Tower of Hanoi

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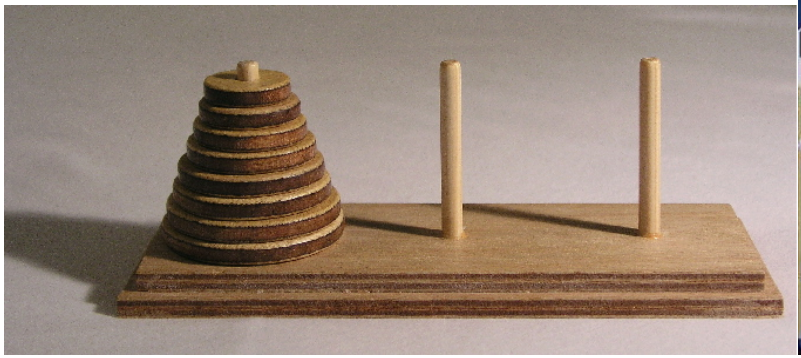
Department of Computer Science
University of Miami

CSC220 Programming II – Spring 2019

Outline

Tower of Hanoi



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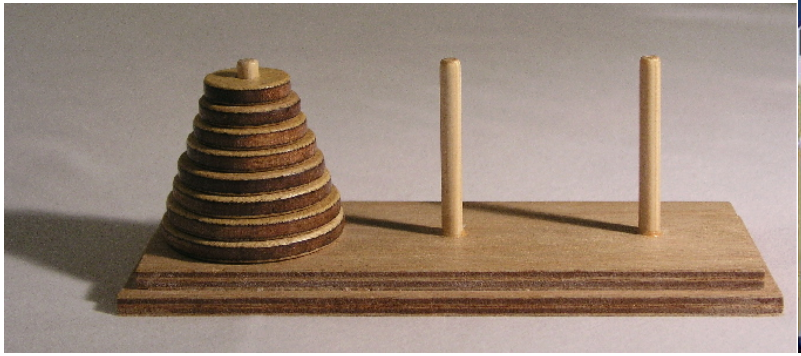

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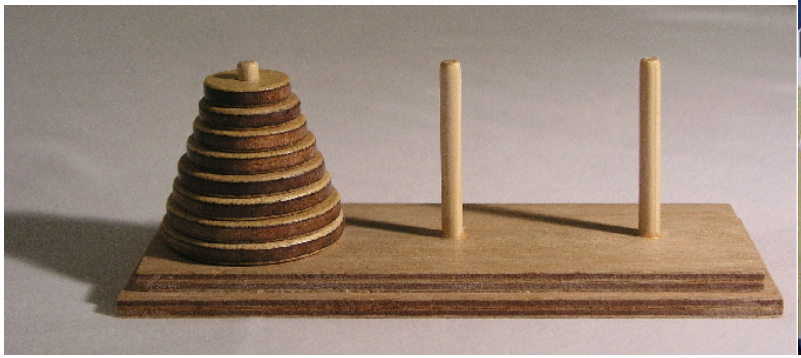

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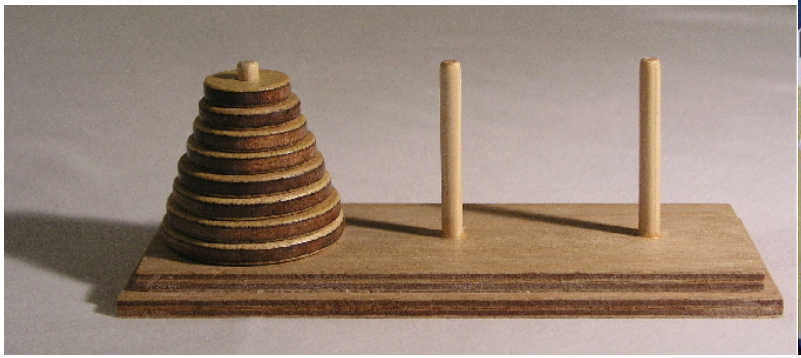

