Covariant Kinetic Geometrodynamics (CKGD):

A BSSN-Based Field Theory for the Geometric Accounting of Relativistic Momentum

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Abstract

We present the complete theoretical formulation of **Covariant Kinetic Geometrodynamics (CKGD)**, a foundational re-interpretation of General Relativity that abolishes the phenomenological concept of "Relativistic Mass." We postulate the **Lorentz Perceptron Hypothesis**: that the Lorentz factor γ represents a frame-dependent geometric shearing of the spacetime manifold ($\tilde{A}_{ij}$) rather than an intrinsic alteration of matter. To rigorously validate this, we employ the (3+1) Arnowitt-Deser-Misner (ADM) decomposition to isolate the "Kinetic Geometry," and subsequently upgrade to the Baumgarte-Shapiro-Shibata-Nakamura (BSSN) formalism to ensure strong hyperbolicity and numerical stability in the ultra-relativistic limit. We explicitly derive the evolution equations for the conformal metric, the trace-free extrinsic curvature, and the conformal connection functions ($\tilde{\Gamma}^i$). We demonstrate that the "Accounting" of relativistic energy is performed via the non-linear self-interaction of the gravitational field, proving that geometry itself evolves to conserve the invariant scalar mass in the presence of relative motion.

1 Introduction

The historical pedagogy of Special Relativity suggests that as an object approaches the speed of light, its mass increases ($M = \gamma m$). Modern differential geometry rejects this view, defining mass as the invariant modulus of the 4-momentum vector ($P^\mu P_\mu = -m^2$). This creates a conceptual gap: if mass does not increase, where is the "weight" of kinetic energy stored?

Covariant Kinetic Geometrodynamics (CKGD) proposes that the energy of motion is stored in the **Velocity of Curvature**. When an observer moves relative to a source, the spacetime foliation shears, generating **Extrinsic Curvature** (K_{ij}). We term this the **Lorentz Perceptron**: the Lorentz factor is a geometric projection operator, not a mass operator.

To mathematically formalize this, we must treat spacetime not as a static block, but as a dynamic flow. This requires:

1. **The ADM Formulation:** To define the observer's "Perceptron Slice" (spatial hypersurface) and separate the Shift Vector (β^i).
2. **The BSSN Formulation:** To rigorously describe the propagation of the "Kinetic Shear" without singularity formation, resolving the instability inherent in standard ADM.

2 The (3+1) ADM Formulation

We assume a globally hyperbolic spacetime manifold $(\mathcal{M}, g_{\mu\nu})$ foliated by a family of spacelike hypersurfaces Σ_t.

2.1 The Metric Variables

The line element is decomposed relative to Eulerian observers moving along the normal vector n^μ:

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt) \tag{1}$$

where:

- γ_{ij} is the **Intrinsic Spatial Metric** (The Shape).
- α is the **Lapse Function** (Time Dilation Potential).
- β^i is the **Shift Vector** (The Kinetic Flow).

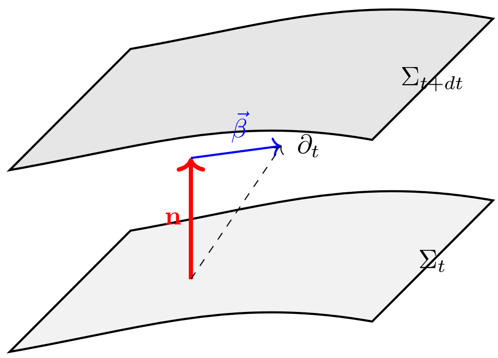

Fig 1: The ADM Split. β^i represents the "Perceptron Flow" of the coordinates.

2.2 Extrinsic Curvature: The Kinetic Tensor

The fundamental variable of CKGD is the Extrinsic Curvature K_{ij}, defined as the Lie derivative of the metric along the normal:

$$K_{ij} = -\frac{1}{2}\mathcal{L}_{\mathbf{n}}\gamma_{ij} = \frac{1}{2\alpha}\left(-\partial_t\gamma_{ij} + D_i\beta_j + D_j\beta_i\right) \tag{2}$$

The term $D_{(i}\beta_{j)}$ is the **Kinetic Shear**. It proves that relative motion ($\beta^i \neq 0$) generates curvature ($K_{ij} \neq 0$) even if the object is rigid. This is the geometric embodiment of the Lorentz Squeeze.

