XJCO3910 – Combinatorial Optimisation

Coursework 2: Modelling; Strong Formulations; B&B

Deadline:	1700 GMT on Wednesday 12 April 2023
Award:	This piece of summative coursework is worth 10% of your grade
Submission:	1) Write or type your answers inside the boxes of the current document. 2) Scan **pages 1-4** using the MyPrint portal, including p. 1 (see instructions at https://it.leeds.ac.uk/it?id=kb_article&sysparm_article=KB0012731) 3) Upload one pdf-file containing **pages 1-4** to Gradescope (see instructions at https://help.gradescope.com/article/ccbpppziu9-student-submit-work#submitting_a_pdf)

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(same as in Gradescope)

A furniture company needs to make a decision about the mix of products to be manufactured next week. It has seven types of desks, each with a profit (£) per unit and a production time (man-hours) per unit as shown below: **[12 marks]**

	Desk 1	Desk 2	Desk 3	Desk 4	Desk 5	Desk 6	Desk 7
Profit (£ per unit)	10	22	35	19	55	10	115
Production time (man-hours per unit)	1.5	2.0	3.7	2.4	4.5	0.7	9.5

The company has 720 man-hours available in the coming week.

The following restrictions are also placed on the production.

R1: If any amount of Desk 7 is to be produced, then an additional fixed cost of £2000 is incurred.

R2: If any amount of Desk 6 is to be produced, then 80 man-hours are needed for production line set-up and hence the (effective) number of man-hours available falls to 720-80=640.

R3: If any amount of Desk 1 is to be produced, then Desks 2 and 3 cannot be produced.

R4: If Desk 4 is to be produced, then at least 10 desks of that type should be produced.

Formulate an ILP to determine the quantities of desks to be produced in order to maximise the profit. Use the following variables:

- x_i ($x_i \geq 0$ and integer) for the number desks of type i to be produced ($i = 1,2,\dots,7$),
- δ_i ($\delta_i \in \{0,1\}$) to indicate whether desk i is to be produced ($\delta_i = 1$) or not ($\delta_i = 0$).

Present an ILP, without explaining how it is derived. If the model uses some numbers which are calculated based on the problem input, then present the formulae (no need to explain). As a sample answer, consider the final model for Problem 2 from Worksheet 5-6 (p. 11 of the solutions file). Do not include unnecessary constraints and unnecessary variables.

maxmise $z = 10x_1 + 22x_2 + 35x_3 + 19x_4 + 55x_5 + 10x_6 + 115x_7 - 2000\delta_7$

$1.5x_1 + 2.0x_2 + 3.7x_3 + 2.4x_4 + 4.5x_5 + 0.7x_6 + 9.5x_7 \leq 720 - 80\delta_6$

$\delta_1 + \delta_2 \leq 1$ $\quad\quad$ $\delta_1 + \delta_3 \leq 1$ $\quad\quad$ $x_4 \geq 10\delta x_4$

$x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0$, and is integer.

$\delta_1, \delta_2, \delta_3, \delta_4, \delta_5, \delta_6, \delta_7 \in \{0,1\}$

(do not solve the formulated ILP)

Question 2

Consider the Production Planning problem (Example 7) from Chapter 6, and the corresponding MILP: **[5 marks]**

$$
\begin{array}{llllllllll}
\max\ z = & 5x_1 & +7x_2 & +3x_3 & & & & & & \\
\text{s.t.} & 3x_1 & +4x_2 & +2x_3 & -36\delta & & & & \leq & 30, \\
 & 4x_1 & +6x_2 & +2x_3 & +36\delta & & & & \leq & 76, \\
 & x_1 & & & & & & & \leq & 7, \\
 & & x_2 & & & & & & \leq & 5, \\
 & & & x_3 & & & & & \leq & 9, \\
 & x_1 & & & & -9\delta_1 & & & \leq & 0, \\
 & & x_2 & & & & -9\delta_2 & & \leq & 0, \\
 & & & x_3 & & & & -9\delta_3 & \leq & 0, \\
 & & & & & \delta_1 & +\delta_2 & +\delta_3 & \leq & 2, \\
 & x_1, x_2, x_3 \geq 0, & & & & & & & & \\
 & \delta, \delta_1, \delta_2, \delta_3 \in \{0,1\}. & & & & & & & &
\end{array}
$$

The disjunctive constraint

$$3x_1 + 4x_2 + 2x_3 \leq 30$$
or
$$4x_1 + 6x_2 + 2x_3 \leq 40$$

are modelled as

$$3x_1 + 4x_2 + 2x_3 \leq 30 + M'\delta, \quad (1)$$
$$4x_1 + 6x_2 + 2x_3 \leq 40 + M'(1 - \delta). \quad (2)$$

One possible choice is $M' = 36$ (see Chapter 6). In fact a smaller value of M' can be selected. Find such a value. Provide a justification.

