Homework 3 Report Problem Set

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Problem 1. (1%)請說明你實作的 CNN model，其模型架構、訓練過程和準確率為何？

Layer (type)	Output Shape	Param #
conv2d_1 (Conv2D)	(None, 48, 48, 64)	640
batch_normalization_1 (Batch	(None, 48, 48, 64)	256
leaky_re_lu_1 (LeakyReLU)	(None, 48, 48, 64)	0
conv2d_2 (Conv2D)	(None, 48, 48, 64)	36928
batch_normalization_2 (Batch	(None, 48, 48, 64)	256
leaky_re_lu_2 (LeakyReLU)	(None, 48, 48, 64)	0
max_pooling2d_1 (MaxPooling2	(None, 24, 24, 64)	0
dropout_1 (Dropout)	(None, 24, 24, 64)	0
conv2d_3 (Conv2D)	(None, 24, 24, 128)	73856
batch_normalization_3 (Batch	(None, 24, 24, 128)	512
leaky_re_lu_3 (LeakyReLU)	(None, 24, 24, 128)	0
conv2d_4 (Conv2D)	(None, 24, 24, 128)	147584
batch_normalization_4 (Batch	(None, 24, 24, 128)	512
leaky_re_lu_4 (LeakyReLU)	(None, 24, 24, 128)	0
max_pooling2d_2 (MaxPooling2	(None, 12, 12, 128)	0
dropout_2 (Dropout)	(None, 12, 12, 128)	0
conv2d_5 (Conv2D)	(None, 12, 12, 192)	221376
batch_normalization_5 (Batch	(None, 12, 12, 192)	768
leaky_re_lu_5 (LeakyReLU)	(None, 12, 12, 192)	0
conv2d_6 (Conv2D)	(None, 12, 12, 192)	331968
batch_normalization_6 (Batch	(None, 12, 12, 192)	768
leaky_re_lu_6 (LeakyReLU)	(None, 12, 12, 192)	0
max_pooling2d_3 (MaxPooling2	(None, 6, 6, 192)	0
dropout_3 (Dropout)	(None, 6, 6, 192)	0
conv2d_7 (Conv2D)	(None, 6, 6, 256)	442624
batch_normalization_7 (Batch	(None, 6, 6, 256)	1024
leaky_re_lu_7 (LeakyReLU)	(None, 6, 6, 256)	0
conv2d_8 (Conv2D)	(None, 6, 6, 256)	590080
batch_normalization_8 (Batch	(None, 6, 6, 256)	1024
leaky_re_lu_8 (LeakyReLU)	(None, 6, 6, 256)	0
max_pooling2d_4 (MaxPooling2	(None, 3, 3, 256)	0
dropout_4 (Dropout)	(None, 3, 3, 256)	0
conv2d_9 (Conv2D)	(None, 3, 3, 512)	1180160
batch_normalization_9 (Batch	(None, 3, 3, 512)	2048
leaky_re_lu_9 (LeakyReLU)	(None, 3, 3, 512)	0
conv2d_10 (Conv2D)	(None, 3, 3, 512)	2359808
batch_normalization_10 (Batc	(None, 3, 3, 512)	2048
leaky_re_lu_10 (LeakyReLU)	(None, 3, 3, 512)	0
max_pooling2d_5 (MaxPooling2	(None, 1, 1, 512)	0
dropout_5 (Dropout)	(None, 1, 1, 512)	0
flatten_1 (Flatten)	(None, 512)	0
dense_1 (Dense)	(None, 512)	262656
batch_normalization_11 (Batc	(None, 512)	2048
dropout_6 (Dropout)	(None, 512)	0
dense_2 (Dense)	(None, 512)	262656
batch_normalization_12 (Batc	(None, 512)	2048
dropout_7 (Dropout)	(None, 512)	0
dense_3 (Dense)	(None, 7)	3591

Total params: 5,927,239
Trainable params: 5,920,583
Non-trainable params: 6,656

模型架構:

前面 Convolution 2D 每層架構為 conv2D (3,3) -> batch normalization -> activation (‘leaky relu’, alpha=0.1) -> conv2D (3,3) -> batch normalization -> activation (‘leaky relu’, alpha=0.1) -> max pooling2D (2,2) -> drop out (0.3)，總共有 5 層，對應的 filter 數為 64、128、192，256 和 512。

