Performance Analysis of OFDMA-based Multi-Channel Random Access over IEEE 802.11ax

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***Abstract*—With the progressive increase of dense WiFi networks deployment, Quality-of-Experience (QoE) and power saving are becoming critical issues. IEEE 802.11ax, a task group aimed at High Efficient WLAN (HEW), is adapted for such dense scenario. The 802.11ax makes revolutionary modifications on both the MAC and PHY layer, especially for the OFDMA-based random access mechanism. We here extend the Markov chain model to precisely describe the steady state behavior of the OFDMA-based random access mechanism, and derive the theoretic formula of system efficiency and access delay. Finally, we evaluate the effects of certain system parameters, including the number of resource units (RUs) for random access, initial and maximum contention window length.**

***Index Terms*—MCRA, Multi-User PHY, OFDMA, IEEE 802.11ax**

I. INTRODUCTION

During last decades, the IEEE 802.11 has achieved tremendous success, enormous WiFi networks have been deployed due to its high throughput and relative simple implementation.According to Cisco Visual Network index [2], the mobile traffic size is supposed to increase by 53% at CAGR from 2015 to 2020, i.e. reaching 30.6 EB per month by 2020.Consequently, WiFi networks will become the *dense scenarios*, where excessive number of stations (STA) or access points (AP) or both exist in a limited area.

So far, a series of standards (suc as 802.11b, g, n, ac) have evolved gradually to handle the increasing WLAN data demand, and they mainly raise data rate on physical layer (PHY) from 2 Mbps to 7 Gbps [3], while few on MAC layer. However, the performance or user experience of WiFi networks still cannot catch up with the data rate, especially in dense scenarios. We find the bottleneck of dense WiFi networks gradually relocate at MAC layer, which relies on a random access protocol named distributed coordination function (DCF). The DCF further brings significant collisions that may consume much resource and lower the efficiency in dense scenario.

MAC efficiency is reduced by the overhead of control signaling and collisions. Much effort has been paid in legacy 802.11 to reduce the overhead of control signaling, such as RIFS, frame aggregation, etc [3]. In dense scenario, collisions will be the major cause of spectrum waste. More specifically, they comes from instable distributed coordination function (DCF) problem and unfair queueing problem. Firstly, DCF is inherently instable as it is a random access protocol, and the resulting collisions will consume even more resource in dense scenarios. Secondly, as we can see from the queueing model of the WiFi network, WiFi network will be an unfair queue, which will worsen the effect of instability of DCF. On the one hand, each STA and AP (each queue) has the same chance to access the channel (server). On the other hand, because WiFi network operates in a star topology, AP as the center and STAs surrounding the AP logically, AP shares more than $1/2$ down-link (DL) traffic loading, while AP only has $1/(n+1)$ chance to access channel, where n is the number of STAs. The queue model of WiFi network is, therefore, an unfair queue between DL and up-link (UL) transmissions.

Study group 802.11, target high efficiency WLAN (HEW) in dense scenario, thus modifies the MAC thoroughly, change DCF to central control. On PHY layer, OFDMA is issued to implement both DL and UL multi-user (MU) channel, which means a STA could communicate with multiple STAs at a time. On MAC layer, a brand new control frame called trigger frame (TF) is created to implement TF-based UL transmission. The AP thus could schedule both DL and UL transmissions, which means DCF will be replaced by a scheduled MAC protocol. And instability of DCF will be mitigated very well and unfair queueing problem will not exist any more as AP does not need to contend with STAs. Moreover, a OFDMA-based multi-channel random access (MCRA) is proposed in 802.11ax as random access is highly efficient to transmit short frames, like bandwidth request. In this mechanism, AP initiates the MCRA by transmit a TF at first. STAs receiving the TF then contends with Aloha and binary exponential backoff mechanism. AP at last responds with ACK telling which STAs succeed in contending. A random access procedure is thus composed of three-way handshake. Details will be illustrated in Section II.

As for the OFDMA-based MCRA, we are concerned about number of stations succeeding in accessing the channel and access delay. Actually, MCRA has been supported by cellular networks to perform initial association to the network and to request transmission resources. This is the first time for 802.11 to apply OFDMA shifting SU PHY to MU PHY. However, cellular networks have adopted OFDMA for a while. Also OFDMA-based MCRA is solicited by cellular network for initial association and bandwidth request etc. The OFDMA-based MCRA in cellular networks is conceptually the same with that of WiFi networks. In recent years, related works have proposed some models to derive the throughput [4] [5] [6], the collision probability [7] [8], and the access delay [4] [7] [8] [9]. [4] gives a closed-form expression of throughput for OFDMA system. [5] proposes a stabilized multi-channel slotted Aloha algorithm. [6] designs a 1-persistent

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type retransmission, i.e., no exponential backoff, to achieve a fast access. Some other works compare performance of two kinds of backoff mechanism, binary exponential backoff and uniform backoff [4] [8] [7], which are implemented by IEEE 802.16 and 3GPP LTE respectively. [10] specifies a model estimating transient behavior of OFDMA system. In addition, [11] generalizes CSMA/CA to OFDMA system for 802.11, which is much different from [12].

In this paper, we estimate the steady state behavior of the OFDMA-based MCRA by extending Bianchi's Markov chain model. Bianchi's model has never been shifted to model the OFDMA-based MCRA. Though the MCRA has big difference from DCF, including SU to MU channel, distributed scheme to central control scheme and listening before talk to Aloha, Bianchi's Markov chain model will have been validated that it precisely model the MCRA by some modifications. Also, we evaluate system efficiency, access delay and effects of important system parameters.

The paper is organized as follows. More explanations of 802.11ax features are given in Section II, including MU UL/DL transmission procedure and OFDMA-based random access procedure. Section III contains the system model and performance analysis, system efficiency and access delay of the MCRA mechanism. Then Section IV shows simulation results compared with analysis results, which validates the model. Additional considerations on optimal performance are carried out in Section V. After that, Section VI evaluates the performance and effects of important system parameters. Conclusion remark is given in Section VII.

II. 802.11AX FEATURES

In this section, necessary features of 802.11ax will be illustrated to understand the OFDMA-based MCRA mechanism. As stated above, collisions in dense scenario degrade performance of WiFi networks as the bottleneck locates at MAC. Study group 802.11ax thus proposes TF-based UL procedure, which means AP schedules both DL/UL transmissions. In respect of DL/UL, 802.11ax shifted the distributed coordination legacy 802.11 to centralized coordination. on the other hand, OFDMA is issued in IEEE 802.11ax to realize MU PHY. Additionally, OFDMA-based multi-channel random access (MCRA) is proposed. For more features of 802.11ax, [13] is a good reference.

A. PHY

With MU PHY, the original SU 20 MHz channel could be specified more fine-grained and also aggregated to a wider channel. The resource unit (RU), which can be accessed by one STA, is specified as Figure 1. The smallest RU is 26-tone, with which a 20 MHz could be separated into 9 subchannels. Also, coarse-grained RU is accepted. And as specified in Figure 1, multiple 20 MHz channels can be aggregated, which is called *Channel Bonding*. This will help improve system performance, since one BSS is not restricted to a single 20 MHz channel. It is worth mentioning that every transmission of MU should end at the same time. That means padding is required for shorter packets.

