Sorting

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Sorting Algorithms

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- We will study four sorting algorithms:

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    - Insertion Sort

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  - Insertion Sort
  - Quick Sort
  - Heap Sort
  - Merge Sort
- Each is useful in a different way.

Insertion Sort

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1 1 3 4 5 9 2 | 6
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- Take the 2 out and put it here: 2
- Copy elements forward until you get to where the 2 goes:

```
1 1 3 4 5 9 9 | 6
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1 1 3 4 4 5 9 | 6
1 1 3 3 4 5 9 | 6
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- Now put the 2 back in:

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- We are ready to insert the 6.

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  - STABLE: doesn’t flip elements if it doesn’t have to
  - IN PLACE: doesn’t require a second array
- Bad:
  - $O(n^2)$ running time in general, so slow on large n when input is not nearly sorted

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  ```
- Pick 3 and partition the others

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- Swap!

```
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    i       j
```

```
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```

Partitioning continued

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- Eventually they pass each other!

```
3 1 2 1 5 9 4 6
        j
        i

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      j i
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3 1 2 1 5 9 4 6
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- Now we swap [0] and [j]

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1 1 2 3 5 9 4 6
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```

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- Recursively sort 0 to j-1 and i to size-1

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- Done!

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- Easy to fix $O(n^2)$ case:
  - just swap first element with random element
  - or just the middle element

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- Remember: even though I am writing it like a tree, it is still just an array.

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1         4
1    5    9    2
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- The 6 has no kids, and neither do 2, 9, nor 5.
- 1 has 6 as a kid, which is o.k.

Heapifying in Place continued

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- 4 has 9 and 2, not good.

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- 4 has 9 and 2, not good.
- Swap 4 and 2.

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- Still not good, swap with 1 again:

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1       2
3   5   9   4
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- Now it is a heap.

Heap Sort Polling Phase

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- Now, let's remove the root and put it in the last element.

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- Now, let's remove the root and put it in the last element.
- We were going to put the 6 at the root for the removal process anyway

Heap Sort Polling Phase

- Now, let's remove the root and put it in the last element.
- We were going to put the 6 at the root for the removal process anyway
- So swap them:

```
6
1         2
3    5    9    4
1
```

Heap Sort Polling Phase

- Now, let's remove the root and put it in the last element.
- We were going to put the 6 at the root for the removal process anyway
- So swap them:

```
6
1         2
3    5    9    4
1
```

- Now swap down the 6, but ignore the 1 at the bottom. (Decrement size.)

```
1
6         2
3    5    9    4
1
```

```
1
3         2
6    5    9    4
1
```

Polling Phase continued

Polling Phase continued

- Swap the 1 and the last element, which is the 4 now, and ignore that 1 thereafter (decrement size):

```
4
3           2
6     5     9     1
1
```

Polling Phase continued

- Swap the 1 and the last element, which is the 4 now, and ignore that 1 thereafter (decrement size):

```
4
3         2
6    5    9    1
1
```

- Fix the 4:

```
2
3         4
6    5    9    1
1
```

Polling Phase continued

- Swap the 1 and the last element, which is the 4 now, and ignore that 1 thereafter (decrement size):

```
4
3         2
6    5    9    1
1
```

- Fix the 4:

```
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```

- Can you continue?

Polling Phase continued

- Swap the 1 and the last element, which is the 4 now, and ignore that 1 thereafter (decrement size):

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- Fix the 4:

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- Can you continue?
- The result is the array sorted in reverse order.

Polling Phase continued

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4
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1
```

- Fix the 4:

```
2
3       4
6   5   9   1
1
```

- Can you continue?
- The result is the array sorted in reverse order.
- But if you can do that, you can do it right!

Heap Sort Properties

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- Good:

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  - Guaranteed $O(n \log n)$

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  - apparently slower than quick sort in practice

Merge Sort

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- Just split the array in two:

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- Just split the array in two:

  `3 1 4 1`
  `5 9 2 6`
- Sort each recursively:

  `1 1 3 4`
  `2 5 6 9`

Merging

Merging

- Now merge them. You only have to look at the front of each list:

