Homework 3

Due Date

2017/06/14 (Wednesday), 23:59 ◦ Late submission will not be accepted.

In this assignment, you should be able to accomplish an algorithm to solve differential equation numerically with specified initial conditions. Please be advised that no grade will be given if any MATLAB's embedded function for solving differential equation is found.

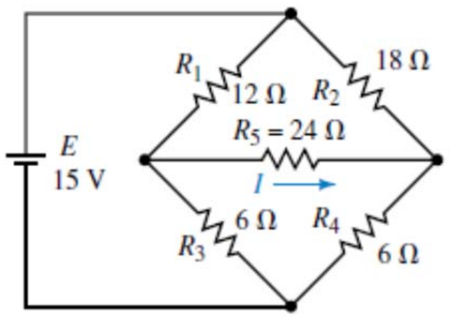
<u>Problem1: Circuit analysis</u>

[Principal m-file name: F<your student id >_hw3_prob1.m]

Given the following network

(1) Write down the equations of the circuit analysis by either mesh current or

(2) Solve the equation to find I5 using Gauss elimination, Jacobi Method or Gauss Seidel Method. Specify which method is used to solve this problem.



Problem2: Numerical Solution for a Damped Harmonic Oscillator

[Principal m-file name: F<your student id >_hw3_prob2.m]

A $1.200 \times 10^3 kg$ brick is mounted on a spring of constant $k = 58.00 \times 10^3$ *N/m* then submerged into an oil container. The damped harmonic oscillator can be modelled by

$$m\frac{d^2}{dt^2}\vec{x}(t) + \beta\frac{d}{dt}\vec{x}(t) + k\vec{x}(t) = \vec{F}(t)$$

The damping constant $(\beta)$for the oil is 4.000×10^3 kg/s. Let x = 0 refer to the equilibrium location of the brick and ignore buoyancy.

(1) [Fig.1] At t = 0, the system is at rest and is compressed by 10.00 cm from its equilibrium. Plot the temporal change of brick's displacement from the equilibrium point.

(2) Based on the result of (2), find the oscillation frequency of the system.

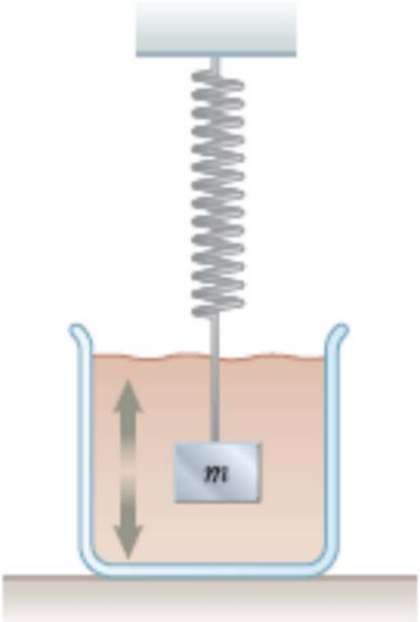
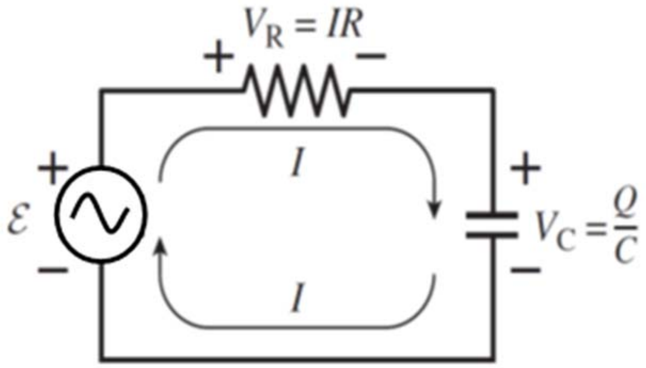


Fig. 1. A damped harmonic oscillator

Problem 3: an RC circuit with a source / Integrator / Differentiator

[Principal m-file name: F<your student id >_hw3_prob3.m]

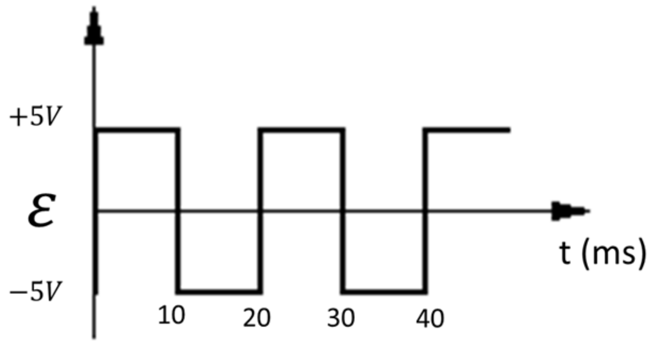
There is a RC circuit with elements in series. Assume that the voltage source provides time-varying electric potential $\varepsilon(t)$



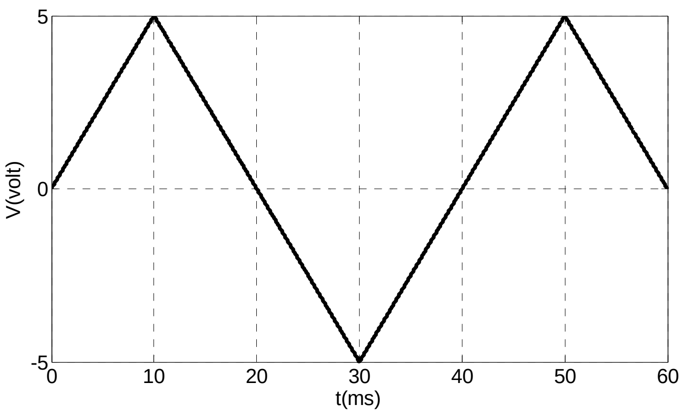
(a) Derive the loop equation in terms of $V_c(t)$, ε (t), R and C for the circuit.

