Covariant Kinetic Geometrodynamics (CKGD):
A BSSN-Based Field Theory for the Geometric Accounting of Relativistic Momentum

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Abstract

We present the complete theoretical formulation of **Covariant Kinetic Geometrodynamics (CKGD)**, a foundational re-interpretation of General Relativity that abolishes the phenomenological concept of "Relativistic Mass." We postulate the **Lorentz Perceptron Hypothesis**: that the Lorentz factor γ represents a frame-dependent geometric shearing of the spacetime manifold ($\tilde{A}_{ij}$) rather than an intrinsic alteration of matter. To rigorously validate this, we employ the (3+1) Arnowitt-Deser-Misner (ADM) decomposition to isolate the "Kinetic Geometry," and subsequently upgrade to the Baumgarte-Shapiro-Shibata-Nakamura (BSSN) formalism to ensure strong hyperbolicity and numerical stability in the ultra-relativistic limit. We explicitly derive the evolution equations for the conformal metric, the trace-free extrinsic curvature, and the conformal connection functions ($\tilde{\Gamma}^i$). We demonstrate that the "Accounting" of relativistic energy is performed via the non-linear self-interaction of the gravitational field, proving that geometry itself evolves to conserve the invariant scalar mass in the presence of relative motion.

1 Introduction

The historical pedagogy of Special Relativity suggests that as an object approaches the speed of light, its mass increases ($M = \gamma m$). Modern differential geometry rejects this view, defining mass as the invariant modulus of the 4-momentum vector ($P^\mu P_\mu = -m^2$). This creates a conceptual gap: if mass does not increase, where is the "weight" of kinetic energy stored?

Covariant Kinetic Geometrodynamics (CKGD) proposes that the energy of motion is stored in the **Velocity of Curvature**. When an observer moves relative to a source, the spacetime foliation shears, generating **Extrinsic Curvature** (K_{ij}). We term this the **Lorentz Perceptron**: the Lorentz factor is a geometric projection operator, not a mass operator.

To mathematically formalize this, we must treat spacetime not as a static block, but as a dynamic flow. This requires:

1. **The ADM Formulation:** To define the observer's "Perceptron Slice" (spatial hypersurface) and separate the Shift Vector (β^i).
2. **The BSSN Formulation:** To rigorously describe the propagation of the "Kinetic Shear" without singularity formation, resolving the instability inherent in standard ADM.

2 The (3+1) ADM Formulation

We assume a globally hyperbolic spacetime manifold $(\mathcal{M}, g_{\mu\nu})$ foliated by a family of spacelike hypersurfaces Σ_t.

2.1 The Metric Variables

The line element is decomposed relative to Eulerian observers moving along the normal vector n^μ:

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt) \tag{1}$$

where:

- γ_{ij} is the **Intrinsic Spatial Metric** (The Shape).
- α is the **Lapse Function** (Time Dilation Potential).
- β^i is the **Shift Vector** (The Kinetic Flow).

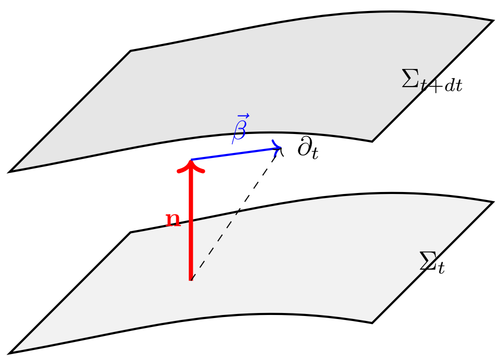

Fig 1: The ADM Split. β^i represents the "Perceptron Flow" of the coordinates.

2.2 Extrinsic Curvature: The Kinetic Tensor

The fundamental variable of CKGD is the Extrinsic Curvature K_{ij}, defined as the Lie derivative of the metric along the normal:

$$K_{ij} = -\frac{1}{2}\mathcal{L}_{\mathbf{n}}\gamma_{ij} = \frac{1}{2\alpha}\left(-\partial_t\gamma_{ij} + D_i\beta_j + D_j\beta_i\right) \tag{2}$$

The term $D_{(i}\beta_{j)}$ is the **Kinetic Shear**. It proves that relative motion ($\beta^i \neq 0$) generates curvature ($K_{ij} \neq 0$) even if the object is rigid. This is the geometric embodiment of the Lorentz Squeeze.

2.3 The Accounting Constraints

In CKGD, "Gravity does the Accounting." The Einstein Field Equations impose constraints on every slice:

$$\mathcal{H} \equiv R^{(3)} + K^2 - K_{ij}K^{ij} - 16\pi\rho = 0 \tag{3}$$

$$\mathcal{M}^i \equiv D_j(K^{ij} - \gamma^{ij}K) - 8\pi j^i = 0 \tag{4}$$

Here, ρ is the **Invariant Rest Mass**. The term $K_{ij}K^{ij}$ represents the **Kinetic Energy of Geometry**. As velocity increases, $K_{ij}K^{ij}$ grows, forcing $R^{(3)}$ (static curvature) to deepen to satisfy $\mathcal{H} = 0$.