後面 Dense 架構為 dense (512, ‘relu’) -> batch normalization -> drop out (0.5) -> dense (512, ‘relu’) -> batch normalization -> drop out (0.5) -> dense (7, ‘softmax’)

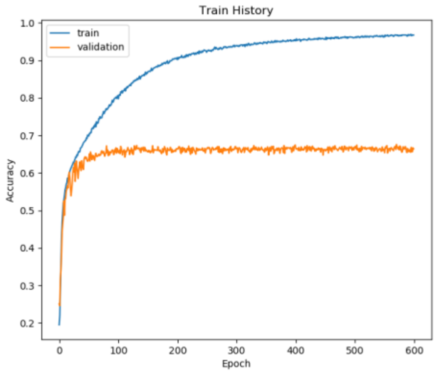
整體模型的 loss 是由 categorical crossentropy 決定，optimizer 是 adam。

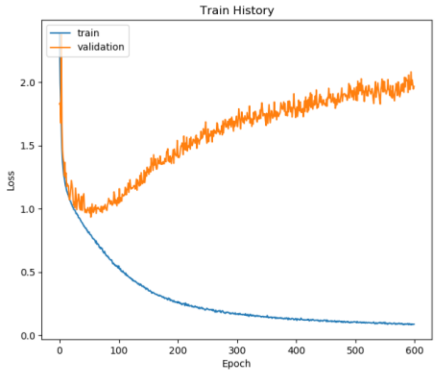
訓練過程:

在資料前處理的部分，將 5000 筆訓練資料當作 validation data，而剩餘資料當作 training data。訓練過程中使用 Image Generator 把資料做旋轉(10 度以內)和水平上下移(10%以內)，用來增加訓練資料量。並在訓練的過程中使用 callbacks 函數中的 ModelCheckpoints，根據每個 epoch 訓練出來的 validation accuracy 將最好的 model 存取下來。

準確率:

此模型訓練 600 個 epoch 後，訓練中最好的 validation accuracy 為 0.6840，而 kaggle public score 為 0.68459，從下圖可以看到 validation accuracy 到最後會趨於飽和，但是 loss 會上升，有 overfitting 的現象。





Problem 2. (1%)承上題，請用與上述 CNN 接近的參數量，實做簡單的 DNN model，其模型架構、訓練過程和準確率為何？試與上題結果做比較，並說明你觀察到了什麼？

Layer (type)	Output Shape	Param #
dense_1 (Dense)	(None, 1024)	2360320
batch_normalization_1 (Batch	(None, 1024)	4096
dropout_1 (Dropout)	(None, 1024)	0
dense_2 (Dense)	(None, 1024)	1049600
batch_normalization_2 (Batch	(None, 1024)	4096
dropout_2 (Dropout)	(None, 1024)	0
dense_3 (Dense)	(None, 1024)	1049600
batch_normalization_3 (Batch	(None, 1024)	4096
dropout_3 (Dropout)	(None, 1024)	0
dense_4 (Dense)	(None, 1024)	1049600
batch_normalization_4 (Batch	(None, 1024)	4096
dropout_4 (Dropout)	(None, 1024)	0
dense_5 (Dense)	(None, 512)	524800
batch_normalization_5 (Batch	(None, 512)	2048
dropout_5 (Dropout)	(None, 512)	0
dense_6 (Dense)	(None, 7)	3591

Total params: 6,055,943
Trainable params: 6,046,727
Non-trainable params: 9,216

模型架構:

DNN 架構為 Dense (1024, ‘relu’) -> batch normalization -> drop out (0.5) -> Dense (1024, ‘relu’) -> batch normalization -> drop out (0.5) -> Dense (1024, ‘relu’) -> batch normalization -> drop out (0.5) -> Dense (1024, ‘relu’) -> batch normalization -> drop out (0.5) -> Dense (512, ‘relu’) -> batch normalization -> drop out (0.5) -> Dense (7, ‘softmax’)