(b) [Fig.2] If R = $10k\Omega$, C = $100\mu F$, the source input $\varepsilon(t)$ becomes $\varepsilon(t) = 5\sin(20\pi t)$ with the same resistor and capacitor, plot $V_c(t)$.

(c) [Fig.3] Suppose R = $10k\Omega$, C = $100\mu F$ and the source input $\varepsilon(t)$ changes as the depicted square wave. Plot $V_c(t)$.



(d) [Fig.4] Suppose R = 100Ω, C = $1\mu F$ and the source input $\varepsilon(t)$ changes as the depicted triangular wave. Plot $V_R(t)$, the voltage wave across the resistor.



Problem4: an RLC circuite with a source to look for resonant frequency.

[Principal m-file name: F<your student id >_hw3_prob4.m]

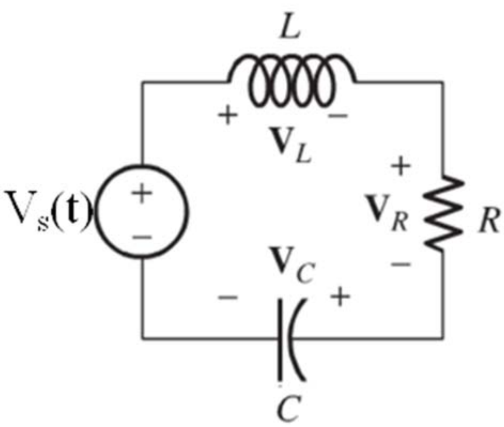
In the following figure, there is an RLC circuit with elements in series. Assume that the voltage provided is a function of time $V_s(t)$ with $L = 2mH$, $R = 8\ \Omega$, and $C = 5\mu F$. Before the voltage source is turned on, there is no charge stored in the capacitor and there is no current flowing on the circuit.

(a) Prove that the current of the loop, $I(t)$, can be described by $\frac{d}{dt}V_s(t) = L\frac{d^2}{dt^2}I(t) + R\frac{d}{dt}I(t) + \frac{1}{C}\ I(t)$

(b) [Fig. 5] If $V_s(t) = cos(6000\ t)$, solve the differential equation numerically and plot $V_R(t)\ and\ V_s(t)$ on the same figure, for $0 \le t \le 10ms$

(c) [Fig. 6] If $V_s(t) = cos(10000\ t)$, solve the differential equation numerically and plot $V_R(t)\ and\ V_s(t)$ on the same figure, for $0 \le t \le 10ms$

(d) [Fig. 7] If $V_s(t) = cos(20000\ t)$, solve the differential equation numerically and plot $V_R(t)\ and\ V_s(t)$ on the same figure, for $0 \le t \le 10ms$



<u>Problem5: Motion of Planet (Numerical Proof of Kepler's Laws of Planet from Newtonian Dynamics)</u>.

[Principal m-file name: F<your student id >_hw3_prob5.m]

Assume that a $2 \times 10^{30}\, kg$ Sun rests at the origin and a $6 \times 10^{24}\, kg$ planet is found at distance $r_0 = 1.5 \times 10^{11}\, m$ from the sun while the planet is moving at speed v toward the direction perpendicular to the segment connecting the center of the two objects. As the mass of the sun is much larger than the planet, the gravitational effect towing the Sun is negligible while you are encouraged to include the force imposed on the Sun.

(a) Write down the equation of motion of the planet. $(ex\text{: } \frac{d^2x_p}{dt^2} = \cdots, \frac{d^2y_p}{dt^2} = \cdots.)$

(b) You should be familiar to calculate the speed for the planet to remain on a circular orbit around the sun if the displacement of the sun can be ignored. Assume that the speed is v_c, find v_c

(c) [Fig 8-10] Use $v = v_c$ as the initial condition in addition to the distance provided then solve the differential equation of problem (a) numerically for at least one orbital period and generate three plots: (1) the trajectory (x-y), (2) the x-location (x-t) and (3) the y-location y(t) of the planet. Find the period of the planet.

(d) [Fig 11-13] If the initial conditions become $v(0) = 0.7\, v_c\hat{\jmath}$, re-calculate the location of the planet from the beginning for at least an orbital period and plot the results similar to those of (c).

(e) Verify that the trajectory of the planet in (d) is an ellipse and find the length of the semi-major axis and the period of the planet in (d).

(f) [Fig 14] You should have notice that the trajectory and the period of the planet changes with the initial condition $v(0) = kv_c\hat{\jmath}$. Now change k from k = 1/5 to k = 1 and record the length of the semimajor axis (R) and the orbital period (T) of each condition and verify that $R^3 \propto T^2$(i.e. make a plot of $R^3 - T^2$ from the data and check if they can be fit on a line);

Contents to submit:

1. All the m-files you compose for the assignment.
2. All the m-files should include proper COMMENTS.
   (No comment, no score)
3. <u>A word document or a PDF</u> includes <u>Your Name</u>, <u>Your Student ID Number</u>,
   In the document, you will need to include all the written answers, plots and the following contents: (1) Which method do you use to solve the differential problem. (2) The step length you choose for each problem. (3) Which coordinate do you use to analyze the system (Which direction is the "POSITIVE" direction)?

Notice:

1. DO NOT PLAGIARIZE. You are encouraged to ask and to discuss the homework content with your fellow classmates, the TAs and the instructor. But identical core program wording is NEVER ACCEPTABLE.
2. Upload all the files without archiving. Do not upload files that don't work well. Any missing file or function that leads to fail of the execution will be regarded as a program that never works.