

# XJCO3910 – Combinatorial Optimisation

## Coursework 1: Modelling; Simplex Method

**Deadline:** 2300 GMT on Wednesday 29 March 2023

**Award:** This piece of summative coursework is worth 10% of your grade

**Submission:**

- 1) Write or type your answers inside the boxes of the current document.
- 2) Scan **pages 1-4** using the MyPrint portal, including p. 1 (see instructions at [https://it.leeds.ac.uk/it?id=kb\\_article&sysparm\\_article=KB0012731](https://it.leeds.ac.uk/it?id=kb_article&sysparm_article=KB0012731))
- 3) Upload one pdf-file containing **pages 1-4** to Gradescope (see instructions at [https://help.gradescope.com/article/ccbpppziu9-student-submit-work#submitting\\_a\\_pdf](https://help.gradescope.com/article/ccbpppziu9-student-submit-work#submitting_a_pdf))

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**Question 1.****[10 marks]**

A manufacturer needs to produce a health-drink satisfying the following vitamin requirements: 1 ml of the drink should contain at least 6 mg of vitamin A, 7 mg of vitamin B and 10 mg of vitamin C.

There are two concentrates, concentrate X and concentrate Y, that can be mixed with water. The contents of vitamins A, B and C in concentrate X are 15, 14, 50 mg per ml, respectively. The contents of the vitamins in concentrate Y are 15, 28, 20 mg per ml, respectively. Concentrate X costs 0.4 pence per ml, and concentrate Y costs 0.5 pence per ml. We assume that water does not cost anything. See the table below for the summary of the input data.

	Concentrate X	Concentrate Y	Minimum quantity of vitamins required
Vitamin A	15 mg/ml	15 mg/ml	6 mg/ml
Vitamin B	14 mg/ml	28 mg/ml	7 mg/ml
Vitamin C	50 mg/ml	20 mg/ml	10 mg/ml
Cost	0.4 pence/ml	0.5 pence/ml	N/A

Model the problem of finding the quantities of concentrates X and Y and the quantity of water needed to produce 1 bottle of 1000 ml of the drink, so that the cost is minimised.

The variables introduced and their meaning:

The LP model: (please present the model in the structured way)

(do not solve the formulated LP)

Provide an explanation for the constraint which ensures that "1 ml of the drink should contain at least 6 mg of vitamin A"

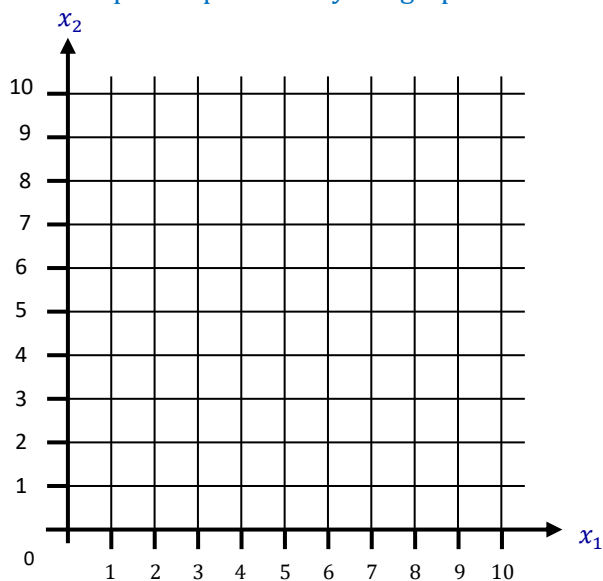
**Question 2** deals with a pair of LP problems A and B, primal and dual. Instructions are on p. 5.

**[20 marks]**

**Problem A (primal):**

$$\begin{array}{llll} \max & z = & 20x_1 & -10x_2 \\ \text{s. t.} & & 3x_1 & -x_2 \leq 8 \\ & & 4x_1 & -3x_2 \leq 4 \\ & & x_1, & x_2 \geq 0 \end{array}$$

Solve the primal problem by the graphical method<sup>a,c</sup>



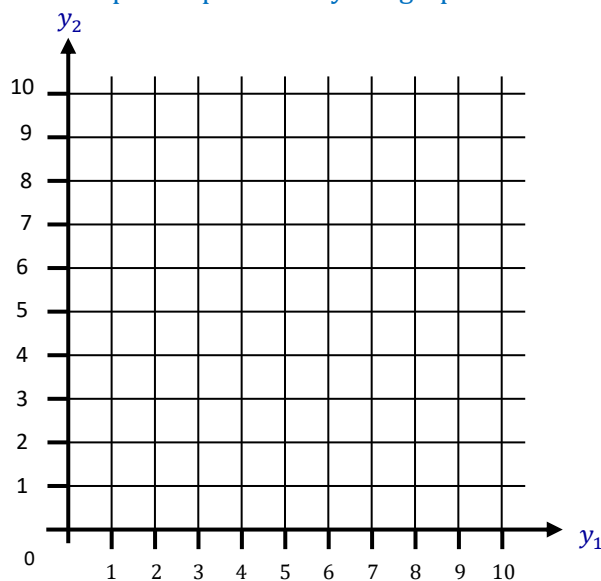
Solve Problem A by the simplex method<sup>b</sup>

The optimal solution is .....  
The optimal value of the objective function is .....

**Problem B (dual):**

$$\begin{array}{llll} \min & v = & 8y_1 & +4y_2 \\ \text{s. t.} & & 3y_1 & +4y_2 \geq 20 \\ & & -y_1 & -3y_2 \geq -10 \\ & & y_1, & y_2 \geq 0 \end{array}$$

Solve the primal problem by the graphical method<sup>a,c</sup>



Solve problem B by the dual simplex method<sup>b</sup>

The optimal solution is .....  
The optimal value of the objective function is .....

**Question 3****[10 marks]**

Consider again Problem B from Question 2:

**Problem B**

min	$v =$	$8y_1$	$+4y_2$		
s. t.		$3y_1$	$+4y_2$	$\geq$	20
		$-y_1$	$-3y_2$	$\geq$	-10
		$y_1,$	$y_2$	$\geq$	0

It can be solved by the two-phase simplex method.

Derive the first tableau of the 1<sup>st</sup> phase. Show all your working. *Do not solve the produced LP.*

Deriving the first tableau:

The tableau:

## Instructions for Question 2.

- <sup>a</sup> For the graphical solutions to Problem A and problem B,
- mark the solution region;
  - for each line, specify two points it passes through;
  - specify coordinates of the corner points of the feasible region;
  - present the line for the objective function, together with the equation used to plot it, and indicate the direction in which the line should be moved;
  - mark the point which defines the optimum and specify its coordinates.
- <sup>b</sup> Use the primal simplex method for problem A in the tableau format and the dual method for problem B, also in the tableau format. Present all tableaux, indicating the choice of the pivot row, pivot column, and the pivot element. For each problem, state the final answer.
- <sup>c</sup> After solving the problems analytically, reconsider the solutions found graphically.
- On the plot for the primal problem, mark the solutions obtained at all iterations of the primal simplex method and indicate the order in which they are obtained.
  - On the plot for the dual problem, mark the solutions obtained at all iterations of the dual simplex method and indicate the order in which they are obtained.

**END**