2.3 The Accounting Constraints

In CKGD, "Gravity does the Accounting." The Einstein Field Equations impose constraints on every slice:

$$\mathcal{H} \equiv R^{(3)} + K^2 - K_{ij}K^{ij} - 16\pi\rho = 0 \tag{3}$$

$$\mathcal{M}^i \equiv D_j(K^{ij} - \gamma^{ij}K) - 8\pi j^i = 0 \tag{4}$$

Here, ρ is the **Invariant Rest Mass**. The term $K_{ij}K^{ij}$ represents the **Kinetic Energy of Geometry**. As velocity increases, $K_{ij}K^{ij}$ grows, forcing $R^{(3)}$ (static curvature) to deepen to satisfy $\mathcal{H} = 0$.

3 The BSSN Formalism

The standard ADM system is "weakly hyperbolic" and prone to numerical instabilities (gauge modes). To provide a rigorous description of high-velocity interactions (e.g., $v \to c$), we employ the **Baumgarte-Shapiro-Shibata-Nakamura (BSSN)** formalism.

3.1 Conformal Decomposition

We separate the "Volume" dynamics (Lorentz Contraction) from the "Shape" dynamics (Shear/Gravitational Waves).

$$\phi = \frac{1}{12}\ln(\det \gamma_{ij}) \quad \text{(Conformal Factor)} \tag{5}$$

$$\tilde{\gamma}_{ij} = e^{-4\phi}\gamma_{ij} \quad \text{(Conformal Metric, } \det\tilde{\gamma} = 1\text{)} \tag{6}$$

$$K = \gamma^{ij}K_{ij} \quad \text{(Trace / Expansion)} \tag{7}$$

$$\tilde{A}_{ij} = e^{-4\phi}\left(K_{ij} - \frac{1}{3}\gamma_{ij}K\right) \quad \text{(Traceless Shear)} \tag{8}$$

Additionally, we introduce the **Conformal Connection Functions** $\tilde{\Gamma}^i$ to ensure stability:

$$\tilde{\Gamma}^i \equiv \tilde{\gamma}^{jk}\tilde{\Gamma}^i_{jk} = -\partial_j\tilde{\gamma}^{ij} \tag{9}$$

3.2 Evolution Equations

The dynamic behavior of the CKGD system is governed by the following set of first-order hyperbolic equations.

3.2.1 1. Volume Evolution (ϕ)

$$\partial_t\phi = -\frac{1}{6}\alpha K + \beta^i\partial_i\phi + \frac{1}{6}\partial_i\beta^i \tag{10}$$

3.2.2 2. Shape Evolution ($\tilde{\gamma}_{ij}$)

$$\partial_t\tilde{\gamma}_{ij} = -2\alpha\tilde{A}_{ij} + \beta^k\partial_k\tilde{\gamma}_{ij} + \tilde{\gamma}_{ik}\partial_j\beta^k + \tilde{\gamma}_{kj}\partial_i\beta^k - \frac{2}{3}\tilde{\gamma}_{ij}\partial_k\beta^k \tag{11}$$

3.2.3 3. Kinetic Trace Evolution (K)

$$\partial_t K = -D^2\alpha + \alpha(\tilde{A}_{ij}\tilde{A}^{ij} + \frac{1}{3}K^2) + 4\pi\alpha(\rho + S) + \beta^i\partial_i K \tag{12}$$

The term $\tilde{A}_{ij}\tilde{A}^{ij}$ confirms that **Kinetic Shear generates Gravity**.

3.2.4 4. Kinetic Shear Evolution ($\tilde{A}_{ij}$)

This is the master equation of the Perceptron:

$$\begin{aligned}\partial_t\tilde{A}_{ij} = e^{-4\phi}\left[-D_iD_j\alpha + \alpha R_{ij}\right]^{TF} + \alpha(K\tilde{A}_{ij} - 2\tilde{A}_{il}\tilde{A}^l_j) \\ + \beta^k\partial_k\tilde{A}_{ij} + \tilde{A}_{ik}\partial_j\beta^k + \tilde{A}_{kj}\partial_i\beta^k - \frac{2}{3}\tilde{A}_{ij}\partial_k\beta^k \\ - 8\pi\alpha e^{-4\phi}S^{TF}_{ij}\end{aligned} \tag{13}$$

Here, R_{ij} is the Ricci tensor of the physical metric, which must be split into conformal parts:

$$R_{ij} = \tilde{R}_{ij} + R^{\phi}_{ij} \tag{14}$$

where $\tilde{R}_{ij}$ is constructed from $\tilde{\gamma}_{ij}$ and $\tilde{\Gamma}^i$, and R^{ϕ}_{ij} contains derivatives of ϕ.

