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JEL Classification: G11, H26, H42
Key Words: dynamic tax evasion; asset allocation;

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We model tax evasion using a portfolio approach in a dynamic framework
Optimal tax evasion is affected neither by return nor by volatility of the risky asset
Evasion reduces the investment in the risky asset and increases consumption
Evasion increases the investment in the riskless asset
By increasing the tax rate on the riskless asset evasion can be reduced
The effect of an increase in the tax rate on tax evasion depends on the audit rules

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Most tax evasion models are set in a timeless environment, though this is not suitable for a study of revenues from financial activities where tax evasion occurs in a dynamic environment. This study examines a representative agent aiming to maximize the expected utility from intertemporal consumption and could invest in both riskless and risky assets, through tax evasion is possible only in the latter case. The investor must pay a fine when his/her evasion is detected (with a given probability). We show that: (i) optimal consumption is higher with tax evasion, (ii) optimal evasion is affected by neither the return nor the volatility of the risky asset, (iii) evasion reduces investment in the risky asset and increases investment in the riskless asset, (iv) evasion can be reduced more efficiently by increasing the fine rather than by increasing the frequency of controls, (v) for a sufficiently high tax rate on the riskless asset, the optimal evasion is zero, and (vi) if the fine is proportional to the amount of taxes, for only 'sufficiently' low fines the Yitzhaki (1974) paradox is confirmed.

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1 Introduction

Tax evasion is one of the most studied and the least desired effects of government intervention in the economy. Since the seminal papers from Allingham and Sandmo [1972] and Yitzhaki [1974], researchers have offered several explanations and possible solutions for this phenomenon. Despite these efforts, tax evasion seems to be increasing. Schneider [2011] and Buehn and Schneider [2007] show that the shadow economy, a good proxy for tax evasion, is rising both in OECD countries (from 13.2% in 1990 to 16.7% in 2010) and in transition economies. The most recent estimates (Feige and Cebula, 2011) show that intentional under-reporting of income is about 18-19% of the total reported income in the US, leading to a tax gap of about 500 billion USD. According to IRS estimates (Slemrod and Gillitzer, 2013), about 8% of income from financial activities is under-reported.[1] Furthermore, tax evasion related to financial activities is important both in terms of lost income and because of its role in supplying the means to evade taxes to implement tax noncompliance (Blackburn et al., 2012). The effects of taxation on household portfolios have long been debated in the literature. Theoretical models predict that under different tax systems, the optimal portfolio allocation depends not only on the risk/return profile, but also on tax characteristics including compensating capital gains with capital losses and whether tax is due upon maturity or realization (Poterba, 2002; Alan et al., 2010). Surprisingly, tax evasion has not received the same attention despite its policy implications (Landskroner et al., 1990, Lee, 1995 present two of the few exceptions). Risk hedging and dynamic decisions are largely relevant for tax evasion on financial assets because evasion increases the riskiness of the investor's portfolio because evasion increases the uncertainty of revenues from the concealed assets, it is usually only possible to evade taxes on the income from riskier assets, and under-reporting of income is often systematic. Tax evasion and auditing are certainly dynamic processes (Allingham and Sandmo, 1972, Engel and Hines, 1999), though dynamic tax evasion has been approached only recently. Lin and Yang [2001], Dzhumashev and Gahramanov [2011], Levaggi and Menoncin [2012, 2013] investigate the relationship between the tax rate, tax evasion, and economic growth using a macroeconomic capital accumulation framework for a risk averse agent, while Niepelt [2005] presents the case of a risk neutral agent.

This study examines an optimal consumption and asset allocation problem between a riskless and a risky asset, where it is possible to evade taxes only on the revenue from the risky asset. Thus, the asset allocation decision considers three asset classes: a riskless asset whose income cannot be evaded, a non-concealed risky asset, and a concealed risky asset. In the event of a tax audit, the probability of which is independent of the other financial stochastic variables, the investor must pay a fine that may be a function of both the tax rate and the value of the concealed assets. Such a fine dampens the asset price accumulation path. Evasion does not alter the risky asset's price and return since this takes

[1] These data exclude under-reporting associated with total evasion.

place within a partial equilibrium framework where the representative agent is a price taker whose behaviour does not affect the prices on the financial market.

This paper makes several contributions to the existing literature on tax evasion and portfolio allocation. In particular, it shows that: (i) optimal consumption is higher with tax evasion; (ii) optimal evasion depends on neither the return nor the volatility of the risky asset; (iii) evasion reduces investment in the risky asset and increases investment in the riskless asset; (iv) an increase in the fine reduces evasion more than an increase in the frequency of controls; (v) evasion is inversely correlated with the tax rate on the riskless asset; thus, if the tax rate is 'sufficiently high', the optimal evasion is zero; and (vi) the relationship between the tax rate on the risky asset and the optimal tax evasion depends on the tax audit rules, particularly if the fine is proportional to the value of the evaded asset, an increase in the tax rate will lead to more tax evasion, as found in Allingham and Sandmo [1972]. However, if the fine is proportional to the amount of taxes evaded, the sign depends on the level of the fine itself; Yitzhaki [1974]'s paradox is confirmed only for a 'sufficiently low' value of the fine. On the other hand, when the tax rate is uniform, evasion is negatively correlated with the tax rate. Finally, there is an interesting trade-off regarding the tax rates: to reduce the excess burden of taxation, the risky asset should be taxed at a higher rate than the riskless one. However, to reduce tax evasion, the riskless asset should be taxed more than the risky one.