RU type	CBW20	CBW40	CBW80	CBW80+80 and CBW160
26-tone RU	9	18	37	74
52-tone RU	4	8	16	32
106-tone RU	2	4	8	16
242-tone RU	1-SU/MU-MIMO	2	4	8
484-tone RU	N/A	1-SU/MU-MIMO	2	4
996-tone RU	N/A	N/A	1-SU/MU-MIMO	2
2×996 tone RU	N/A	N/A	N/A	1-SU/MU-MIMO

Fig. 1: Maximum number of RUs for each channel width

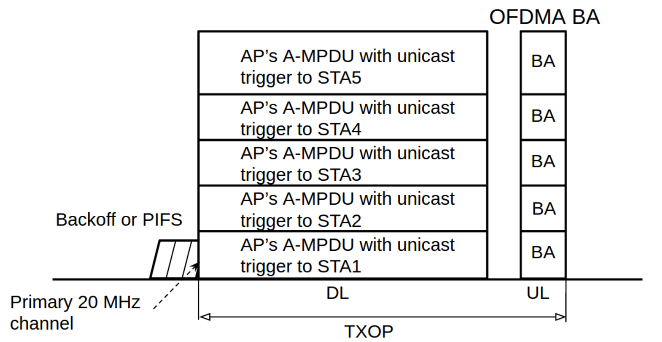


Fig. 2: MU DL of 802.11ax

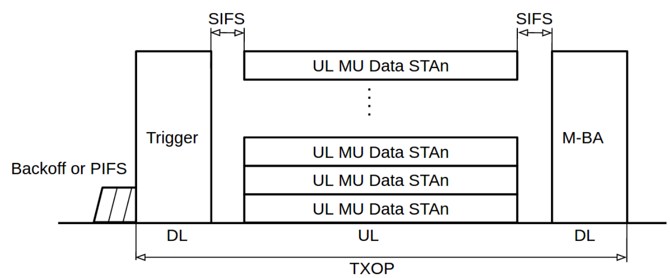


Fig. 3: Trigger-based MU UL of 802.11ax

Actually, MU PHY has been implemented in 802.11n and 802.11ac with MU-MIMO, which only realizes MU DL transmission. It is an absolutely different method from OFDMA and beyond the scope of this paper.

B. MAC

For MU DL transmission, AP will transmit DL packets to multiple stations simultaneously as in Figure 2. The difficulty of OFDMA MU is MU UL transmission. Since 802.11 is not a well synchronous system, preamble and even two-way handshaking are required before a data transmission. A trigger-based MU UL is issued as in Figure 3. A brand new control frame, trigger frame (TF), is created to be transmitted by AP at first. STAs could not transmit UL packets until they receive a TF which allocates RU for the STA or for random access. The trigger frame format is as in Figure 4. Since the standard is in progress, many fields remain to be determined (TBD). In the field *User Info*, *AID* represents the STA and *RU Allocation* represents the RU allocated to the STA. Especially, *AID* of value 0 means the RU for random access.

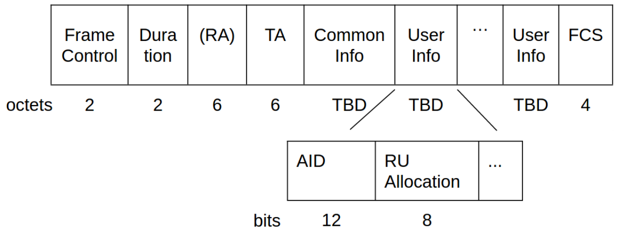


Fig. 4: Trigger Frame format

C. OFDMA-based MCRA

We first check a use case of OFDMA-based MCRA as in Figure 5. When a STA attempts to transmit data packets, the STA will send buffer state report (BSR) frame with OFDMA-based MCRA. After successful contention, AP will allocate RUs for the STA by another type of trigger frame. Then the STA transmits data packets on the allocated RUs.

Different from legacy 802.11, where all the parameters are predefined in each STA's hardware, system parameters of 802.11ax are configured dynamically by AP. The parameters includes OCW_{min}, OCW_{max}, M, where OCW_{min}, OCW_{max} represent the minimum and the maximum contention window. And M is the number of RUs for random access. OCW_{min}, OCW_{max} are given in an element called RAPS (random access parameter set) contained in beacon frame sent by AP. M is obtained from TF where some *AID* of RUs equals 0. OFDMA-based MCRA is thus more flexible compared with DCF.

Now let's check details by referring Figure 6. To initialize a random access procedure, AP first transmits a TF. The TF announces some RUs for random access by setting the AID of those RUs value of 0. Attempting STAs maintain a backoff counter, called OFDMA Backoff (OBO), which are randomly generated among range $[0, OCW]$. Then the OBO subtracts the value of M when STA receives a TF for random access. In other periods, the OBO will be frozen. When the OBO reaches 0, the STA will randomly select a RU from those for random access to transmit a packet after short inter-frame spacing (SIFS). After that, AP responds with a block ACK indicating which STAs succeed in contending. The whole three-way handshaking is called a *stage*. A successful stage means at least one STA succeed in transmitting a request in a stage. It is worth reminding that the stage in this paper is a concept of time interval, not the backoff stage in other papers. It is specified from standard [12]. To distinguish the two meanings, we use *backoff level* in this paper replacing backoff stage. When STAs fail to contend, the OCW will be doubled until OCW reaches OCW_{max}. It means the backoff level increases until the highest level.

We look deeply into the implementation of the mechanism by checking the RAPS element as in Figure 7. The two critical parameters OCW_{min}, OCW_{max} are specified in field *OCW Range*. The value is defined by $OCW_{min} = 2^{EOCW_{min}} - 1$, $OCW_{max} = 2^{EOCW_{max}} - 1$. In the following analysis, we issue another parameter m, *maximum backoff level*, so that $OCW_{max} = (OCW_{min} + 1) * 2^m - 1$. Then we later specify OCW_{max} with m and OCW_{min} in following section, which simplifies analysis.

III. System Model

Bianchi's Markov chain model could accurately depict the steady state behavior of DCF. It models all the details of DCF based on the assumption that at each request transmission, and regardless of the number of retransmission suffered, each request frame collides with constant and independent probability p. Although there are some great differences between OFDMA-based MCRA and DCF, the differences could be modeled by modifying the original Markov chain. Therefore, we modify this model to conduct the saturation analysis of 802.11ax OFDMA-based MCRA also with the assumption of constant and independent conditional collision probability.

The analysis is divided into two parts. First is the Markov chain model to estimate the packet transmission probability τ and conditional collision probability p. Secondly, we evaluate the metrics given τ, including number of stations who succeed in contending in a stage n_s, self-defined system efficiency *eff*, and expected access delay of a STA D. Table I is a list of all parameters and notations.

TABLE I: Parameters and Notations

n	# of stations
OCW_{min} (W_0)	minimum OFDMA contention window
OCW_{max} (W_m)	maximum OFDMA contention window
M	# of RUs for random access
m	maximum backoff level
p	packet collision probability
τ	station's transmission probability
n_s	# of successful stations in a stage
D	access delay, # of stages for a station to contend successfully

A. Packet Transmission Probability

Consider a fixed number n of stations under saturation condition, which means each station is an attempting station. For saturation analysis, stage of MCRA is one after another. Thus DL transmissions does not need to be considered here. Also, ideal channel is assumed so that collision happens only if more than one station transmit at the same RU.

