Causal Distillation for Language Models

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Abstract

Distillation efforts have led to language models that are more compact and efficient without serious drops in performance. The standard approach to distillation trains a student model against two objectives: a task-specific objective (e.g., language modeling) and an imitation objective that encourages the hidden states of the student model to be similar to those of the larger teacher model. In this paper, we show that it is beneficial to augment distillation with a third objective that encourages the student to imitate the *causal* computation process of the teacher through *interchange intervention training* (IIT). IIT pushes the student model to become a *causal abstraction* of the teacher model – a simpler model with the same causal structure. IIT is fully differentiable, easily implemented, and combines flexibly with other objectives. Compared with standard distillation of BERT, distillation via IIT results in lower perplexity on Wikipedia (masked language modeling) and marked improvements on the GLUE benchmark (natural language understanding), SQuAD (question answering), and CoNLL-2003 (named entity recognition).

1 Introduction

Large pretrained language models have improved performance across a wide range of tasks in NLP, but they have also brought increased costs due to their very large size. *Distillation* seeks to reduce these costs while maintaining the performance improvements by training a simpler student model from a larger teacher model (Hinton et al., 2015; Sun et al., 2019; Sanh et al., 2019; Jiao et al., 2019).

Hinton et al. (2015) propose model distillation with an objective that encourages the student to produce output logits similar to those of the teacher while also supervising with a task-specific objective (e.g., sequence classification). Sanh et al. (2019), Sun et al. (2019), and Jiao et al. (2019) adapt this method, strengthening it with additional supervision to align internal representations between the two models. However, these approaches may push the student model to match all aspects of internal states of the teacher model irrespective of their causal roles (Geiger et al., 2021a). This motivates us to develop a method that focuses on aligning the *causal* role of representations between the student and the teacher model.

We propose augmenting standard distillation with a new objective that pushes the student model to become a *causal abstraction* of the teacher model – a simpler model that has approximately the same overall *causal* structure (Beckers and Halpern, 2019; Beckers et al., 2020; Geiger et al., 2020). To achieve this, we employ the *interchange intervention training* (IIT) method of Geiger et al. (2021b). In IIT, one aligns a high-level causal model with a low-level neural model and then performs *interchange interventions* (swapping of aligned internal states) on the neural model during training, guided by the output behavior of the high-level model. This has the effect of pushing the low-level model to conform to the causal dynamics of the high-level model.

Adapting IIT to model distillation is straightforward: in addition to standard distillation objectives, we use IIT to push the student model to match the causal dynamics of the teacher model. Figure 1 shows a schematic example of how this happens. Here, hidden layer 2 of the student model (bottom) is aligned with layers 3 and 4 of the teacher model. The figure depicts a single interchange intervention replacing aligned states in the left-hand models with those from the right-hand models. This is akin to creating two new examples that are shaped partially by the original inputs and partially by the interchanged hidden states. It can be interpreted as a certain kind of counterfactual as shown in Figure 1: what would the output be for the sentence "I ate some ⟨MASK⟩." if the activation values for