The paper is organized as follows. Section 2 presents the model. In Section 3, the optimal consumption and portfolios with and without evasion are computed and the key results presented and discussed. Section 4 discusses some policy implications and Section 5 concludes. The Appendixes elaborate on some technical results.

2 The Model

We model a partial equilibrium, frictionless, and complete financial market in continuous time (from time t_0 to time T) where two assets are listed:

1. A riskless asset whose (constant) return is r and whose price $G(t)$ solves the deterministic differential equation

$$\frac{dG(t)}{G(t)} = rdt, \tag{1}$$

   with an initial value in t_0 given by $G(t_0) = G_0 > 0$. This investment could be considered a government bond or a liquid bank/deposit account. Income from this riskless asset cannot be evaded.

2. A risky asset whose (constant) expected return is μ and whose price $S(t)$ follows a geometric Brownian motion

$$\frac{dS(t)}{S(t)} = \mu dt + \sigma dW(t), \tag{2}$$

where σ measures the instantaneous standard deviation of risky return and $W(t)$ is a Wiener process whose normal density has zero mean and t variance. This asset may be declared to the tax authority or may be concealed. Its return is taxed in the former case but not in the latter. Here, we assume $\mu > r$.[2] This asset does not pay coupons/dividends and the investor's gain (loss) coincides with the increase (decrease) in the asset's value.

This financial market is arbitrage-free and complete; there exists a unique market price of risk that coincides with the Sharpe ratio $\frac{\mu-r}{\sigma}$. The agent under consideration is a price-taker and his behaviour will not affect the assets' prices on the financial market.

2.1 Tax system

We assume that the government taxes invested income but not its use; that is, consumption is not taxed. Income is taxed in a symmetric manner through an accrual-based capital income tax such that the investor pays tax if the change in asset value is positive and receives a refund if it is negative. In other words, the government shares the risk of loss in asset value with the investor.[3]

Given that the average OECD tax rate on government bonds is 27% and the average tax on risky assets is 23.5% (Harding, 2013), we allow for different tax rates between assets in order to capture this common feature of most tax systems (some implications of a uniform tax system will, however, be investigated):

1. $\tau_G \in [0,1]$ is levied on the riskless payoff $dG(t)$. The riskless investment cannot be concealed from the tax authority; hence, its net payoff is

$$w_G(t)(1-\tau_G)\,dG(t),$$

where $w_G(t)$ is the number of riskless assets held in portfolio at time t;

2. $\tau \in [0,1]$ is levied on the risky payoff $dS(t)$; without tax evasion the net payoff would be

$$w(t)(1-\tau)\,dS(t),$$

where $w(t)$ is the number of risky assets held in portfolio at time t. Since $dS(t)$ may take either positive or negative values, the amount of tax due $(w(t)\,\tau dS(t))$ is actually a refund in case of a loss. In this model, taxes are due at maturity, even if the asset $S(t)$ is not sold. This model includes two considerations. First, for the concealed asset, taxation is at maturity if audited; otherwise, the individual is liable to sanctions for the illegal behaviour (concealing assets) only when the latter are sold. Actual tax

[2]Even if this hypothesis is not necessary for a complete and arbitrage-free financial market, it is more in line with the empirical data.

[3]In many tax systems, the (negative) taxes on financial losses are not immediately refunded, but are deducted from the taxes on future returns. For simplicity, this study does not account for this time lag.

systems use the maturity criterion for evaded assets (the fee is due when the concealed asset is audited and not when it is sold). Second, the tax rate is fixed throughout the model, taxation is symmetric for gains and losses, and the expected growth rate of the risky asset is positive. These elements reduce the scope for the lock-in effect (Constantinides, 1983).

Since investors may decide to conceal part of the investment in the risky asset, they are actually faced with a choice between three asset classes. Thus, we can set

1. $w_G(t)G(t)$ as the amount of money invested in the riskless asset;

2. $w(t)S(t)$ as the amount of money invested in the non-concealed risky asset;

3. $w_0(t)S(t)$ as the amount of money invested in the concealed risky asset (which coincides with evasion).

If the investor has wealth equal to $R(t)$, then his/her budget constraint is:

$$R(t) = w_G(t)G(t) + w(t)S(t) + w_0(t)S(t). \tag{3}$$

An agent must then decide how to allocate his portfolio across these three assets, so the decision variables are given by w_G, w, and w_0.

The differential of investor's wealth can be written as:

$$\begin{aligned} dR(t) &= w_G(t)dG(t) + w(t)dS(t) + w_0(t)dS(t) \\ &\quad + dw_G(t)G(t) + dw(t) \times (S(t) + dS(t)) + dw_0(t) \times (S(t) + dS(t)), \end{aligned}$$

where the sum of all terms denoting the differentials of portfolio allocation (dw_G, dw and dw_0) must equal the portfolio inflows and outflows. In our framework, the agent must finance:

- consumption ($c(t)dt$);

- taxes on the riskless asset payoff ($\tau_G w_G(t)dG(t)$) and on the non-concealed risky asset payoff ($\tau w(t)dS(t)$);

- the fine on the concealed asset value, which must be paid when the evasion is discovered. The fine is defined as a percentage of the total amount of revenue evaded as $\theta(\tau) \in [0,1]$; if we model the audit using a Poisson process (whose differential is $d\Pi(t)$ and whose intensity is λ), the total fee is given by $\theta(\tau)w_0(t)S(t)d\Pi(t)$ where w_0 is the number of concealed risky assets. [4]

The fee function $\theta(\tau)$ allows to model the following three most relevant cases:

[4] Audit as a Poisson process is presented in Levaggi and Menoncin [2012, 2013], Bernasconi et al. [2015] Details about the Poisson processes used in finance are summarized in Cont and Tankov [2004] and Øksendal and Sulem [2007].

