

Project Euler Problem Fifteen

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1 Problem

Starting in the top left corner of a 22 grid, and only being able to move to the right and down, there are exactly 6 routes to the bottom right corner. How many such routes are there through a 2020 grid?

2 Formulas to Note

We can calculate the total routes of a lattice path using factorials.

$$\frac{(n+m)!}{n!m!} \tag{1}$$

3 Formula for Sample

We are given the results for a 2x2 grid.

$$\frac{(2+2)!}{2!2!} => \frac{4!}{2!2!} \tag{2}$$

Now expand to find terms that cancel.

$$\frac{4x3x2!}{2!2!} => \frac{4x3}{2!} = 6 \tag{3}$$

4 Main Problem

We will now move on to using this sequence for the main problem.

$$\frac{(20+20)!}{20!20!} => \frac{40!}{20!20!} \tag{4}$$

Expand to find terms that cancel.

$$\frac{40x39x38x37x36x35x34x33x32x31x30x29x28x27x26x25x24x23x22x21x20!}{20!20!} \tag{5}$$

Cancel terms and expand the denominator.

$$\frac{40x39x38x37x36x35x34x33x32x31x30x29x28x27x26x25x24x23x22x21}{20x19x18x17x16x15x14x13x12x11x10x9x8x7x6x5x4x3x2x1} \quad (6)$$

Calculate the answer from the leftover terms.

$$\frac{40x39x38x37x36x35x34x33x32x31x30x29x28x27x26x25x24x23x22x21}{20x19x18x17x16x15x14x13x12x11x10x9x8x7x6x5x4x3x2x1} = 137846528820 \quad (7)$$