Multichannel ALOHA Data Networks for Personal Communications Services (PCS)

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Abstract

In this paper, we study two wireless data network access protocols, *multichannel ALOHA with and without reservation*, for personal communications services. A Markov model is used to analyze the system throughput and average message delay. The combined effect of Rayleigh fading, log-normal shadowing, and spatial distribution of the mobile users to the system throughput is also included.

Numerical results show that the throughput of such a slotted multichannel ALOHA system with capture is found to be much higher than that without capture, as expected. The results also indicate that multichannel ALOHA protocol with reservation has much higher throughput and less delay than the protocol without reservation, if the set-up time is negligible. The analysis presented here can be easily used to analyze the Reservation Random Access (RRA) schemes for voice transmission.

1 Introduction

Current trends in the US and abroad show a steadily rising demand for mobile/personal communications services through wireless data networks. Mobile users within the service areas can access resources through a centrally located base station by using radio links. Due to limited radio spectrum, however, only a finite number of radio communication channels can be shared by the mobile users. Access control protocols must be implemented to efficiently utilize the resource among all the users.

Slotted ALOHA is one of the simplest random access schemes which have been successfully implemented in practical packet radio systems. In an ALOHA system, each mobile user with a packet ready to transmit simply transmits the packet at will regardless of whether or not other users are transmitting their packets. An unsuccessful transmission will be detected by the user, possibly at a later time, if no acknowledgment has been received. The user then assumes that the packet is lost and retransmits the packet after a random amount of time. In a slotted ALOHA system, all packets have the same length, and each packet requires one time unit (called a slot) for transmission. All transmitters are synchronized so that all packets are transmitted at the start of a slot. The advantage of this scheme in data networks is that its implementation does not require any sophisticated control mechanism.

Much research has already been devoted to the slotted ALOHA protocol. However, most of the previous results are limited to single-channel slotted ALOHA networks. In a recent publication [1], a multichannel slotted ALOHA system was analyzed; however, only one packet per message was allowed. Data traffic generally requires a short channel transmission time, one or two packets per connection, except for very long files. If the data file is short, slot reservation is generally not necessary. However, if the data file is long, slot reservation should be more efficient.

In this paper, we extend the results in [1] to both data and voice applications. We consider two wireless ALOHA access protocols, *multichannel ALOHA with and without reservation*, taking into consideration the effects of near-far effects, log-normal shadowing, and Rayleigh fading, typically encountered in a mobile radio communication environment. A Markov model is used to obtain the system throughput and average message delay. The effect of the retransmission probability on system throughput is investigated and it is found that the retransmission probability has a significant impact on the system throughput. Application of the analysis presented in this paper to voice transmission is discussed. In voice transmission, each user is modeled to generate packets according to a two state on/off Markov chain, alternating between "talkspurt" and "silence" periods. Packets that cannot be transmitted during a slot are assumed to be lost. An analytical expression of the packet dropping probability is easily derived.

This paper is organized as follows. Section 2 describes the system model. Section 3 analyzes the system throughput and message delay. Section 4 presents numerical results. Section 5 discusses application of the analysis, presented in Section 3, to voice transmission systems. Finally, Section 6 concludes the paper.

2 System Description

2.1 System Model

Let's consider a multichannel packet radio slotted ALOHA data network for personal communications services over land mobile radio channels, shown in Fig. 1. For simplicity, we focus only on single-cell systems. N mobile data terminals share M parallel, equal-capacity channels for transmission to one base station. The M channels can be implemented based on either Frequency Division Multiplexing (FDM) or Time Division Multiplexing (TDM) approaches.

If we assume that the available transmission bandwidth is B (in hertz), in an FDM approach, the available bandwidth is partitioned into M subchannels, each with B/M and a transmission rate equal to $\alpha B/M$, where α is a constant depending on the modulation schemes being used. The transmission time of each channel is divided into time slots. Each slot duration is equal to the transmission time of one packet. The packet transmission can only start at the beginning of the time slot. A special name of this scheme is called Slotted Frequency-Division Multiplexing (SFDM). Fig. 2 shows the SFDM scheme with M subchannels.

In a TDM approach, a single high speed communication channel, with information transmission rate αB, is implemented. The transmission time is partitioned into frames, and each frame is further divided into M time slots. The duration of each time slot is also equal to the transmission duration of one packet. The M slots within the frame are competed for by the users. Each user can only compete for one slot within one frame. Therefore, it is equivalent to saying that there are M subchannels available for the mobile users to compete and each subchannel with data rate equal to $\alpha B/M$. Fig. 3 shows the TDM schemes with M time slot/frame (M subchannels).

