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Mechanical Vibrations

System identification of a 3 DOF system

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Contents

Chapter 1

Introduction

1.1 System and the experimental setup

For this experiment, we are going to identify the parameters of the Rectilinear Control System (Model 210). The experimental control system is comprised of the three subsystems shown in Figure 1.1. The first of these is the electromechanical plant which consists of the spring/mass mechanism, its actuator and sensors. The design features a brushless DC servomotor, high resolution encoders, adjustable masses, and reconfigurable plant type.

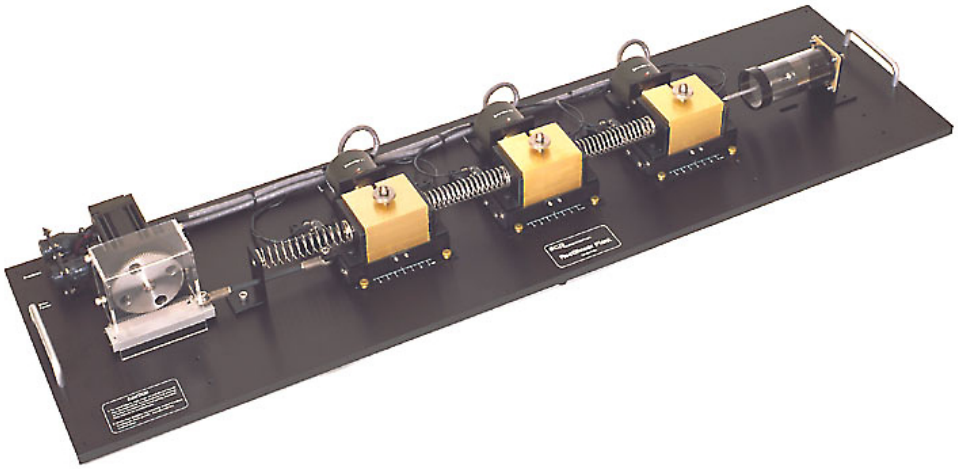


Figure 1.1: Dynamical system

1.1.1 Parameters

The system is configured with three bodies above the mass carriage suspension is an anti-friction ball bearing type with approximately ± 3 [cm] of available travel. The linear drive is comprised of a gear rack suspended on an anti-friction carriage and pinion (pitch

diameter 7.62 [cm]) coupled to the brushless servo-motor shaft. Optical encoders measure the mass carriage positions - also via a rack and pinion with a pinion pitch about 3.18 [cm]. The bodies are connected by known stiffness springs, and a spring connects the third mass to the frame. Instead, the first body is rigidly connected to a pinion gear with a live-powered motor with a PC interface. The position of each body is provided by an encoder. The position zeros are at the equilibrium positions of the springs. For the springs we use the nominal values provided:

- $k_1 = k_2 = 800$ [$\mathrm{N\,m^{-1}}$] between m_1 and m_2, m_2 and m_3;
- $k_3 = 400$ [$\mathrm{N\,m^{-1}}$] between m_3 and the ground.

The shifts x_1, x_2, x_3 are provided in encoder counts, where the relationship (1.1) between the measured counts and the displacement was used. Where r_e is the radius of the encoder and $2\pi r_e = 0.0706$ [m]; 16000 is the number of counts per encoder revolution.

$$\Delta x = 2\pi \cdot r_e \cdot \frac{\Delta count}{16000} \tag{1.1}$$

The input data are given by the voltage V. The following relation between the applied voltage and the applied force holds: $f = (k_a \cdot k_t \cdot k_{mp}) \cdot v$.
Where:

k_a is the Servo Amp gain:

$$k_a \approx 2 \quad [\mathrm{A\,V^{-1}}]$$

k_t is the Servo Motor Torque constant:

$$k_t \approx 0.1 \quad [\mathrm{N\,m\,A^{-1}}]$$

k_{mp} is the Motor Pinion pitch radius inverse:

$$k_{mp} = 26.25 \quad [\mathrm{m^{-1}}]$$

1.2 The dynamical model

1.2.1 Assumption

The system described in the previous chapter is modelled as a linear system and for this reason some simplifications are made. It is considered that all the bodies move on the same axis, assuming therefore that the rack meshed by the pinion plots the force on this

axis, so that a straight motion is assumed. In the model there are only viscous frictions. The block containing the engine with the attachment unit and rack is considered rigidly connected to the mass m, according to the equation:

$$\begin{cases} m_1 & = m_{11} + \frac{J_{\text{motor}}}{r^2} \\ c_1 & = c_{11} + \frac{c_{\text{motor}}}{r^2} \end{cases} \tag{1.2}$$

In equation (1.2): r is the radius of the pinion-rack coupling, J_{motor} the inertia of the motor, c_{motor} the rotational damping. While m_{11} and c_{11} are respectively the mass and damping of the first body.

1.2.2 Equation of motion

We describe the equations of motion for each body according to the embodiments reported in (1.3), the schematic representation is observable in the figure 1.2

$$\begin{aligned} m_1\ddot{x}_1 &= k_1(x_2 - x_1) - c_1\dot{x}_1 + g_{\text{v}} \cdot v \\ m_2\ddot{x}_2 &= k_1(x_1 - x_2) + k_2(x_3 - x_2) - c_2\dot{x}_2 \\ m_3\ddot{x}_3 &= k_2(x_2 - x_3) - c_3\dot{x}_3 - k_3x_3 \end{aligned} \tag{1.3}$$

The equation in matrix form is shown below (1.4):

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \ddot{x} + \begin{bmatrix} c_1 & 0 & 0 \\ 0 & c_2 & 0 \\ 0 & 0 & c_3 \end{bmatrix} \dot{x} + \begin{bmatrix} k_1 & -k_1 & 0 \\ -k_1 & k_1 + k_2 & -k_2 \\ 0 & -k_2 & k_2 + k_3 \end{bmatrix} x = \begin{bmatrix} g_{\text{v}} \\ 0 \\ 0 \end{bmatrix} \cdot v \tag{1.4}$$

