Hash Tables

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We have done six implementations:

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- Unbalanced binary search tree: add, find, remove, all O(log n) expected time for input in random order but all O(n) worst case time.





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- Balanced binary search tree (DLLTree): add and remove in O(log n) amortised time and find in O(log n) worst case time.





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- Sometimes it is important that the entries be in alphabetical or numerical order.
- When you look up a name, it is nice to be able to go forward or back a few names in case you misspelled it.
- If you want Milenkovic in a hash table, you better not look for Milenkovich because it will be far away.









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- We will look at the one for String:

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public int hashCode()
Returns a hash code for this string. The hash code for a String object is computed as s[0]*31^{(n-1)} + s[1]*31^{(n-2)} + ... + s[n-1]
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using int arithmetic, where s[i] is the ith character of the string, n is the length of the string, and $^{\circ}$ indicates exponentiation. (The hash value of the empty string is zero.)





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WHAT???





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So the letter 'M' is really just a small integer.
If I run:

 String name = "Milenkovic";
 for (int i = 0; i < name.length(); i++) {
 char c = name.charAt(i);
 int n = c;
 System.out.println(c + " " + n);
 }
I get:</pre>



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Hash code of Milenkovic is -1110834957 It's negative? How can that be?

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int code = 0;
for (int i = 0; i < name.length(); i++) {
  char c = name.charAt(i);
  code = 31 * code + c;
  System.out.println("code = " + code);
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Let's do the same trick with "Milenkovic" and print out each step:

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An int can only hold integers in the range from -2147483648 to 2147483647.









Why not just add up the characters? Why the powers of 31?

If we just added up the characters,





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- If we just added up the characters,
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- ▶ But it is worse than that.





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- Using powers of 31 makes these all different.







We start with "Milenkovic" and we get a seemly random 32 bit integer.

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```
int hashIndex (String name, int m) {
  int code = name.hashCode();
  int index = code % m;
  if (index < 0)
     index += m;
  return index;
}</pre>
```





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- -19 divided by 7 is -2 remainder -5, right?
- ▶ But it is also -3 remainder 2. Check -3*7+2=-19.









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- Don't just move the lists!!
- Entries in the same list in the first table will be in different lists in the second table.









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Here is one problem with this scheme.

▶ If I need to find your car, I go to your assigned space.





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- ▶ The latter tells me you are not there.





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- stores a linked list at each index.
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- Item cycles through the array until it finds an empty spot.
- Removed items need to leave a "traffic cone".
- Needs to rehash and possibly reallocate when less than m/2 spaces are empty.