DCF and OFDMA-based MCRA seem quite different from each other. One of the most important differences is the concept of time. In DCF, time is measured by slot. Here, we measure time by stage, the three-way handshake. However, they are both discrete and integer time scale. And stage does not directly reflect real time. The overhead of IFS and length

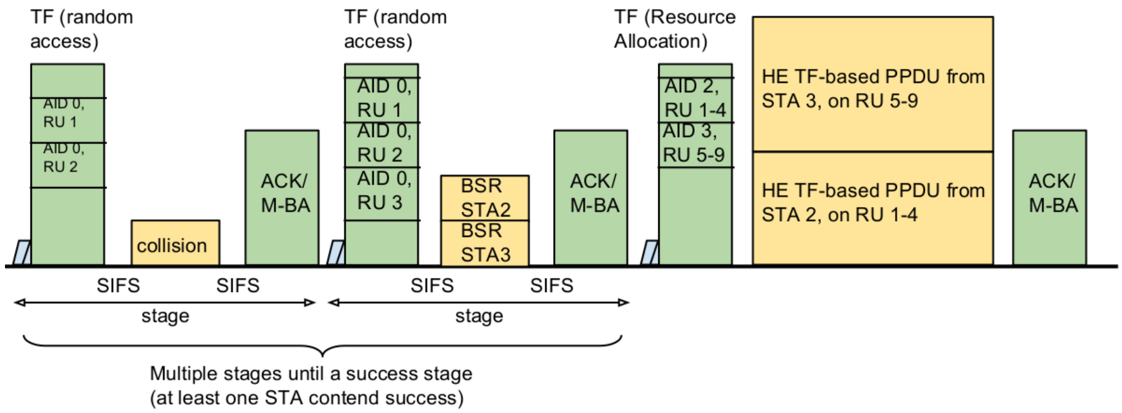


Fig. 5: An example of OFDMA-based MCRA for UL in 802.11ax

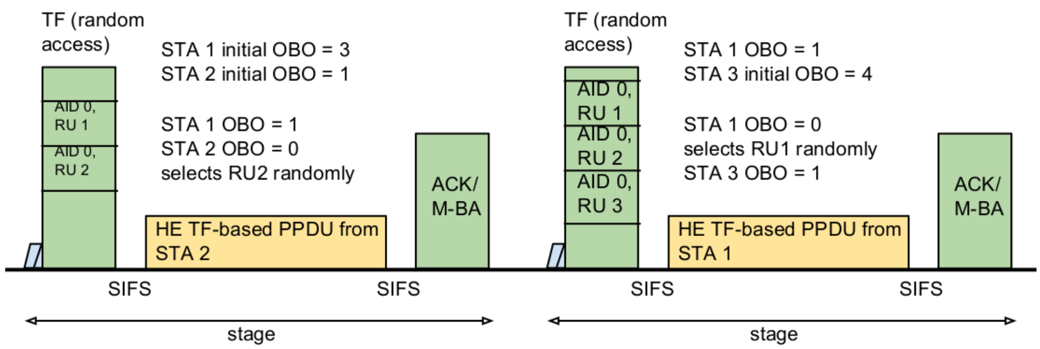


Fig. 6: Illustration of OFDMA-based MCRA

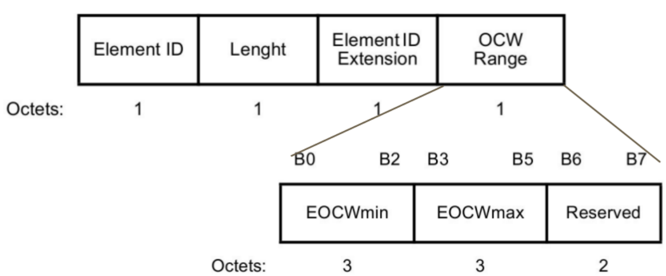


Fig. 7: Random Access Parameter Set (RAPS) Element

of the request or BSR packet is not our concern. We only care about number of stages.

Similar to Bianchi's work, let $\{s(t), b(t)\}$ be the bi-dimension process, where $s(t)$ denotes the backoff level $(0, \cdots, m)$, and $b(t)$ denotes backoff counter $(0, W_i)$. With the discrete and integer time scale, t and $t+1$ corresponds to beginnings of two consecutive stages. $\{b(t)\}$ is not Markov process, because state of the current stage does not only depend on the state of last stage. The bi-dimensional process $\{s(t), b(t)\}$ will be modeled with Markov chain. The key assumption is still necessary that at each request transmission, and regardless of the number of retransmission suffered, each request frame collides with constant and independent probability p. With the independence assumption, p will be a constant. The Markov chain is able to be conducted as in Figure 8.

To understand both Markov chain models of the two different mechanisms, OFDMA-based MCRA and DCF, one important concept is clarified here. Since stations of DCF senses the carrier before transmitting, stations may stay at a state for multiple slots. In OFDMA-based MCRA, a stage, a three-way handshake, contains exactly a period for a packet transmission. Stations will subtract M from the OBO only if they receive a TF for random access. Stations thus stay at one state for a period of exactly single stage.

Some modifications are mentioned here to adapt to differences between OFDMA-based MCRA and DCF. First, in a row of states, as OBO subtracts value of M rather than 1, stations transfer to states M-step ahead. Second, since states with $b(t) \leq M$ will decrease to 0 at current state, which means stations could access RUs, we could merge these states into one state, denoted by $\{i, T\}$. T is an integer set of $[0, M]$.

Let's assume $P\{i_1, k_1 | i_0, k_0\} = P\{s(t+1) = i_1, b(t+1) = k_1 | s(t) = i_0, b(t) = k_0\}$. In this Markov Chain, the only non

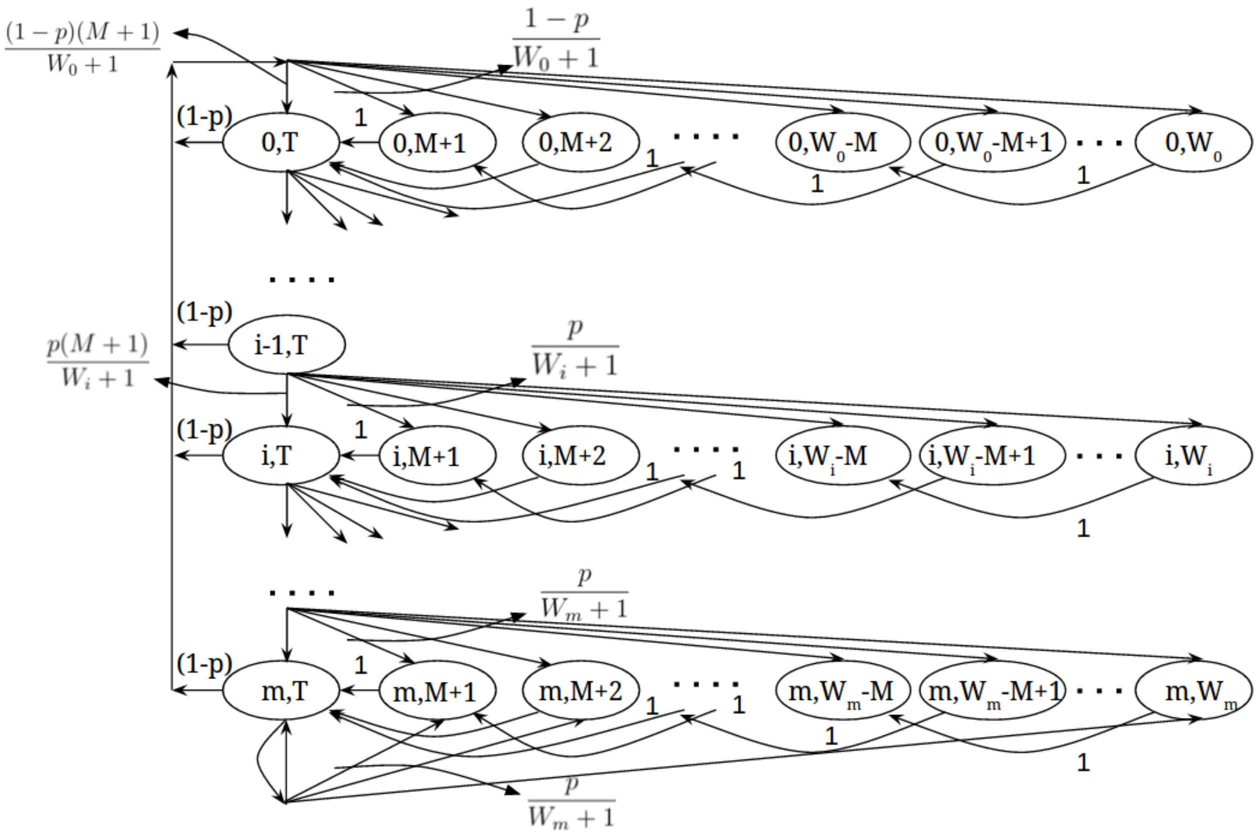


Fig. 8: Markov Chain model for the backoff window size

null one-step transition probabilities are

$$\begin{cases} P\{i,T|i,k\}=1 & k\in[M+1,2M] & i\in[0,m] \\ P\{i,k-M|i,k\}=1 & k\in[2M+1,W_i] & i\in[0,m] \\ P\{0,k|i,T\}=\frac{1-p}{W_0+1} & k\in[M+1,W_0] & i\in[0,m] \\ P\{0,T|i,T\}=\frac{(1-p)(M+1)}{W_0+1} & & i\in[0,m] \\ P\{i,k|i-1,T\}=\frac{p}{W_i+1} & k\in[M+1,W_i] & i\in[1,m] \\ P\{i,T|i-1,T\}=\frac{p(M+1)}{W_i+1} & & i\in[1,m] \\ P\{m,k|m,T\}=\frac{p}{W_m+1} & k\in[M+1,W_m] & \\ P\{m,k|m,T\}=\frac{p(M+1)}{W_m+1} & & \end{cases} \tag{1}$$