1. a fine proportional to the value of the evaded asset, as in Allingham and Sandmo, 1972; that is, $\theta(\tau) = \beta$;

2. a fine proportional to the tax evaded, as in Yitzhaki, 1974; that is, $\theta(\tau) = \alpha\tau$;

3. a fine with both a component proportional to the evaded asset value and a component proportional to the evaded tax; that is, $\theta(\tau) = \alpha\tau + \beta$.

Tax audits are performed with an intensity λ that determines the probability of the audit itself. In the case of an audit, assets that have been concealed from the tax authority are detected and the investor has to pay a fine.

If N_t is the number of audits that have taken place until time t, the probability of having n audits is $\mathbb{P}(N_t = n) = e^{-\lambda t}(\lambda t)^n / n!$. Accordingly, the probability of having at least one audit is

$$\mathbb{P}(N_t \geq 1) = 1 - \mathbb{P}(N_t = 0) = 1 - e^{-\lambda t}. \tag{4}$$

Despite the country specific characteristics (Harding, 2013) of tax on financial activities and its impact on the optimal portfolio allocation (Poterba, 2002, Bergstresser and Poterba, 2004), we believe that our assumptions capture the most relevant features of this tax within a dynamic setting. The total revenue on which tax is evaded between t_0 and t is the sum of all revenues $dS(s)$ for $s \in [t_0, t]$:

$$\int_{t_0}^{t} w_0(s)\, dS(s).$$

Recall that the risky asset does not pay dividends and, accordingly, its revenue is equal to its price increments. If we assume $w_0(s)$ to remain constant through time ($w_0(s) = \hat{w}_0, \forall s \in [t_0, t]$), then the evasion revenue is

$$\hat{w}_0 \int_{t_0}^{t} dS(s) = \hat{w}_0 (S(t) - S(t_0)).$$

In computing the fee, if the audit authority assumes that the number of evaded assets $w_0(s)$ has been equal to $w_0(t)$ for any $s \in [t_0, t]$, the evaded revenue is:

$$\int_{t_0}^{t} w_0(t)\, dS(s) = w_0(t) \int_{t_0}^{t} dS(s) = w_0(t) (S(t) - S(t_0)).$$

The model contains the assumption $S(t_0) = 0$; that is, it does not account for the investor's costs to buy the risky asset at time 0. Thus, evasion is

$$w_0(t)\, S(t).$$

Our assumption is in line with the literature on uncertainty and tax evasion (Slemrod and Yitzhaki, 2002); thus, even if the value of λ does not depend on past evasion, we are able to consider a tax system based on the compounded revenues obtained from evasion.

There are no costs associated with concealing or emerging assets. This hypothesis does not seem to be strong as it is easier to conceal financial capital than other income sources. For symmetry, we have assumed zero cost of revealing capital; this assumption is not relevant in the model since optimal tax evasion is a constant proportion of wealth whose expected value is constantly increasing. The investor's differential wealth is given by

$$dR = (1-\tau_G)\, w_G dG + (1-\tau)\, w dS + w_0 dS - c dt - w_0 S\theta d\Pi, \tag{5}$$

where, for simplicity, the model omits all functional dependencies on time.

When the differentials dG and dS are plugged into (5) from (1) and (2), we finally obtain

$$\begin{aligned} dR \;=\;& (R(1-\tau_G)\, r + wS\,((1-\tau)\,\mu - (1-\tau_G)\, r) & (6)\\ & + w_0 S\,(\mu - (1-\tau_G)\, r) - c)\, dt & (7)\\ & + (w(1-\tau) + w_0)\, S\sigma dW - w_0 S\theta d\Pi. & (8) \end{aligned}$$

2.2 The investor's choices

The representative investor maximizes the expected utility of both inter-temporal consumption $c(t)$ and final wealth $R(T)$, where T is the time horizon. Investor's preferences belong to the constant relative risk aversion family (CRRA) wherein the utility of consumption is given by

$$U(c(t)) = \begin{cases} \frac{c(t)^{1-\delta}}{1-\delta}, & \delta > 1, \\ \ln c(t), & \delta = 1, \end{cases}$$

where δ is the (constant) Arrow-Pratt relative risk aversion index. All results obtained for $\delta > 1$ can be traced back to the logarithm case by setting $\delta = 1$.[5] Utility is discounted at a subjective constant discount rate ρ. The inter-temporal optimization problem is:

$$\max_{w(t), w_0(t), c(t)} \mathbb{E}_{t_0}\left[\int_{t_0}^{T} \frac{c(t)^{1-\delta}}{1-\delta} e^{-\rho(t-t_0)} dt + \chi \frac{R(T)^{1-\delta}}{1-\delta} e^{-\rho(T-t_0)}\right] \tag{9}$$

where $\mathbb{E}_{t_0}$ is the expected value operator (conditional on information at time t_0), and final wealth is weighted by a constant $\chi \geq 0$. The higher the value of χ, the stronger the investor's preference for final wealth (bequest motivations) with respect to inter-temporal consumption.

Investor's wealth $R(t)$ solves (6).

[5] Note that $\ln c(t)$ is the limit, for $\delta = 1$, of the utility function $\left(c(t)^{1-\delta} - 1\right)/(1-\delta)$ that gives the same maximum as $c(t)^{1-\delta}/(1-\delta)$.