The SFDM system has the same throughput rate as the TDM system. However, the delay is more favorable in the TDM than the SFDM system. It has been shown in [2] that for M independent Poisson arrival streams,

$$T_{\text{TDM}} = T_{\text{SFDM}} - M + 1, \tag{1}$$

where T_{TDM} is the total delay time for the TDM system and T_{SFDM} is the total delay time for the SFDM system. Throughout the paper, we assume that M parallel channels are using M different frequency bands (FDM) for easier explanation of the derivation.

Each mobile terminal has one buffer, which can store at most one *message*. We assume that the messages are segmented into fixed-length packets. The transmission time of a packet is equal to one slot. The number of packets in a message is geometrically distributed with parameter q_1. When the buffer is empty, a new message is generated by the user during a slot with probability λ. We consider the following two access protocols:

1. Multichannel slotted ALOHA without reservation.
2. Multichannel slotted ALOHA with reservation.

2.2 Multichannel Slotted ALOHA without Reservation

Under this scheme, messages are transmitted on a packet by packet basis. In other words, users compete for the channel on a slot-by-slot basis. At the beginning of each time slot, the user will first "tune" its transmitter to one of the available frequency bands and then transmit one packet. After the user successfully transmits one packet in a slot, it will transmit another packet in the message if there are any in the following slot, competing with other users. This model is similar to the one studied in [1], with a different packet arrival process.

In [1], if a user transmits its packet successfully in one slot, it will generate a *new* packet in the following slot with probability λ; while in the model studied in this paper, if a user transmits one packet successfully and if the packet is not the last one in the message, the user will generate another packet with probability one. The number of packets in a message is assumed to be geometrically distributed with parameter q_1. If a user fails to transmit its packet successfully, it enters a backlogged state and retransmits its packet at a later time, geometrically distributed with parameter p_r. If a user transmits its packet successfully and if the packet is not the last one in the message, it is said to be in a busy state.

2.3 Multichannel Slotted ALOHA with Reservation

Under this scheme, each mobile user keeps a list of channels which are available for transmission. At the beginning of each time slot, the base station will broadcast all the available channels at the current slot within a very short duration δt, as shown in Figure 2 and 3. The mobile users will also adjust their lists accordingly.

When a mobile user has a message to transmit, it will "tune" to one of the available frequency bands and compete for the subchannel according to the ALOHA scheme. If the user succeeds in obtaining a subchannel, then the user transmits all the packets of the message contiguously by using that subchannel. The base station will update the list of all the available channels at the end of each time slot. Therefore, no other mobile users will compete for the subchannels which have been occupied in the next time slot.

It is equivalent to saying that the mobile users make a reservation for transmission by competing with each other according to the multichannel ALOHA scheme. After all the packets of the message have been transmitted, the user releases the channel. If the user fails to make the reservation, it enters a backlogged state and tries to make the reservation at a later time, geometrically distributed with parameter p_r. In the following analysis, we assume that $\delta t = 0$ for simplicity. The effect of δt is therefore ignored.

3 Performance Analysis

3.1 Multichannel Slotted ALOHA without Reservation

Let x_t and y_t be the number of busy channels and the number of backlogged states, respectively, at the end of slot t. The state of the system, defined as $(x_t,\ y_t)$ and $\{x_t,\ y_t\}$, is a two-dimensional finite state Markov chain. The first step transition probability, from $(x_t = i,\ y_t = j)$ to $(x_{t+1} = l,\ y_{t+1} = k)$, which is derived in a manner similar to [1, 3] from the conditional probability that n active users simultaneously transmit packets and c_s packets are successfully received, is given by, for $0 \le i \le M,\ 0 \le j \le N - i,\ \ 0 \le l \le M,\ 0 \le k \le N - l$,

$$P_{ij\ lk} = \sum_{n=i}^{N} \sum_{c_s=l}^{\min(M,n)} \binom{j}{n-i-a} p_r^{n-i-a}(1-p_r)^{j-n+a+i} \binom{N-i-j}{a} \lambda^a (1-\lambda)^{N-i-j-a} \binom{c_s}{l} q_1^{c_s-l}(1-q_1)^l S(c_s|n, M) \tag{2}$$

where $a = k - j + i + c_s$, c_s is the number of successful packets, and the conditional probability $S(c_s|n, m)$ is defined as

$S(c_s|n, m) =$ Prob(c_s packets are successfully transmitted | n packets are simultaneously transmitted over m channels).

