7.5 *G is m-colorable if and only if $\alpha(G\Box K_m) \geq n(G)$.*

Proof. *Necessity* $\chi(G\Box K_m) = max\{\chi(G), \chi(K_m)\} = m$ since G is m-colorable. And since $\chi(H) \geq \frac{n(H)}{\alpha(H)}$, $\alpha(G\Box K_m) \geq \frac{n(G)m}{m} = n(G)$.

Sufficiency

□

7.7

(a) *Proof.* Construct a graph G, area as vertex, adjacent if areas share a common edge. Because no three intersecting at a point, $\omega(G) = 2$ and no odd cycle exists; i.e. G is a bipartite. We can find an orientation D (from partite X to Y) such that $l(D) = 1$. According to **Gallai's Thm**, then $\omega(G) \leq \chi(G) \leq 1 + l(D) = 2$. □

(b) We can always map the vertices to a line disjointly, forming an ordering of vertex. Since no three intersects at a point, each vertex has at most 2 neighbors earlier in the ordering. Thus, $\chi(G) \leq 3$.

7.8

7.10 $\chi(G_{n,k}) = k + 1$ *if* $k + 1$ *divides* n *and* $\chi(G_{n,k})$ *does not divides* n.

Proof. According to the construction of the graph $G_{n,k}$, cliques of size $k + 1$ are formed. Then color the graph $G_{n,k}$ with $k+1$ colors. If $k+1$ divides n, $1, 2, 3..., k+1, 1, 2, 3..., k+1$ will be a proper coloring of graph $G_{n,k}$.

If $k+1$ does not divides n, let $n = q(k+1)+r = (q-r)(k+1)+r(k+2), 1 \leq r < (k+1)$. Since $n \geq k(k+1)$, $q \geq k \geq r$. Therefore, a proper coloring exists that color $q - r$ times with $k + 1$ colors and r times with $k + 2$ colors. $\chi(G_{n,k}) = k + 2$. □

7.12

Bibliography