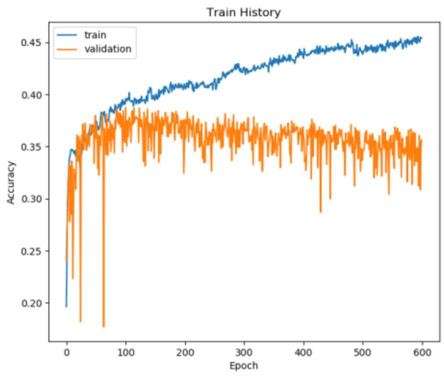
整體模型的 loss 是由 categorical crossentropy 決定，optimizer 是 adam。

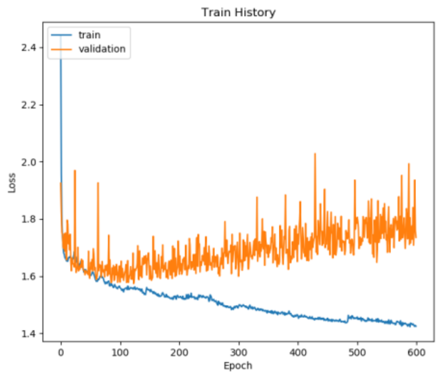
訓練過程:

在資料前處理的部分，將 5000 筆訓練資料當作 validation data，而剩餘資料當作 training data。並在訓練的過程中使用 callbacks 函數中的 ModelCheckpoints，根據每個 epoch 訓練出來的 validation accuracy 將最好的 model 存取下來。

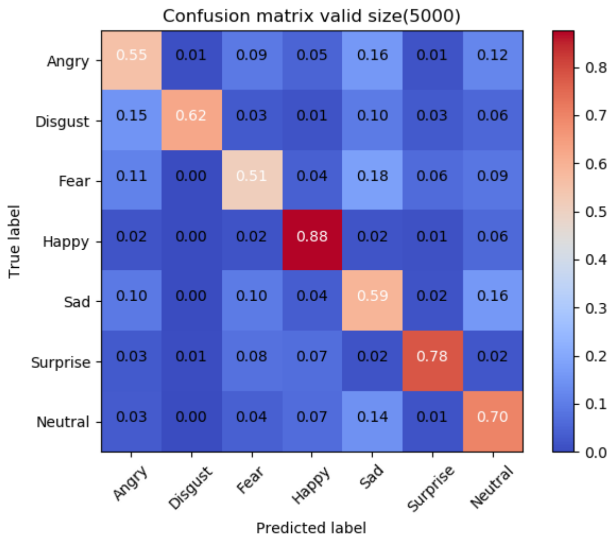
準確率:

此模型訓練 600 個 epoch 後，訓練中最好的 validation accuracy 為 0.3874，下圖顯示 validation accuracy 和 error 震盪都蠻嚴重的。可以知道在差不多參數量的情況下，DNN 的預測能力非常差，我覺得是因為 DNN 相對 CNN 而言並沒有將圖片分區塊辨識的能力，所以當圖片歧異度很高時，純粹用數值的訓練不容易成功。





Problem 3. (1%)觀察答錯的圖片中，哪些 class 彼此間容易用混？ 並說明你觀察到了什麼？ [繪出 confusion matrix 分析]



如同訓練 CNN 時的假設，將前 5000 筆資料當作 validation set，所以這邊是用 validation set 的 data 去做 confusion matrix。

根據 confusion matrix 可以看出 angry 中最容易被錯誤判斷成 sad，disgust 中最容易被錯誤判斷成 angry，fear 中最容易被錯誤判斷成 sad，sad 中最容易被錯誤判斷成 angry，從這四個觀察點可以

發現，我們人類對於這四個情緒的分類比較屬於「負面情緒」，因此可能 CNN 在學習的時候會比較容易將這四種情緒搞混。這種現象也可以在「正面情緒」happy 和 surprise 中觀察到。

同時根據 confusion matrix 可以看出 happy 和 surprise 的準確率最高，推測可能是因為正面情緒的表情較為誇張，例如微笑的嘴角上揚等特徵，且正面情緒選擇較少，使得 CNN 比較容易能分辨這兩個情緒。而負面情緒較低，可能是因為負面情緒除了選擇多，加上表情小還可能跟 neutral 搞混的原因。