3 Optimal tax evasion and portfolio allocation

In this section, we present the optimal consumption and asset allocation that solves Problem (9). The following proposition summarizes the key results:

Proposition 1. *The optimal consumption, concealed risky asset (evasion), non-concealed risky asset, and riskless asset allocation that solve (9), given the wealth differential (6), are:*

$$\frac{c^*(t)}{R(t)} = \frac{1}{\frac{1-e^{-\phi(T-t)}}{\phi} + \chi^{\frac{1}{\delta}} e^{-\phi(T-t)}}, \tag{10}$$

$$\frac{w_0^*(t)\,S(t)}{R(t)} = \frac{1}{\theta(\tau)}\left(1-\left(\frac{(1-\tau)\,\theta(\tau)\,\lambda}{(1-\tau_G)\,r\tau}\right)^{\frac{1}{\delta}}\right), \tag{11}$$

$$\frac{w^*(t)\,S(t)}{R(t)} = \frac{1}{\delta}\frac{(1-\tau)\,\mu-(1-\tau_G)\,r}{(1-\tau)^2\,\sigma^2} - \frac{1}{1-\tau}\frac{w_0^*(t)\,S(t)}{R(t)}, \tag{12}$$

$$\frac{w_G^*(t)\,G(t)}{R(t)} = 1-\frac{1}{\delta}\frac{(1-\tau)\,\mu-(1-\tau_G)\,r}{(1-\tau)^2\,\sigma^2} + \frac{\tau}{1-\tau}\frac{w_0^*(t)\,S(t)}{R(t)}, \tag{13}$$

where

$$\phi \equiv \frac{\rho+\lambda}{\delta} + \frac{\delta-1}{\delta}(1-\tau_G)\,r + \frac{1}{2}\frac{\delta-1}{\delta^2}\left(\frac{(1-\tau)\,\mu-(1-\tau_G)\,r}{(1-\tau)\,\sigma}\right)^2 \tag{14}$$
$$+\frac{\delta-1}{\delta}\lambda\left(\frac{(1-\tau_G)\,r\tau}{(1-\tau)\,\theta(\tau)\,\lambda}\right) - \lambda\left(\frac{(1-\tau_G)\,r\tau}{(1-\tau)\,\theta(\tau)\,\lambda}\right)^{1-\frac{1}{\delta}} < 1.$$

Proof. See Appendix A. □

The following corollary describes the optimal behaviour of an agent who does not evade taxes:

Corollary 2. *The optimal consumption and asset allocation that solve (9), given the wealth differential (6) for an agent who does not evade, are:*

$$\frac{\hat{c}^*(t)}{\hat{R}(t)} = \frac{1}{\frac{1-e^{-\hat{\phi}(T-t)}}{\hat{\phi}} + \chi^{\frac{1}{\delta}} e^{-\hat{\phi}(T-t)}}, \tag{15}$$

$$\frac{\hat{w}^*(t)\,S(t)}{\hat{R}(t)} = \frac{1}{\delta}\frac{(1-\tau)\,\mu-(1-\tau_G)\,r}{(1-\tau)^2\,\sigma^2}, \tag{16}$$

$$\frac{\hat{w}_G^*(t)\,G(t)}{\hat{R}(t)} = 1-\frac{1}{\delta}\frac{(1-\tau)\,\mu-(1-\tau_G)\,r}{(1-\tau)^2\,\sigma^2}, \tag{17}$$

where

$$\hat{\phi} \equiv \frac{\rho}{\delta} + \frac{\delta-1}{\delta}(1-\tau_G)\,r + \frac{1}{2}\frac{\delta-1}{\delta^2}\left(\frac{(1-\tau)\,\mu-(1-\tau_G)\,r}{(1-\tau)\,\sigma}\right)^2.$$

Proof. It is sufficient to substitute $\lambda\theta(\tau) = \frac{(1-\tau_G)r\tau}{(1-\tau)}$ in the results for Proposition 1. □

The next subsections examine the key features of the representative agent's optimal behaviour.

3.1 Optimal consumption

The optimal consumption is given by the inverse of an annuity providing 1 monetary unit at any instant from t to T and $\chi^{\frac{1}{\delta}}$ monetary units at T, and whose discount rate is ϕ (as defined in (14)). For the log investor with $\delta = 1$, the discount rate is equal to the subjective discount rate ρ (set $\delta = 1$ in (14)). For an infinitely risk-averse agent ($\delta \to \infty$), the discount rate is equal to the net riskless return $(1-\tau_G)\,r$.

The optimal consumption as a percentage of wealth may increase or decrease over time according to the value of χ, in fact

$$\frac{\partial\left(\frac{c^*(t)}{R(t)}\right)}{\partial t} \gtreqless 0 \Longleftrightarrow \chi \lesseqgtr \phi^{-\delta}.$$

This result is fairly intuitive. If χ is sufficiently high (that is, higher than $\phi^{-\delta}$), the agent values the utility of the final wealth more than inter-temporal consumption and will try to keep the later as low as possible, decreasing it over time to preserve the final wealth. On the contrary, if χ is sufficiently low (that is, lower than $\phi^{-\delta}$), the agent's utility primarily depends on the level of inter-temporal consumption and the investor will consume as much as possible by increasing consumption over time.

In other words, χ may be interpreted as a measure of the relevance of the bequest motivation: the higher the value of χ, the higher the bequest motivation.

The level of risk aversion determines the value of $\phi^{-\delta}$. In particular, for an infinitely risk-averse agent, $\phi^{-\delta}$ approaches infinity and, accordingly, consumption increases through time.