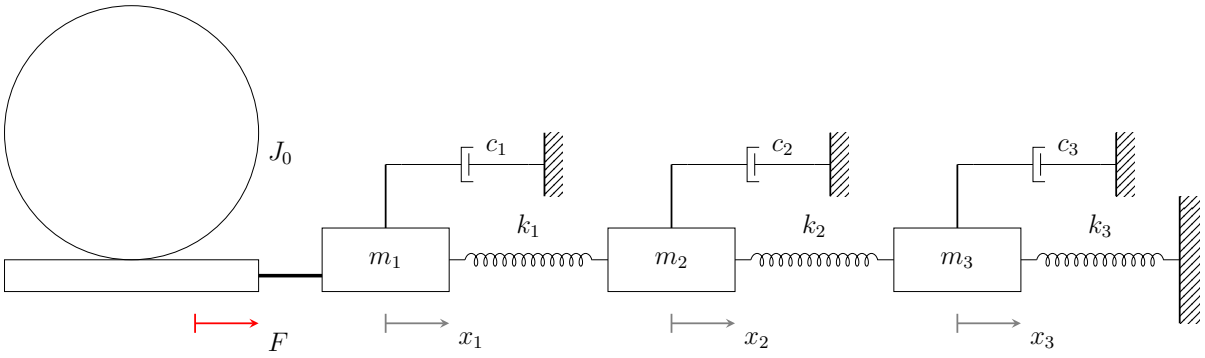


Figure 1.2: model system

Chapter 2

Parameters identification

2.1 Steady state analysis

Using the data of the file *data_steps.mat*, we analysed the step response of the system by going to check the relationships between the stiffness of the springs. And estimating a new coefficient for the voltage-to-force. The estimate of a new voltage-to-force coefficient is carried out using the equation (1.4):

$$[K]x = \begin{bmatrix} g_{\mathrm{v}} \\ 0 \\ 0 \end{bmatrix} \cdot v \tag{2.1}$$

and therefore from (2.1) it is possible to derive:

$$g_{\mathrm{v}} = [K]^{-1} \cdot b \tag{2.2}$$

Using the steady state value of the three output and the equation (2.2) it is possible to evaluate the ratio of the stiffness of the springs with respect to the nominal value of the third spring indicated with k_3, so it is possible to rewrite:

$$k_3 \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = v \cdot \begin{bmatrix} g_{\mathrm{v}} + g_{\mathrm{v}} \cdot \frac{k_3}{k_1} + g_{\mathrm{v}} \cdot \frac{k_3}{k_2} \\ g_{\mathrm{v}} + g_{\mathrm{v}} \cdot \frac{k_3}{k_2} \\ g_{\mathrm{v}} \end{bmatrix} \tag{2.3}$$

From the above equation (2.3) we obtain the results: nominal and estimated ratios between the stiffness of the springs as well as the coefficient voltage-to-force. The value of steady state response x_i are picked from the data directly in a point far from the transient, as shown in Figure 2.1.

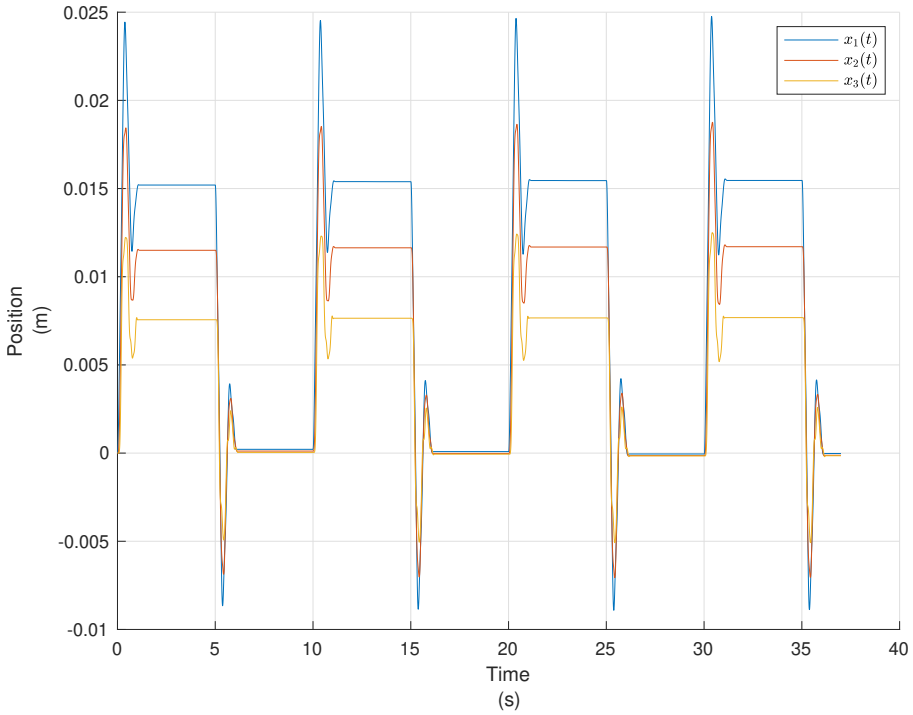


Figure 2.1: Steady state value pick

To reduce the zero-mean noise an average with the previous 400 samples, i.e. 2 [s]. The table 2.1 shows values and it is possible to appreciate in the last column the error made on the estimate of these values, according to the formulation:

$$\mathbf{err} = 100 \cdot \frac{\text{computed}}{\text{nominal}} \tag{2.4}$$

	nominal	computed	error [%]
ratio k_3/k_1	0.50000	0.48935	2.17651
ratio k_3/k_2	0.50000	0.52086	4.00560
gain ratio g_v	5.25000	6.04866	15.21248

Table 2.1: Stiffnesses ratios and voltage-to-force coefficients

2.2 System identification

It is necessary to estimate the parameters of three DOF system at simple identification, so that the impulse response is used to suppose that a model is based on a set of parameters, identifying the inputs as the initial force and conditions and the output of the system.

Thus comparing the system response measured with that predicted by the model.
Defined a model for the system: depends on a set of parameters θ. Identify the input (strength and initial conditions) and output ($x(t)$) of the system. By giving the same input to the model and comparing the measured response $x(t)$ to that expected by the model ($x(t, \theta)$).

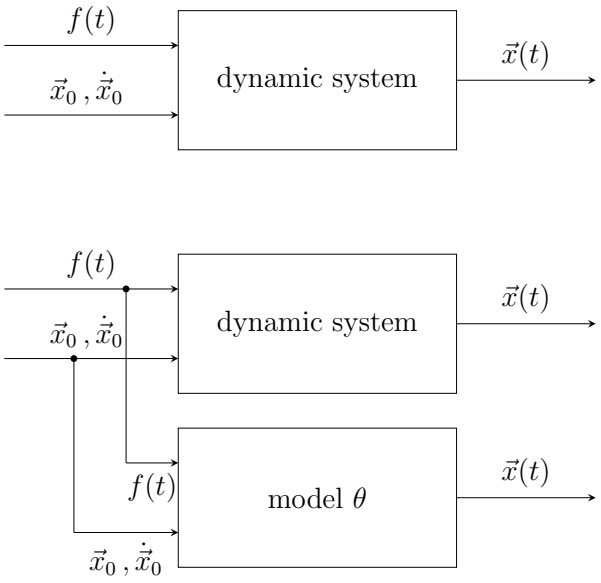


Figure 2.2: System identification: real system and model

We define the residue as the difference between the two signals as in the equation (2.5).