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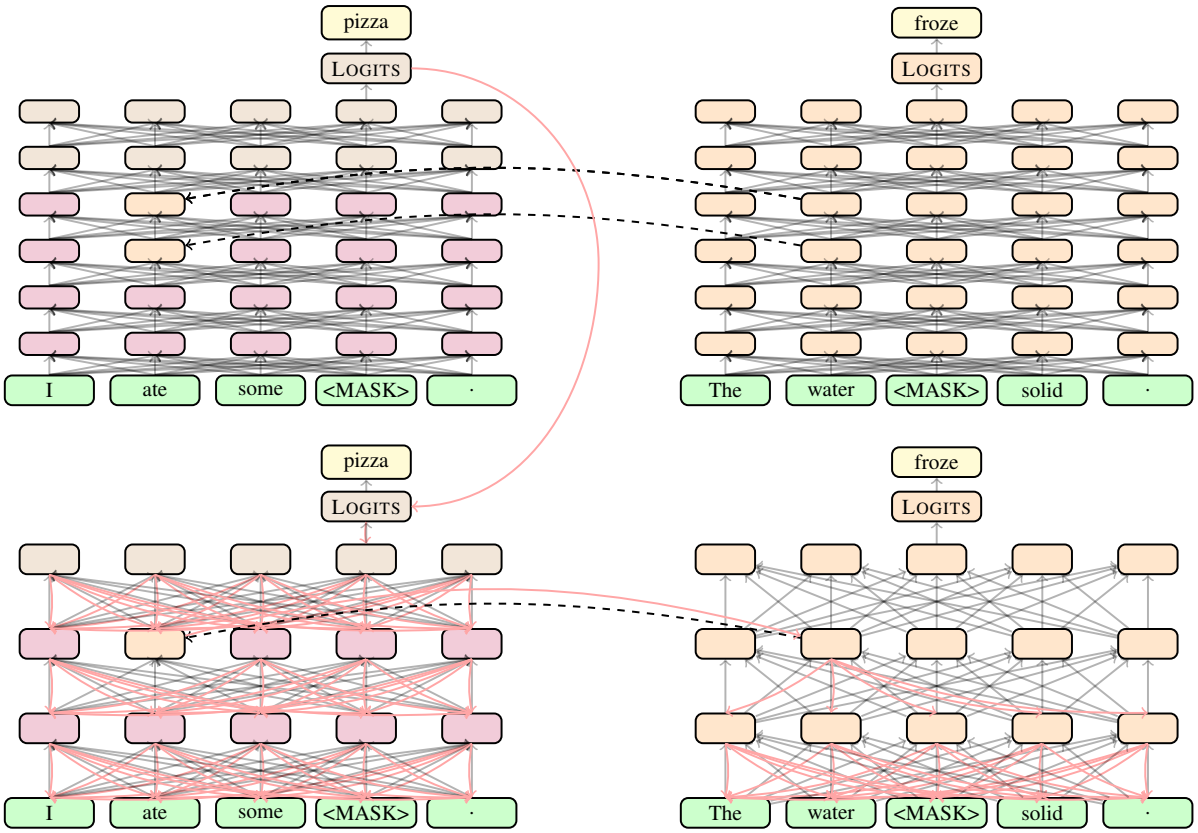

Figure 1: A IIT update in the context of masked language modelling (MLM). The teacher network (top) has 6 layers and the student (bottom) has 3 layers, and we align layer 2 in the student with layers 3-4 in the teacher. Solid lines are feed-forward connections, red lines show the flow of backpropagation, and dashed lines indicate interchange interventions. In this case, the student originally predicted some logits $\mathbf{y}$ and, in turn, some other token (say, "lettuce") after the intervention. IIT trains the student to minimize the divergence between $\mathbf{y}$ and the logits from the teacher model after the intervention. This has the effect of updating the student to conform to causal dynamics of the teacher. The weights below the interchange intervention are updated twice (i.e, once for each input).

the second token at the middle two layers were set to the values they have for the input "The water ⟨MASK⟩ solid."? The IIT distillation objective then pushes the student model to output the same logits as the teacher, i.e., matching the teacher's output distribution under the counterfactual setup.

To assess the contribution of distillation with IIT, we begin with BERT-base (Devlin et al., 2019a) and distill it under various alignments between student and teacher, and we assess the results in a language modeling task (Wikipedia), the GLUE benchmark, SQuAD, and CoNLL-2003 name-entity recognition. All of the alignments we explore lead to marked improvements across all these tasks as compared to standard distillation, and we see the best results with the richest alignment we explore.[1]

[1]We release our code at https://github.com/frankaging/Causal-Distill.

2 Related Work

Pretrained Model Compression Numerous methods have been developed for compressing large-scale pretrained language models, including architecture pruning (Cui et al., 2019; McCarley, 2019), weight sharing and compression (Dehghani et al., 2018; Ma et al., 2019), knowledge distillation (Sun et al., 2019; Sanh et al., 2019; Jiao et al., 2019), and quantization (Shen et al., 2020).