The first and second equations in (1) accounts for the fact that the backoff counter maintained by stations will subtract M, the number of RUs for random access. The third and fourth equations represent that after a successful contention, stations will reset the contention window size to initial window size and uniformly generate a backoff value among $[0,W_0]$, since T is an integer set $[0,M]$, the transition probability to states $\{i,T\}$ is $M+1$ times of that to states $\{i,k\}$. For the fifth and sixth equations, they represents when a failure contention occurs, the contention window size will be doubled, $W_i=2W_{i-1}+1$. The last two equations are situation of failure contention at the maximum backoff level. We assume no packets are discarded, repeating retransmitting until success.

Let $b_{i,k}=\lim_{t\to\infty}P\{s(t)=i,b(t)=k\},\ i\in[0,m],\ k\in[0,W_i]$, be the stationary distribution of the Markov chain. Then we show how to obtain transmit probability τ and conditional probability p. First, for states with $b(t)=T$, in which stations will transmit request or BSR in current stage,

$$\begin{aligned} b_{i-1,T}\cdot p=b_{i,T}\to b_{i,T}=p^i b_{0,T}, \quad 0\le i<m \\ b_{m-1,T}\cdot p=(1-p)b_{m,T}\to b_{m,T}=\frac{p^m}{1-p}b_{0,T}. \end{aligned} \tag{2}$$

Then, other states are expressed with states of $b(t)=T$,

$$b_{i,k}= \begin{cases} (\lfloor\frac{W_0-k}{M}\rfloor+1)\frac{(1-p)}{W_0+1}\sum_{i=0}^m b_{i,T}, & M+1\le k\le W_0,\ i=0 \\ (\lfloor\frac{W_i-k}{M}\rfloor+1)\frac{p}{W_i+1}b_{i-1,T}, & M+1\le k\le W_i,\ 0<i<m \\ (\lfloor\frac{W_m-k}{M}\rfloor+1)\frac{p}{W_m+1}(b_{m-1,T}+b_{m,T}), & M+1\le k\le W_m, \quad i=m \end{cases} \tag{3}$$

From (2), we have $\sum_{i=0}^m b_{i,T}=\frac{b_{0,T}}{1-p}$; with which, sum (3) respectively; we obtain (4).

Each subequation in (4) has the same term: $-\frac{M}{2}\left\lfloor\frac{W_i}{M}\right\rfloor^2+\left(W_i-\frac{M}{2}\right)\left\lfloor\frac{W_i}{M}\right\rfloor$. To simplify the expression, let $X_i=-\frac{M}{2}\left\lfloor\frac{W_i}{M}\right\rfloor^2+\left(W_i-\frac{M}{2}\right)\left\lfloor\frac{W_i}{M}\right\rfloor$. Here, we sum steady state probability of all states to get (6).

Now we can express τ, the probability of a station transmit a request at a randomly selected stage, as a function of p.

$$\begin{cases} \sum_{k=M+1}^{W_0} b_{0,k} = \frac{b_{0,T}}{W_0+1} \left(-\frac{M}{2} \left\lfloor \frac{W_0}{M} \right\rfloor^2 + \left(W_0 - \frac{M}{2} \right) \left\lfloor \frac{W_0}{M} \right\rfloor \right) \\ \sum_{i=1}^{m-1} \sum_{k=M+1}^{W_i} b_{i,k} = \frac{b_{0,T}}{W_0+1} \left(\frac{p}{2} \right)^i \left(-\frac{M}{2} \left\lfloor \frac{W_i}{M} \right\rfloor^2 + \left(W_i - \frac{M}{2} \right) \left\lfloor \frac{W_i}{M} \right\rfloor \right) \\ \sum_{k=M+1}^{W_m} b_{m,k} = \frac{b_{0,T}}{W_0+1} \frac{(\frac{p}{2})^m}{1-p} \left(-\frac{M}{2} \left\lfloor \frac{W_m}{M} \right\rfloor^2 + \left(W_m - \frac{M}{2} \right) \left\lfloor \frac{W_m}{M} \right\rfloor \right) \end{cases} \tag{4}$$

$$1 = \sum_{i=0}^{m} \sum_{k=0}^{W_i} b_{i,k} = \frac{b_{0,T}}{W_0+1} \left(X_0 + \sum_{i=1}^{m-1} X_i \left(\frac{p}{2} \right)^i + X_m \frac{\left(\frac{p}{2}\right)^m}{1-p} \right) + \frac{b_{0,T}}{1-p} \tag{5}$$

$$= b_{0,T} \left(\frac{(1-p)X_0 + (1-p)\sum_{i=1}^{m-1} X_i \left(\frac{p}{2}\right)^i + X_m \left(\frac{p}{2}\right)^m + W_0 + 1}{(W_0+1)(1-p)} \right) \tag{6}$$

$$\tau = \sum_{i=0}^{m} b_{i,T} = \frac{b_{0,T}}{1-p} = \frac{W_0+1}{W_0 + 1 + (1-p)X_0 + (1-p)\sum_{i=1}^{m-1} X_i \left(\frac{p}{2}\right)^i + X_m \left(\frac{p}{2}\right)^m} \tag{7}$$

For $m = 0, M = 1$ which is SU PHY with a fixed contention window, check (5), the terms containing $X_i, i > 0$ will disappear, and $b_{0,T}/(1-p)$ degrades to $b_{0,T}$. As a result, (6) degrades to

$$1 = b_{0,T} \left(\frac{W_0 + 1 + X_0}{W_0 + 1} \right), \tag{8}$$

thereby,

$$\tau = b_{0,T} = \frac{W_0+1}{W_0+1+X_0} = \frac{2(W_0+1)}{W_0^2 + W_0 + 2}. \tag{9}$$

It is a little surprising at first that it is different from [14], where $\tau = \frac{2}{W_0+1}$. After thinking for a while, it is reasonable, because the simplified OFDMA case $M = 1$ does not require carrier sensing before transmission. It is certainly different from [14] which adopted CSMA/CA.

On the other hand, conditional collision probability p has another relation with transmit probability τ. With ideal channel assumption, collision happens only if at least one of other stations select the same RU. Thus we have

$$p = 1 - \left(1 - \frac{\tau}{M}\right)^{n-1}. \tag{10}$$

Rewrite (10) as $\tau^\star = \left(1 - (1-p)^{\frac{1}{n-1}}\right) M$. To obtain transmission probability τ and conditional collision probability p, we need to find solutions to group of equations 7 and 10. $\tau^\star(p)$ is a monotonically increasing function. Though $\tau(p)$ is hard to determine the monotonicity from the expression of equation 7 with respect to p. We justify the monotonic decrease of function 7 with numerical method. Also, $\tau(0) = \frac{W_0+1}{W_0+1+X_0} > \tau^\star(0) = 0$. And $\tau(1) < \tau^\star(1) = M$. We then find the only solution with numerical method.