$$\varepsilon(t, \theta) = x(t) - x(t, \theta) \tag{2.5}$$

The best choice for the set of parameter θ is the one that minimizes the residue integer (2.6) of the residue.

$$\theta_{optim} = \text{argmin}\left(\int \varepsilon(t, \theta)^2 \, dt\right) \tag{2.6}$$

In this case, as the discrete signals with N samples, the optimal value N calculated as (2.7), this correspond to solving a least square problem.

$$\theta_{optim} = \text{argmin}\left(\sum_{n=1}^{N} \varepsilon(t, \theta)^2\right) \tag{2.7}$$

A linear model is adopted as described above, refer 1.2.1, assuming the stiffness matrix $[K]$ given. Then quantifying the $[M]$ and $[C]$ parameters are used to minimize the problem. Using the *fmincon* algorithm, the problem is resolved. In addition, the selected algorithm is required to specify the lower and upper search limits; as well as the first guess conditions

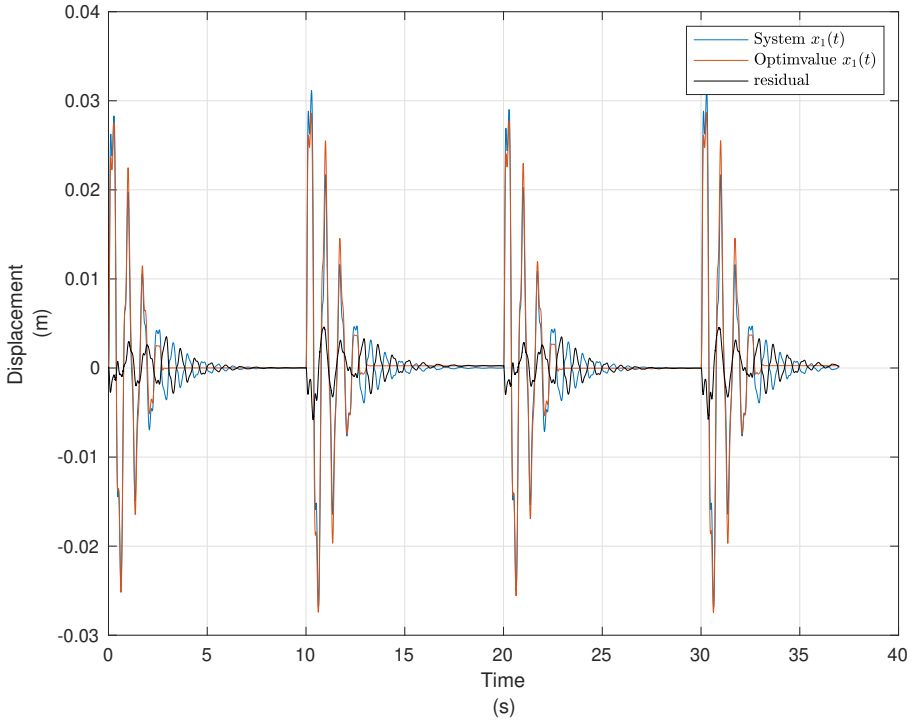
from which the result is strictly dependent.

2.2.1 Free damping case

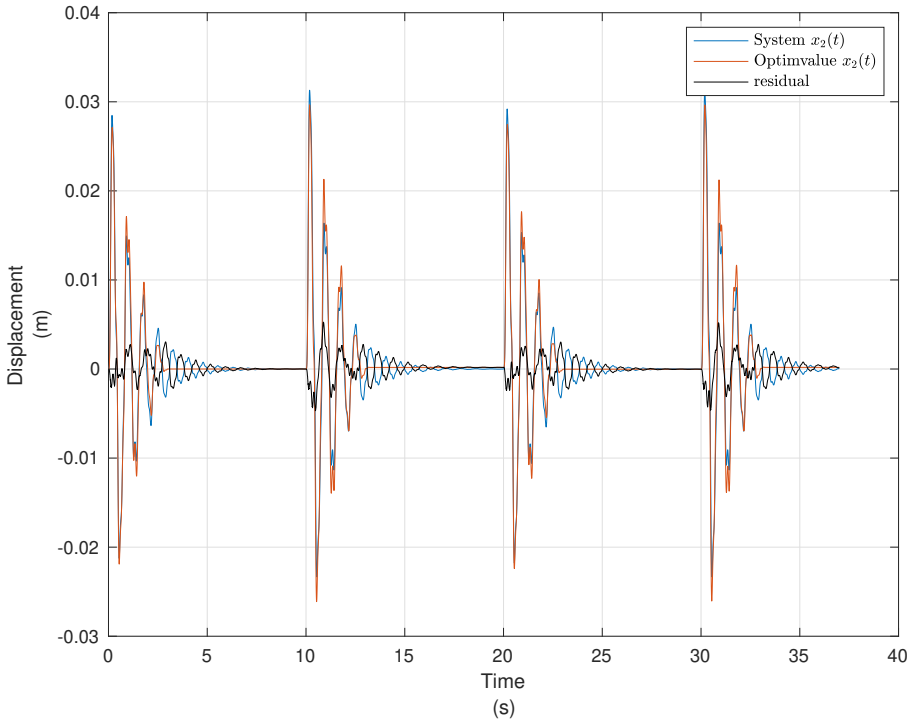
Using the data of the file *data_impulse.mat* identify the seven parameters previously identified in the model and described by the equation (1.4). In this case, since the response to impulse force, two to approximation of the force estimation, will be analysed, the voltage-to-force coefficient will be studied again.
The results of this optimization are shown in table 2.2.

m_1	m_2 [kg]	m_3	c_1	c_2 [N s m^{-1}]	c_3	g_v [V]
1.5712	1.5020	1.2018	2.7956	1.9978	2.1950	1.2096

