Automatic Control - Final Project

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I. Exercise 1

The vibration absorber [1][Ch. 9.11], also called dynamic vibration absorber, is a mechanical device which is used to reduce or eliminate unwanted vibration. It consists of a mass and a spring attached to the main (or original) mass that needs to be protected from vibration. Vibration absorbers are commonly used in a variety of applications which include sanders, saws, internal combustion engines and high-voltage transmission lines. A sketch of the system is pictured in Figure 1.

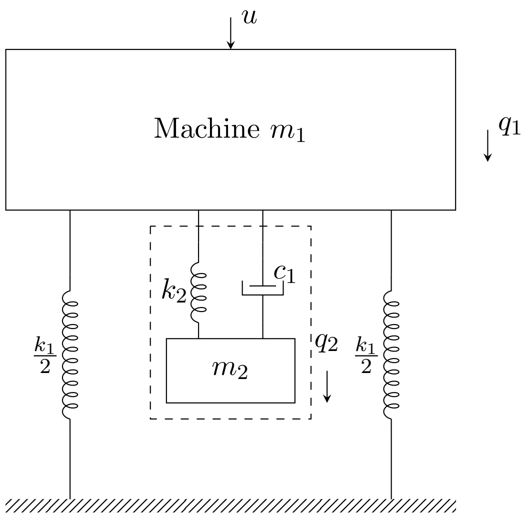

Figure 1: Representation of the harmonic absorber.

The equations of motion for this two-degrees of freedom system can easily obtained using a Lagrange formulation and can be conveniently written in the following matrix form:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{Bmatrix} + \begin{bmatrix} c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix} \begin{Bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = \begin{bmatrix} u \\ 0 \end{bmatrix} \tag{1}$$

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II. Answer

i. 1

Starting from the equation of motion, seen in the equation (1), we obtain the state space representation as below:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-(k_1+k_2)}{m_1} & \frac{k_2}{m_1} & \frac{-c_2}{m_1} & \frac{c_2}{m_1} \\ \frac{k_2}{m_2} & \frac{-k_2}{m_2} & \frac{c_2}{m_2} & \frac{-c_2}{m_2} \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{m_1} \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$$

ii. 2

Setting $u = 0$ through the help of the "*sdpt3* " solver the solution was solved numerically, obtaining the solution from the Lyapunov inequality for $\mathbf{P}$ given in the matrix (2) is obtained.

$$\begin{bmatrix} 603.3417 & -8.5265 & 0.1252 & -5.0157 \\ -8.5265 & 128.7847 & 2.8368 & 0.6276 \\ 0.1252 & 2.8368 & 3.9082 & 0.5233 \\ -5.0157 & 0.6276 & 0.5233 & 1.1363 \end{bmatrix} \tag{2}$$

from the stability theorem linked to the eigenvalues that reports "*the linear and stationary system is asymptotically stable if and only if all its eigenvalues have a real negative part*".[2] From the vector (3) containing the eigenvectors it can be said that the system is stable.

$$\begin{bmatrix} -0.7983 + 14.2721i \\ -0.7983 - 14.2721i \\ -0.3517 + 9.8873i \\ -0.3517 - 9.8873i \end{bmatrix} \tag{3}$$

iii. 3

Using the LMI formulation[3] and using the "*sdpt3*" solver the $\mathcal{L}_2$ value of the γ system is estimated. Where the minimum value assumed by γ is 0.5456×10^{-3}, obtaining the solution from the Lyapunov inequality for $\mathbf{P}$ given in the matrix (4).

$$\mathbf{P} = 1.0 \times 10^3 \cdot \begin{bmatrix} 2.5019 & -0.4262 & -0.0144 & -0.0281 \\ -0.4262 & 0.2806 & 0.0218 & 0.0038 \\ -0.0144 & 0.0218 & 0.0137 & -0.0007 \\ -0.0281 & 0.0038 & -0.0007 & 0.0020 \end{bmatrix} \tag{4}$$

Moreover, the same result was obtained using the matlab toolbox.

iv. 4

From the calculation of the transfer matrix (5), calculated in, we obtain the Bode diagram in the figure 2, where we can see that the peak has a value of $0.544\,001\,5 \times 10^{-3}$.

$$\begin{aligned} G(s) &= \frac{y(s)}{u(s)} = C(sI - A)^{-1}B \\ &= \frac{0.01s^2 + 0.02s + 1.333}{s^4 + 2.3s^3 + 303.3s^2 + 300s + 20000} \end{aligned} \tag{5}$$

It is observed that the value of $\mathcal{L}_2 = 0.5456 \times 10^{-3}$ is similar to the peak present in the Bode diagram in Figure 2. $\mathcal{L}_2$ gain is a popular measure of smallness for system approximation errors

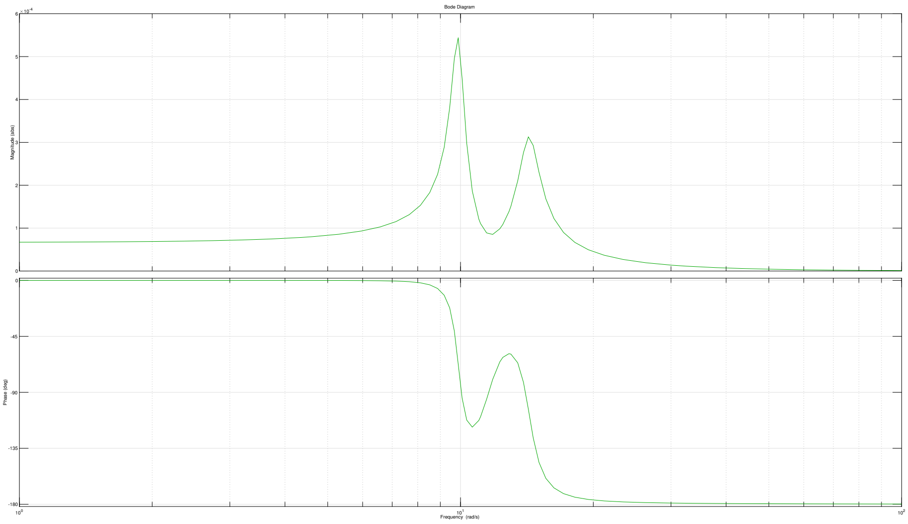

Figure 2: Bode diagram of tranfer matrix.

and dynamical perturbations. Consider a causal convolution model of an LTI system, possibly of an infinite order, defined by the impulse response matrix $g = g(t)$ or by a transfer matrix $G = G(s)$. Remember that such system defines the output y as a convolution integral.

$$y(t) = \int_0^{\infty} h(\tau) f(t-\tau) d\tau \tag{6}$$

where f is assumed to vanish fast enough as $t \to -\infty$.
Let $\hat{g} = \hat{g}(t)$ and $\hat{G} = \hat{G}(s)$ be the impulse response and the transfer matrix of another convolution model, intended to serve as a simplified approximation of the original one. One common way of measuring approximation quality is by comparing system responses $y_0 = y_0(t)$ and $\hat{y}_0 = \hat{y}_0(t)$ to a particular "testing" input $f_0 = f_0(t)$. This leads to the so-called H-Infinity norm of the difference $H(s) = G(s) - \hat{G}(s)$ as the approximation error measure of choice.[4]

References

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[3] A. S. Luca Zaccarian, "Analysis and synthesis of feedback systems: quadratic functions and lmis," April 22, 2018.

[4] A. Megretski, "L2 gains and system approximation quality," vol. Fall 2004: MODEL REDUCTION, Septmber 19 2004.