```
1 3 4
2 5 6 9
1

3 4
2 5 6 9
1 1

3 4
5 6 9
1 1 2

4
5 6 9
1 1 2 3


5 6 9
1 1 2 3 4
```

Merging continued

Merging continued

- Since the first list is empty, we can just copy the rest of the second list:

  `1 1 2 3 4 5 6 9`

Merge Sort Properties

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  - These are $O(1)$ operations for a linked list.
- So the running time is the same.

External Sorting

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- Open both files and read elements from the files, merging them and writing them out.

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  - Read in 3 and 1 and write out 13.

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- “Deal out” the elements of the file to two different files on different hard disks.
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- “Deal out” the elements of the file to two different files on different hard disks.
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  - 1196
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  - Read in 3 and 1 and write out 13.
  - Read in 4 and 1 and write out 14.
  - Read in 5 and 9 and write out 59.

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  - Read in 2 and 6 and write out 26.

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- “Deal out” to different files:

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- “Deal out” to different files:
  - 1359

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- “Deal out” the elements of the file to two different files on different hard disks.
  - 3452
  - 1196
- Open both files and read elements from the files, merging them and writing them out.
  - Read in 3 and 1 and write out 13.
  - Read in 4 and 1 and write out 14.
  - Read in 5 and 9 and write out 59.
  - Read in 2 and 6 and write out 26.
- “Deal out” to different files:
  - 1359
  - 1426

External Sorting continued

External Sorting continued

- Next we will merge groups of two.

External Sorting continued

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  - Read in 1 and 1.

External Sorting continued

- Next we will merge groups of two.
  - Read in 1 and 1.
  - The first 1 wins the tie so write it out and read in 3.

External Sorting continued

- Next we will merge groups of two.
  - Read in 1 and 1.
  - The first 1 wins the tie so write it out and read in 3.
  - The second 1 is smaller so write it out and read in 4.

External Sorting continued

- Next we will merge groups of two.
  - Read in 1 and 1.
  - The first 1 wins the tie so write it out and read in 3.
  - The second 1 is smaller so write it out and read in 4.
  - The 3 is smaller so write it out, but don't read any more

External Sorting continued

- Next we will merge groups of two.
  - Read in 1 and 1.
  - The first 1 wins the tie so write it out and read in 3.
  - The second 1 is smaller so write it out and read in 4.
  - The 3 is smaller so write it out, but don't read any more
  - because that group of two is done.

External Sorting continued

- Next we will merge groups of two.
  - Read in 1 and 1.
  - The first 1 wins the tie so write it out and read in 3.
  - The second 1 is smaller so write it out and read in 4.
  - The 3 is smaller so write it out, but don't read any more
  - because that group of two is done.
  - The 4 is all we have left, so write it out.

External Sorting continued

- Next we will merge groups of two.
  - Read in 1 and 1.
  - The first 1 wins the tie so write it out and read in 3.
  - The second 1 is smaller so write it out and read in 4.
  - The 3 is smaller so write it out, but don't read any more
  - because that group of two is done.
  - The 4 is all we have left, so write it out.
- So now we have 1134 in one file.

External Sorting continued

- Next we will merge groups of two.
  - Read in 1 and 1.
  - The first 1 wins the tie so write it out and read in 3.
  - The second 1 is smaller so write it out and read in 4.
  - The 3 is smaller so write it out, but don't read any more
  - because that group of two is done.
  - The 4 is all we have left, so write it out.
- So now we have 1134 in one file.
  - Read in the 5 and 2.

External Sorting continued

- Next we will merge groups of two.
  - Read in 1 and 1.
  - The first 1 wins the tie so write it out and read in 3.
  - The second 1 is smaller so write it out and read in 4.
  - The 3 is smaller so write it out, but don't read any more
  - because that group of two is done.
  - The 4 is all we have left, so write it out.
- So now we have 1134 in one file.
  - Read in the 5 and 2.
  - The 2 is smaller so write it out and read in the 6.

External Sorting continued

- Next we will merge groups of two.
  - Read in 1 and 1.
  - The first 1 wins the tie so write it out and read in 3.
  - The second 1 is smaller so write it out and read in 4.
  - The 3 is smaller so write it out, but don't read any more
  - because that group of two is done.
  - The 4 is all we have left, so write it out.
- So now we have 1134 in one file.
  - Read in the 5 and 2.
  - The 2 is smaller so write it out and read in the 6.
  - The 5 is smaller so write it out and read in 9.

External Sorting continued

- Next we will merge groups of two.
  - Read in 1 and 1.
  - The first 1 wins the tie so write it out and read in 3.
  - The second 1 is smaller so write it out and read in 4.
