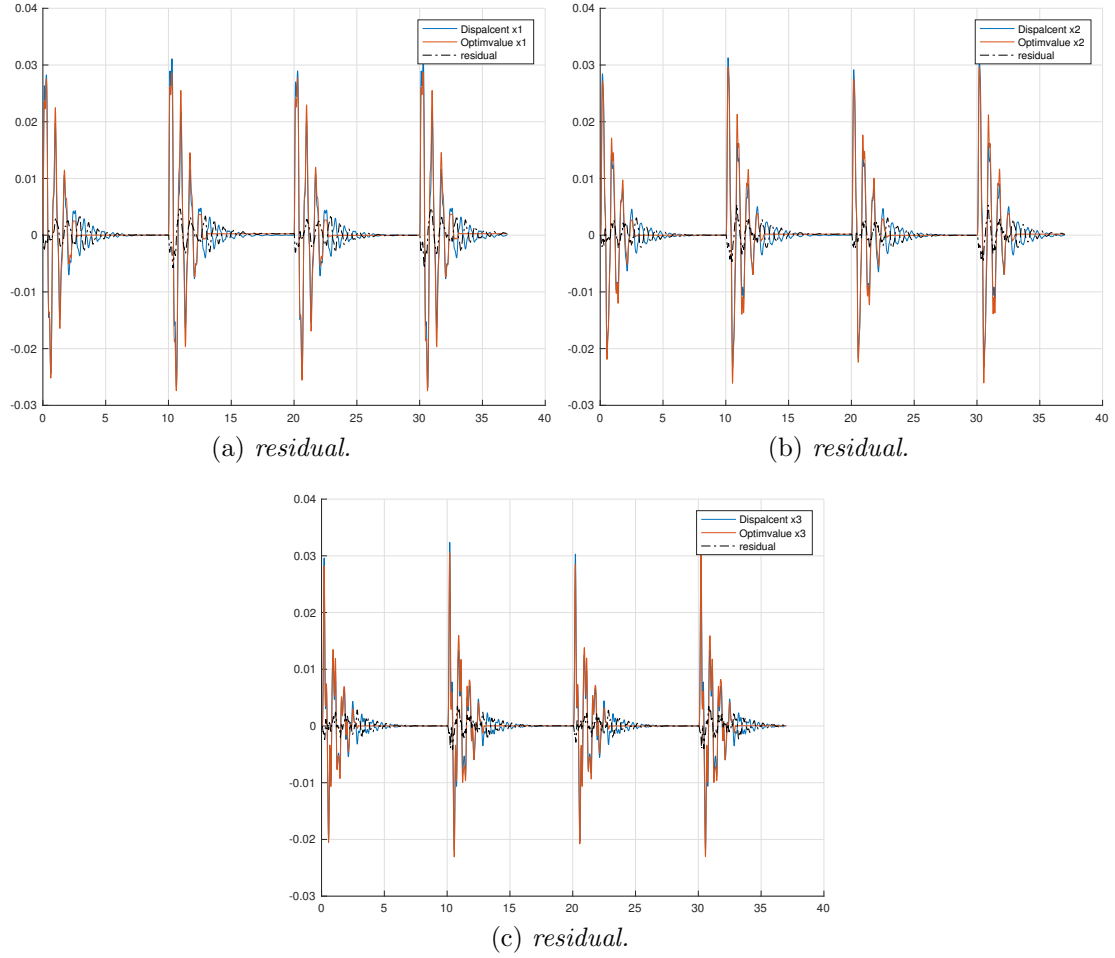
(a) *residual.*

(b) *residual.*

(c) *residual.*

Figure 2.3: Comparison between the response of the model and the response of the system proportional damping case

2.2.3 Comparison

After fitting data with a model, you should evaluate the goodness of fit. The goodness of fit is calculated using the normalized root mean square error as the cost function. In table 2.4 are aviable the percentages the measured output. This method to assess goodness of fit for both linear and non linear parametric fits. As is common in statistical literature, the term goodness of fit is used here in sense: "good fit" might be a model where the data could reasonably have come from, given the assumptions of least-squares fitting.

	x_1 [%]	x_2 [%]	x_3 [%]
free damping case	81.36	81.17	81.71
proportinal damping case	81.36	81.17	81.56

Table 2.4: Result *goodness of fit* measured output

2.2.4   Multiple single DOF system case

Chapter 3

Modal analysis

3.1 Eigenvalue problem

Full method: frequencies 8.27843 Hz, 27.40935 Hz, 41.87035 Hz modes:

$$\begin{bmatrix} 1.00000 & 1.00000 & 1.00000 \\ 0.86540 & -0.47555 & -2.44325 \\ 0.61945 & -1.28034 & 2.15533 \end{bmatrix} \tag{3.1}$$

Proportional damping: frequencies 8.27799 Hz, 27.40863 Hz, 41.91576 Hz modes:

$$\begin{bmatrix} 1.00000 & 1.00000 & 1.00000 \\ 0.86499 & -0.48006 & -2.46147 \\ 0.61907 & -1.28528 & 2.16954 \end{bmatrix} \tag{3.2}$$

3.2 Rayleigh

MODAL ANALYSIS - Rayleigh: Full method: frequencies 8.27843 Hz, 27.40935 Hz, 41.87035 Hz modes:

$$\begin{bmatrix} 1.00000 & 1.00000 & 1.00000 \\ 0.86540 & -0.47555 & -2.44325 \\ 0.61945 & -1.28034 & 2.15533 \end{bmatrix} \tag{3.3}$$

Proportional damping: frequencies 8.27799 Hz, 27.40863 Hz, 41.91576 Hz modes:

$$\begin{bmatrix} 1.00000 & 1.00000 & 1.00000 \\ 0.86499 & -0.48006 & -2.46147 \\ 0.61907 & -1.28528 & 2.16954 \end{bmatrix} \tag{3.4}$$

3.3 Matrix Iteration Method

MODAL ANALYSIS - Matrix Iteration Method: Full method: frequencies 8.27843 Hz, 27.41224 Hz, 41.85486 Hz modes:

$$\begin{bmatrix} 1.00000 & 1.00000 & 1.00000 \\ 0.86540 & 0.21230 & -0.85499 \\ 0.61945 & -0.30643 & -0.30390 \end{bmatrix} \tag{3.5}$$

Proportional damping: frequencies 8.27799 Hz, 27.40863 Hz, 41.91576 Hz modes:

$$\begin{bmatrix} 1.00000 & 1.00000 & 1.00000 \\ 0.86499 & -0.48006 & -2.46147 \\ 0.61907 & -1.28528 & 2.16954 \end{bmatrix} \tag{3.6}$$

Chapter 4

Conclusioni

Spero che la guida possa servirvi ragazzuoli!

Ringrazio di cuore Giordano Cardillo e Matteo Merola per avermi aiutato (e sopportato) a capire come funziona questo maledetto LaTeX! :P
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Guida scritta da Lorenzo Valente [1]

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Bibliography