B. Random Access Efficiency

With the transmit probability, we could easily estimate efficiency of OFDMA-based MCRA mechanism. First, find expected number of stations who contend successfully to transmit request at a stage, which is denoted with $E[n_s]$. Extending n_s, we define a system efficiency as an important metric. Secondly, we are interested in the access delay of request frame. In another word, say how many stages are required for a station to contend successfully, denoted by D.

1) n_s and System Efficiency: What we are concerned about in the MCRA is that how many stations contend successfully during a stage, denoted by n_s. Given τ and p, we could obtain probability that a station contend successfully in a stage, $P_{s\_station} = \tau(1-p)$. Then, with (10), $E[n_s]$ is easily expressed as follows.

$$E[n_s] = nP_{s\_station} = n\tau(1-p) = n\tau(1 - \frac{\tau}{M})^{n-1} \tag{11}$$

Furthermore, normalizing n_s, *system efficiency* here is defined as

$$\textit{eff}\ (\tau) = \frac{E[\text{number of successful stations in a given stage}]}{\text{number of RUs for random access in a stgae}} = \frac{E[n_s]}{M} = \frac{n\tau(1-\frac{\tau}{M})^{n-1}}{M}. \tag{12}$$

Both two metrics are our concerns. Another metric, access delay, is derived in next subsection. With all these metrics, we could evaluate the performance later.

2) Access Delay: D, a random variable represents how many stages are required for a station to contend successfully in a stage. Because of saturation assumption, new request arrives once one previous request is transmitted successfully. Thus queueing waiting time is not considered here. In this way, access delay D follows geometric distribution with parameter $P_{s\_station}$, which is obtained just now. Then the expected value of access delay of request frame, $E[D]$, is

$$E[D] = \frac{1}{\tau(1-\frac{\tau}{M})^{n-1}}. \tag{13}$$

In a word, we focus on three metrics: number of successful stations in a stage n_s, system efficiency $E[n_s]/M$ and access delay given by D.

IV. Model Validation

Since we are only concerned about the OFDMA-based MCRA, to simplify the simulation, we assume STAs only solicit the OFDMA-based MCRA to transmit requests as in Figure 6 rather than in Figure 5. The system parameters are set according to Figure 1. We run simulations for 100,000 stages with variety of parameter sets $\{M, OCW_{min}, OCW_{max}\}$ and collects the information of the two variables, number of successful attempt STAs n_s and expected access delay D. The values of results from both analysis and simulation are given in figure 9 and table II. The results show that the Markov model precisely predict the steady state behavior of the OFDMA-based MCRA mechanism.

TABLE II: Analysis versus simulation: n_s and access delay with $m = 3, M = 9, OCW_{min} = 15$

n_s	analysis	simulation
$n = 1$	0.72727	0.72728
$n = 5$	2.23001	2.22335
$n = 10$	2.88954	2.88546
$n = 20$	3.29798	3.29857
delay	analysis	simulation
$n = 1$	1.37500	1.37499
$n = 5$	2.24214	2.24886
$n = 10$	3.46075	3.46565
$n = 20$	6.06432	6.06323

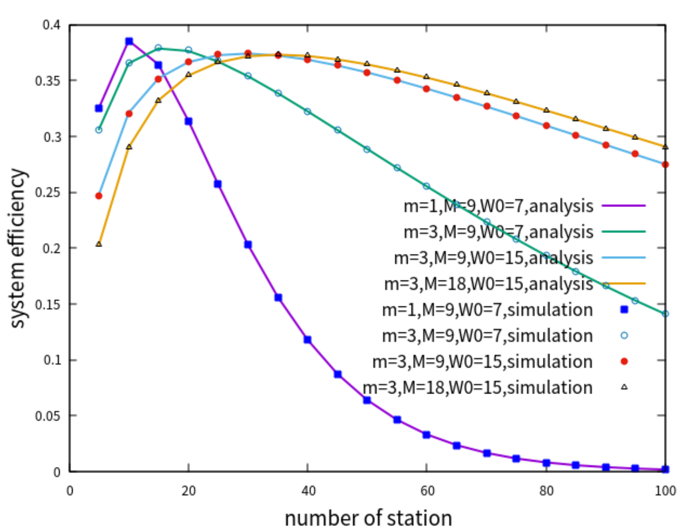


Fig. 9: System efficiency: Analysis versus Simulation

V. Maximum System Efficiency and Minimum Access Delay

With the system efficiency given in (12), we take the derivative with respect to τ, and find the extreme point, $\tau^\star = M/n$. Since $\tau \in [0, 1]$, $\tau^\star = min\{1, M/n\}$. In dense scenario, i.e., the number of contending stations n is large, then $\tau^\star = M/n$. The system efficiency thus is

$$eff\ (\tau^\star) = (1 - \frac{1}{n})^{n-1} \quad (14)$$

Then the maximum n_s is

$$E[n_s]^\star = M \cdot eff\ (\tau^\star) = M(1 - \frac{1}{n})^{n-1}. \quad (15)$$

The limit of system efficiency, based on infinite n, is

$$\lim_{n\to\infty} eff\ (\tau^\star) = \lim_{n\to\infty} (1 - \frac{1}{n})^{n-1} = \frac{1}{e} \quad (16)$$

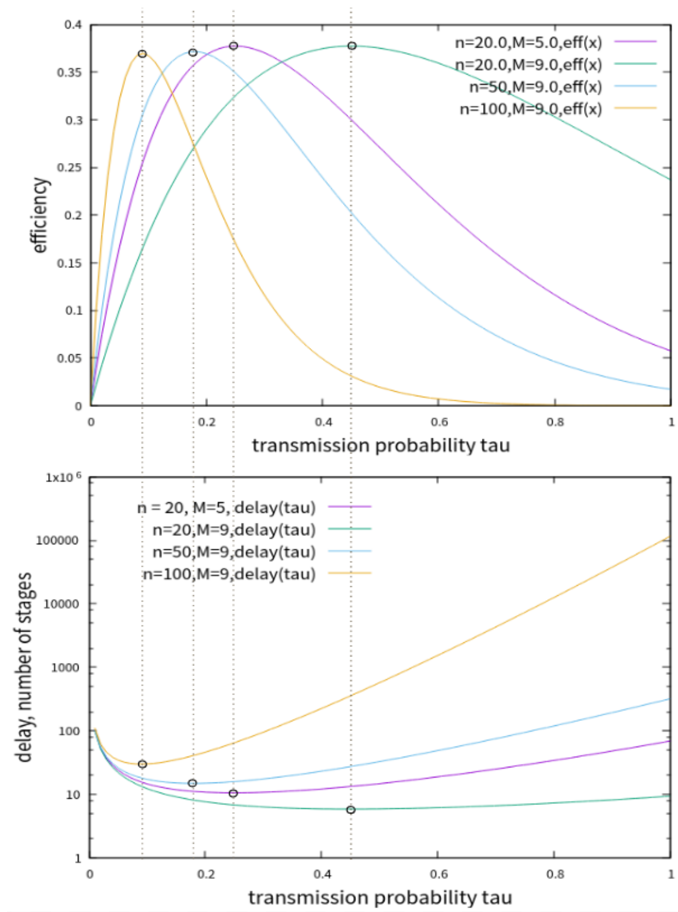


Fig. 10: Efficiency and access delay versus transmission probability τ

With the delay analysis given in (13), we also take the derivative with respect to τ, and find the extreme point, $\tau^\star = M/n$. Again, $\tau^\star = min\{1, M/n\}$. When $n \geq M$, the minimum access delay is

$$D(\tau^\star) = \frac{n}{M(1 - \frac{1}{n})^{n-1}}. \quad (17)$$

From above analysis, we find that the maximum system efficiency and minimum access delay are both obtained by the $\tau^\star = min\{1, M/n\}$. What's more, optimal system efficiency is independent with M, while M affects access delay. The larger M is, the shorter the access delay will be. It indicates that when AP allocates RUs for random access, the AP could