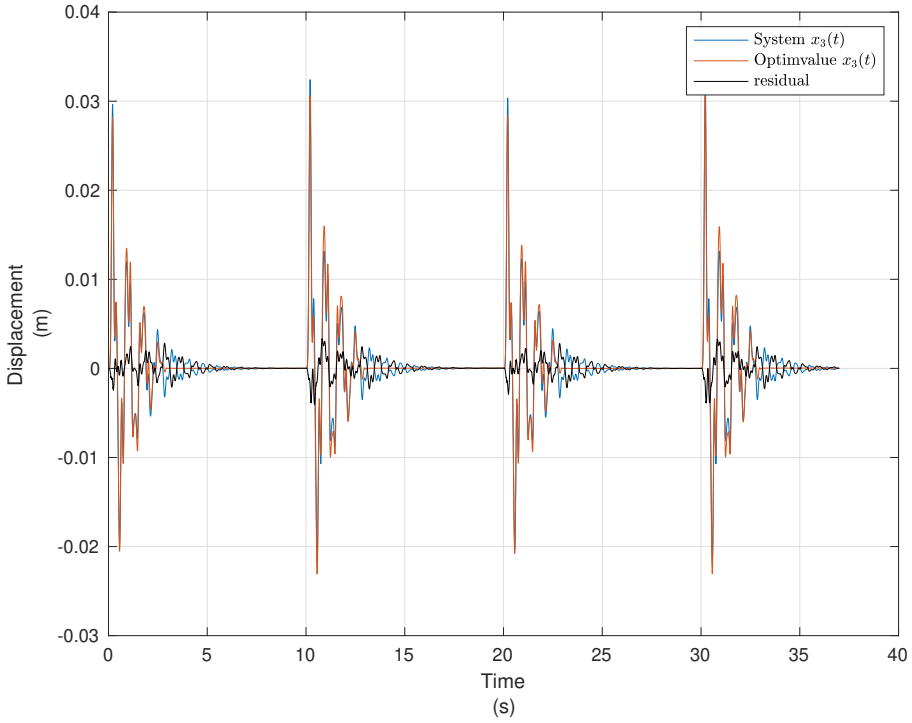
Table 2.2: Optimizations results in free damping case



(a) *mass vs displacement 1.*



(b) *mass vs displacement 2.*



(c) *mass vs displacement 3.*

Figure 2.2: Comparison between the response of the model and the response of the system free damping case

2.2.2 Proportional damping case

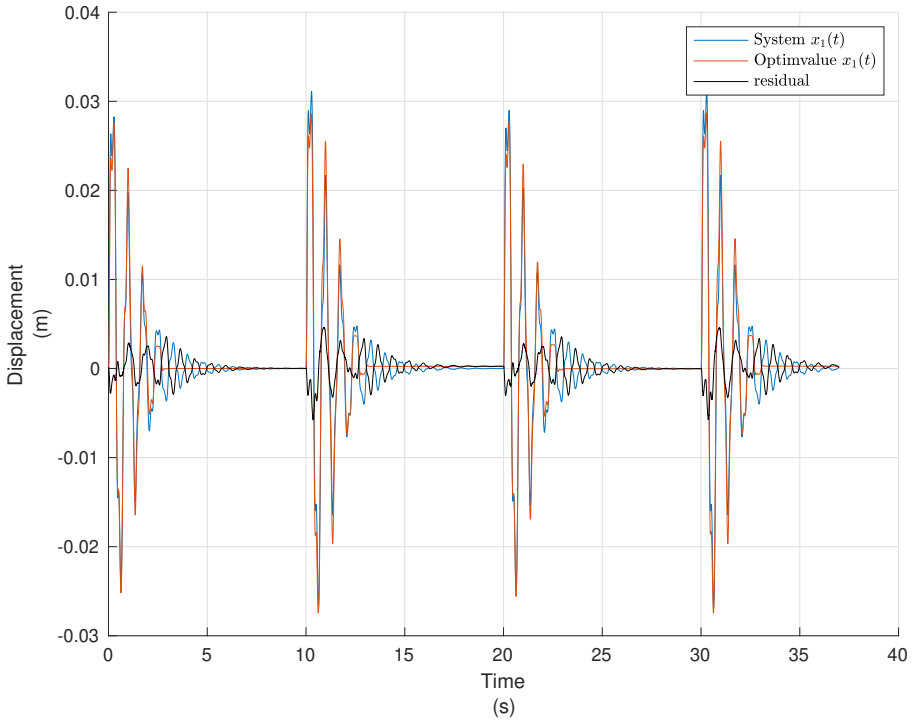
In this case, the procedure used to solve the problem raised in the case of free damping to obtain the parameters is used; changing some conditions. We define damping through the equation (2.8) where it is possible to observe that this depends on the linear combination of the mass matrix multiply by constant and the stiffness matrix multiply by the another constant. Then the search limits and first guess will be changed to search the mass values and the unknown constants. The interesting property of this representation is the ability to perform modal decomposition on the system.

$$[C] = \alpha \cdot [M] + \beta \cdot [K] \tag{2.8}$$

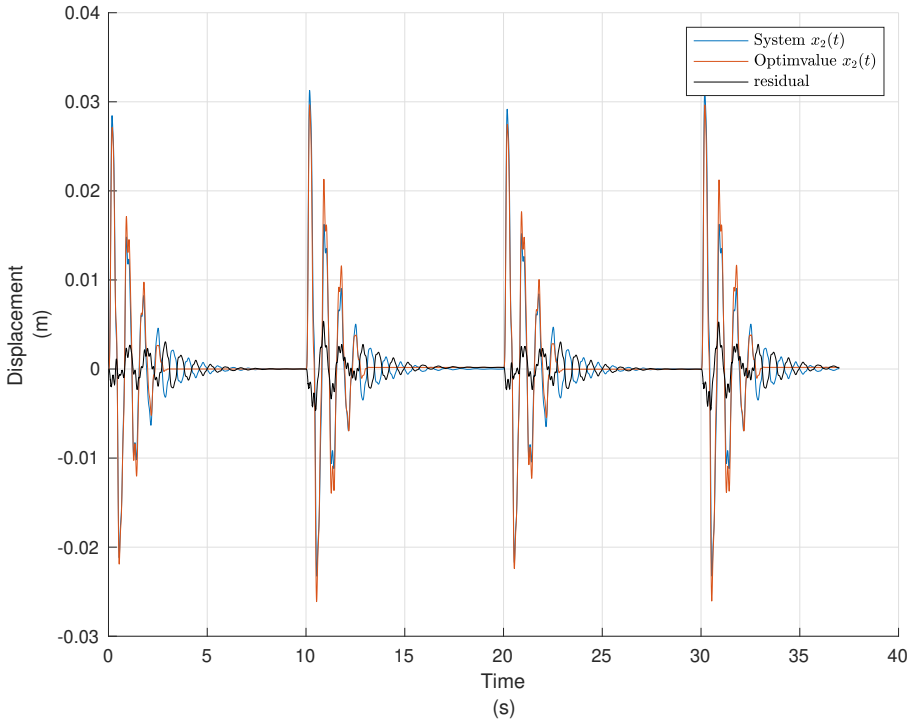
The optimization results are available in the table 2.3.

m_1	m_2 [kg]	m_3	g_v [V]	α [N s m^{-1}]	β
1.5761	1.4970	1.1996	1.2094	1.6209	0.0001