Problem 4. (1.5%, each 0.5%) CNN time/space complexity

For a. b. Given a CNN model as

```
model = Sequential()
model.add(Conv2D(filters=6,
                 strides=(3, 3),
"""Layer A"""    padding ="valid",
                 kernel_size=(2,2),
                 input_shape=(8,8,5),
                 activation='relu'))
model.add(Conv2D(filters=4,
                 strides=(2, 2),
"""Layer B"""    padding ="valid",
                 kernel_size=(2,2),
                 activation='relu'))
```

And for the c. given the parameter as:

kernel size = (k,k);

channel size = c;

filter size = f;

input shape = (n,n);

padding = 1;

strides = (s,s);

a. **How many parameters are there in each layer(Hint: you may consider whether the number of parameter is related with)**

Parameter: $(\text{kernel size} \times \text{kernel size} \times \text{number of channels} + 1) \times \text{number of filters}$

Layer A: $(2 \times 2 \times 5 + 1) \times 6 = 126$

Layer B: $(2 \times 2 \times 6 + 1) \times 4 = 100$

b. **How many multiplications/additions are needed for a forward pass (each layer).**

Multiplications:

$$\text{kernel size} \times \text{kernel size} \times \text{number of channels} \times \text{output size} \times \text{number of filters}$$

Additions:

kernel size × kernel size × number of channels × output size × number of filters

Layer A:

Multiplications: $2 \times 2 \times 5 \times 9 \times 6 = 1080$

Additions: $2 \times 2 \times 5 \times 9 \times 6 = 1080$

Layer B:

Multiplications: $2 \times 2 \times 6 \times 1 \times 4 = 96$

Additions: $2 \times 2 \times 6 \times 1 \times 4 = 96$

c. **What is the time complexity of convolutional neural networks?(note: you must use big-O upper bound, and there are l layer, you can use C_l,C_{l-1} as lth and l-1th layer)**

假設每一層 layer 的參數不一樣，將k_l表示成代表第 l 層的 kernel size，以此類推

Time Complexity of CNN

$$= O\left(\sum_{i=1}^{total\ layers} (number\ of\ input\ channel) \times (length\ of\ filter)^2 \times (number\ of\ filters) \times (length\ of\ output\ feature\ map)^2\right)$$

$$= O\left(\sum_{i=1}^{l} (f_{l-1}) \times (k_l)^2 \times (f_l) \times \left(\left\lfloor \frac{n_l - k_l}{s_l} \right\rfloor\right)^2\right)$$

Ref: https://arxiv.org/pdf/1412.1710.pdf

Problem 5. (1.5%, each 0.5%) PCA practice: Given 10 samples in 3D space

(1,2,3),(4,8,5),(3,12,9),(1,8,5),(5,14,2), (7,4,1),(9,8,9),(3,8,1),(11,5,6),(10,11,7)

(1) What are the principle axes?

a. Compute the covariance matrix

$$C = \frac{1}{N} X X^T = \frac{1}{N} \sum_{i=1}^{N} (\boldsymbol{x_i} - \boldsymbol{\mu})(\boldsymbol{x_i} - \boldsymbol{\mu})^T = U \Lambda U^T$$

$$X = \begin{bmatrix} 1 & 4 & 3 & 1 & 5 & 7 & 9 & 3 & 11 & 10 \\ 2 & 8 & 12 & 8 & 14 & 4 & 8 & 8 & 5 & 11 \\ 3 & 5 & 9 & 5 & 2 & 1 & 9 & 1 & 6 & 7 \end{bmatrix} = [\boldsymbol{x_1}\ \boldsymbol{x_2}\ \boldsymbol{x_3}\ \boldsymbol{x_4}\ \boldsymbol{x_5}\ \boldsymbol{x_6}\ \boldsymbol{x_7}\ \boldsymbol{x_8}\ \boldsymbol{x_9}\ \boldsymbol{x_{10}}]$$

b. 用 numpy.linalg.eig 去對 covariance matrix 做 eigenvalue decomposition

$$C \approx \begin{bmatrix} 0.39985541 & -0.67817891 & -0.6165947 \\ 0.33758926 & 0.73439013 & -0.58881629 \\ -0.85214385 & -0.02728563 & -0.52259579 \end{bmatrix} \begin{bmatrix} 5.47203291 & 0 & 0 \\ 0 & 11.63052369 & 0 \\ 0 & 0 & 15.2974434 \end{bmatrix}$$

