Homework 2

Due Date

2017/05/31, PM 11:59。No LATE Submission will be accepted.

Create a matlab script and change the filename to F7xxxxxxx_hw2.m. Link all the programs to solve following problems to this script. Make sure once type the filename' F7xxxxxxx_hw2', the results of the following problems will pop-up automatically in order. Remember not to type any 'clear all ', 'close all' command in any of the codes.

<u>Problem 1:[F7xxxxxxx_hw2_prob1.m]</u>

Use "Finite Riemman Sum", "Trapezoid Method", or "Simpson's Method" to find out following integral. The precision should be at least 8 significant digits.

(i) $\int_0^{\pi} \frac{1}{\sqrt{1+2\sin(x)}}\,dx$

(ii) $\int_0^{1} \frac{\sin(50x)}{1+x^2}\,dx$

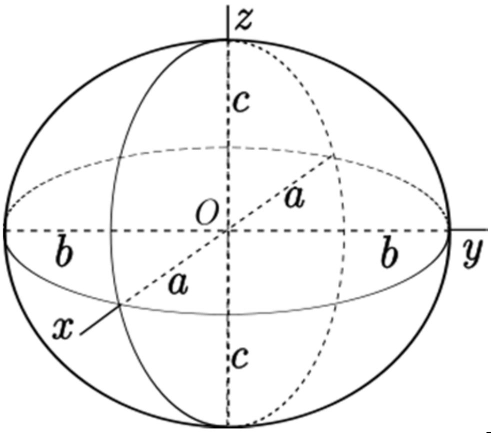
<u>Print out the result in the following format after executing the principal m file</u>

(i) xxxxxx

(ii) xxxxxx

<u>Problem 2:[F7xxxxxxx_hw2_prob2.m]</u>

Use Monte Carlo Integration to estimate the volume of an ellipsoid enclosed by $\frac{x^2}{4}+y^2+\frac{z^2}{9}=1$. Justify the precision of the answer.



<u>Print out the result in the following format after executing the principal m file</u>

A total of xxxxx random points are used; the volume of the ellipsoid is xxxxxxx.

<u>Problem3: Numerical Proof of Gauss's Law of Electrostatics [F7xxxxxxx_hw2_prob3.m]</u>

The Gauss's Law of Electrostatics states that the electrical flux through a close surface should be proportional to the total amount of charge enclosed by the surface:

$$\oint_S \vec{E}\cdot d\vec{A} = Q_{enc}/\epsilon_0 \quad \text{or} \quad \oiint_S \vec{E}\cdot(d\vec{x}\cdot d\vec{y}) = Q_{enc}/\epsilon_0\,.$$

Assume that there is a charged particle of 1 Coulomb placed at the origin (0,0,0) of a space . From homework1, you must have been familiar with calculating the electrical field of the charge in the space.

(1) Suppose there is a closed surface of square cube centered at (0,0,0) and the length of the edge is 1m enclosing the charge. Calculate the total electric flux through the square cube.
(2) Find the total flux if each edge of the square cubic grows to 2m while the center remains at (0,0,0)
(3) Following (2), find the total flux if the charge moves to (0.4m, 0.4m,0.4m).
(4) Following (3), find the total flux if the charge moves to (2m, 2m, 2m).

Print out the result in the following format after executing the principal m file [at least 6 significant digits]

(1) The total flux is xxxxxxx $(Volt \cdot m)$.
(2) The total flux is xxxxxxx $(Volt \cdot m)$.
(3) The total flux is xxxxxxx $(Volt \cdot m)$.
(4) The total flux is xxxxxxx $(Volt \cdot m)$.

Problem4: Solving 1D equations [F7xxxxxxx_hw2_prob4.m]

Try to find both the smallest positive root and the biggest negative root of the following equations. The accuracy of the answer should at least up to 8 significant digits.

(a) $\frac{1}{2} + \frac{1}{4}x - xsin(x) - \frac{1}{2}\cos(2x) = 0$

(b) $e^{3x} - 27x^6 + 27x^4e^x - 9x^2e^{2x} = 0$

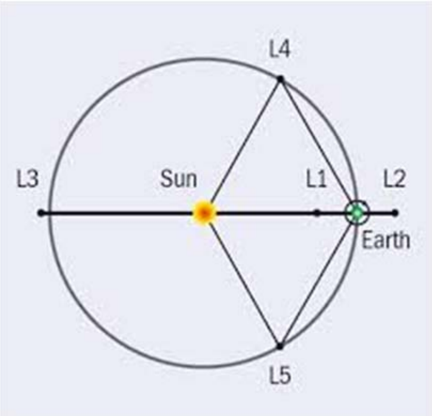
Print out the result in the following format [at least 6 significant digits]

The smallest positive root of Prob.4(a) is xxxxxxxxx
The largest negative root of Prob.4(a) is xxxxxxxxx
The smallest positive root of Prob.4(b) is xxxxxxxxx
The largest negative root of Prob.4(b) is xxxxxxxxx

Problem5: Find a position to place an eclipse-free satellite or to hide an object from the Earth [F7xxxxxxx_hw2_prob5.m]

As shown in the figure, the Lagrange points are the five positions in an orbital configuration where a small object (m) affected only by gravity of two larger celestial bodies (such as a satellite, the Sun (M_s) and the Earth(M_e)). The combined gravitational force from the two larger bodies acting on the small object at the Lagrange points provides precisely the centripetal force for the small object required to maintain a stable position relative to the two larger objects. That is the angular velocity of an object at any of the Lagrange point possesses identical angular velocity of the Earth with respect to the sun and hence the distances between these satellites and the two bodies will remain constant. However, without the earth's gravitational influence, a satellite at any of the Lagrange Points, would have to move at a different angular velocity than that of the earth. Now try to follow the instructions/questions below to find the Lagrange points.

Note: You are allowed to assume that $M_s \gg M_e \gg m$. Thus all the centers of the orbits lie right at the center of the Sun to simplify the analysis.



(a) Write down the equations to find the three points L1, L2 and L3 in terms of $\mathrm{M_s}, \mathrm{M_e}, \mathrm{R_{M_s m}}$ and $\mathrm{R_{M_s M_e}}$.

(b) Find distance ratios ($x = \mathrm{R_{M_s m}}/\mathrm{R_{M_s M_e}}$) of the Lagrange points L1, L2 and L3 with at least 6-digit precision using your program that solves single variable equations with $\mathrm{M_S} = 1.98892 \times 10^{30}$kg and $\mathrm{M_e} = 5.97219 \times 10^{24}$ kg.

Print out the result in the following format [at least 6 significant digits]

The distance ratio of L1: xxxxx

The distance ratio of L2: xxxxx

The distance ratio of L3: xxxxx

Contents to submit:

1. All the m-files you compose for the assignment. PLEASE DO NOT COMPRESS.
2. All the m-files should include proper COMMENTS.
   (No comment, no score)
3. A PDF includes Your Name, Your Student ID Number,
   In the document, you will need to include all the contents described above including: (1) Which method do you use to solve numerical integration and find the roots. (2) How should TA use the programs to solve any specific integrations or equations when needed (3) The answers including the

plots, the step length you choose for each problem and the explanation of how you justify the precision of the answers should be included.

Notice:

1. DO NOT PLAGIARIZE. You are encouraged to ask and to discuss the homework content with your fellow classmates, the TAs and the instructor. But identical core program wording is NEVER ACCEPTABLE.
2. Upload all the files without archiving. Do not upload files that don’t work well. Any missing file or function that leads to fail of the execution will be regarded as a program that never works.