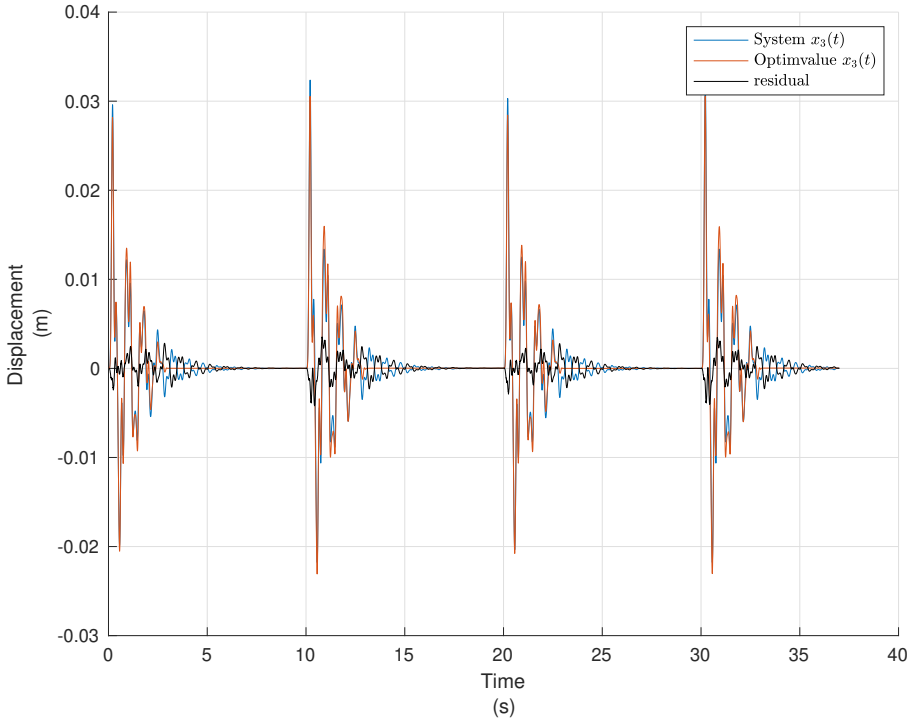
Table 2.3: Optimizations results in proportional damping case



(a) *mass vs displacement 1.*



(b) *mass vs displacement 2.*



(c) *mass vs displacement 3.*

Figure 2.2: Comparison between the response of the model and the response of the system proportional damping case

$$[C] = \begin{bmatrix} 2.64026 & -0.08550 & 0.00000 \\ -0.08550 & 2.59748 & -0.08550 \\ 0.00000 & -0.08550 & 2.07270 \end{bmatrix} \tag{2.9}$$

2.2.3 Comparison

After fitting data with a model, you should evaluate the goodness of fit. The goodness of fit is calculated using the normalized root mean square error as the cost function. In table 2.4 are available the percentages the measured output. This method to assess goodness of fit for both linear and non linear parametric fits. As is common in statistical literature, the term goodness of fit is used here in sense: "good fit" might be a model where the data could reasonably have come from, given the assumptions of least-squares fitting.

	x_1 [%]	x_2 [%]	x_3 [%]
free damping case	81.36	81.17	81.71
proportional damping case	81.36	81.17	81.56

Table 2.4: Result *goodness of fit* measured output

2.2.4 Multiple single DOF system case

Chapter 3

Modal analysis

When external forces act on a multi degree-of-freedom system, the system undergoes forced vibration. For a system with three coordinates or degrees of freedom, the governing equation of motion are a set of three coupled ordinary differential equations of second second order. The equations of motion of a multi degree-of-freedom system under external force are given by

$$[\mathbf{M}]\ddot{\vec{x}} + [\mathbf{K}]\vec{x} = \vec{\mathbf{F}} \tag{3.1}$$

The equation (3.1) represents the system in the version without damping, where $\vec{\mathbf{F}}$ is the vector of the external force $g_{\mathrm{v}} \cdot v$.

3.1 Forced vibration of undamped system

To solve the equation (3.1) by modal analysis, it is necessary first to solve the eigenvalue problem and find the natural frequencies $\omega_1, \ldots, \omega_n$ as reported below: $\omega_1 = 8.27843$ [rad s^{-1}], $\omega_2 = 27.40935$ [rad s^{-1}], $\omega_3 = 41.87035$ [rad s^{-1}]; and the corresponding normal modes:

$$[\mathbf{U}] = \begin{bmatrix} 1.00000 & 1.00000 & 1.00000 \\ 0.86540 & -0.47555 & -2.44325 \\ 0.61945 & -1.28034 & 2.15533 \end{bmatrix} \tag{3.2}$$

3.2 Forced vibration of viscous damped system

Modal analysis, as presented in previous section 3.1, applies only to undamped systems. In many cases, the influence of damping upon the response of a vibratory system is minor and can be disregarded. However, it must be considered if the response of the system is required for a relatively long period of time compared to the natural periods of the system. Further, if the frequency of excitation is at or near one of the natural frequencies

of the system, damping is of primary importance and must be taken into account. In general, since the effects are not known in advance, damping must be considered in the vibration analysis of any system. Using the equation of motion:

$$[\mathbf{M}]\ddot{\vec{x}} + [\mathbf{c}]\dot{\vec{x}} + [\mathbf{K}]\vec{x} = \vec{\mathbf{F}} \tag{3.3}$$

Then we can consider for which the damping matrix can be expressed as a linear combination of the mass and stiffness matrices:

$$[C] = \alpha \cdot [\mathbf{M}] + \beta \cdot [\mathbf{K}] \tag{3.4}$$

where α and β are constant. By substituting in (3.3), we obtain:

$$[\mathbf{M}]\ddot{\vec{x}} + [\alpha \cdot [\mathbf{M}] + \beta \cdot [\mathbf{K}]]\dot{\vec{x}} + [\mathbf{K}]\vec{x} = \vec{\mathbf{F}} \tag{3.5}$$

By expressing the solution vector $\vec{x}$ as a linear combination of the natural modes oft he undamped system $x(t) = [\mathbf{X}]q(t)$ Equation (3.5) can be rewritten as

$$[\mathbf{X}]^\top[\mathbf{M}][\mathbf{X}]\ddot{\vec{x}} + [\alpha \cdot [\mathbf{X}]^\top[\mathbf{M}][\mathbf{X}] + \beta \cdot [\mathbf{X}]^\top[\mathbf{K}]][\mathbf{X}]\dot{\vec{x}} + [\mathbf{X}]^\top[\mathbf{K}][\mathbf{X}]^\top\vec{x} = [\mathbf{X}]^\top\vec{\mathbf{F}} \tag{3.6}$$

