5.11 Do degree ear decomposition; then no link exists between neighbor of joint vertex on cycle or previous ear and the neighbor on current ear,i.e. no edge ac, cd, eh, gh as in Figure 1. Otherwise, a large cycle $a - bca$ exists.

What's more, since G is claw-free, ad, eg must exist. Then we can find disjoint graph P_3 as follow.

Select graph P_3 from ear P_n to P_0. $k = 0, 1, 2, ...,$ For ear P_i, if $n(P_i) = 3(k+1)$, we can select k disjoint graph P_3. Then the neighbor of joint vertex will help hold 2-connectivity for remaining graph.

If $n(P_i) = 3k+1$, select k graph P_3 from one endpoint. The other endpoint remains.

If $n(P_i) = 3k+2$, select k graph P_3 among the middle vertices of ear P_i.

Use the method above, 2-connectivity holds after selecting one ear. No vertex is "wasted". Therefore, we can find $\lfloor n(G)/3 \rfloor$ graph P_3.

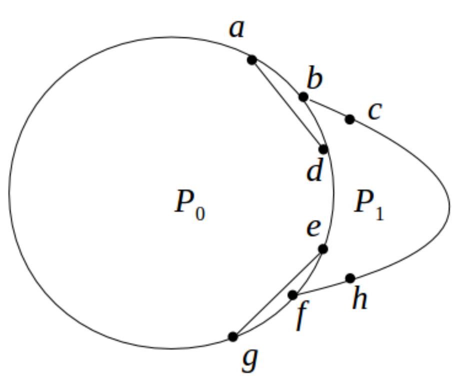

Figure 1: Greedy ear decomposition

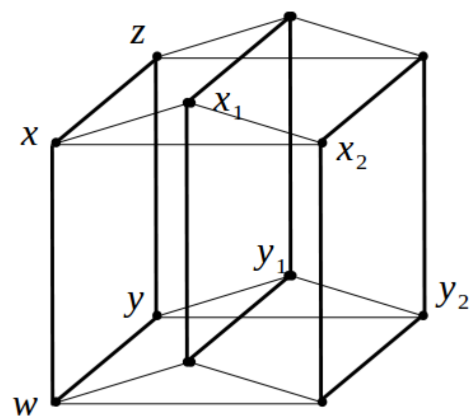

Figure 2: Q_4

5.13 *Proof.* We prove the contrary that *if each block of G is not an edge or odd cycle, even cycle exits in G.* Proof are in two parts.

If G is 1-connected, such an edge exists that it is the only edge connecting two blocks. This edge is the block of edge. It contradicts the condition.

G must be 2-connected; then it has an ear decomposition. For ear P_0, P_1, two uncover cycles are odd cycles. The big cycle will be an even cycle.

□

5.14 *Proof.* We prove by find k disjoint paths between arbitrary x, y.

$k = 1$ is trivial. For $k \leq 2$, we can draw the Q_k as $k - 1$ slices. The slice is a cycle of 4 vertices, and each slice is connected with all other slices by links between corresponding vertices, Q_4 in Figure 3 as an example.

For x, y are in the same slice. We can find two disjoint paths among the slice. And find other $k - 2$ disjoint paths by going from x to other $k - 2$ slices then going through the slice to the counterpart vertex of y at last reaching y. Total in k paths.

For x, y not in the same slice, we can find three disjoint paths between the two slices of x, y reaching y. And find another $k - 3$ disjoint paths by going from x to other slices first then do as former situation.

Above all, Q_k is k-connected. ¡¡¡¡¡¡¡ HEAD

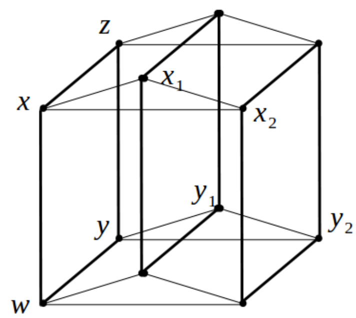

Figure 3: Q_4

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5.17

(1) *Proof.* Since G is two-connected, $\delta(G) \leq 2$. We prove when $\delta(G) \leq 3$, G is not minimally 2-connected.
Do ear decomposition. In last ear P_n, for $v \in P_n \backslash endpoints$, since $\delta(G) \leq 3$, an v-induced edge e exists that $e \notin P_i$. So $G - e$ is still 2-connected, which contradicts definition of minimal 2-connected. □

(2) *Proof.* Graph G is minimally k-edge-connected. So $\delta(G) \leq k$. Then we find the vertex whose degree is k.
We can find a pair vertice x, y with k edge-disjoint paths between x, y. Add vertice x', y' connected with x, y respectively. Then draw a corresponding graph G' of G with vertex as edge of G and edge existing if edges in G are connected.
In this way, G' will be a minimally k-connected graph. According to **5.17(1)**, $\delta(G') = k$. We claim $\delta_{G'}(xx') = k$. Because if another pair u, v which has more than k edge-disjoint paths, $\delta_{G'}(u) = k + 1$. Then in graph G, $deg(x) = k$, i.e. $\delta(G) = k$. □

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5.18

(1) ¿¿¿¿¿¿¿ ce9f6862c61c1b636c2d4ae0ab1a535a0fee5675