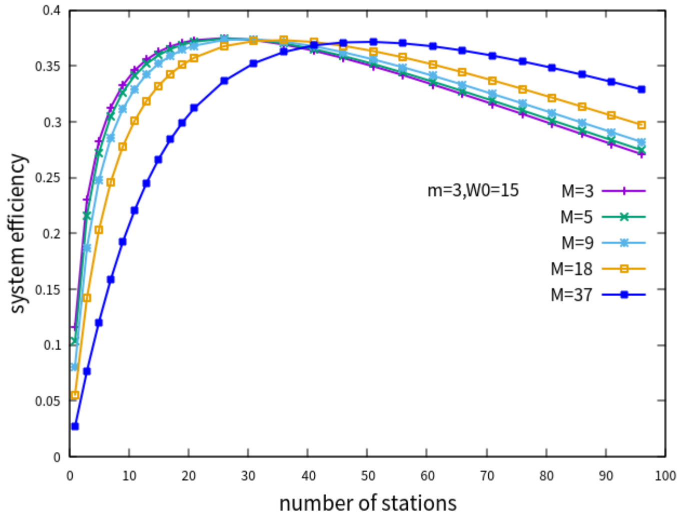


Fig. 11: System efficiency versus number of stations

allocates as many as possible, only if the channel of the RU is sensed idle by AP.

Figure 10 is plotted corresponding to (12) and (13). Consistent to the analysis above, the figure shows that the maximum system efficiency is independent of number of RUs for random access when $n \geq M$, and approaching to $1/e$ with n increasing. What's more, the optimal transmission probability τ of system efficiency and access delay is consistent with each other, which also validates the analysis.

Obtaining τ and p by solving group equations (7) and (10), it's hard to give a closed-form of τ depending on system parameters, M, W_0, m or W_m and n. We could only tune the system parameter set $\{M, W_0, W_m\}$ based on a strong assumption that n is known.

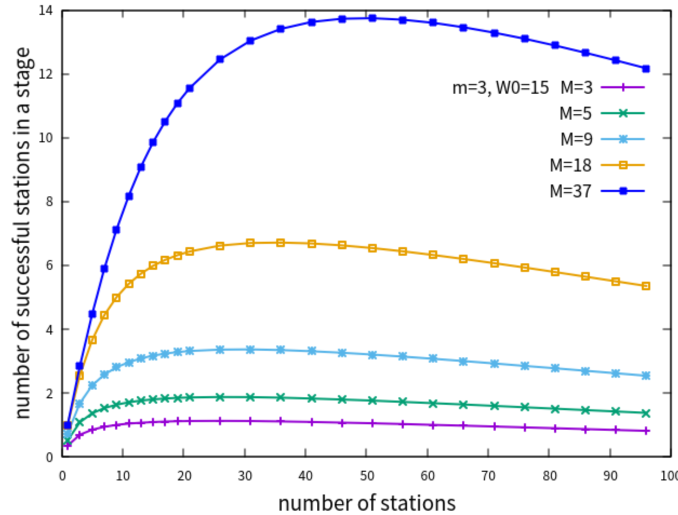
VI. Performance Evaluation

With above analysis, we could conveniently evaluate system efficiency and access delay by evaluating τ. Now that it is hard to directly find the relation between τ and system parameters, we could tune system parameters with a variety of system parameter sets to get some insight of the relation.

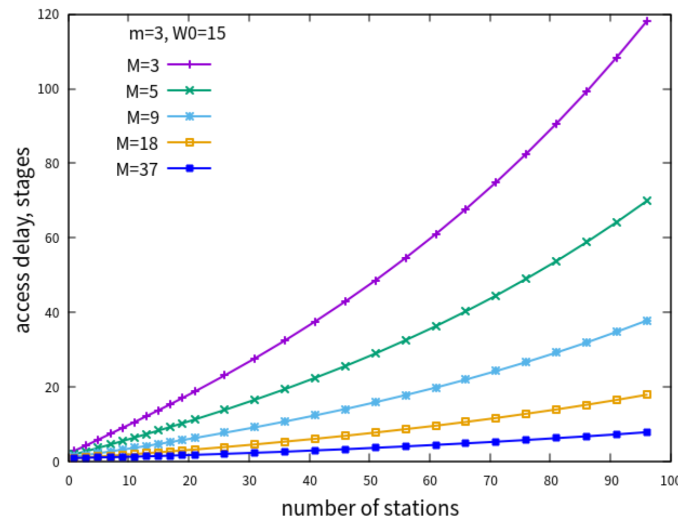
A. RUs for Random Access M

(14) indicates that M, has nothing to do with optimal system efficiency. However, n_s and D are proportional and inversely proportional to M respectively according to (15) and (17). Following analysis results validate the statement.

In Figure 11, the maximum system efficiency is almost the same, approaching $1/e$. The only difference is "when" the optimal point will be. However, more practically, how many stations contend successfully in a stage? Figure 12a shows that larger M results more stations contend successfully in a stage. Moreover, Figure 12b shows that larger M markedly decrease the access delay. Above all, when AP allocates RUs for random access, AP will allocate as many RUs as possible.



(a) Number of successful stations in a single stage versus number of stations



(b) Access delay versus number of stations

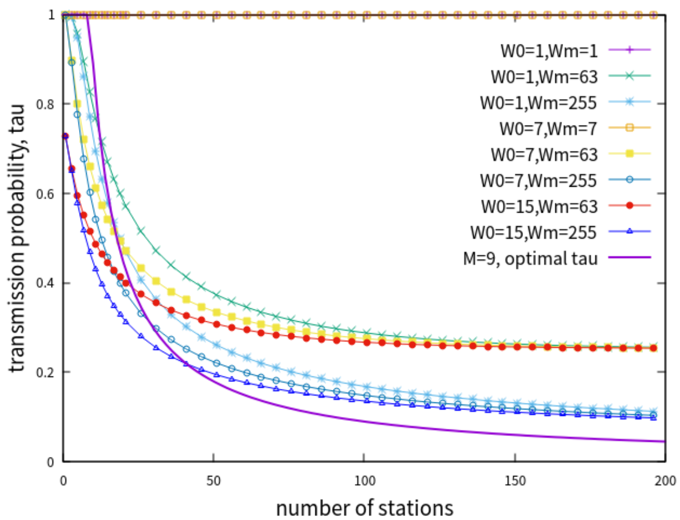
Fig. 12: Configure M

B. OCW_{min}, OCW_{max}

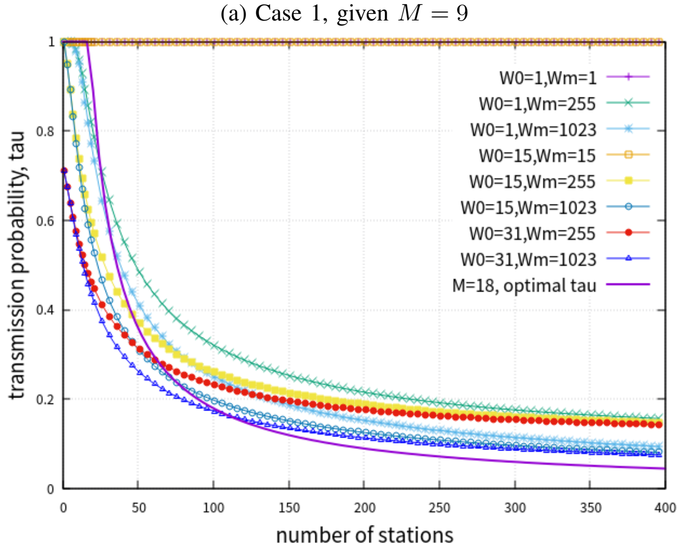
Figure 13 shows case 1 ($M = 9$) and case 2 ($M = 18$), so that the rules we get are more convinced. In the figure, the purple line without point depicts the optimal τ, which is given according to $\tau^{\star} = min\{1, M/n\}$. With above analysis and effects of M, we tune remaining parameters OCW_{min}, OCW_{max} so that τ approaches the optimal line. To see the trend of line, we give larger n. Then some rules could obtained as follows.