If the eigenvectors $\vec{\mathbf{X}}^{(j)}$ are normalized we can obtain:

$$\ddot{q}_i(t) + (\alpha + \omega_i^2\beta)\dot{q}_i(t) + \omega_i^2 q(t) = Q_i(t) \tag{3.7}$$

where: $\omega_1 = 8.27799$ $[\mathrm{rad\,s^{-1}}]$, $\omega_2 = 27.40863$ $[\mathrm{rad\,s^{-1}}]$, $\omega_3 = 41.91576$ $[\mathrm{rad\,s^{-1}}]$ are the natural frequencies of the undamped system. The eigenvector are reported in (3.8).

$$[\mathbf{U}] = \begin{bmatrix} 1.00000 & 1.00000 & 1.00000 \\ 0.86499 & -0.48006 & -2.46147 \\ 0.61907 & -1.28528 & 2.16954 \end{bmatrix} \tag{3.8}$$

From equation $\alpha + \omega_i^2\beta = 2\,\zeta_i\,\omega_i$, where ζ_i is called *modal damping ratio* for the i_{th} normal mode, equation (3.7) can be written:

$$\ddot{q}_i(t) + 2\zeta_i\omega_i\dot{q}_i(t) + \omega_i^2 q(t) = Q_i(t) \tag{3.9}$$

It can be seen that each of then equations represented by this expression is uncoupled from all of the others. Hence we can find the response of the i_{th} mode in the same manner as that of a viscously damped single-degree-of-freedom system. The identification of the sources and magnitude of damping is difficult in most practical problems. The condition

given by equation (3.4) is sufficient but not necessary for the existence of normal modes in damped systems. The necessary condition is that the transformation that diagonalizes the damping matrix also uncouples the coupled equations of motion. This condition is less restrictive than equation (3.4) and covers more possibilities. In the general case of damping, the damping matrix cannot be diagonalized simultaneously with the mass and stiffness matrices. A common procedure for finding the solution of the eigenvalue problem of a damped system involves the transformation of the coupled second-order equations of motion into $2n$ uncoupled first-order equation.

3.3 Rayleigh quotient method

Rayleigh's method can be extended to find the approximate value of the fundamental natural frequency of a discrete system. It is possible derive an expression for the approximate value of the first natural frequency of a multidegree-of-freedom system according to Rayleigh's method. The kinetic and potential energies of an n-degree-of-freedom discrete system can be expressed as

$$T = \frac{1}{2}\dot{\vec{x}}^{\top}[\mathbf{M}]\dot{\vec{x}} \tag{3.10}$$

$$V = \frac{1}{2}\dot{\vec{x}}^{\top}[\mathbf{K}]\dot{\vec{x}} \tag{3.11}$$

To find the natural frequencies, we assume harmonic motion to be $\vec{x} = \vec{\mathbf{X}}\cos(\omega t)$, where $\vec{\mathbf{X}}$ denotes the vector of amplitudes and ω represents the natural frequency of vibration. If the system is conservative, the maximum kinetic energy is equal to the maximum potential energy. By equating T and V obtain:

$$\omega^2 = \frac{\dot{\vec{x}}^{\top}[\mathbf{K}]\dot{\vec{x}}}{\dot{\vec{x}}^{\top}[\mathbf{M}]\dot{\vec{x}}} \tag{3.12}$$

The right-hand side of (3.12) is known as *Rayleigh's quotient* and is denoted as $R(\vec{X})$, then the Rayleigh's quotient can be expressed as:

$$\omega^2 = R(\vec{X}) = \frac{c_1^2\,\omega_2^2\,\dot{\vec{x}}^{\top}\,[\mathbf{M}]\,\dot{\vec{x}} + c_2^2\,\omega_2^2\,\dot{\vec{x}}^{\top}\,[\mathbf{M}]\,\dot{\vec{x}} + \ldots}{c_1^2\,\dot{\vec{x}}^{\top}\,[\mathbf{M}]\,\dot{\vec{x}} + c_2^2\,\dot{\vec{x}}^{\top}\,[\mathbf{M}]\,\dot{\vec{x}} + \ldots} \tag{3.13}$$

$$R(\vec{X}) = \frac{c_1^2\,\omega_2^2 + c_2^2\,\omega_2^2 + \ldots}{c_1^2 + c_2^2 + \ldots} \tag{3.14}$$

The results obtained with regard to the natural frequencies are as follows: $\omega_1 = 8.27843$ [rad s^{-1}], $\omega_2 = 27.40935$ [rad s^{-1}], $\omega_3 = 41.87035$ [rad s^{-1}]. Instead the values obtained for the mode shapes are:

$$[\mathbf{U}] = \begin{bmatrix} 1.00000 & 1.00000 & 1.00000 \\ 0.86540 & -0.47555 & -2.44325 \\ 0.61945 & -1.28034 & 2.15533 \end{bmatrix} \tag{3.15}$$

The results obtained for the proportionally damped system are the natural frequencies: $\omega_1 = 8.27799$ [rad s^{-1}], $\omega_2 = 27.40863$ [rad s^{-1}], $\omega_3 = 41.91576$ [rad s^{-1}]. So the values obtained for the mode shapes are

$$[\mathbf{U}] = \begin{bmatrix} 1.00000 & 1.00000 & 1.00000 \\ 0.86499 & -0.48006 & -2.46147 \\ 0.61907 & -1.28528 & 2.16954 \end{bmatrix} \tag{3.16}$$

Rayleigh's quotient has a stationary value in the neighbourhood of an eigenvector. The stationary value is actually a minimum value in the neighbourhood of the fundamental mode. Which means that Rayleigh s quotient is never higher than the highest eigenvalue. Thus Rayleigh's quotient provides an upper bound for ω_1^2 lower bound for ω_n^2.

3.4 Matrix Iteration Method

the matrix iteration method assumes that the natural frequencies are distinct and well separated such that $\omega_1 < \omega_2 < \cdots < \omega_n$. The iteration starting by selecting a trial vector $\vec{\mathbf{X}}_1$, which is then premultiplied by the dynamical matrix $[D]$. The resulting column vector is then normalized, usually by making one of its components equal to unity. The normalized column vector is premultiplied by $[D]$ to obtain a third column vector, which is normalized in the same way as before and become still another trial column vector. The process is repeated until the successive column vectors converge to a common vector: the fundamental eigenvector. According to the expansion theorem, any arbitrary n-dimensional vector $\vec{\mathbf{X}}_1$ can be expressed as linear combination of the n orthogonal eigenvectors of the system $\vec{\mathbf{X}}^{(i)}$.

$$\sum_{i=1}^{n} = c_i \, \vec{\mathbf{X}}^{(n)} \tag{3.17}$$

where c_i are unknown constant number to be determined. As view before, it is possible premultiply $\vec{\mathbf{X}}_1$ by matrix $[D]$ obtaining:

$$[D]\vec{\mathbf{X}}_1 = c_1 [D] \vec{\mathbf{X}}_{(1)} + c_2 [D] \vec{\mathbf{X}}_{(2)} + \cdots + c_n [D] \vec{\mathbf{X}}_{(n)} \tag{3.18}$$