Corollary 3. *The consumption share with evasion is always greater than the consumption share without evasion; that is,*

$$\frac{c^*(t)}{R(t)} \geq \frac{\hat{c}^*(t)}{\hat{R}(t)}.$$

Proof. The optimal consumption shares $c^*(t)/R(t)$ and $\hat{c}^*(t)/\hat{R}(t)$ have identical structures with different discount rates (ϕ and $\hat{\phi}$, respectively). Accordingly, we must determine whether $\phi \gtreqless \hat{\phi}$ by checking

$$\phi \gtreqless \hat{\phi} \Longleftrightarrow \frac{1}{\delta} + \frac{\delta-1}{\delta}\frac{(1-\tau_G)\,r\tau}{(1-\tau)\,\theta(\tau)\,\lambda} - \left(\frac{(1-\tau_G)\,r\tau}{(1-\tau)\,\theta(\tau)\,\lambda}\right)^{1-\frac{1}{\delta}} \gtreqless 0.$$

Now, we must consider the function

$$f(x) \equiv \frac{1}{\delta} + \frac{\delta - 1}{\delta} x - x^{1-\frac{1}{\delta}}$$

with

$$x \equiv \frac{(1-\tau_G)\, r\tau}{(1-\tau)\,\theta(\tau)\,\lambda} > 1$$

where the inequality holds if evasion is expedient. Since it is easy to show that $f(1) = 0$ and $\frac{\partial f(x)}{\partial x} > 0$, we conclude that

$$\phi \geq \hat{\phi}.$$

The higher the discount rate ϕ, the lower the value of the following annuity:

$$\int_t^T e^{-\phi(s-t)} ds + \chi^{\frac{1}{\delta}} e^{-\phi(T-t)}$$

and, accordingly, the higher the optimal consumption. □

This implies that the income effect caused by tax evasion outweighs the substitution effect. In fact, tax evasion increases not only the investor's expected total income, but also the relative price of consumption. This increase in consumption implies that the effect of tax evasion on total wealth is uncertain and less wealth will be invested in financial assets, reducing the amount of total wealth. In contrast, tax evasion increases the expected net return on risky assets that, in turn, may increase investment. The uncertainty regarding the effect on total wealth implies that it is not possible to determine the impact of tax evasion on economic growth.

3.2 Investment (asset allocation)

An investor will optimize a portfolio in the following manner:

1. Investment in the non-concealed risky asset is proportional to the Sharpe ratio (whose elements are taken net of taxation) weighted by the relative risk tolerance index ($\frac{1}{\delta}$), minus the amount $\frac{1}{1-\tau}\frac{w_0^*(t)S(t)}{R(t)}$.

2. Investment in the riskless asset increases by $\frac{\tau}{1-\tau}\frac{w_0^*(t)S(t)}{R(t)}$.

Thus, when evasion is expedient, the amount of wealth invested in the non-concealed risky asset shifts ($\frac{1}{1-\tau}\frac{w_0^*(t)S(t)}{R(t)}$): a percentage τ of this amount is invested in the riskless asset, while a percentage $1-\tau$ is reinvested in the concealed risky asset.

Corollary 4. *Evasion (11) reduces investment in the non-concealed risky asset, that is,*

$$\frac{\hat{w}^*(t)\,S(t)}{\hat{R}(t)} > \frac{w^*(t)\,S(t)}{R(t)},$$

and increases the investment in the riskless asset, that is,

$$\frac{\hat{w}_G^*(t)\,G(t)}{\hat{R}(t)} < \frac{w_G^*(t)\,G(t)}{R(t)}.$$

Proof. The proof comes directly from a comparison between (16) and (12), and between (17) and (13). □

This result is rather intuitive. Tax evasion increases the risk associated with investing in the non-concealed risky asset; thus, to reduce portfolio variance, the investor increases the proportion invested in the riskless asset. This result has important implications, as it may provide empirical evidence that portfolios are often more liquid than they should optimally be. The literature contains many attempts to explain this phenomenon. The psychological theory of expected utility (Caplin and Leahy, 2001) argues that high investments in riskless asset could be the result of a need to hedge against the negative feelings of relevant losses on risky assets.

In the model, we show that the choice of a highly liquid portfolio is completely rational. From a policy point of view, high liquidity may be interpreted as a sign of tax evasion and may be used to target audits. Thus, a portfolio with a high proportion of riskless asset may imply either high-risk aversion or a significant amount of evasion. The following subsections provide more interpretations and implications of these results.

3.3 Tax evasion

The optimal allocation to the concealed risky asset is given in (11), where we see that the higher the risk aversion (δ) the lower the evasion (and the investment in the non-concealed risky asset).

Tax evasion is expedient for the investor ($w_0^*(t)\,S(t)\,/R(t) \in [0,1]$) if

$$\frac{(1-\tau)\,\theta(\tau)\,\lambda}{(1-\tau_G)\,\tau} \le r \le (1-\theta(\tau))^{-\delta}\,\frac{(1-\tau)\,\theta(\tau)\,\lambda}{(1-\tau_G)\,\tau}. \tag{18}$$

Accordingly, a 'sufficiently high' interest rate will lead to total evasion, while a 'sufficiently low' interest rate will ensure zero evasion; these two thresholds depend only on fiscal parameters ($\theta(\tau)$, τ, τ_G and λ), respectively. This result is consistent with our previous argument about the role of the riskless asset in the optimal portfolio. If investing in the riskless asset is less profitable (r is too low), then evasion is also less profitable. However, if the riskless asset generates a sufficiently high return, the amount of money invested in the riskless asset results in high profits and evasion reaches 100%.