Distillation Distillation was first introduced in the context of computer vision (Hinton et al., 2015) and has since been widely explored for language models (Sun et al., 2019; Sanh et al., 2019; Jiao et al., 2019). For example, Sanh et al. (2019) propose to extract information not only from output probabilities of the last layer in the teacher model, but also from intermediate layers in fine-tuning stage. Recently, Rotman et al. (2021) adapt causal analysis methods to estimate the effects of inputs

on predictions to compress models for better domain adaption. In contrast, we focus on learning the causal structure of the teacher through interventions on hidden representations.

Causal Interventions on Neural Networks Causal interventions on neural networks were originally developed as a structural analysis method aimed at illuminating neural representations and their role in network behavior (Feder et al., 2021; Pryzant et al., 2021; Vig et al., 2020; Elazar et al., 2020; Giulianelli et al., 2020; Geiger et al., 2020, 2021a). Geiger et al. (2021b) extend these methods to the optimization process. The central contribution of the current paper is adapting this optimization method to language model distillation.

3 Causal Distillation via Interchange Intervention Training

We first describe the standard distillation method for BERT compression, adapted from Sanh et al. (2019), and then we present our new distillation objective with IIT.

3.1 Standard Distillation

We adopt the three standard distillation objectives from Sanh et al. 2019 (defined formally in Appendix A.1):

$\mathcal{L}_{\text{MLM}}$ The task-specific loss for the student model. We use the masked language modeling (cross-entropy) loss of the student model calculated over all examples. This represents model performance on masked token predictions.

$\mathcal{L}_{\text{CE}}$ Following Hinton et al. (2015), the smoothed cross-entropy loss measuring the divergence between the student and teacher outputs on masked tokens. This pushes the student model to imitate the teacher model with output logits.

$\mathcal{L}_{\text{Cos}}$ The cosine embedding loss defined in terms of the cosine similarities between the contextualized representations of the teacher and the student on masked tokens in the last layer. This pushes the student model to align the directions of its late representations with the ones produced by the teacher model.

3.2 Interchange Intervention Training

In this section, we formally define our distillation training procedure and provide a simplified implementation in Algorithm 1.

The GETVALS Operator The GETVALS operator is an activation value retriever for a neural model. Given a neural model $\mathcal{M}$ containing a set of neurons $\mathbf{N}$ (a set of internal representations or subparts of representations) and an appropriate input $\mathbf{x}_i$, $\text{GETVALS}(\mathcal{M}, \mathbf{x}_i, \mathbf{N})$ is the set of values that $\mathbf{N}$ takes on when processing $\mathbf{x}_i$. In the case that $\mathbf{N}^{\mathbf{y}}$ represents the neurons corresponding to the final output, $\text{GETVALS}(\mathcal{M}, \mathbf{x}_i, \mathbf{N}^{\mathbf{y}})$ is the output of model $\mathcal{M}$ when processing $\mathbf{x}_i$ (i.e., output from a standard forward call of a neural model).

The SETVALS Operator The SETVALS operator is a function generator that defines a new neural model with a computation graph that specifies an intervention on the original model $\mathcal{M}$ Pearl (2001). $\text{SETVALS}(\mathcal{M}, \mathbf{N}, \mathbf{v})$ is the new neural model where the neurons $\mathbf{N}$ are set to constant values $\mathbf{v}$. In practice, we overwrite neurons with $\mathbf{v}$ in-place, which allows gradients to back-propagate through $\mathbf{v}$.

