CIS 515

Project 3: Curve Interpolation

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Part 1.

$$\boldsymbol{C(t) = (1-t)^3 b_0 + 3(1-t)^2 t^2 b_1 + 3(1-t)^1 t b_2 + 3t^3 b_3}$$

Second derivative:

$$\boldsymbol{C''(t) = 6(1-t) b_0 + (-12+18t) b_1 + (6-18t) b_2 + 6t b_3}$$

Second derivative at $\boldsymbol{b_0}$: t = 0, at $\boldsymbol{b_3}$: t = 1

$$\boldsymbol{C''(0) = 6(b_0 - 2b_1 + b_2)}$$

$$\boldsymbol{C''(1) = 6(b_1 - 2b_2 + b_3)}$$

From the constraints of $\boldsymbol{C^2}$-continuity, we know $\boldsymbol{C_i'(1) = c_{i+1}'(0), c_i''(1) = c_{i+1}''(0)}$, thus there is N-1 equations. And reduce and organize these equations to the form of the matrix, we would get

$$b_2^i + b_{12}^{i+1} = 2b_0^{i+1} = 2x_i$$

$$b_1^i - 2b_2^i = b_2^{i+1} - 2b_1^{i+1}$$

if i is between 2 and N-1,

$$d_{i-1} + 4d_i + d_{i+1} = 6x_i$$

If i == 1 or i == N, using the fact of natural end condition, $d_0 = \frac{2}{3}x_0 + \frac{1}{3}d_1$ and $d_N = \frac{1}{3}d_{N-1} + \frac{2}{3}x_N$

$$4d_1 + d_2 = 6x_1 - x_0$$

$$d_{N-2} + d_{N-1} = 6x_{N-1} - x_N$$

$$\begin{pmatrix} 4 & 1 & 0 & \cdots & & \cdots \\ 1 & 4 & 1 & 0 & & \\ \vdots & \ddots & \ddots & \ddots & & \vdots \\ 0 & \vdots & \ddots & \ddots & & \\ & & \cdots & 1 & 4 & 1 \\ & \cdots & \cdots & 0 & 1 & 4 \end{pmatrix} \begin{pmatrix} d_1 \\ d_2 \\ \vdots \\ \vdots \\ d_{N-2} \\ d_{N-1} \end{pmatrix} = \begin{pmatrix} 6x_1 - x_0 \\ 6x_2 \\ \vdots \\ \vdots \\ 6x_{N-2} \\ 6x_{N-1} - x_N \end{pmatrix}$$

From the natural end conditions, $C_1''(\boldsymbol{0}) = \boldsymbol{0}, C_N''(\boldsymbol{1}) = \boldsymbol{0}$, we can conclude that

$$d_0 = \frac{2}{3}x_0 + \frac{1}{3}d_1$$

$$d_N = \frac{1}{3}d_{N-1} + \frac{2}{3}x_N$$

If N == 3, matrix would reduce to the this

$$\begin{pmatrix} 4 & 1 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} d1 \\ d2 \end{pmatrix} = \begin{pmatrix} 6x_1 - x_0 \\ 6x_2 - x_3 \end{pmatrix}$$