This result is an outcome of the riskless asset's hedging role. When the agent invests in the concealed risky asset, the risk of an audit arises. In order to hedge against such a risk, the investor must suitably increase the amount of wealth invested in the riskless asset which is capable of providing a guaranteed return.

There are a number of other interesting results, from a policy point of view:

- Neither the risky asset's return μ nor its variance σ affects optimal evasion. This result is driven by two factors: (i) from a financial point of view, there is no alternative to the risky asset, because evasion $(w_0(t)S(t))$ has return and volatility (μ and σ, respectively) equal to those of the risky asset, and accordingly, the only relevant return on evasion is the riskless interest rate r; and (ii) consumption also hedges against the risk of being audited as it tends to absorb the uncertainty on the fiscal parameters (as demonstrated by Bernasconi et al., 2015).
- Optimal evasion is a negative function of risk aversion; when the investor is infinitely risk averse (that is, $\delta \to +\infty$), evasion is zero; optimal evasion is 100% when
$$\delta = \frac{\ln\left(\frac{(1-\tau)\theta(\tau)\lambda}{(1-\tau_G)r\tau}\right)}{\ln(1-\theta(\tau))}.$$
- Optimal evasion is a negative function of the tax rate on the riskless asset (τ_G); thus, increasing this tax rate is an effective tool for reducing evasion. Investment in the riskless asset can be thought of as insurance against an audit. In fact, the riskless return guarantees stable (albeit low) gains which, in case of audit, can make the penalty more affordable. When τ_G increases, the fiscal cost to hold the riskless asset increases, decreasing the average expected return on evasion.
- A higher audit frequency (λ) reduces evasion.

A uniform tax on both assets (that is, $\tau_G = \tau$) does not alter the market price of risk, but affects the optimal asset allocation. In fact, without evasion, the optimal investment in the risky asset is

$$\left.\frac{\hat{w}^*(t)S(t)}{\hat{R}(t)}\right|_{\tau_G=\tau} = \frac{1}{\delta}\frac{\mu-r}{\sigma^2}\frac{1}{1-\tau},$$

which is higher than the optimal investment in the risky asset without taxation:

$$\left.\frac{\hat{w}^*(t)S(t)}{\hat{R}(t)}\right|_{\tau_G=\tau=0} = \frac{1}{\delta}\frac{\mu-r}{\sigma^2}.$$

Corollary 2 shows that, without evasion, the government can set τ and τ_G to remove the effect of taxes on portfolio choice.

Corollary 5. *Without evasion, taxation does not alter the optimal portfolio allocation if*

$$\tau_G = \tau - (1-\tau)\tau\frac{\mu-r}{r} < \tau.$$

Proof. It is sufficient to solve the following equation

$$\frac{1}{\delta}\frac{(1-\tau)\mu-(1-\tau_G)r}{(1-\tau)^2\sigma^2} = \frac{1}{\delta}\frac{\mu-r}{\sigma^2},$$

for τ_G. □

We highlight that this neutrality result can be obtained with $\tau_G < \tau$, in line with the traditional literature on the effects of taxation on risk taking behaviour (Domar and Musgrave, 1944; Stiglitz, 1969). Since the government participates in both positive and negative returns, it shares the risk of investing in asset $S(t)$, thus reducing risk for an investor who can then invest a higher amount in the asset.

Finally, it is important to note that all portfolio shares are constant percentages of wealth as an outcome of the hypothesis that all financial (μ, σ, r), preference (ρ, δ, χ), and fiscal $(\tau, \tau_G, \lambda, \theta)$ parameters are constant and do not depend on the time span.

[Figure 1 about here]

In Figure 1, we show six simulations of the optimal investment paths during $T = 20$ years (5,000 daily values with $dt = \frac{1}{250}$) with evasion (in bold) and without evasion (dotted line) beginning with a value of 100. Risky asset parameters are $\mu = 0.08$ and $\sigma = 0.2$. The fiscal parameters are: $\lambda = 0.1$, $\theta = 0.08$, $\tau = 0.235$, and $\tau_G = 0.27$. Considering a frequency of 0.1 number of jumps every 250 days ($\lambda = 0.1$), we expect to have more or less 2 jumps on 5,000 simulations. The riskless interest rate is $r = 0.04$ and the preference parameters are $\rho = 0.04$, $\chi = 1$, and $\delta = 2.5$. With these parameters, the optimal investments in the concealed asset $(w_0(t) S(t) / R(t))$ is almost 56%.

On average, there are about two fiscal audits (with a range between 1 and 4). The number of audits and their timing determines whether tax evasion is profitable. When audits happen early, there is enough time to recover from the fee (see the bottom two graphs).

3.4 Government revenue

The general idea is that evasion reduces the government's revenue and forces honest taxpayers to shoulder an unfair burden of the cost of public activities. This is certainly true for the amount of tax evasion that goes undetected, but to evaluate the impact on the government budget, we must consider the (net) revenue that can be derived from tax audits and the evolution of income through time. In fact, if evaded income was used to increase wealth, such an increase would be greater than under tax compliance; in the long run, this may even imply an increase in government revenues. However, this does not happen in practice. Tax evaders increase consumption (which is tax free in our model) and the expected government revenue is lower than under tax compliance (see Appendix B).

4 Discussion and policy implications

The analysis presented in the previous sections shows the richness of the dynamic framework in describing investor's decisions. This section discusses and highlights the findings that are especially relevant from a policy point of view.