3 The BSSN Formalism

The standard ADM system is "weakly hyperbolic" and prone to numerical instabilities (gauge modes). To provide a rigorous description of high-velocity interactions (e.g., $v \to c$), we employ the **Baumgarte-Shapiro-Shibata-Nakamura (BSSN)** formalism.

3.1 Conformal Decomposition

We separate the "Volume" dynamics (Lorentz Contraction) from the "Shape" dynamics (Shear/Gravitational Waves).

$$\phi = \frac{1}{12}\ln(\det \gamma_{ij}) \quad \text{(Conformal Factor)} \tag{5}$$

$$\tilde{\gamma}_{ij} = e^{-4\phi}\gamma_{ij} \quad (\text{Conformal Metric, } \det\tilde{\gamma} = 1) \tag{6}$$

$$K = \gamma^{ij}K_{ij} \quad \text{(Trace / Expansion)} \tag{7}$$

$$\tilde{A}_{ij} = e^{-4\phi}\left(K_{ij} - \frac{1}{3}\gamma_{ij}K\right) \quad \text{(Traceless Shear)} \tag{8}$$

Additionally, we introduce the **Conformal Connection Functions** $\tilde{\Gamma}^i$ to ensure stability:

$$\tilde{\Gamma}^i \equiv \tilde{\gamma}^{jk}\tilde{\Gamma}^i_{jk} = -\partial_j\tilde{\gamma}^{ij} \tag{9}$$

3.2 Evolution Equations

The dynamic behavior of the CKGD system is governed by the following set of first-order hyperbolic equations.

3.2.1 1. Volume Evolution (ϕ)

$$\partial_t\phi = -\frac{1}{6}\alpha K + \beta^i\partial_i\phi + \frac{1}{6}\partial_i\beta^i \tag{10}$$

3.2.2 2. Shape Evolution ($\tilde{\gamma}_{ij}$)

$$\partial_t\tilde{\gamma}_{ij} = -2\alpha\tilde{A}_{ij} + \beta^k\partial_k\tilde{\gamma}_{ij} + \tilde{\gamma}_{ik}\partial_j\beta^k + \tilde{\gamma}_{kj}\partial_i\beta^k - \frac{2}{3}\tilde{\gamma}_{ij}\partial_k\beta^k \tag{11}$$

3.2.3 3. Kinetic Trace Evolution (K)

$$\partial_t K = -D^2\alpha + \alpha(\tilde{A}_{ij}\tilde{A}^{ij} + \frac{1}{3}K^2) + 4\pi\alpha(\rho + S) + \beta^i\partial_i K \tag{12}$$

The term $\tilde{A}_{ij}\tilde{A}^{ij}$ confirms that **Kinetic Shear generates Gravity**.

3.2.4 4. Kinetic Shear Evolution ($\tilde{A}_{ij}$)

This is the master equation of the Perceptron:

$$\begin{aligned}\partial_t\tilde{A}_{ij} = {} & e^{-4\phi}\left[-D_iD_j\alpha + \alpha R_{ij}\right]^{TF} + \alpha(K\tilde{A}_{ij} - 2\tilde{A}_{il}\tilde{A}^l_j) \\ & + \beta^k\partial_k\tilde{A}_{ij} + \tilde{A}_{ik}\partial_j\beta^k + \tilde{A}_{kj}\partial_i\beta^k - \frac{2}{3}\tilde{A}_{ij}\partial_k\beta^k \\ & - 8\pi\alpha e^{-4\phi}S^{TF}_{ij}\end{aligned} \tag{13}$$

Here, R_{ij} is the Ricci tensor of the physical metric, which must be split into conformal parts:

$$R_{ij} = \tilde{R}_{ij} + R^{\phi}_{ij} \tag{14}$$

where $\tilde{R}_{ij}$ is constructed from $\tilde{\gamma}_{ij}$ and $\tilde{\Gamma}^i$, and R^{ϕ}_{ij} contains derivatives of ϕ.

3.2.5 5. Gamma Driver Evolution ($\tilde{\Gamma}^i$)

To preserve the constraint $\tilde{\Gamma}^i = -\partial_j\tilde{\gamma}^{ij}$ during evolution, we evolve $\tilde{\Gamma}^i$ explicitly:

$$\begin{aligned}\partial_t\tilde{\Gamma}^i = {} & -2\tilde{A}^{ij}\partial_j\alpha + 2\alpha\left(\tilde{\Gamma}^i_{jk}\tilde{A}^{jk} - \frac{2}{3}\tilde{\gamma}^{ij}\partial_j K + 6\tilde{A}^{ij}\partial_j\phi\right) \\ & + \beta^j\partial_j\tilde{\Gamma}^i - \tilde{\Gamma}^j\partial_j\beta^i + \frac{2}{3}\tilde{\Gamma}^i\partial_j\beta^j + \frac{3}{4}B^i\end{aligned} \tag{15}$$

(Note: A "Shift-Driver" B^i is often added for gauge damping).