Interchange Intervention An interchange intervention is a simple combination of GETVALS and SETVALS operations. First, for a training dataset $\mathcal{D}$, we randomly shuffle the dataset to form our $\mathcal{D}'$, which contains the same set of examples but with a different order. We then draw two examples in sequence from these two sets independently as $\{\mathbf{x}_i, \mathbf{y}_i\}$ and $\{\mathbf{x}_j, \mathbf{y}_j\}$ for $i, j \in [1, |\mathcal{D}|]$.[2] Next, where $\mathbf{N}$ is the set of neurons that we are targeting for intervention, we use $\mathcal{M}_{\mathbf{N}}^{\mathbf{x}_i}$ to abbreviate the new neural model as follows:

$$\text{SETVALS}\big(\mathcal{M}, \mathbf{N}, \text{GETVALS}(\mathcal{M}, \mathbf{x}_i, \mathbf{N})\big) \quad (1)$$

This is the version of $\mathcal{M}$ obtained from setting the values of $\mathbf{N}$ to be those we get from processing input $\mathbf{x}_i$. The interchange intervention targeting $\mathbf{N}$ with $\mathbf{x}_i$ as the source input and $\mathbf{x}_j$ as the base input[3] is then defined as follows:

$$\text{INTINV}(\mathcal{M}, \mathbf{N}, \mathbf{x}_i, \mathbf{x}_j) \stackrel{\text{def}}{=} \text{GETVALS}(\mathcal{M}_{\mathbf{N}}^{\mathbf{x}_i}, \mathbf{x}_j, \mathbf{N}^{\mathbf{y}}) \quad (2)$$

where $\mathbf{N}^{\mathbf{y}}$ is the output neurons for $\mathcal{M}$. In other words, $\text{INTINV}(\mathcal{M}, \mathbf{N}, \mathbf{x}_i, \mathbf{x}_j)$ is the output state we get from $\mathcal{M}$ for example $\mathbf{x}_j$ but with the neurons $\mathbf{N}$ set to the values obtained when processing input $\mathbf{x}_i$.

[2] It is possible but highly unlikely that $\{\mathbf{x}_i, \mathbf{y}_i\}$ is the same as $\{\mathbf{x}_j, \mathbf{y}_j\}$ for $i, j \in [1, |\mathcal{D}|]$ as $|\mathcal{D}|$ is large. The expected probability of this event is $\frac{1}{|\mathcal{D}|}$.

[3] For simplicity, standard distillation objectives are not calculated over the base input.

Algorithm 1 Causal Distillation via Interchange Intervention Training

Require: Student model $\mathcal{S}$, teacher model $\mathcal{T}$, student neurons $\mathbf{N}_\mathcal{S}$, student output neurons $\mathbf{N}^\mathbf{y}_\mathcal{S}$, alignment Π, shuffled training dataset $\mathcal{D}$.

$\mathcal{S}$.train()
$\mathcal{T}$.eval()
$\mathcal{D}'$ = random.shuffle($\mathcal{D}$)
$\mathbf{N}_\mathcal{T} = \Pi(\mathbf{N}_\mathcal{S})$
$\mathbf{N}^\mathbf{y}_\mathcal{T} = \Pi(\mathbf{N}^\mathbf{y}_\mathcal{S})$
while not converged **do**
  // input and output pairs
  for $\{\mathbf{x}_i, \mathbf{y}_i\}$, $\{\mathbf{x}_j, \mathbf{y}_j\}$ **in** iter($\mathcal{D}$, $\mathcal{D}'$) **do**
    with no_grad:
      $\mathcal{T}_a = \textsc{SetVals}($
        $\mathcal{T}, \mathbf{N}_\mathcal{T}, \textsc{GetVals}(\mathcal{T}, \mathbf{x}_i, \mathbf{N}_\mathcal{T}))$
      $o_\mathcal{T} = \textsc{GetVals}(\mathcal{T}_a, \mathbf{x}_j, \mathbf{N}^\mathbf{y}_\mathcal{T})$
    $\mathcal{S}_a = \textsc{SetVals}($
      $\mathcal{S}, \mathbf{N}_\mathcal{S}, \textsc{GetVals}(\mathcal{S}, \mathbf{x}_i, \mathbf{N}_\mathcal{S}))$
    $o_\mathcal{S} = \textsc{GetVals}(\mathcal{S}_a, \mathbf{x}_j, \mathbf{N}^\mathbf{y}_\mathcal{S})$
    $\mathcal{L}_{\text{Causal}} =$ **get_loss**$(o_\mathcal{T}, o_\mathcal{S})$
    Calculate $\mathcal{L}_{\text{MLM}}$, $\mathcal{L}_{\text{CE}}$, $\mathcal{L}_{\text{Cos}}$
    $\mathcal{L}$ = $\mathcal{L}_{\text{MLM}}$ + $\mathcal{L}_{\text{CE}}$ + $\mathcal{L}_{\text{Cos}}$ + $\mathcal{L}_{\text{Causal}}$
    $\mathcal{L}$.backward()
    Step optimizer
  end while

