3.1 Because (1),(2),(3),(4) are equivalent, we only need to prove that (5),(6) are equivalent with one of (1) $\sim$ (4).

Proof. (1) $\Rightarrow$ (5) : G is a tree, i.e. G is connected and has no cycle. Because G is connected, a path $x - y$ exists between x and y. We can add an arbitrary edge $e : xy$. Then a cycle will be formed. (5) $\Rightarrow$ (1) : G has no cycle. With an arbitrary edge $e : xy$, a cycle is formed. That means a path $x - y$ exits in G. $\Rightarrow G$ is connected $\Rightarrow$ G is a tree. □

Proof. (4) $\Rightarrow$ (6) : Delete an arbitrary edge $e : xy$. Because only one path exists between two arbitrary vertexes x and y. The path between x and y will be xey. So deleting the edge e, no path exists between x and y. G will be unconnected.
(6) $\Rightarrow$ (4) : First prove (6) $\Rightarrow$ (1),then because (1) and (4) are equivalent, (6) $\Rightarrow$ (4) is proved.Following is proof of (6) $\Rightarrow$ (1).
Deleting an arbitrary edge $e : xy$, G will be unconnected. $\Rightarrow G$ has no cycle(**Property 3.4**). What's more, G is connected, so G is a tree. So(6) $\Rightarrow$ (1) is proved. □

3.3 Deleting an arbitrary vertex, G will be a tree. $\Rightarrow G$ has a cycle.(**Thm 3.7(5)**).
If G has other vertexes except cycles, remove these vertexes, G still have cycles, not a tree. That will be a contradiction. Therefore, G is just a cycle, containing n edges.

3.5 *Proof.* "$\Rightarrow$": $d_1, d_2, ...d_n$ is a degree sequence of a tree. The tree has n-1 edges (textbfThm 3.7(2)). Therefore, $\sum_{i=1}^{n} d_i = 2(n-1)$.
"$\Leftarrow$": $\sum_{i=1}^{n} d_i = 2(n-1) \Rightarrow G$ has n-1 edges. Prove with induction on n.
$n = 2$, because $d_i \leq 1$, $d_1 = d_2 = 1$. G with two vertexes and one edge, is of course a tree.
Assume $n = k$, there is a tree T satisfying sequence d, then when $n = k+1$, G has one more vertex and one more edge than tree T. Adding a vertex and an edge to tree T will form a new tree T'. Therefore, G is T', a tree. □

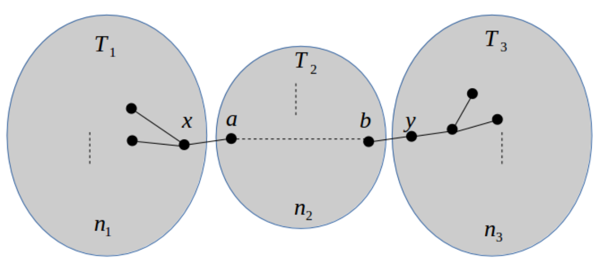

Figure 1: Three components of Tree T

3.10 *Proof.* If there exist two non-adjacent medians, x, y. tree T can be grouped into three groups, two groups rooting at x, y respectively and a group between x and y, T_1, T_2, T_3 as in Figure 1. T_1, T_2, T_3 have n_1, n_2, n_3 vertexes respectively, and $n_2 \geq 2$.

In tree T_2, vertexes a, b are directly connected with x, y respectively. We have

$$\begin{aligned} S_T(x) &= S_{T_1}(x) + S_{T_2}(a) + S_{T_3}(y) + n_2 + n_3(2 + d(a,b)) \\ S_T(y) &= S_{T_1}(x) + S_{T_2}(b) + S_{T_3}(y) + n_2 + n_1(2 + d(a,b)) \\ S_T(a) &= S_{T_1}(x) + S_{T_2}(a) + S_{T_3}(y) + n_1 + n_3(1 + d(a,b)) \\ S_T(b) &= S_{T_1}(x) + S_{T_2}(b) + S_{T_3}(y) + n_3 + n_1(2 + d(a,b)) \end{aligned}$$

Then

$$\begin{aligned} &(S_T(x) + S_T(y)) - (S_T(a) + S_T(b)) = 2n_2 > 0 \\ &S_T(x) + S_T(y) > S_T(a) + S_T(b) \end{aligned}$$

which is a contradiction of definition of median. Therefore x, y are adjacent.
When $n_2 = 1$, in the same way, we can find that vertex a in T_2, $S_T(a) < S_T(x) = S_T(y)$. So only one median exists.
Above all, only one median or two adjacent medians exit in tree T. □

3.11 *Proof.* Assume $|e(G)| = m$. To find a spanning tree T_i, $|e(T_i)| = n - 1$, we need to remove m-n+1 edges of G. Two spanning trees are adjacent if they share $n - 2$ edges. That means if two spanning trees are adjacent, they have only one different edge.
Let $\triangle\ (V_x, V_y)$ denote the number different edges between two spanning trees. When $\triangle\ (V_x, V_y) = 1$, two spanning trees V_x, V_y are adjacent. Now, prove $d(V_x, V_y) =\triangle\ (V_x, V_y)$ by inducting on $\triangle$.
When $\triangle\ (V_x, V_y) = 1$, $d(V_x, V_y) = 1$. It is true. Assume $\triangle\ (V_x, V_y) = k$ is right, $d(V_x, V_y) = k$. When $\triangle\ (V_x, V_y) = k+1$, according to induction, we can find a vertex V_z, such that $\triangle\ (V_x, V_z) = k, \triangle\ (V_y, V_z) = 1$, and V_x, V_z are connected, $d(V_x, V_z) = k$. V_z, V_y are directly connected. Then $d(V_x, V_y) = k + 1$. Therefore, G' is connected.
From above definition, a spanning tree is formed by removing $m - n + 1$ edges from G. Therefore, the longest distance will be $m-n+1$, i.e. diameter will be m-n+1. □