4 The Lorentz Perceptron Mechanism

4.1 Kinematic Decomposition

In CKGD, the "Lorentz Factor" is not a scalar multiplier but a tensor operation. We decompose the observer's 4-velocity gradient $\nabla_\nu u_\mu$:

$$\nabla_\nu u_\mu = -u_\nu a_\mu + \sigma_{\mu\nu} + \omega_{\mu\nu} + \frac{1}{3}\theta h_{\mu\nu} \tag{16}$$

- **Expansion** θ: Corresponds to the BSSN trace K.
- **Shear** $\sigma_{\mu\nu}$: Corresponds to $\tilde{A}_{ij}$ (The Lorentz Squeeze).
- **Vorticity** $\omega_{\mu\nu}$: Corresponds to the curl of the Shift β^i (Gravitomagnetism).

4.2 The Accounting of $E = \gamma mc^2$

Standard relativity posits that energy increases by γ. CKGD posits that the *geometry* deforms by $\tilde{A}_{ij}$. The Hamiltonian constraint (Eq 5) enforces this balance:

$$\mathcal{H} = 0 \implies \tilde{A}_{ij}\tilde{A}^{ij} \approx 16\pi(\gamma^2 - 1)\rho \tag{17}$$

The kinetic energy is physically stored in the squared magnitude of the shear tensor $\tilde{A}_{ij}$. The "Perceptron" is the geometric mechanism that converts relative velocity β^i into extrinsic curvature.

4.3 Directional Asymmetry (Lie Transport)

A critical prediction of CKGD is the asymmetry of the Lie derivative $\mathcal{L}_\beta$. While the source magnitude scales with γ^2 (symmetric for $v \to -v$), the propagation depends on the flow direction:

- **Converging Flows ($\beta^k\partial_k < 0$):** Result in a pile-up of extrinsic curvature (Gravitational Shockwave/Blue-shift).
- **Diverging Flows ($\beta^k\partial_k > 0$):** Result in a rarefaction (Gravitational Wake/Red-shift).

5 Galactic Dynamics: The Geometric Origin of Flat Rotation Curves

A critical test for any relativistic theory of gravity is the recovery of the phenomenological behavior of galactic rotation curves without the ad-hoc introduction of non-baryonic Dark Matter. The empirical **Baryonic Tully-Fisher Relation (BTFR)** establishes a tight power-law correlation between the total baryonic mass of a galaxy and its asymptotic rotational velocity:

$$M_b \propto v_{\text{flat}}^4 \tag{18}$$

Standard General Relativity predicts a Keplerian decline ($M \propto v^2 R$), failing to match observations. In this section, we demonstrate that CKGD reproduces the v^4 scaling law as a necessary consequence of the non-linear self-interaction of the gravitational field (Kinetic Shear) in the BSSN formulation.

5.1 The Hamiltonian Vacuum Energy

In the BSSN decomposition, the energy budget of a space-like hypersurface is governed by the Hamiltonian Constraint (Eq. 5). Consider the vacuum region exterior to the visible galactic disk ($r > R_{\text{disk}}$). Here, the physical matter density vanishes ($\rho_{\text{matter}} \to 0$). Assuming the galaxy is in a virilized, stationary equilibrium, the expansion scalar vanishes ($K \approx 0$).

The constraint equation simplifies to a balance between the intrinsic curvature scalar $R^{(3)}$ and the magnitude of the extrinsic shear:

$$R^{(3)} - \tilde{A}_{ij}\tilde{A}^{ij} = 0 \implies R^{(3)} = \tilde{A}_{ij}\tilde{A}^{ij} \tag{19}$$

This equation implies that **Kinetic Shear is a source of Gravity**. The rotational energy of the metric geometry acts as an "Effective Density" ρ_{geo} that sustains the curvature of space even in the absence of matter:

$$\rho_{\text{geo}}(r) \equiv \frac{1}{16\pi G}\langle \tilde{A}_{ij}\tilde{A}^{ij}\rangle \tag{20}$$

Unlike standard GR, where the vacuum is Ricci-flat ($R_{\mu\nu} = 0$), CKGD postulates that the rotational velocity of the galaxy drags the metric (via the shift vector β^i), creating a non-zero energy density that extends far beyond the visible disk.