IIT Objective The IIT objective employs $\mathcal{T}$ as the teacher model, $\mathcal{S}$ as the student model, $\mathcal{D}$ as the training inputs to both models, and Π as an alignment that maps sets of student neurons to sets of teacher neurons. For each set of student neurons $\mathbf{N}_S$ in the domain of Π, we define the IIT objective as follows:

$$\mathcal{L}_{\text{Causal}} \overset{\text{def}}{=} \sum_{\mathbf{x}_i, \mathbf{x}_j \in \mathcal{D}} \text{CE}_\text{S}\Big(\textsc{IntInv}(\mathcal{S}, \mathbf{N}_\mathcal{S}, \mathbf{x}_i, \mathbf{x}_j), \textsc{IntInv}(\mathcal{T}, \Pi(\mathbf{N}_\mathcal{S}), \mathbf{x}_i, \mathbf{x}_j)\Big) \quad (3)$$

where CE_S is the smoothed cross-entropy loss measuring the divergences of predictions, under interchange, between the teacher and the student model.

Distillation Objectives Our final training objective for the student is a linear combination of the four training objectives reviewed above: $\mathcal{L}_{\text{MLM}}$, $\mathcal{L}_{\text{Actual}}$, $\mathcal{L}_{\text{Cos}}$, and $\mathcal{L}_{\text{Causal}}$.

4 Experimental Set-up

Student and Teacher Models Our student has the standard BERT architecture, with 3 layers and 12 heads with a hidden dimension of 768. [4] Our pretrained teacher has the same architecture, except with 12 layers. Following practices introduced by Sanh et al. (2019), we initialize our student model with weights from skipped layers (i.e., one out of four layers) in the teacher model.

Alignment Our teacher and student BERT models can both be understood as having columns of neural representations above each token, with L rows (layer) and M columns (sequence length), as in Figure 1.[5] For alignment Π, we map student representations at selected row $a \in [1, L_\mathcal{S}]$ to the teacher representations at selected rows $b \in [1, L_\mathcal{T}]$ through $c \in [1, L_\mathcal{T}]$. Additionally, we may select multiple rows in the student model for different alignments in the teacher model. In case of multiple alignments, we randomly select one alignment at each training iteration for intervention.

We experiment with three different alignments:

`FULL` Each layer a in the student is aligned with layers $(a-1) \times 4 + 1$ to $a \times 4 + 1$ in the teacher.

`MIDDLE` The single middle layer $a = L_\mathcal{S} \mathbin{/\!/} 2$ in the student is aligned with the single middle layer $b = L_\mathcal{T} \mathbin{/\!/} 2$ in the teacher.

`LATE` The first layer in the student is aligned with layer $b = L_\mathcal{T} - 2$ in the teacher, and the second layer in the student is aligned with the second to last layer $c = L_\mathcal{T} - 1$ in the teacher.

For each, we align neurons after the feed-forward layer at each Transformer block.[6] For each training iteration, we randomly select one aligned student layer to perform the interchange intervention, and we randomly select 30% of token embeddings for alignment for each sequence. For simplicity, we select consecutive tokens.

Distillation We adapt the open-source Huggingface implementation for model distillation (Wolf et al., 2020).[7] We distill our models on the MLM pretraining task (Devlin et al., 2019b). We use large gradient accumulations over batches as in Sanh

[4] The standard distilBERT introduced by Sanh et al. (2019) has the same architecture but with 6 layers. Here, we experiment with a more extreme case with a smaller model.