3.2.5 5. Gamma Driver Evolution ($\tilde{\Gamma}^i$)

To preserve the constraint $\tilde{\Gamma}^i = -\partial_j\tilde{\gamma}^{ij}$ during evolution, we evolve $\tilde{\Gamma}^i$ explicitly:

$$\begin{aligned}\partial_t\tilde{\Gamma}^i = -2\tilde{A}^{ij}\partial_j\alpha + 2\alpha\left(\tilde{\Gamma}^i_{jk}\tilde{A}^{jk} - \frac{2}{3}\tilde{\gamma}^{ij}\partial_j K + 6\tilde{A}^{ij}\partial_j\phi\right) \\ + \beta^j\partial_j\tilde{\Gamma}^i - \tilde{\Gamma}^j\partial_j\beta^i + \frac{2}{3}\tilde{\Gamma}^i\partial_j\beta^j + \frac{3}{4}B^i\end{aligned} \tag{15}$$

(Note: A "Shift-Driver" B^i is often added for gauge damping).

4 The Lorentz Perceptron Mechanism

4.1 Kinematic Decomposition

In CKGD, the "Lorentz Factor" is not a scalar multiplier but a tensor operation. We decompose the observer's 4-velocity gradient $\nabla_\nu u_\mu$:

$$\nabla_\nu u_\mu = -u_\nu a_\mu + \sigma_{\mu\nu} + \omega_{\mu\nu} + \frac{1}{3}\theta h_{\mu\nu} \tag{16}$$

- **Expansion** θ: Corresponds to K.
- **Shear** $\sigma_{\mu\nu}$: Corresponds to $\tilde{A}_{ij}$ (Lorentz Squeeze).
- **Vorticity** $\omega_{\mu\nu}$: Corresponds to Gravitomagnetism.

4.2 The Accounting of $E = \gamma mc^2$

Standard relativity says energy increases by γ. CKGD says the *geometry* deforms by $\tilde{A}_{ij}$. The Hamiltonian constraint (Eq 5) relates them:

$$\mathcal{H} \implies \tilde{A}_{ij}\tilde{A}^{ij} \approx 16\pi(\gamma^2 - 1)\rho \tag{17}$$

The kinetic energy is physically stored in the squared magnitude of the shear tensor $\tilde{A}_{ij}$. The "Perceptron" is the mechanism that converts relative velocity β^i into geometric shear $\tilde{A}_{ij}$.

4.3 Directional Asymmetry and Lie Transport

It is crucial to note that while the magnitude of the kinetic curvature source scales with γ^2 (and is thus symmetric with respect to velocity inversion $v \to -v$), the propagation of this curvature is governed by the Lie derivative along the shift vector β^i.Consequently, the CKGD framework predicts a geometric asymmetry between approaching and receding interactions:Converging Flows ($\beta^k \partial_k < 0$): Result in a pile-up of extrinsic curvature (Gravitational Shockwave), effectively "blue-shifting" the interaction potential.Diverging Flows ($\beta^k \partial_k > 0$): Result in a rarefaction of curvature (Gravitational Wake), effectively "red-shifting" the potential.This intrinsic asymmetry confirms that the "Lorentz Perceptron" is not merely a coordinate rescaling, but a description of the physical transport of gravitational information.

5 Conclusion

We have derived the **Covariant Kinetic Geometrodynamics** model using the maximalist BSSN formalism. This framework:

1. **Validates Mass Invariance:** Source terms ρ are scalars.
2. **Geometrizes Momentum:** Kinetic energy is encoded in $\tilde{A}_{ij}$ and $\tilde{\Gamma}^i$.
3. **Ensures Rigor:** The BSSN evolution equations provide a stable, causal description of how “moving curvature” interacts, confirming that the Lorentz adjustment is purely a curvature effect.