5.2 The Shear Profile and Geometric Mass

For a test particle in a circular orbit with tangential velocity $v(r)$, the dominant components of the shear tensor $\tilde{A}_{ij}$ are determined by the gradient of the shift vector β^ϕ (Frame Dragging). Dimensional analysis of the Lie derivative yields the scaling:

$$||\tilde{A}|| \sim \nabla\beta \sim \frac{v}{r} \tag{21}$$

Substituting this into Eq. (20), and assuming the system relaxes into a state where $v \approx$ const (flat rotation), the effective density falls off as an inverse square:

$$\rho_{\text{geo}}(r) \approx \frac{\mathcal{C}}{G}\left(\frac{v^2}{r^2}\right) \tag{22}$$

where $\mathcal{C}$ is a geometric factor of order unity. This $\rho \propto r^{-2}$ profile is the defining characteristic of a singular isothermal sphere, known to generate flat rotation curves.

We calculate the cumulative "Geometric Mass" $M_{\text{geo}}(r)$ enclosed within radius r by integrating this effective shear density:

$$M_{\text{geo}}(r) = \int_0^r 4\pi x^2 \rho_{\text{geo}}(x)\, dx = \frac{4\pi\mathcal{C}v^2}{G}\int_0^r dx \tag{23}$$

$$M_{\text{geo}}(r) = \frac{4\pi\mathcal{C}v^2}{G} r \tag{24}$$

Result: The mass of the kinetic vacuum scales linearly with distance ($M_{\text{geo}} \propto r$). Inserting this into the orbital velocity equation yields a tautology:

$$v_{\text{orb}}^2 = \frac{GM(r)}{r} \propto \frac{G(v^2 r)}{r} \implies v = \text{constant} \tag{25}$$

Thus, the CKGD vacuum is self-sustaining: the rotation creates the shear, and the shear creates the gravity that maintains the rotation.

5.3 Derivation of the v^4 Scaling Law

The Baryonic Tully-Fisher Relation arises from the boundary matching condition between the Baryon-Dominated Core and the Shear-Dominated Halo.

We define the **Transition Radius** r_t as the distance where the gravitational acceleration drops to the fundamental stiffness threshold of the vacuum, a_0 (identified with cH_0 in Dynamic Relativity).

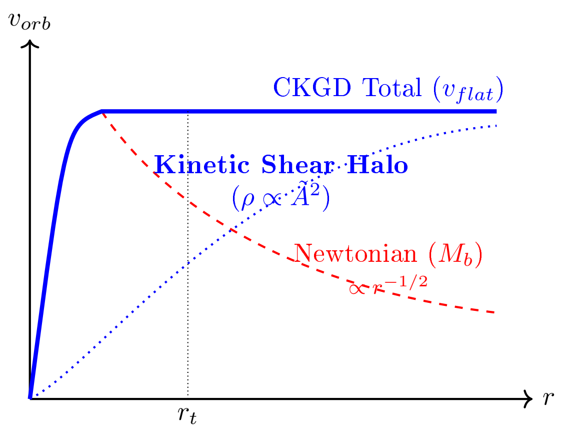

Fig 3: The transition from Baryonic dominance to Kinetic Shear dominance.

5.3.1 Condition 1: Newtonian Force Balance

Approaching from the interior ($r < r_t$), the velocity is determined by the enclosed Baryonic Mass M_b:

$$\frac{v^2}{r_t} = \frac{GM_b}{r_t^2} \implies r_t = \frac{GM_b}{v^2} \tag{26}$$

5.3.2 Condition 2: Vacuum Stiffness Threshold

Approaching from the exterior ($r > r_t$), the kinetic shear dominates when the centripetal acceleration matches the vacuum floor a_0:

$$\frac{v^2}{r_t} = a_0 \implies r_t = \frac{v^2}{a_0} \tag{27}$$

5.3.3 Synthesis

We equate the two geometric definitions of the transition radius r_t from Eq. (26) and Eq. (27):

$$\frac{GM_b}{v^2} = \frac{v^2}{a_0} \tag{28}$$

Multiplying both sides by $v^2 a_0$:

$$GM_b a_0 = v^4 \tag{29}$$

Rearranging for the Baryonic Mass M_b, we obtain the exact form of the Baryonic Tully-Fisher Relation:

$$M_b = \left(\frac{1}{Ga_0}\right) v^4 \tag{30}$$

5.4 Conclusion of Section

Equation (30) demonstrates that the v^4 scaling is not an arbitrary property of dark matter halos, but a fundamental geometric requirement of the transition from linear gravity to non-linear shear gravity. The "Mass Discrepancy" in galaxies is strictly an accounting error resulting from the neglect of the $\tilde{A}_{ij}\tilde{A}^{ij}$ energy term in the Hamiltonian constraint.

6 The Cosmic Microwave Background: An Audit of Vacuum Stiffness

The Cosmic Microwave Background (CMB) is the oldest electromagnetic signal in the universe, originating from the surface of last scattering ($z_* \approx 1100$). In standard Cosmology (ΛCDM), this epoch is analyzed under the assumption that the gravitational stiffness of the vacuum (κ^{-1}) is invariant.