First, our model shows that the level of tax evasion depends on the investor's preference parameters, but it is not affected by uncertainty: tax evaders adjust their consumption path according to uncertainty, not tax evasion. This result, which is in line with previous findings regarding dynamic tax evasion (Bernasconi et al., 2015) shows that government should not increase uncertainty in fiscal parameters, as the traditional literature seems to suggest (Alm, 1988, Slemrod and Yitzhaki, 2002). This policy will simply depress consumption with potentially perverse effects on economic growth.

Our model contributes to the debate on how financial assets bearing a different level of risk should be taxed. Proposition 1 shows that the tax system distorts the optimal portfolio allocation due to the risk sharing mechanism determined by the tax rebates. To reduce such distortion, the tax rate on riskless assets should be lower than that on risky assets; however, equation (11) shows that in order to reduce evasion, the tax rate on the riskless asset should increase. From this point of view, on average, OECD countries seem to have chosen to fight evasion; in fact, the average tax rate on government bonds $\tau_G = 0.27$ is higher than the average tax on risky assets $\tau = 0.235$ (Harding, 2013).

If we turn our attention to the behaviour of optimal evasion (i.e. investment in the concealed risky asset) with respect to the tax rate τ, the following corollary presents the most general results.

Corollary 6. *The value of the elasticity of the fine with respect to τ determines the effect of an increase in the tax rate on the optimal level of tax evasion according to the following rule:*

$$\frac{\partial \frac{w_0^*(t)S(t)}{R(t)}}{\partial \tau} \gtreqless 0 \Longleftrightarrow \frac{\partial \theta(\tau)}{\partial \tau}\frac{\tau}{\theta(\tau)} \lesseqgtr \frac{1}{1-\tau}\frac{1-\theta(\tau)\frac{w_0^*(t)S(t)}{R(t)}}{1+(\delta-1)\theta(\tau)\frac{w_0^*(t)S(t)}{R(t)}}. \tag{19}$$

Proof. It is sufficient to compute the following derivative

$$\begin{aligned}\frac{\partial \frac{w_0^*(t)S(t)}{R(t)}}{\partial \tau} &= -\theta(\tau)^{-2}\frac{\partial \theta(\tau)}{\partial \tau} \\ &\quad + \theta(\tau)^{-2}\left(-\frac{1-\delta}{\delta}\frac{\partial \theta(\tau)}{\partial \tau} + \theta(\tau)\frac{1}{\delta}\frac{1}{1-\tau}\frac{1}{\tau}\right)\left(\frac{(1-\tau)\theta(\tau)\lambda}{(1-\tau_G)r\tau}\right)^{\frac{1}{\delta}},\end{aligned}$$

and to simplify the right-hand side. □

The result in Corollary 6 allows a definition of two relevant cases according to the fee structure.

1. $\theta(\tau) = \beta$ (a fine proportional to the value of the evaded asset with $\frac{\partial \theta(\tau)}{\partial \tau}\frac{\tau}{\theta(\tau)} = 0$): the effect is positive; in fact, the right-hand side of (19) is strictly positive. This result is in line with Lin and Yang [2001], Allingham and Sandmo [1972] and the most recent empirical literature (Feige and Cebula, 2011), and it also holds with $\tau_G = \tau$; in fact, we have

$$\left.\frac{w_0^*(t)S(t)}{R(t)}\right|_{\theta(\tau)=\beta,\tau_G=\tau} = \frac{1}{\beta}\left(1-\left(\frac{\beta\lambda}{r\tau}\right)^{\frac{1}{\delta}}\right).$$

2. $\theta(\tau) = \alpha\tau$ (a fine proportional to the tax evaded with $\frac{\partial\theta(\tau)}{\partial\tau}\frac{\tau}{\theta(\tau)} = 0$): equation (19) can be written as

$$\frac{\partial \frac{w_0^*(t)S(t)}{R(t)}}{\partial\tau} \gtreqless 0 \Longleftrightarrow \alpha \gtreqless \alpha^* \equiv \frac{(1-\tau_G)\, r}{(1-\tau)\,\lambda}\left(1+\frac{1}{\delta}\frac{\tau}{1-\tau}\right)^{-\delta}.$$

The value of α determines the sign of the relationship between tax evasion and the tax rate. In a dynamic context, we show that Yitzhaki [1974]'s counter-intuitive result that tax evasion decreases with a decrease in the tax rate is confirmed only for values of α lower than α^*. In this case, if $\tau_G = \tau$, Yitzhaki [1974]'s paradox is confirmed; in fact,

$$\left.\frac{w_0^*(t)\, S(t)}{R(t)}\right|_{\theta(\tau)=\alpha\tau, \tau_G=\tau} = \frac{1}{\alpha\tau}\left(1-\left(\frac{\alpha\lambda}{r}\right)^{\frac{1}{\delta}}\right),$$

where it is evident that an increase in τ implies reduced evasion. This result illustrates that the difference between τ and τ_G plays a crucial role in the optimal reaction of an investor towards a change in the fiscal parameters. This is a consequence of the presence of the riskless asset in the optimal portfolio. Since the riskless asset is used to invest part of the evaded wealth, the relationship between τ_G and τ makes the riskless asset more or less profitable with respect to the risky asset.

From an empirical point of view, it may be interesting to define a numerical value for the threshold α^*. Figure 2, shows for which pairs of α and the probability of getting caught for five periods (from (4) with $t = 5$), Yitzhaki [1974]'s paradox arises. The same analysis is performed for three values of the risk aversion parameter δ. The paradox is extremely relevant either for high risk aversion, or for low risk aversion combined with a low fine.