[5] We use subscripts to differentiate the rows and columns for the teacher (i.e., $L_\mathcal{T}$ and $M_\mathcal{T}$) and the student (i.e., $L_\mathcal{S}$ and $M_\mathcal{S}$) models.

[6] Different mappings may result in different computational costs for the new gradient computation graph.

[7] https://github.com/huggingface/transformers

Model	**Score**	CoLA	MNLI	MNLI-mm	MRPC	QNLI	QQP	RTE	SST-2	STS-B	WNLI
Standard	67.81	22.78	71.55	72.75	78.17	82.12	84.27	55.43	86.47	56.73	**24.23**
MIDDLE	69.63	23.21	**72.97**	**73.98**	**78.75**	**83.15**	84.85	55.98	86.52	**67.23**	23.94
LATE	69.35	24.12	72.80	73.85	77.96	82.88	**84.88**	**57.29**	**87.31**	63.03	21.41
FULL	**69.66**	**25.01**	72.85	73.78	78.59	83.05	84.85	55.37	86.92	66.51	21.50

Table 1: Model performance results on the development sets of the GLUE benchmark. The GLUE score is the average of all performance scores excluding WNLI. Numbers with the best aggregated performance are bolded.

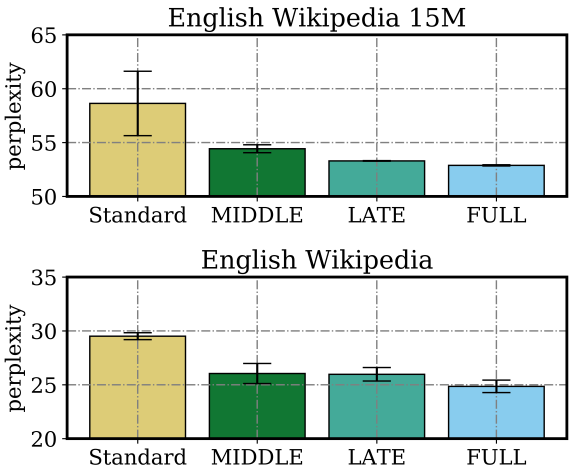

Figure 2: Perplexity scores with standard deviations for the development set of the English Wikipedia corpus after three training epochs. The best models are the ones with the richest alignment structure (FULL), in the full and low-resource distillation settings.

et al. (2019) for better performance. We use the English Wikipedia corpus for distillation. Additionally, we experiment with a low-resource case where we only distill with 15% of the English Wikipedia corpus. For fair comparison, we distill all models for three epochs. We weight all objectives equally for all experiments. With our new objectives, the distillation takes about 90 hours on 4 NVIDIA Titan 12G GPUs.

5 Experiments

In this section, we compare our IIT distilled BERT with standard distilled BERT across multiple benchmarks. To ensure a fair comparison between methods, we distill BERT for each condition with three distinct random seeds. We then fine-tune each model with five distinct random seeds. Consequently, we report results aggregated from three distinct runs for the language modeling task, and 15 distinct runs for others.

Language Modeling We first evaluate our models using perplexity on the held-out evaluation data

Model	CoNLL-2003 acc./F1	SQuAD EM/F1
Standard	97.93/89.15	56.23/68.26
MIDDLE	98.01/89.75	58.64/70.33
LATE	97.98/89.21	58.79/70.56
FULL	**98.01/89.85**	**59.33/71.05**

Table 2: Model performance results on the development sets of the CoNLL-2003 corpus for the name-entity recognition task, and SQuAD v1.1 for the question answering task.

from the English Wikipedia corpus. As shown in Figure 2, our IIT objective brings performance gains for all alignments. IIT training is also beneficial in a low-resource setting (top panel). Additionally, we find that more complete alignments result in lower perplexity (2.85% for the low-resource setting and 13.7% for the full corpus). This suggests that even richer alignments might lead to even larger gains.