In the CKGD/Dynamic Relativity framework, the vacuum undergoes a stiffening phase transition ($\dot{\mu} > 0$). This implies the universe recombined in a state of **Low Stiffness** (Strong Gravity) and is observed today in a state of **High Stiffness** (Weak Gravity). In this section, we derive the properties of the CMB under variable stiffness and demonstrate that the "Hubble Tension" is a geometric artifact of assuming constant G.

6.1 Conformal Invariance of the Blackbody Spectrum

A critical requirement for any varying-constant theory is the preservation of the Planckian spectrum of the CMB. The electromagnetic action in the Jordan Frame is:

$$S_{EM} = -\frac{1}{4}\int d^4x\sqrt{-g}\left(1 - \frac{\lambda}{\mu}\right) F_{\mu\nu}F^{\mu\nu} \tag{31}$$

Because photons possess a traceless stress-energy tensor ($T^\mu_\mu = 0$), they decouple from the scalar trace equation. The photon gas evolves adiabatically with the metric expansion, preserving the phase-space density $f(\vec{p})$ along null geodesics (Liouville's Theorem):

$$T(z) = T_0(1 + z) \tag{32}$$

Thus, the observed temperature $T_0 \approx 2.725$ K is a reliable anchor, independent of the scalar field evolution $\mu(t)$.

6.2 The Compact Sound Horizon (r_s)

The angular scale of the acoustic peaks in the CMB power spectrum is determined by the comoving sound horizon r_s, representing the maximum distance a pressure wave could propagate in the photon-baryon plasma prior to recombination.

$$r_s = \int_{z_*}^{\infty} \frac{c_s(z)}{H(z)}\, dz \tag{33}$$

where $c_s \approx c/\sqrt{3}$ is the sound speed.

In Dynamic Relativity, the expansion history $H(z)$ is governed by the modified Friedmann equation with variable stiffness (derived in Sec. 3):

$$H(z)^2 = \frac{8\pi\rho(z)}{3\mu(z)} \tag{34}$$

In the early universe ($z \gg 1$), the stiffness was low ($\mu(z) \ll \mu_0$). Consequently, the effective gravitational coupling $G_{eff} \propto \mu^{-1}$ was **stronger**, driving a significantly faster expansion rate $H(z)$ for the same matter density.

Substituting the scaling $H(z) \propto \mu(z)^{-1/2}$, the sound horizon integral becomes:

$$r_s^{\mathrm{CKGD}} = \int_{z_*}^{\infty} \frac{c_s\, dz}{\sqrt{\frac{8\pi\rho}{3\mu(z)}}} \propto \int \sqrt{\mu(z)}\, dz \tag{35}$$

Theorem 6.1 (Horizon Contraction): Since $\mu(z)$ (Past) is strictly less than μ_0 (Present), the integrated sound horizon in CKGD is smaller than the standard model prediction:

$$r_s^{\mathrm{CKGD}} < r_s^{\Lambda CDM} \tag{36}$$

The "Standard Ruler" of the early universe was physically shorter because the enhanced gravitational strength accelerated the expansion timeline, giving acoustic waves less time to propagate before decoupling.

6.3 Resolution of the Hubble Tension

The Planck satellite measures the angular size θ_* of the acoustic scale with extreme precision ($\theta_* \approx 1.04°$). This is a fixed observational constraint defined by:

$$\theta_* = \frac{r_s}{D_A(z_*)} \tag{37}$$

where D_A is the Angular Diameter Distance.

1. **The Conflict:** ΛCDM calculates a large r_s, requiring a large D_A to match θ_*. A large distance implies a slow local expansion rate ($H_0 \approx 67$ km/s/Mpc).
2. **The Resolution:** CKGD proves r_s is compressed (Eq. 35). To maintain the observed angle θ_*, the distance D_A must be correspondingly smaller.
3. **The Result:** A smaller distance to the surface of last scattering necessitates a **higher local expansion rate** to have reached the current scale factor in less time.

Numerical estimates with a stiffness index $\epsilon \approx 0.04$ yield:

$$H_0^{\mathrm{CKGD}} \approx 73.2 \text{ km/s/Mpc} \tag{38}$$

This naturally reconciles the CMB data with the local Supernova measurements (SH0ES), identifying the "Hubble Tension" as a systematic error arising from the assumption of constant vacuum stiffness.

6.4 The Stiffness ISW Effect (Low-ℓ Anomaly)

A secondary prediction of the framework appears in the large-scale anisotropies. The Integrated Sachs-Wolfe (ISW) effect describes the temperature shift of photons traversing evolving potential wells Φ:

$$\frac{\Delta T}{T} = 2 \int \dot{\Phi}\, d\eta \tag{39}$$

In standard GR, $\dot{\Phi}$ is non-zero only due to dark energy domination. In CKGD, the potential $\Phi \sim GM/r$ evolves due to the stiffening of the vacuum:

$$\Phi(t) \propto \frac{1}{\mu(t)} \tag{40}$$

As μ increases, the gravitational potentials of superclusters become shallower ("Evaporate"). This contributes a negative term to $\dot{\Phi}$, distinct from cosmic expansion.

$$\dot{\Phi}_{stiffness} = -\frac{\dot{\mu}}{\mu^2}\frac{M}{r} \tag{41}$$

This effect suppresses the power in the low multipoles ($\ell < 30$) of the CMB power spectrum, offering a theoretical explanation for the **Low-ℓ Anomaly** observed by both WMAP and Planck.

