Binary Trees, Binary Search Trees, and Heaps

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CSC220 Programming II – Spring 2019

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- Possible list orders:

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- Possible list orders:
  - unsorted,

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- Possible list orders:
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  - unsorted,
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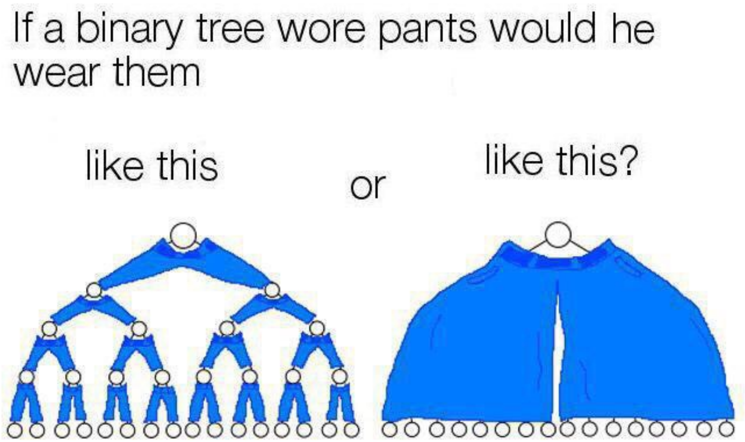
If a binary tree wore pants would he wear them
like this
or
like this?

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  - All levels except the bottom level are full.

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Tree Representations

- Linked
  - **root** variable points to root.
  - **node.left** and **node.right** point to left and right child of **node**.
  - Either one can be null.
- Array
  - **theItems** is an array of items.
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  - For example, at a hospital emergency room serve in order of minutes until death.

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- We will learn about tree balancing in prog08.

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  - The running time is $O(\log n)$.
- Cost of **poll**.
  - Swap with a child up to $\log_2 n$ times
  - The running time is $O(\log n)$.
- **peek** is obviously $O(1)$.

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  - Offer and poll are $\mathrm{O}(\log n)$.