Project Euler Problem One

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1 Problem

Find the sum of all the multiples of 3 or 5 below 1000.

2 Formulas to Note

When working with arithmetic sequences we can write the general term as the following:

$$a_n = a_1 + (n-1)d \tag{1}$$

When working with arithmetic sequences we can find the partial sum of the series using the following:

$$s_n = \frac{n}{2}(a_1 + a_n) \tag{2}$$

3 Sums of the Multiple of 5

We need to sum

$$5 + 10 + 15 + 25... + 995 \tag{3}$$

Thus, we find the general form of the series.

$$a_n = 5 + (n-1)5 \tag{4}$$

We then find the value of, n, which we need to use for the sum.

$$995 = 5 + (n-1)5 => 199 = n \tag{5}$$

We then compute the sum.

$$S_{199} = \frac{199}{2}(5 + 995) = 99500 \tag{6}$$

4 Sums of the Multiple of 3

We need to sum

$$3 + 6 + 9... + 999 \tag{7}$$

Thus, we find the general form of the series.

$$a_n = 3 + (n - 1)3 \tag{8}$$

We then find the value of, n, which we need to use for the sum.

$$999 = 3 + (n - 1)3 => 333 = n \tag{9}$$

We then compute the sum of the series.

$$S_{333} = \frac{333}{2}(3 + 999) = 166833 \tag{10}$$

5 Sums of the Multiple of 15

Since 3 and 5 both multiple to 15, there will be a set of "duplicate" values that have been summed. We will need to remove these from the final sum.

$$15 + 30 + 45... + 990 \tag{11}$$

We then find the general form for the summation series.

$$a_n = 15 + (n - 1)15 \tag{12}$$

We then find the value of, n, which we need to use for the sum.

$$990 + 15 + (n - 1)15 => n = 66 \tag{13}$$

We then compute the sum.

$$S_{66} = \frac{66}{2}(15 + 990) = 33165 \tag{14}$$

6 Final Answer

We now take the sums of the multiples of 3 and 5 and add them together, then remove the sum of the multiples of 15.

$$(166833 + 99500) - 33165 = 233168 \tag{15}$$