  - The 3 is smaller so write it out, but don't read any more
  - because that group of two is done.
  - The 4 is all we have left, so write it out.
- So now we have 1134 in one file.
  - Read in the 5 and 2.
  - The 2 is smaller so write it out and read in the 6.
  - The 5 is smaller so write it out and read in 9.
  - The 6 is smaller so write it out.

External Sorting continued

- Next we will merge groups of two.
  - Read in 1 and 1.
  - The first 1 wins the tie so write it out and read in 3.
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  - The 3 is smaller so write it out, but don't read any more
  - because that group of two is done.
  - The 4 is all we have left, so write it out.
- So now we have 1134 in one file.
  - Read in the 5 and 2.
  - The 2 is smaller so write it out and read in the 6.
  - The 5 is smaller so write it out and read in 9.
  - The 6 is smaller so write it out.
  - Write out the 9.

External Sorting continued

- Next we will merge groups of two.
  - Read in 1 and 1.
  - The first 1 wins the tie so write it out and read in 3.
  - The second 1 is smaller so write it out and read in 4.
  - The 3 is smaller so write it out, but don't read any more
  - because that group of two is done.
  - The 4 is all we have left, so write it out.
- So now we have 1134 in one file.
  - Read in the 5 and 2.
  - The 2 is smaller so write it out and read in the 6.
  - The 5 is smaller so write it out and read in 9.
  - The 6 is smaller so write it out.
  - Write out the 9.
  - We have 2569.

External Sorting continued

- Next we will merge groups of two.
  - Read in 1 and 1.
  - The first 1 wins the tie so write it out and read in 3.
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- So now we have 1134 in one file.
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  - Write out the 9.
  - We have 2569.
- Now we need to merge 1134 and 2569 into a single file.

External Sorting continued

- Next we will merge groups of two.
  - Read in 1 and 1.
  - The first 1 wins the tie so write it out and read in 3.
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- So now we have 1134 in one file.
  - Read in the 5 and 2.
  - The 2 is smaller so write it out and read in the 6.
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  - Write out the 9.
  - We have 2569.
- Now we need to merge 1134 and 2569 into a single file.
  - Read in the 1 and 2.

External Sorting continued

- Next we will merge groups of two.
  - Read in 1 and 1.
  - The first 1 wins the tie so write it out and read in 3.
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- So now we have 1134 in one file.
  - Read in the 5 and 2.
  - The 2 is smaller so write it out and read in the 6.
  - The 5 is smaller so write it out and read in 9.
  - The 6 is smaller so write it out.
  - Write out the 9.
  - We have 2569.
- Now we need to merge 1134 and 2569 into a single file.
  - Read in the 1 and 2.
  - The 1 is smaller so write it out and read in 1.

External Sorting continued

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  - The first 1 wins the tie so write it out and read in 3.
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- So now we have 1134 in one file.
  - Read in the 5 and 2.
  - The 2 is smaller so write it out and read in the 6.
  - The 5 is smaller so write it out and read in 9.
  - The 6 is smaller so write it out.
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- Now we need to merge 1134 and 2569 into a single file.
  - Read in the 1 and 2.
  - The 1 is smaller so write it out and read in 1.
  - The 1 is smaller so write it out and read in 3.

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  - The 1 is smaller so write it out and read in 1.
  - The 1 is smaller so write it out and read in 3.
  - The 2 is smaller so write it out and read in 5.

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  - The 2 is smaller so write it out and read in 5.
  - The 3 is smaller so write it out and read in 4.
  - The 4 is smaller so write it out.

External Sorting continued

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  - Read in 1 and 1.
  - The first 1 wins the tie so write it out and read in 3.
  - The second 1 is smaller so write it out and read in 4.
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  - The 1 is smaller so write it out and read in 3.
  - The 2 is smaller so write it out and read in 5.
  - The 3 is smaller so write it out and read in 4.
  - The 4 is smaller so write it out.
  - Write out the 5 and read in the 6.

External Sorting continued

- Next we will merge groups of two.
  - Read in 1 and 1.
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  - The second 1 is smaller so write it out and read in 4.
  - The 3 is smaller so write it out, but don't read any more
  - because that group of two is done.
  - The 4 is all we have left, so write it out.
- So now we have 1134 in one file.
  - Read in the 5 and 2.
  - The 2 is smaller so write it out and read in the 6.
  - The 5 is smaller so write it out and read in 9.
  - The 6 is smaller so write it out.
  - Write out the 9.
  - We have 2569.
- Now we need to merge 1134 and 2569 into a single file.
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- Result: 11234569.

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