6.2 A torus can be unfolded into a plane that vertex in bottom and top is the same while that in left edge and right edge is the same. Figure 1 is an example.

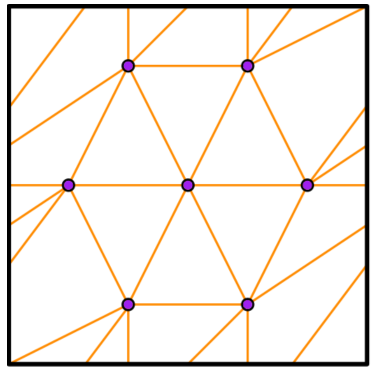

Figure 1: K7 embedded into a torus
ref[1]

6.3 *Proof.* A plane can be written in a form of $ax + by + cz + d = 0$. Substitute $(x, y, z) = (t, t^2, t^3)$ into the equation obtaining a cubic equation. The cubic equation has 3 different solutions at most. Therefore, no such four vertices exist that they all lay on the same plane. □

6.4

(1) *Proof. A polygon with $n \leq 5$, there is a vertex inside G can see all vertices inside the polygon.* It is trivial when $n = 3$. When $n = 4$, vertex on the shorter diagonal line works. When $n = 5$, sum of interior angle is $540°$ degree. If no angle extends $180°$, it is convex. Such vertex exists. If one angle extends $180°$, vertices inside the triangle of the obtuse angle and other two disjoint vertices are such vertices. If two angle extends $180°$, vertices in the shadow of the Figure 2. □

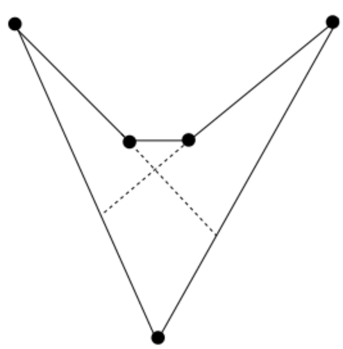

Figure 2: Polygon(n=5) with two obtuse angles

(2) *Proof. Any planar graph has a straight-line drawing.* Prove by induction on the number of vertices. Without loss of generality assume that G is maximally planar(means

adding any edge will make G not planar). It is because if maximally planar graph is can be straight-line drawn, removing any edges the graph can be still straight-line drawn.
According to **Corollary 6.8**, planar graph G always has vertex $\delta(v) \leq 5$. Remove v and triangulate the face that is created by it's removal to create G'. G' is a maximally planar graph which has fewer vertices than G. With induction hypothesis, G' can be straight-line drawn. Then to turn G' into a straight line drawing of G first erase allthe edges which were used to triangulate the face the removal v created. Next by **(1)**, we know there is a vertex "see" all vertices in the face, since there are 5 vertices on the face. We add v at such a location, and add the lines between it and the vertices of the face. This gives a straight line drawing of G.

□

6.6 *Proof.* If a graph is isomorphic with its dual, it is connected, because any graph's dual graph is connected. With Euler polyhedral formula, we have $n - e + f = 2$. Since isomorphic, $n = f$. Then we have $e = 2n - 2$. □

Figure 3 is an example of $n = 5$ that the original plane graph is isomorphic with its dual. Its $n - 1$ nodes form a cycle and the left one is in the center joining all other nodes.

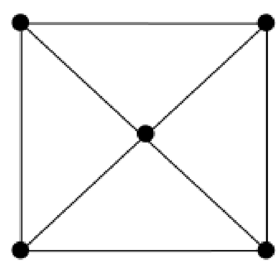

Figure 3: $n = 5$, Plane graph isomorphic with its dual

6.7 *Proof.* According to **Corollary 6.8**, for any planar graph G, $\delta(G) \leq 5$. Therefore, There is no 6-connected planar graph. □

6.9 *Proof. A planar graph with n nodes and girth of k has at most* $(n-2)\frac{k}{k-2}$. Each edge is calculated by both sides of faces, while each face has at least k edges. We have $2e \geq kf$. Substitute $n - e + f = 2$ into the inequality. We will have $e \geq (n-2)\frac{k}{k-2}$.

Petersen graph is not planar graph. Petersen graph G, $n = 10, e = 15, k = 5$. According to above, $e \leq 13$, which results in contradiction. Petersen graph is not planar graph. □

Reference

[1] http://www3.math.tu-berlin.de/geometrie/Lehre/WS12/MathVis/resources/projects /loeweSiegSlides.pdf