First, the OCW_{min} or W_0, determines τ of loose scenario. The larger W_0 is, the lower τ is at $n = 1$. That's why cases in Figure 13 have two different start points.

Secondly, $m = 0$ results in constant τ, which is consistent with (9) that τ does not depend on n. A special case is $W_0 < M$ for scenario $n \leq M$, it results in constant transmission probability equals to 1 regardless of n. It perfectly matches $\tau^{\star}$ at $n \leq M$ as $\tau^{\star} = 1$ for the scenario $n \leq M$. Thus, if given $n \leq M$, the optimal configuration will be $OCW_{max} =$



(a) Case 1, given $M = 9$



(b) Case 2, given $M = 18$

Fig. 13: Transmission probability versus number of stations

$OCW_{min} < M$.

Thirdly, W_m affects more on dense scenario. It determines the limit of the τ, i.e., where the line of τ will converge. Lines with the same W_m will converge to a same value. And both the two figures in Figure 13 correspond with the above statement. And larger W_m causes lower τ, which is closer to $\tau^\star$ when n is large.

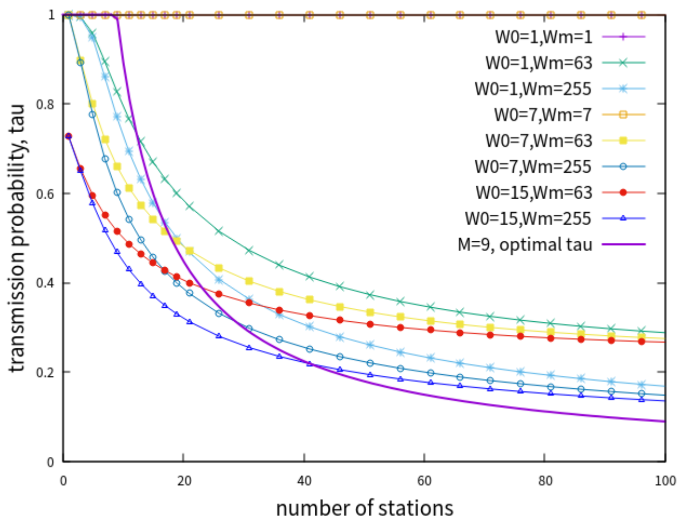
Then, we draw two subfigures with x axis range $[0, 100]$ to observe more clearly as in Figure 14. From the two subfigures 14a and 14b we find another rule that when $W_0 = 1$, line of τ will have a flat start. It happens that the flat start matches better with $\tau^\star$ when $n \leq M$.

Based on all above observations, we conclude that W_0 has significant influence on scenario of $n \leq M$, while W_m counts when n is large or $n > M$. In the next two subsections, we practically check the system efficiency and access delay under different parameter configurations of $\{W_0, W_m\}$, with which we could find explicit relation between the parameter and performance.

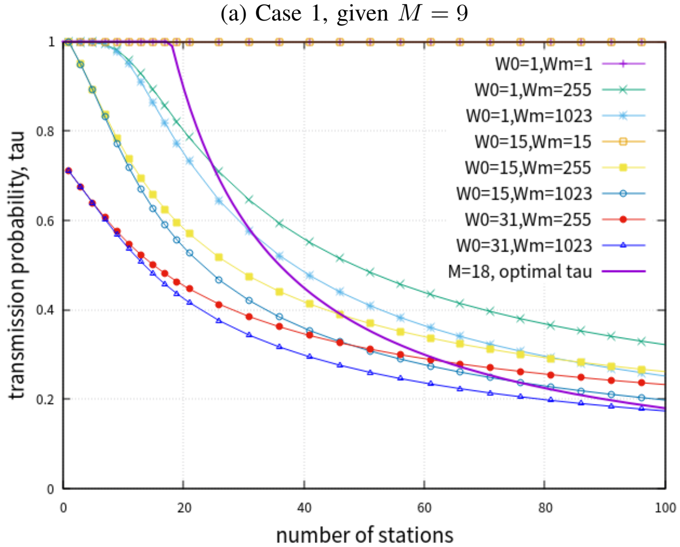
1) Configure OCW_{max}: With above rough rules, we estimate the effects of OCW_{max} first by setting different OCW_{max} while given $OCW_{min} = 1$ and $M = 9$. Actually the data we use to generate the figures are the same with that for Figure 13a. In Figure 15a, we select three of them to clearly display the effect of OCW_{max} on system efficiency. From the figure, it is obvious that larger OCW_{max} is much better for system efficiency when $n \geq M$. The result corresponds to the above rules we obtain from effect of parameters on τ. Additionally, OCW_{max} has the same effect on access delay . And since we use the same data with that in figure 13a, we find that the lines which converge in figure 13a also have the same tendency in figure 15b.

As stated in previous section that OCW_{max} has significant influence on scenario of $n \geq M$. With increasing n, larger OCW_{max} will obtain larger gain.

2) Configure OCW_{min}: To estimate the effect of OCW_{min}, we compare the performance of different configurations of OCW_{min} while given $OCW_{max} = 1023$, which has



(a) Case 1, given $M = 9$



(b) Case 2, given $M = 18$

Fig. 14: Details of transmission probability versus number of stations when $n \leq 100$

been validated that larger OCW_{max} is better, and $M = 18$. First, as we claim in Section VI-B that $W_0 = W_m \leq M$ is the perfect configuration in scenario of $n \leq M$, it is validated here that maximum system efficiency and minimum access delay are achieved with the configuration in scenario of $n \leq M$. However, configuration of small OCW_{min} and large OCW_{max} almost achieves as good performance as the perfect configuration. Secondly, OCW_{min} determines the start of line, and it has significant influence on scenario of $n \leq M$. From Figure 16a and 16b, we find larger $OCW_{min} = 127$ has lower system efficiency and larger access delay.

Now, we summarize above rules as follows.

1 For M: the larger the better
2 For OCW_{min}, OCW_{max}:
   - OCW_{min} counts when $n \leq M$. Smaller OCW_{min} is better.
   - OCW_{max} counts when $n > M$. Larger OCW_{max} is better.

- Special case: for $n \leq M$
  $OCW_{max} = OCW_{min} < M$

VII. Conclusion

For the new appeared OFDMA-based MCRA mechanism of IEEE 802.11ax networks, we extend classic model to the MCRA and conduct a saturation analysis, as well as evaluate its performance. After the precisely depiction of the steady state behavior of OFDMA-based MCRA with the refined Markov model, we derive the close-form mathematics expressions of system efficiency and access delay time.

We find the performance relies much on certain system parameters: the number of RUs or subchannels can grow as large as possible, the initial contention window length matters only where few stations exist, and the maximum contention window length is important in dense scenarios.

Different from the restricted DCF mechanisms of legacy 802.11, the OFDMA-based MCRA mechanism tends to be more flexible with configurable system parameters. Our work take the first step forward to grasp some insight from the steady state behavior of the MCRA mechanism, and lay foundation for transient analysis and its corresponding real-time algorithm that configures the system parameters dynamically.

REFERENCES

[1] G. Bianchi, "Performance analysis of the ieee 802.11 distributed coordination function," *IEEE Journal on selected areas in communications*, vol. 18, no. 3, pp. 535–547, 2000.

[2] "Cisco visual networking index:forecast and methodology, 20152020," http://www.cisco.com/c/en/us/solutions/collateral/service-provider/visual-networking-index-vni/complete-white-paper-c11-481360.pdf, [Online; accessed on 13-June-2016].

[3] E. Perahia and R. Stacey, *Next Generation Wireless LANS: 802.11 n and 802.11 ac*. Cambridge university press, 2013.