The conditional probability $S(c_s|n, m)$ is derived in [1]:

$$S(c_s|n, m) = \sum_{j=0}^{n} \binom{n}{j} (1 - \frac{1}{m})^{n-j} (\frac{1}{m})^j [\gamma_j S(c_s - 1|n - j, m - 1) + (1 - \gamma_j) S(c_s|n - j, m - 1)] \tag{3}$$

where $\gamma_0 = 0$ and γ_j, $j \geq 1$ is the probability that one packet is transmitted successfully if j packets are transmitted simultaneously over a given channel and is given in [1].

Let $\mathbf{P} = (P_{ij\ lk})$ be the probability transition matrix and $\Pi = \{\pi_{ij}\}$ denote the steady-state probability distribution. Then,

$$\Pi = \Pi\mathbf{P}$$

and

$$\sum_{ij}^{N} \pi_{ij} = 1.$$

Note that the size of the linear system is $(M+1)(N+1) - M(M+1)/2$. The system message throughput θ_1, which is defined as the average number of *messages* transmitted per slot, can be obtained as follows:

$$\theta_1 = \sum_{ij}(N - i - j)\lambda\pi_{ij}. \qquad (4)$$

The system packet throughput θ_2, which is defined as the average number of *packets* transmitted per slot, is then equal to

$$\theta_2 = \frac{1}{q_1}\theta_1. \qquad (5)$$

The message delay is defined as the interval from the time the message is generated to the time the last packet of the message is transmitted. The mean message delay can be obtained by noting that $\frac{1}{\lambda} - 1 + E(D)$ is the expected time to generate one message and, at steady state, the input and output balance equation

$$\frac{N}{\frac{1}{\lambda} + E(D) - 1} = \theta_1. \qquad (6)$$

From the above equation, we obtain the message delay time

$$E(D) = \frac{N}{\theta_1} - \frac{1}{\lambda} + 1. \qquad (7)$$

3.2 Multichannel Slotted ALOHA with Reservation

Let x_t and y_t be the number of busy states and the number of backlogged states, respectively, at the end of slot t. The state of the system, defined as $(x_t,\ y_t)$ as in the previous section, and $\{x_t,\ y_t\}$ is also a two-dimensional finite state Markov chain. The first step transition probability, from $(x_t = i,\ y_t = j)$ to $(x_{t+1} = l,\ y_{t+1} = k)$, which is derived in a manner similar to that given in (2), is given by, for $0 \leq i \leq M,\ 0 \leq j \leq N - i,\ \ 0 \leq l \leq M,\ 0 \leq k \leq N - l$,

$$P_{ij\ lk} = \sum_{n=0}^{N-i} \sum_{c_s=0}^{\min(M-i,n)} \binom{j}{n-a} p_r^{n-a}(1-p_r)^{j-n+a} \binom{N-i-j}{a}$$

$$\lambda^a(1-\lambda)^{N-i-j-a} \binom{c_s+i}{l} q_1^{c_s+i-l}(1-q_1)^l S(c_s|n, M-i), \quad (8)$$

where $a = k - j + c_s$. A recursive formula for $S(c_s|n, m)$ is defined in (3). It should be noted that in this scheme, c_s is the number of successful reservations.

The message and packet throughputs as well as the message delay can be obtained using (4), (5), and (7), respectively, with the transition matrix given in (8).

3.3 Effect of Retransmission Probability on Throughput

In this section, we investigate the effect of the retransmission probability on the throughput. We consider three retransmission probabilities:

1. p_r is a constant.

2. p_r is a function of the available channels, $M - i$, and the number of users without reserved channels (both active and inactive), $N - i$, for example,

$$p_r = \frac{M - i}{N - i}, \qquad (9)$$

where i is the number of reserved channels.

3. p_r is a function of the available channels, $M - i$, and the number of active users without reservation, n, for example,

$$p_r = \min\{1,\ \frac{M - i}{n}\}, \qquad (10)$$

where i is the number of reserved channels.

The scheme with retransmission probability given in (10) is not easy to implement because one must know the number of users who have the potential to transmit. However, the throughput obtained using (10) is an upper bound. The scheme with retransmission probability given in (9) is implementable because the only information needed is the number of channels reserved by other active users.

4 Numerical Results

Two parameters of interest are used in this section to evaluate the system performance: channel throughput and message delay. The average channel throughput is given by

$$U = \theta_1/M. \qquad (11)$$

The message delay D is defined as the time interval from a message arrival to the completion of its transmission and is given in (7).