$$\begin{bmatrix} 0.39985541 & -0.67817891 & -0.6165947 \\ 0.33758926 & 0.73439013 & -0.58881629 \\ -0.85214385 & -0.02728563 & -0.52259579 \end{bmatrix}^T = U\Lambda U^T$$

c. 總共有三條 principal axes，依照 eigenvalue 大小順序分別為(大到小)

$$\boldsymbol{u_1} = [-0.6165947 \quad -0.58881629 \quad -0.52259579]$$

$$\boldsymbol{u_2} = [-0.67817891 \quad 0.73439013 \quad -0.02728563]$$

$$\boldsymbol{u_3} = [0.39985541 \quad 0.33758926 \quad -0.85214385]$$

(2) Compute the principal components for each sample.

$$\mathbf{y} = \boldsymbol{U^T x} = [\boldsymbol{u_1}\ \boldsymbol{u_2}\ \boldsymbol{u_3}\ ]^T[\boldsymbol{x_1}\ \boldsymbol{x_2}\ \boldsymbol{x_3}\ \boldsymbol{x_4}\ \boldsymbol{x_5}\ \boldsymbol{x_6}\ \boldsymbol{x_7}\ \boldsymbol{x_8}\ \boldsymbol{x_9}\ \boldsymbol{x_{10}}]$$

$$\mathbf{y} = \begin{bmatrix} -0.6165947 & -0.67817891 & 0.39985541 \\ -0.58881629 & 0.73439013 & 0.33758926 \\ -0.52259579 & -0.02728563 & -0.85214385 \end{bmatrix}^T \begin{bmatrix} 1 & 4 & 3 & 1 & 5 & 7 & 9 & 3 & 11 & 10 \\ 2 & 8 & 12 & 8 & 14 & 4 & 8 & 8 & 5 & 11 \\ 3 & 5 & 9 & 5 & 2 & 1 & 9 & 1 & 6 & 7 \end{bmatrix}$$

$$\approx \begin{bmatrix} -3.362 & -9.79 & -13.62 & -7.940 & -12.37 & -7.194 & -14.96 & -7.083 & -12.86 & -16.30 \\ 0.709 & 3.026 & 6.533 & 5.062 & 6.836 & -1.837 & -0.474 & 3.813 & -3.952 & 1.106 \\ -1.481 & 0.039 & -2.419 & -1.160 & 5.021 & 3.297 & -1.370 & 3.048 & 0.973 & 1.747 \end{bmatrix}$$

$$= [\boldsymbol{y_1}\ \boldsymbol{y_2}\ \boldsymbol{y_3}\ \boldsymbol{y_4}\ \boldsymbol{y_5}\ \boldsymbol{y_6}\ \boldsymbol{y_7}\ \boldsymbol{y_8}\ \boldsymbol{y_9}\ \boldsymbol{y_{10}}]$$

$[\boldsymbol{y_1}\ \boldsymbol{y_2}\ \boldsymbol{y_3}\ \boldsymbol{y_4}\ \boldsymbol{y_5}\ \boldsymbol{y_6}\ \boldsymbol{y_7}\ \boldsymbol{y_8}\ \boldsymbol{y_9}\ \boldsymbol{y_{10}}]$ 分別為 $[\boldsymbol{x_1}\ \boldsymbol{x_2}\ \boldsymbol{x_3}\ \boldsymbol{x_4}\ \boldsymbol{x_5}\ \boldsymbol{x_6}\ \boldsymbol{x_7}\ \boldsymbol{x_8}\ \boldsymbol{x_9}\ \boldsymbol{x_{10}}]$ 對應$[\boldsymbol{u_1}\ \boldsymbol{u_2}\ \boldsymbol{u_3}\ ]$的 principal components

(3) Reconstruction error if reduced to 2D. (Calculate the L2-norm)

另$\widetilde{\boldsymbol{x}}$為從 2D 到 3D 的 reconstruction samples

$$\widetilde{\boldsymbol{x}} = \mathbf{U}[:,:\mathbf{2}]\boldsymbol{y}[:2,:]$$

Reconstruction error

$$= \sum_{i=1}^{10} (\boldsymbol{x_i} - \widetilde{\boldsymbol{x}}_i)^2 \approx 60.644$$