GLUE The GLUE benchmark (Wang et al., 2018) covers nine different NLP tasks. We report scores on the development sets for each task by fine-tuning our distilled models. We fine-tune for 15 epochs for the smaller datasets (WNLI, RTE and CoLA) and 3 epochs for the others. We use Matthew's Correlation for CoLA, the mean of accuracy and F1 for MRPC and QQP, the mean of Pearson and Spearman correlation for STS-B, and accuracy for all the other datasets.

Our GLUE results are summarized in Table 1 along with the macro-score (average of individual scores, with WNLI left out to allow comparisons with Sanh et al. 2019). The results suggest that distilled models with IIT lead to consistent improvements over standard distillation, except for the WNLI task. Overall, IIT with the FULL mapping brings an average of 2.72% improvement.

Named Entity Recognition We also evaluate our models on the CoNLL-2003 Named Entity Recog-

nition task (Tjong Kim Sang and De Meulder, 2003). We report accuracy and Macro-F1 scores along with precision and recall on the development sets. We fine-tune our models for three epochs. Overall, IIT brings small but consistent improvements, as seen in Table 2.

Question Answering Finally, we evaluate on a question answering task, SQuAD v1.1 (Rajpurkar et al., 2016). We report Exact Match and Macro-F1 on the development sets as our evaluation metrics. We fine-tune our models for two epochs. IIT again yields marked improvements (Table 2).

6 Conclusion

In this paper, we explored distilling a teacher by training a student to capture the *causal* structure of its computations. Across a wide range of NLP tasks, we find that IIT training leads to improvements, with the largest gains coming from the models that use the richest alignment between student and teacher. These findings suggest that IIT is a promising tool for effective model distillation.

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A Appendix

A.1 Standard Distillation Objectives

In our setting, our teacher model $\mathcal{T}$ is a BERT model, and our student model $\mathcal{S}$ is a shallow BERT model with fewer layers.

Assume that we randomly draw a training example $\{\mathbf{x}_i, \mathbf{y}_i\}$ for $i \in [1, |\mathcal{D}|]$ from the training dataset $\mathcal{D}$, where $\mathbf{x}_i$ is the i-th input to our models and $\mathbf{y}_i$ is the corresponding ground truth (the token prediction at each masked position). We denote the model predictions (output logits) as $\mathcal{T}(\mathbf{x}_i)$ and $\mathcal{S}(\mathbf{x}_i)$. Additionally, we denote the contextualized representation for tokens for $\mathbf{x}_i$ at the last layer as $\text{BERT}_{\mathcal{T}}(\mathbf{x}_i)$ and $\text{BERT}_{\mathcal{S}}(\mathbf{x}_i)$.

We adopt the three standard distillation objectives of Sanh et al. (2019):

$\mathcal{L}_{\text{MLM}}$ The masked language modeling loss of the student model calculated over all examples using the cross-entropy loss as follows:

$$\sum_{\{\mathbf{x}_i, \mathbf{y}_i\} \in \mathcal{D}} \text{CE}(\mathcal{S}(\mathbf{x}_i), \mathbf{y}_i) \tag{4}$$

$\mathcal{L}_{\text{CE}}$ Following Hinton et al. (2015), the smoothed cross-entropy loss measuring the divergence between the student and teacher outputs as follows:

$$\sum_{\mathbf{x}_i \in \mathcal{D}} \text{CE}_{\text{S}}(\mathcal{S}(\mathbf{x}_i), \mathcal{T}(\mathbf{x}_i)) \tag{5}$$

$\mathcal{L}_{\text{Cos}}$ The cosine embedding loss defined in terms of the final hidden states of the teacher and the student as follows:

$$\sum_{\mathbf{x}_i \in \mathcal{D}} \text{COS}(\text{BERT}_{\mathcal{S}}(\mathbf{x}_i), \text{BERT}_{\mathcal{T}}(\mathbf{x}_i)) \tag{6}$$