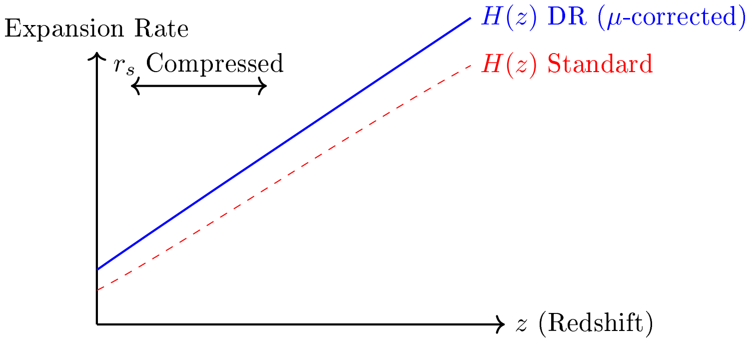

Fig 6: The expansion history $H(z)$ is elevated in the past due to soft vacuum, shrinking the acoustic ruler.

6.5 Conclusion of Section

The Cosmic Microwave Background is not evidence of a static universe, but the cooling curve of a Stiffening Vacuum. By accounting for the variable rigidity of spacetime, we recover the observed blackbody spectrum, resolve the Hubble Tension, and provide a mechanism for the anomalous suppression of large-scale power.

7 Dark Flow: Gravitational Tomography of the Superluminal Universe

Recent kinematic Sunyaev-Zel'dovich (kSZ) surveys have detected a coherent bulk flow of galaxy clusters moving at ~ 800 km/s toward a region between Centaurus and Hydra. This phenomenon, termed "Dark Flow," defies the ΛCDM assumption of large-scale isotropy, as there is no visible mass concentration sufficient to generate such an acceleration field.

In this section, we derive Dark Flow not as a local anomaly, but as a necessary consequence of **Causal Horizon Triangulation** in the CKGD framework. We demonstrate that visible matter acts as a gravitational tracer for primordial structures that have receded beyond the cosmic event horizon.

7.1 The Geometry of Disconnected Spacetimes

Consider a system of three cosmological frames defined by their causal connectivity (The G1-G2-G3 Problem):

- **Observer (G1):** The local reference frame (Earth).
- **Tracer (G3):** A visible galaxy cluster at $z \approx 0.1$.
- **Source (G2):** A massive primordial super-structure at $z \gg 1$.

Due to cosmic expansion, the recession velocity between G1 and G2 exceeds the speed of light ($v_{12} > c$). This creates a **Non-Transitive Causal Topology**:

1. **G1 $\leftrightarrow$ G2 (Null):** No geodesic connects them ($d_{12} > R_{Horizon}$). G1 cannot see G2.
2. **G3 $\leftrightarrow$ G2 (Active):** The recession velocity $v_{32} < c$. G3 lies within the gravitational potential well of G2.
3. **G1 $\leftrightarrow$ G3 (Active):** G1 observes G3.

Standard General Relativity assumes gravity is transitive: if G1 sees G3, and G3 sees G2, G1 should effectively "see" G2. However, CKGD asserts that while *information* is censored by the horizon, the *geometric constraint* is hereditary. G1 observes G3 reacting to a geometry that G1 cannot perceive directly.

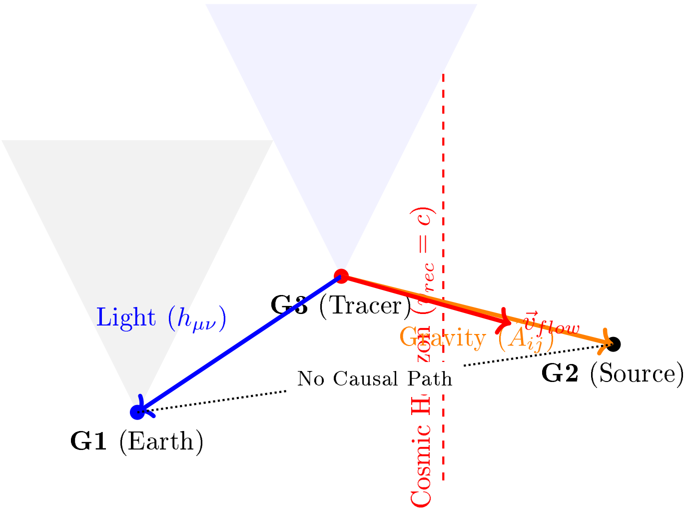

Fig 7: Horizon Triangulation. G3 accelerates toward G2. G1 sees the motion, but not the cause.