In according with the equation $\lambda [I] \vec{X} = [D] \vec{X}$, then we obtain $\vec{X}^{(n)} = \lambda [I] \vec{X}^{(n)} = \frac{1}{\omega_n^2} \vec{X}^{(n)}$, thus after having substituted it in the equation (3.18) one comes to the:

$$[D] \vec{\mathbf{X}}_1 = \vec{\mathbf{X}}_2 \tag{3.19}$$

$$= \frac{c_1}{\omega_1^2} \vec{\mathbf{X}}^{(1)} + \frac{c_2}{\omega_2^2} \vec{\mathbf{X}}^{(2)} + \cdots + \frac{c_n}{\omega_n^2} \vec{\mathbf{X}}^{(n)} \tag{3.20}$$

By repeating the process we obtain, after r_{th} iteration,

$$[D] \vec{\mathbf{X}}_r = \vec{\mathbf{X}}_{r+1} \tag{3.21}$$

$$= \frac{c_1}{\omega_1^{2r}} \vec{\mathbf{X}}^{(1)} + \frac{c_2}{\omega_2^{2r}} \vec{\mathbf{X}}^{(2)} + \cdots + \frac{c_n}{\omega_n^{2r}} \vec{\mathbf{X}}^{(n)} \tag{3.22}$$

Since the natural frequencies are are assumed $\omega_1 < \omega_2 < \cdots < \omega_n$, a sufficiently large value of r yields $\frac{1}{\omega_1^{2r}} >> \frac{1}{\omega_2^{2r}} >> \cdots >> \frac{1}{\omega_n^{2r}}$. Thus the first term in right-hand side of equation (3.21) becomes the only significant one. Which means that the $(r+1)_{th}$ trial vector becomes identical to the fundamental modal vector to within a multiplicative constant $\vec{\mathbf{X}}_r = \frac{c_1}{\omega_1^{2(r-1)}} \vec{\mathbf{X}}^{(1)}$. Then the fundamental natural frequency ω_1 can be found by taking the ratio of any two corresponding components in the vectors $\vec{\mathbf{X}}_r$ and $\vec{\mathbf{X}}_{r+1}$:

$$\omega_1^2 \simeq \frac{\vec{\mathbf{X}}_{n,r}}{\vec{\mathbf{X}}_{n,r+1}} \tag{3.23}$$

where $\vec{\mathbf{X}}_{n,r}$ and $\vec{\mathbf{X}}_{n,r+1}$ are the n_{th} elements of vector $\vec{\mathbf{X}}_r$ and $\vec{\mathbf{X}}_{r+1}$, respectively.

3.4.1 Intermediate natural frequencies

Once the first natural frequency $omega_1$ and the corresponding eigenvector $\vec{\mathbf{X}}^{(1)}$ are determined, it is possible proceed to find the higher natural frequencies and the corresponding mode shapes. To find the eigenvector $\vec{\mathbf{X}}^{(i)}$, the previous eigenvector $\vec{\mathbf{X}}^{(i-1)}$ is normalized with respect to the mass matrix such that $\vec{\mathbf{X}}^{(i-1)^\top} [m] \vec{\mathbf{X}}^{(i-1)} = 1$. The deflated matrix $[D_i]$ is the constructed as is used.

$$[D_i] = [D_i] - \lambda_{i-1} \vec{\mathbf{X}}^{(i-1)} \vec{\mathbf{X}}^{(i-1)^\top} [m] \tag{3.24}$$

where $[D_1] = [D]$. Once $[D_i]$ is constructed the iterative scheme

$$\vec{\mathbf{X}}^{(r+1)} = [D_i]\,\vec{\mathbf{X}}^{(r)} \tag{3.25}$$

3.4.2 Result

The whole procedure is performed for the case of free damping where the natural frequencies are: $\omega_1 = 8.27843\ \mathrm{rad\,s^{-1}}$, $\omega_2 = 27.41224\ \mathrm{rad\,s^{-1}}$, $\omega_3 = 41.85486\ \mathrm{rad\,s^{-1}}$. The mode shapes results are shown in (3.26).

$$\begin{bmatrix} 1.00000 & 1.00000 & 1.00000 \\ 0.86540 & 0.21230 & -0.85499 \\ 0.61945 & -0.30643 & -0.30390 \end{bmatrix} \tag{3.26}$$

In the case of proportional damping the natural frequencies are: $\omega_1 = 8.27799\ \mathrm{rad\,s^{-1}}$, $\omega_2 = 27.40863\ \mathrm{rad\,s^{-1}}$, $\omega_3 = 41.91576\ \mathrm{rad\,s^{-1}}$. The mode shapes results are shown in (3.27).

$$\begin{bmatrix} 1.00000 & 1.00000 & 1.00000 \\ 0.86499 & -0.48006 & -2.46147 \\ 0.61907 & -1.28528 & 2.16954 \end{bmatrix} \tag{3.27}$$

3.4.3 Observation

Although it is theoretically necessary to have $r \to \infty$ for the convergence of the method, in practice only a finite number of iterations suffices to obtain a reasonably good estimate of ω_1. The actual number of iterations necessary to find the value of ω_1 to within a desired degree of accuracy depends on how closely the arbitrary trial vector $\vec{\mathbf{X}}_1$ resembles the mode and how well ω_1 and ω_2 are separated. The required number of iterations is less if ω_2 is very large compared to ω_1. The method has a distinct advantage in that any computational errors made do not yield incorrect results. Any error made in premultiplying $\vec{\mathbf{X}}_i$ by $[D]$ results in a vector other than the desired one, $\vec{\mathbf{X}}_{i+1}$. But this wrong vector can be considered as a new trial vector. This may delay the convergence but does not produce wrong results.

Chapter 4

Transfer Function

The transfer function was calculated from the system equation (1.3) analysed in the section 1.2.2, where the input is g_{v}. To calculate the transfer function between the force and the positions, the voltage-to-force coefficient gain has to be removed.

$$G(s) = \left([M]s^2 + [C]s + [K]\right)^{-1} \tag{4.1}$$

Given $F(s)$ we obtain the Laplace transform of the force signal. The transfer function is a column vector with three entries, one for each output, they share the denominator as in eq.(4.1).

$$\frac{X(s)}{F(s)} = \frac{1}{g_{\mathrm{v}}} \frac{X(s)}{V(s)} \tag{4.2}$$

The bode diagram are shown in Figure 4.1.