Figs. 4 and 5 show the channel throughput and average message delay vs. input traffic load for $N = 30$, $M = 5$, and $p_r = 0.5$. We assume that the message contains one packet with probability one. It is noted that the channel throughput always increases as the input traffic load increases. We observe that the maximum throughput of the system with capture ratio equal to 30 dB, 20 dB and 10 dB is 10%, 35%, and 80%, respectively, higher than that of the same system without capture, which indicates that reducing the capture ratio can significantly improve the performance of the system.

Figs. 6 and 7 show the channel throughput and average message delays vs. input traffic load for $(N, M, q_1) = (30, 5, 0.1)$ with two different retransmission rate, 0.1 and 0.5, at $E_s/N_o = 10$ dB. We observe that the ALOHA protocol with reservation has much higher throughput and less delay than that without reservation. This is due to the fact that fewer collisions occur with the reservation scheme. It should be noted that if the message is very short, for example, one or two packets in each message, and the overhead δt is relatively long, then the scheme without reservation is expected to outperform the one with reservation.

In Fig. 8 and 9, we plot the throughput and average message delay versus message arrival rate for different retransmission probabilities, respectively. The throughput (average message delay, respectively) with p_r given in (10) is the highest (lowest, respectively) among the values of p_r considered. When the arrival rate is higher ($\lambda > 0,5$), the throughout with p_r given in (9) is almost the same as the one with p_r given in (10), which implies that the throughput with p_r given in (9) is a good approximation for the optimal value of p_r. Similar results are observed for $N = 30, M = 10$, and $q_1 = .05$ and as such are omitted.

5 Application to Voice Transmission

In this section, we discuss application of the analysis presented in the previous sections to voice transmission systems. In a voice transmission system, we assume that there are N users, each user transmiting fixed-length voice packets over M channels. The transmission time of a voice packet is equal to one slot. An active user generates packets according to a two-state on/off Markov chain, alternating between "talkspurt" and "silence" periods. During a talkspurt period, the user generates packets at a rate of one per slot, at the start of a slot. No packets are generated during the silence period. We assume that the talkspurt period and the silence period are exponentially distributed with parameters α and β, respectively. Let the slot duration be τ seconds. Then, the probability that an active user becomes silent during a slot is equal to

$$q_1 = 1 - e^{-\alpha\tau},$$

and the probability that a silent user becomes active during a slot is

$$\lambda = 1 - e^{-\beta\tau}.$$

There are various Reservation Random Access (RRA) schemes for voice transmission [6] which are similar to the multichannel ALOHA reservation system. Since voice cannot tolerate any excessive delay, we assume that any packet that stays in the buffer for one slot is lost.

Following the same approach as in the previous sections, define x_t and y_t to be the number of active users with a reserved slot and active users without a reserved slot, respectively. Then $\{x_t, y_t\}$ is a two-dimensional Markov chain. The one step transition can be obtained as

$$P_{ij\ lk} = \sum_{n=0}^{N-i} \sum_{c_s=0}^{\min(M-i,n)} \sum_{a=0}^{N-i-j} f(i,j,l,k,n,c_s,a)$$

where

$$f(i,j,l,k,n,c_s,a) = \binom{j}{n-a} p_r^{n-a}(1-p_r)^{j-n+a} \binom{j+a-c_s}{k}$$

$$q_1^{j+a-c_s-k}(1-q_1)^k \binom{N-i-j}{a} \lambda^a (1-\lambda)^{N-i-j-a}$$

$$\binom{c_s+i}{l} q_1^{c_s+i-l}(1-q_1)^l S(c_s|n, M-i) \qquad (12)$$

and $S(c_s|n, M)$ is given in (3).

A key measure of the scheme performanceis the packet dropping probability and is given by

$$P(\text{packet dropping}) = \sum_{i,j,l,k} \sum_{n=0}^{N-i} \sum_{c_s=0}^{\min(M-i,n)}$$

$$\sum_{a=0}^{N-i-j} (j+a-c_s) f(i,j,l,k,n,c_s,a). \qquad (13)$$

6 Conclusion

In this paper, we have studied two wireless data network access protocols, *multichannel ALOHA with and without reservation*, for personal communications services. The combined effect of Rayleigh fading, log-normal shadowing, and spatial distribution of the mobile users to the system throughput has been included. A Markov model is used to analyze the system throughput and average message delay. Numerical results show that the throughput of such a slotted multichannel ALOHA system with capture is much higher than that without capture. The results also indicate that multichannel ALOHA protocol with reservation has much higher throughput and less delay then the protocol without reservation, if the set-up time is ignored.

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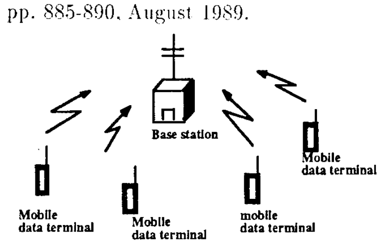

Figure 1: Wireless data network for personal communications service.