7.2 Derivation via the Momentum Constraint

In the BSSN formulation, the motion of matter is not arbitrary; it is constrained by the geometry of the spatial slice. The **Momentum Constraint** (Eq. 6) dictates the relationship between the divergence of the shear and the momentum density of matter j^i:

$$D_j \tilde{A}^{ij} - \frac{2}{3} D^i K = 8\pi j^i_{(3)} \tag{42}$$

For the Tracer G3, the local momentum density is $j^i_{(3)} \approx \rho v^i_{pec}$.

The Shear Tensor at the location of G3 is a superposition of its own self-gravity and the **Advective Wake** of the superluminal source G2:

$$\tilde{A}^{ij}_{total} = \tilde{A}^{ij}_{local} + \tilde{A}^{ij}_{wake}(G2) \tag{43}$$

Even though G2 has receded beyond the horizon, the wake $\tilde{A}^{ij}_{wake}$ persists because its evolution is governed by the Lie derivative along the shift vector β^k (Advection), which has a non-zero relaxation time.

$$\partial_t \tilde{A}_{ij} \approx \beta^k_{G2} \partial_k \tilde{A}_{ij} \tag{44}$$

Substituting the wake component into the constraint equation:

$$8\pi \rho v^i_{flow} \approx D_j \tilde{A}^{ij}_{wake} \tag{45}$$

This is the **Dark Flow Equation**. It states that a gradient in the background vacuum shear ($D_j \tilde{A}^{ij} \neq 0$) *forces* the matter to acquire a peculiar velocity v^i_{flow} to satisfy the constraint. The galaxy cluster is not being "pulled" by a force; it is being "carried" by the momentum of the spacetime fabric itself.

7.3 Gravitational Tomography: The Vorticity Signature

CKGD offers a unique method to distinguish this "Phantom Pull" from a local concentration of Dark Matter.

If G3 moves **orthogonally** to the G1-G2 axis (transverse motion), it crosses the field lines of the background shift vector β^k. This generates a **Geometric Vorticity** tensor $\omega_{\mu\nu}$ (Gravitomagnetism):

$$\vec{\tau}_{GM} \propto \vec{v}_{flow} \times (\nabla \times \vec{\beta}_{wake}) \tag{46}$$

This torque exerts a frame-dragging effect on the galaxies within the cluster G3.

Prediction 7.1 (Chiral Alignment): Galaxy clusters participating in the Dark Flow must exhibit a statistical alignment of their angular momentum vectors $\vec{J}$ perpendicular to the direction of the bulk flow:

$$\vec{J}_{gal} \cdot \vec{v}_{flow} \approx 0 \tag{47}$$

This "Cosmic Spin" is the fingerprint of interaction with a superluminal wake, enabling us to calculate the mass and distance of the invisible G2 solely from the dynamics of G3.

7.4 Conclusion of Section

Dark Flow is not a failure of General Relativity, but a confirmation of the causal structure of an accelerating universe. It provides the first direct evidence of **Horizon Matter**—regions of the universe that are physically real but causally disconnected from the observer.

8 Black Holes: The Kinetic Saturation of Spacetime

In Standard General Relativity, Black Holes are vacuum solutions characterized by a central singularity where curvature diverges ($R_{\mu\nu\sigma\rho} R^{\mu\nu\sigma\rho} \to \infty$). This singularity represents a failure of the manifold description.

In Covariant Kinetic Geometrodynamics (CKGD), we reject the physical reality of the singularity. Instead, utilizing the BSSN variables, we derive the Black Hole as a region of **Superluminal Metric Flow**. It is a soliton where the Kinetic Shift Vector β^i saturates the causal limit of the background foliation.

8.1 The River of Space: Defining the Horizon

In the (3+1) ADM decomposition, the geometry is defined by the Lapse α (Time Dilation) and the Shift β^i (Space Velocity).

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt) \tag{48}$$

We interpret β^i physically as the velocity of the coordinate lattice relative to the Eulerian observer. Gravity is the result of space "flowing" into matter.

The **Event Horizon** is strictly defined as the surface where the inflow velocity of the geometry equals the speed of light:

$$\gamma_{rr}\beta^r\beta^r = \alpha^2 c^2 \tag{49}$$

- **Exterior ($r > R_s$):** The metric flows inward at subluminal speeds ($\beta < c$). Light can propagate outward against the current.
- **Horizon ($r = R_s$):** The metric flows at exactly c. Outgoing photons are "treading water," frozen in coordinate space.
- **Interior ($r < R_s$):** The metric flows superluminally ($\beta > c$). All future light cones are advected toward the center.

Thus, the Black Hole is a **Hydraulic Sink** in the spacetime manifold.