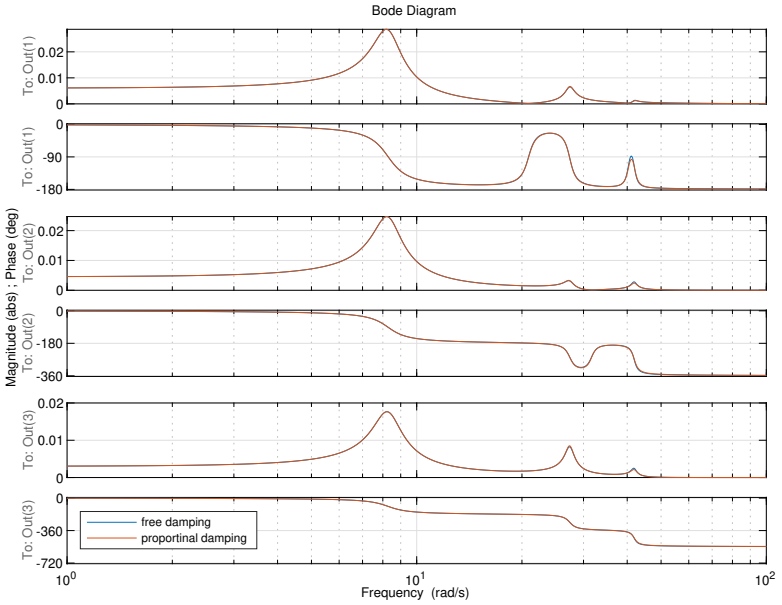
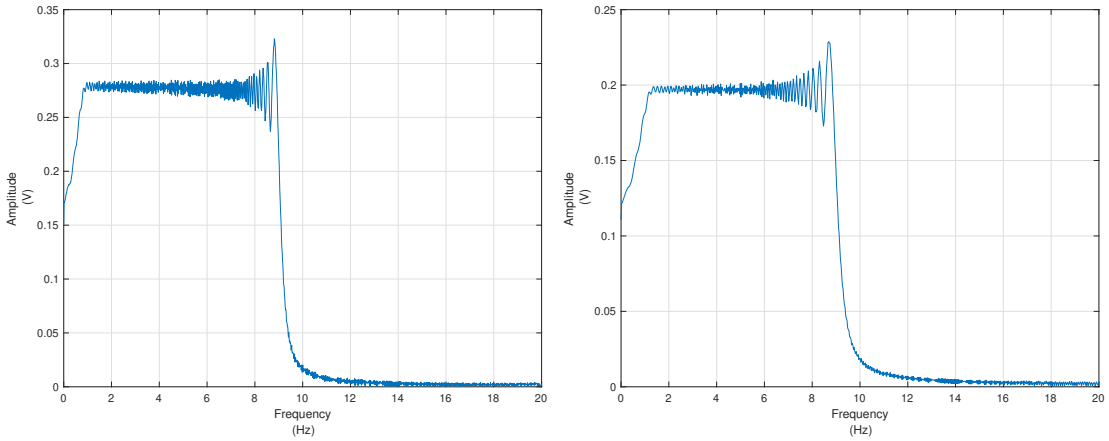


Figure 4.1: Bode diagram of the transfer functions as in (4.2)

Chapter 5

Sine-sweep analysis

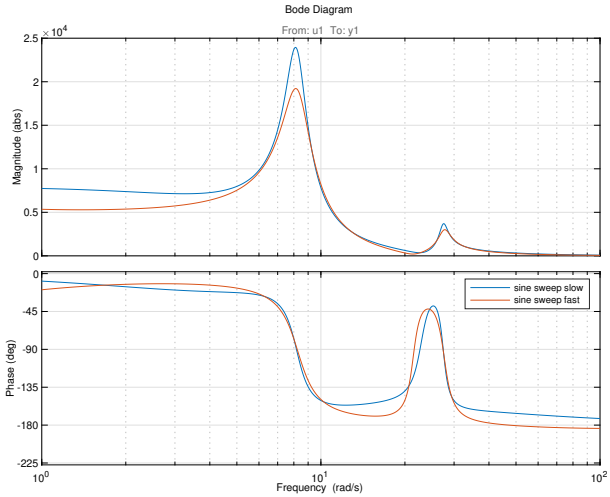
The inputs provided are two sine sweep equal signal with different sample frequencies 5 [ms] and 10 [ms] respectively. To visualize the spectra of the signals, by *fft* is performed on both the signal. The spectrum is plotted with the real frequencies in figure 5.1.



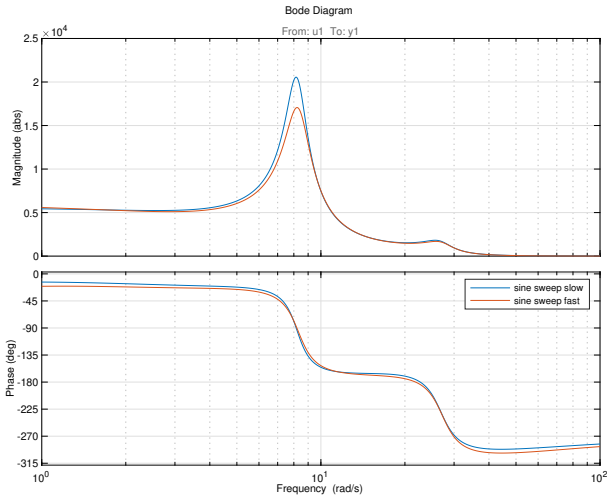
(a) *slow* (b) *fast*

Figure 5.1: Sine sweep spectrum

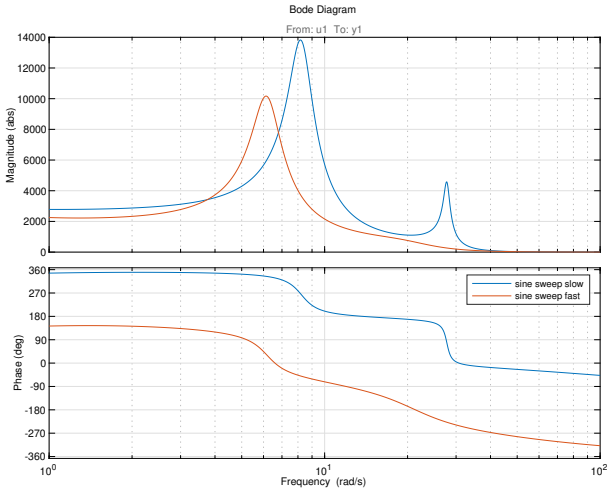
Although the two signals sweep the same frequency range and the difference is only in the sampling time. These include the resonance peaks of the transfer function. On the other hand, the spectrum of the sinusoidal scan excites each frequency for a limited period of time so it is possible to notice that the third mode is not clearly visible.



(a) *plot respect mass 1*



(b) *plot respect mass 2*



(c) *plot respect mass 3*

Figure 5.2: Comparison sine sweep outputs of the system

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