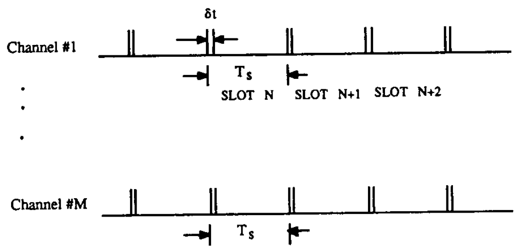

Figure 2: Time slot for multichannel ALOHA system with SFDM approach.

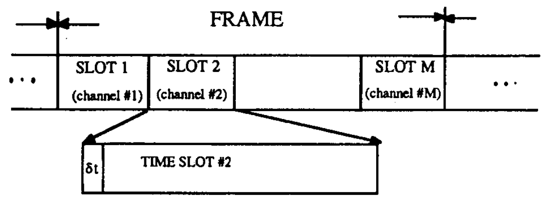

Figure 3: Time slot for multichannel ALOHA system with TDM approach.

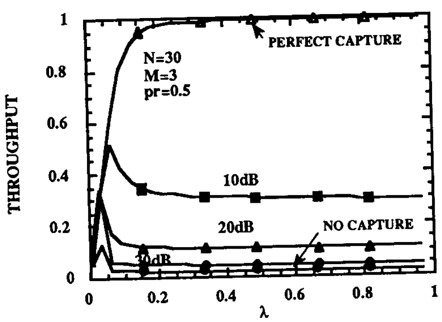

Figure 4: Average channel throughput versus packet arrival rate

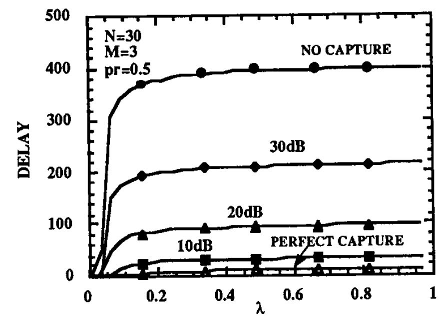

Figure 5: Average packet delay versus packet arrival rate

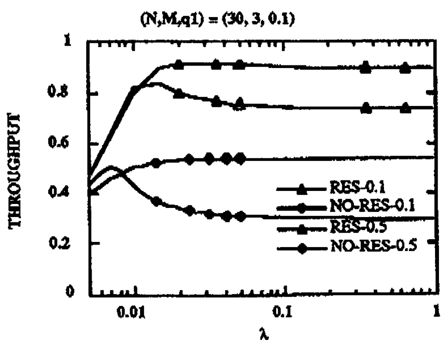

Figure 6: Average packet channel throughput versus message arrival rate

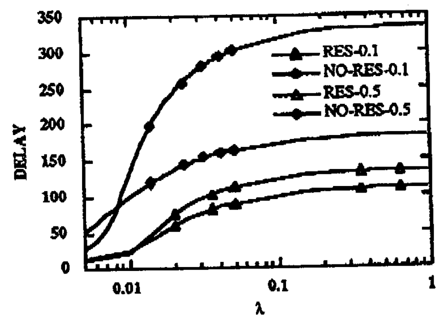

Figure 7: Average message delay versus message arrival rate

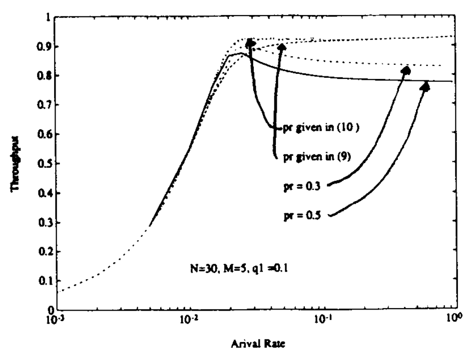

Fig. 8 Throughput versus Arrival Rate

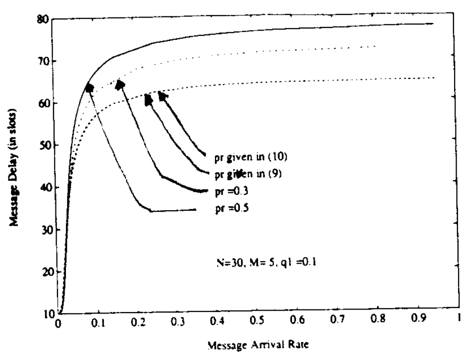

Fig 9. Average Message Delay versus Message Arrival Rate