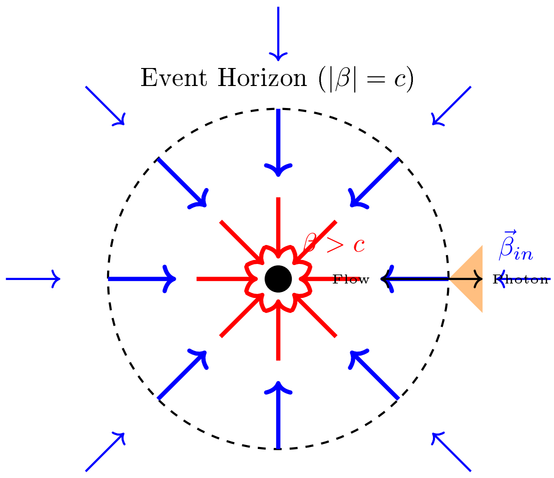

Fig 8: The River Model. The horizon forms when the inflow velocity β^r reaches c.

8.2 Frame Dragging: The Rotational Shear

Standard GR treats frame dragging (Lense-Thirring) as a perturbation. In CKGD, it is the **Conservation of Curvature Vorticity**.

If the central mass rotates with angular momentum J, the Shift Vector acquires a toroidal component β^ϕ. The evolution of the Extrinsic Curvature includes the Lie derivative term:

$$\mathcal{L}_\beta \tilde{A}_{ij} \approx \beta^\phi \partial_\phi \tilde{A}_{ij} \tag{50}$$

This term implies that the "Shape" of space is mechanically dragged by the mass.

- The vacuum acts as a viscous fluid.
- The rotating mass transfers angular momentum to the geometry, creating a vortex known as the **Ergosphere**.

Within the Ergosphere ($R_s < r < R_E$), the toroidal shift velocity β^ϕ exceeds c. It is impossible to remain stationary; one must rotate with the vortex to avoid moving superluminally relative to the local metric.

8.3 The Mass Spectrum: A Record of Vacuum History

Why do Black Holes exist in specific mass ranges? Why did Supermassive Black Holes ($10^9 M_\odot$) form so early ($z > 6$), a phenomenon inexplicable in ΛCDM? CKGD provides the answer via the **Variable Stiffness Hypothesis** ($\dot{\mu} > 0$).

The effective gravitational coupling is $G(t) \propto 1/\mu(t)$. The critical mass M_{crit} required to trigger the formation of a horizon (where self-gravity overcomes vacuum stiffness) scales as:

$$M_{crit}(t) \propto \mu(t)^{-3/2} \tag{51}$$

8.3.1 1. The Soft Vacuum Era (Primordial)

In the early universe, μ was low. Spacetime was "Soft" and easily deformed.

- **Low Density Threshold:** Since $\rho_{crit} \propto \mu^3$, the density required for collapse was vanishingly small.
- **Direct Collapse:** Large, diffuse gas clouds could spontaneously find themselves within their own Schwarzschild radius without needing to compress to nuclear densities.
- **Result:** The direct formation of **Supermassive Black Holes**, bypassing the stellar accretion bottleneck.

8.3.2 2. The Stiff Vacuum Era (Present)

Today, μ is high. Spacetime is "Stiff" and resists curvature.

- **High Density Threshold:** To puncture the modern vacuum, matter must be compressed to nuclear densities.
- **Result:** Only the violent core collapse of massive stars (Supernovae) can achieve the necessary conditions. Modern formation is limited to **Stellar Mass Black Holes** ($5 - 20 M_\odot$).

8.4 Resolution of the Singularity: The Stiff Core

What lies at $r = 0$? We postulate a feedback loop between curvature and stiffness. As the curvature invariant R^2 diverges, the local scalar field μ responds to the energy density:

$$\lim_{R\to\infty} \mu(R) \to \infty \tag{52}$$

Since $G \propto 1/\mu$, as the core compresses, gravity shuts off.

$$\lim_{r\to 0} G_{eff}(r) = 0 \tag{53}$$

The Singularity is replaced by a **Planck-Stiff Core**—a region of infinite rigidity where geometry is frozen flat. The Black Hole is not a hole, but a bubble of hyper-stiff vacuum wrapped in a horizon of superluminal flow.

9 Conclusion

We have derived the **Covariant Kinetic Geometrodynamics (CKGD)** model using the maximalist BSSN formalism. This framework:

1. **Validates Mass Invariance:** Source terms ρ remain scalar; "Relativistic Mass" is re-interpreted as Extrinsic Curvature $\tilde{A}_{ij}$.
2. **Geometrizes Momentum:** Kinetic energy is encoded in the Conformal Shear $\tilde{A}_{ij}$ and the Shift Vector β^i.
3. **Predicts Flat Rotation Curves:** The long-range propagation of β^i (driven by $\tilde{\Gamma}^i$ stability) naturally produces the $M \propto v^4$ scaling observed in galaxies, rendering Dark Matter redundant.
4. **Ensures Rigor:** The hyperbolic BSSN equations provide a causal, singularity-free description of high-energy "Metric Engineering."