

Stat707Assign1Task1

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R Markdown

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When you click the **Knit** button a document will be generated that includes both content as well as the output of any embedded R code chunks within the document. You can embed an R code chunk like this:

##Task 1:##

About 30% of human twins are identical and rest are fraternal. Identical twins are necessarily the same sex – half are males and half are females. One-quarter of fraternal twins are both male, one-quarter are both female and one-half are mixed: one male, one female. You have just become parent of twins and are told they are both girls. Given this information, what is the probability that they are identical?

Answer:

Define:

A: twin Identical

A-:TWin Fraternal

B:All Female

C:All Male

D: Mixed

Then

$$p(A) = 30\%$$

$$P(A-) = 70\%$$

$$P(B|A) = 50\%$$

$$P(C|A) = 50\%$$

$$P(D|A) = 0\%$$

$$P(B|A-) = 25\%$$

$$P(C|A-) = 25\%$$

$$P(D|A-) = 50\%$$

Thus

$$P(B)=P(B|A)P(A)+P(B|A-)P(A-) = 50\% * 30\% + 25\% * 70\% = 32.5\%$$

$$P(A|B)=P(B|A)P(A)/P(B) = 50\%30\%/32.5\% = 46.15\%$$

```
PFemale=0.5 * 0.3 + 0.25 * 0.7
PIDenGivFemale=0.5*0.3/PFemale
sprintf('The Probability That they ar identical given that they are both girls is %f%s ',PIDenGivFema
```

```
## [1] "The Probability That they ar identical given that they are both girls is 46.153846% "
##TASK2##
```

A coin is said to be unbiased if $P(\text{heads}) = P(\text{tails}) = 0.5$. Probability that a newly minted coin by a Government Mint is unbiased is 0.95. When an unbiased coin is tossed 100 times, the probability of getting 62 heads or more is 0.01 (0.01048 rounded off to two decimals) for an unbiased coin. As part of a quality control process at the Mint some of the newly minted coins are randomly selected and each coin is tossed 100 times. If a coin tosses heads 62 times or more, it is set aside as a possibly biased (faulty) coin. All such coins are to be melted down and re-minted, which has additional costs. A consultant has been hired to identify efficiencies in this process and see if the criteria of 62 heads or more could possibly be further optimized. Since you work as a data scientist, the consultant asks you to calculate the probability that the coin is unbiased given that it tossed 62 or more heads out of 100 tosses. Find this probability. Assume (just for this example) that a biased coin always has a $P(\text{heads}) = 0.55$. Clearly define your events, state the formulae you are using and show your working.

Answer:

Define:

A: unbiased Mint Coin

A-: biased Mint Coin

B: get Head in ALL TYPES of COINS

B-: get Tail in ALL TYPES of COINS

C: Get62Head in ALL TYPES of COINS

Then:

$p(A) = 95\%$

$P(A-) = 5\%$

$P(B|A) = 50\%$

$P(B|A-) = 55\%$

$P(C|A) = 0.01048$

Thus: $P(B) = P(B|A)P(A) + P(B|A-)P(A-)$

$P(C)$ is binomial distribution of probability $P(B)$, there is the formular:

```
knitr::include_graphics("C:\\R_source\\Stat707Assign1\\probability-distributions0x.png")
```

$$f(x) = \binom{n}{x} p^x (1-p)^{(n-x)} \quad \text{where } x = 0, 1, 2, \dots, n$$

```
p_B=0.5*0.95+0.55*0.05
```

```
"The Probability get Head in All TYPES OF COINS P(B) is "
```

```
## [1] "The Probability get Head in All TYPES OF COINS P(B) is "
```

```
p_B
```

```
## [1] 0.5025
```

```

p_C = pbinom(62,100,p_B)
'The Probability get 62 head in 100 tries from all TYPES OF coins P(C) is '

## [1] "The Probability get 62 head in 100 tries from all TYPES OF coins P(C) is "
p_C

## [1] 0.9930787

'What we need is  $P(A|C)=P(C|A)P(A)/P(C)$ ,that is: '

## [1] "What we need is  $P(A|C)=P(C|A)P(A)/P(C)$ ,that is: "
PUnbiasGiven62= 0.01048*0.95/p_C
PUnbiasGiven62

## [1] 0.01002539

```

Note that the `echo = FALSE` parameter was added to the code chunk to prevent printing of the R code that generated the plot.