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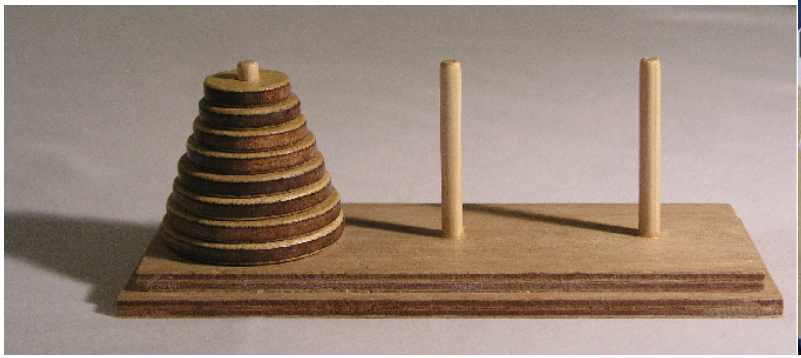

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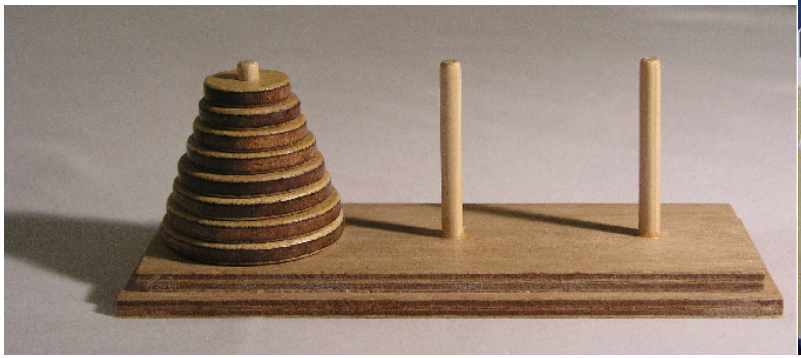

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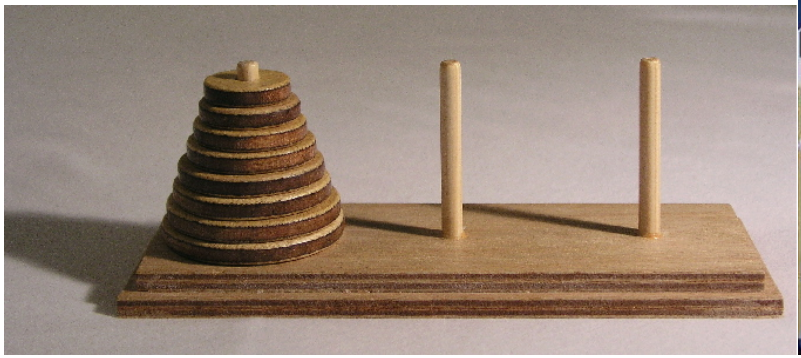

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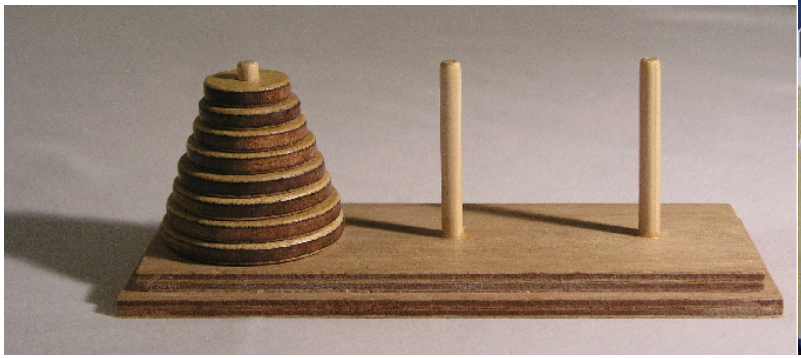

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    - and push the subgoals onto the stack.

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- Any goal with one disk, such as 1ab, is easy, and we can do it right away.
- All others can be broken into three subgoals.

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- Continue on your own!

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- And the stack is StackInt<Goal>.

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- Printing or displaying uses a *helper* stack.
- Solving Tower of Hanoi requires *top-down* planning.
- Which we can implement using a StackInt<Goal>.
- This solution uses $O(2^n)$ time.