[4] P. Zhou, H. Hu, H. Wang, and H.-h. Chen, "An efficient random access scheme for ofdma systems with implicit message transmission," *IEEE transactions on wireless communications*, vol. 7, no. 7, pp. 2790–2797, 2008.

[5] D. Shen and V. O. Li, "Performance analysis for a stabilized multi-channel slotted aloha algorithm," in *Personal, Indoor and Mobile Radio Communications, 2003. PIMRC 2003. 14th IEEE Proceedings on*, vol. 1. IEEE, 2003, pp. 249–253.

[6] Y.-J. Choi, S. Park, and S. Bahk, "Multichannel random access in ofdma wireless networks," *IEEE Journal on Selected Areas in Communications*, vol. 24, no. 3, pp. 603–613, 2006.

[7] S. Kim, J. Cha, S. Jung, C. Yoon, and K. Lim, "Performance evaluation of random access for m2m communication on ieee 802.16 network," in *Advanced Communication Technology (ICACT), 2012 14th International Conference on*. IEEE, 2012, pp. 278–283.

[8] J.-B. Seo and V. C. Leung, "Design and analysis of backoff algorithms for random access channels in umts-lte and ieee 802.16 systems," *IEEE Transactions on Vehicular Technology*, vol. 60, no. 8, pp. 3975–3989, 2011.

[9] A. B. Behroozi-Toosi and R. R. Rao, "Delay upper bounds for a finite user random-access system with bursty arrivals," *IEEE transactions on communications*, vol. 40, no. 3, pp. 591–596, 1992.

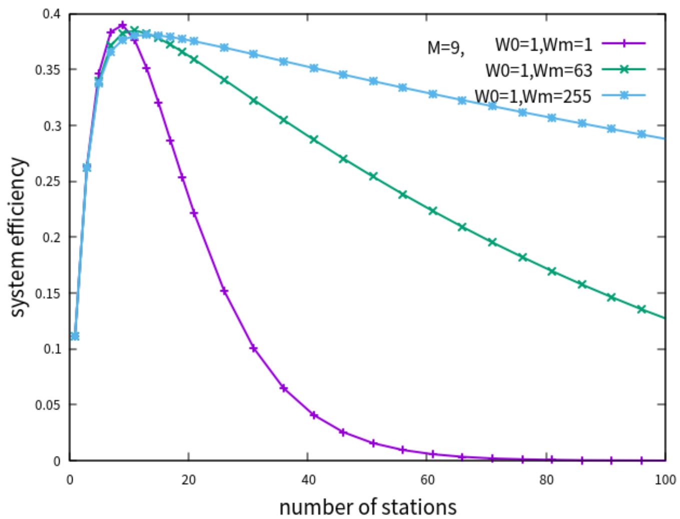
[10] C.-H. Wei, G. Bianchi, and R.-G. Cheng, "Modeling and analysis of random access channels with bursty arrivals in ofdma wireless networks," *IEEE Transactions on Wireless Communications*, vol. 14, no. 4, pp. 1940–1953, 2015.

[11] H. Kwon, H. Seo, S. Kim, and B. G. Lee, "Generalized csma/ca for ofdma systems: protocol design, throughput analysis, and implementation issues," *IEEE Transactions on Wireless Communications*, vol. 8, no. 8, pp. 4176–4187, August 2009.

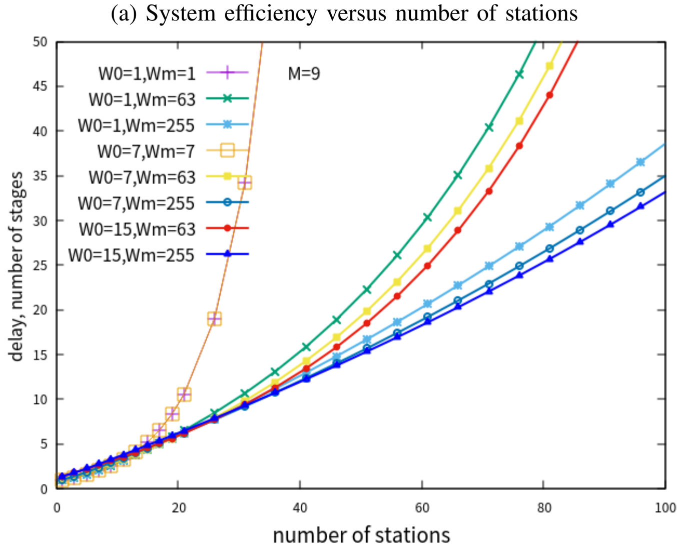
[12] *Draft 1.0, Part 11: Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications, Amendment 6: Enhancements for High Efficiency WLAN, 802.11ax*, pp. 1–376, Sep 2016.

[13] D.-J. Deng, S.-Y. Lien, J. Lee, and K.-C. Chen, "On quality-of-service provisioning in ieee 802.11 ax wlans," *IEEE Access*.

[14] T.-S. Ho and K.-C. Chen, "Performance analysis of ieee 802.11 csma/ca medium access control protocol," in *proc. PIMRC*, vol. 96, 1996, pp. 407–411.

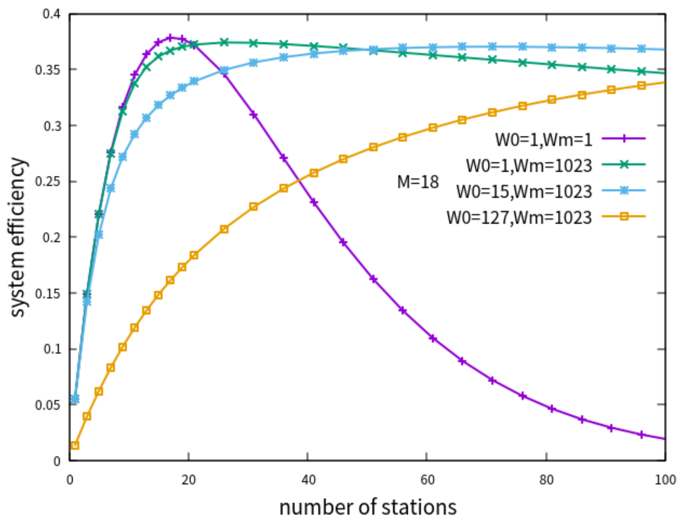


(a) System efficiency versus number of stations

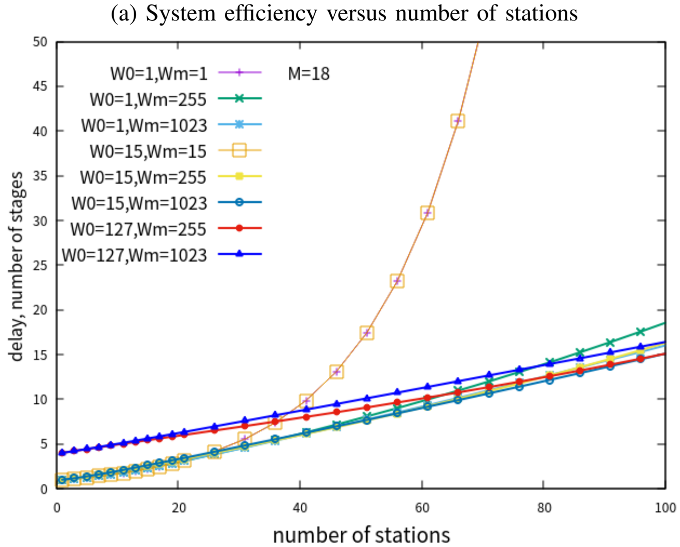


(b) Access delay versus number of stations

Fig. 15: Example of Configuring OCW_{max}, given $M = 9$



(a) System efficiency versus number of stations



(b) Access delay versus number of stations

Fig. 16: Example of Configuring OCW_{min}, given $M = 18$