$x_1 \leq 7$, $x_2 \leq 5$, $x_3 \leq 9$, $\delta_1 + \delta_2 + \delta_3 \leq 2$, the left hand sales satisfy

$$
\begin{cases}
3x_1 + 4x_2 \leq 41 \\
3x_1 + 2x_3 \leq 39 \\
4x_2 + 2x_3 \leq 38 \\
4x_1 + 6x_2 \leq 58 \\
4x_1 + 2x_3 \leq 46 \\
6x_2 + 2x_3 \leq 48
\end{cases}
\quad \text{then} \quad
\begin{cases}
41 \leq 30 + m' \\
39 \leq 30 + m' \\
38 \leq 30 + m'
\end{cases}
\quad
\begin{cases}
58 \leq 40 + m' \\
46 \leq 40 + m' \\
48 \leq 40 + m'
\end{cases}
\Rightarrow
\begin{array}{l}
m' \geq \max(11, 9, 8, 18, 6, 8) \\
\quad = 18 \\
m' \geq 18
\end{array}
$$

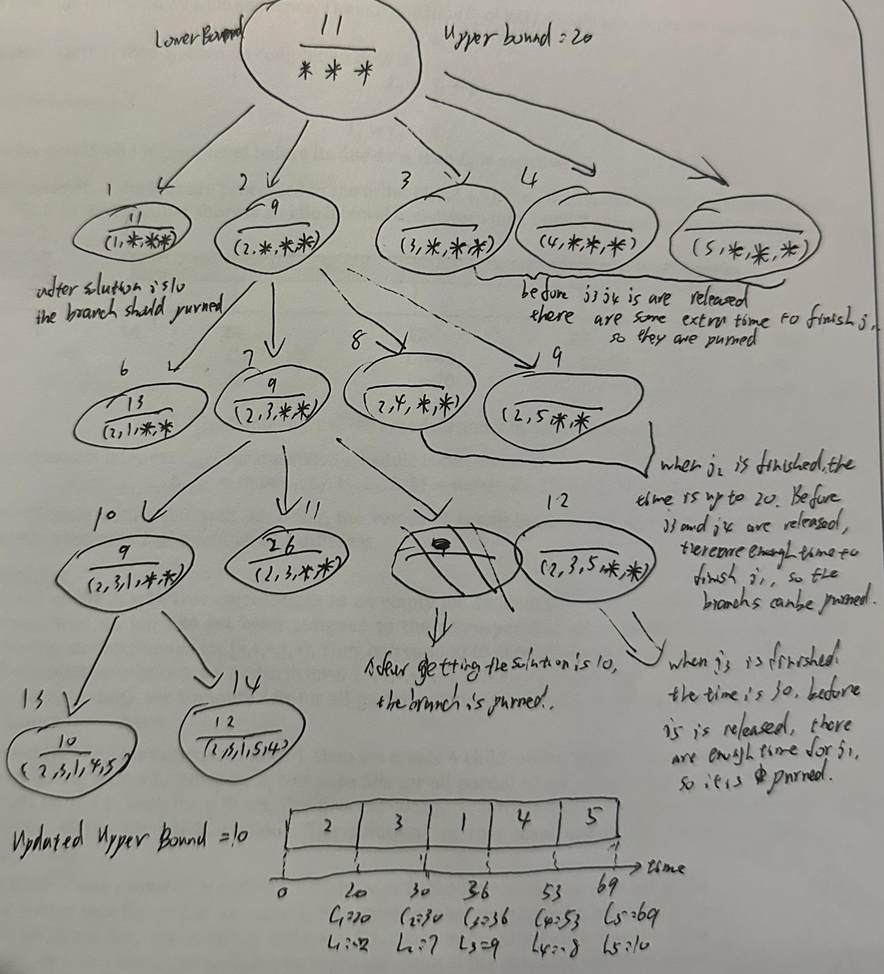
Question 3

Consider an instance of the scheduling problem given by the table: **[23 marks]**

Job j	1	2	3	4	5
Release time r_j	0	2	14	27	40
Processing time p_j	6	18	10	17	16
Due date d_j	27	22	23	61	59

Find an optimal solution by the branch-and-bound algorithm. Present the search tree. Use the depth-first strategy, selecting each time the most promising node among **all** child nodes of the same parent node. Number the nodes of the tree in the order they are obtained. State the optimal solution and the optimal objective value.

The full description of the problem is given on pages 5-6, together with the details of the B&B algorithm. Present the search tree. Number the nodes of the tree in the order they are obtained. Mark the nodes by corresponding partial schedules. For example, the node marked by (*,*,*,*,*) is the root of the tree; the node marked by (1,*,*,*,*) is one of the child nodes. For each node either specify its LB or explain why the node is discarded. State the initial UB and indicate how it is updated. Mark the node corresponding to the optimal schedule. Output a job permutation which defines an optimal schedule and the optimal objective value.



The LowerBound of (2,3,1,4,5) :

$LB = \max\{L', L\} = \max\{-2, 7, 9, -8, 10\} = 10$