[Figure 2 about here]

Our model gives interesting insights also as concerns auditing. The response of the optimal evasion with respect to the auditing parameters θ and λ can be summarized as follows:

Corollary 7. *The (absolute value of the) elasticity of optimal tax evasion to θ is higher than the (absolute value of the) elasticity to λ.*

Proof. The elasticity of $\frac{w_0^*(t)S(t)}{R(t)}$ with respect to θ is

$$\frac{\partial \frac{w_0^*(t)S(t)}{R(t)}}{\partial\theta}\frac{\theta}{\frac{w_0^*(t)S(t)}{R(t)}} = -1 - \frac{1}{\delta}\frac{\left(\frac{(1-\tau)\theta\lambda}{(1-\tau_G)r\tau}\right)^{\frac{1}{\delta}}}{1-\left(\frac{(1-\tau)\theta\lambda}{(1-\tau_G)r\tau}\right)^{\frac{1}{\delta}}} < 0,$$

and the elasticity with respect to λ is

$$\frac{\partial \frac{w_0^*(t)S(t)}{R(t)}}{\partial \lambda} \frac{\lambda}{\frac{w_0^*(t)S(t)}{R(t)}} = -\frac{1}{\delta} \frac{\left(\frac{(1-\tau)\theta\lambda}{(1-\tau_G)r\tau}\right)^{\frac{1}{\delta}}}{1-\left(\frac{(1-\tau)\theta\lambda}{(1-\tau_G)r\tau}\right)^{\frac{1}{\delta}}} < 0.$$

It is obvious that in absolute value

$$\left| \frac{\partial \frac{w_0^*(t)S(t)}{R(t)}}{\partial \theta} \frac{\theta}{\frac{w_0^*(t)S(t)}{R(t)}} \right| > \left| \frac{\partial \frac{w_0^*(t)S(t)}{R(t)}}{\partial \lambda} \frac{\lambda}{\frac{w_0^*(t)S(t)}{R(t)}} \right|.$$

□

This result is due to the different role played by θ and λ in the wealth process. As shown in Figure 1, a sudden fall in wealth due to a fee payment of θ is hard to recover. In fact, the higher the θ, the farther the wealth deviates from its path without an audit. This means that, from an evader's point of view, recovering from frequent (high λ) but minor audits (low θ) is easier than recovering from less frequent (low λ) but more thorough audits (high θ).

From a policy perspective, this implies that the government can fight evasion more effectively by increasing the fine θ rather than the number of controls. This result is found elsewhere in the literature, but is often based on the assumption of costly audit procedures; in our model, this result is confirmed despite assuming a cost-free audit process, and is in line with two recent empirical findings: that tax evasion decreases with the audit rate (Feige and Cebula, 2011), and the number of audits is decreasing over time (Slemrod, 2007). If fines are more effective in reducing tax evasion and less costly than controls, it may make sense to reduce controls. On the other hand, fines should be credible: if they are very high, their social cost may be too high to be enforced. Given this implicit threshold on fines, audits are still necessary.

From an empirical point of view, high liquidity can be used both for targeting audits and as a proxy for tax evasion; without tax evasion, an optimal portfolio is highly liquid only for highly risk averse investors. Nevertheless, when evasion is expedient, high liquidity is a by-product of evasion and the optimal asset allocation cannot be used alone as a measure of an investor's risk aversion. Furthermore, the consumption pattern could possibly serve the same purpose, as our model predicts that the investor uses consumption to hedge against the incremental risk caused by tax evasion.

Our model considers a representative individual and it does not provide policy implications concerning equality and a fair distribution of the tax burden. Nevertheless, some equity issues arise. The tax rate on riskless assets should be increased to counterbalance tax evasion. However, if consumers are heterogeneous, highly risk-averse individuals have a portfolio biased towards riskless assets. These individuals are also less prone to tax evasion and yet they will be taxed at the same rate as evaders. If there is a correlation between risk aversion and income, there is also the risk of creating a regressive tax system.

5 Conclusions and directions for future research

This paper proposes a model to examine tax evasion on financial activities through an extension of the basic portfolio model. We examine the intertemporal allocation problem of an investor who allocates investments between a taxable, risk-free asset where evasion is impossible and a risky asset where evasion is possible.

Without tax evasion, a high investment in the riskless asset implies a high risk aversion. When evasion is expedient, an (apparently) less risky portfolio is a by-product of evasion and the optimal asset allocation cannot be used to measure an investor's risk aversion. In our framework, high liquidity may be used for targeting audits, provided it is coupled with other indicators that prevent audits on highly risk-averse individuals who are very unlikely to evade tax.

Tax evasion has an interesting countervailing effect on the distortion created by a symmetric tax system. Through tax evasion, the government shares the expected losses with investors only for the assets that have been declared. This increases the risk borne by the investor and leads to a re-allocation among financial assets.

An interesting trade-off emerges in this context. The tax system distorts the optimal portfolio allocation due to the risk-sharing mechanism of the tax rebate. To reduce this distortion, the tax rate on the riskless asset should be lower than that on the risky one; however, we have also shown that in order to reduce evasion, the tax rate on the riskless asset should be increased. EU countries have different approaches: in the UK, the tax rate on the riskless assetsì is higher than that on risky assets, while the opposite is true in Italy.

The tax rate on riskless assets should be increased to counterbalance tax evasion, and would reduce the role of riskless assets as a sort of insurance against the negative consequences of tax evasion. The framework we use is symmetric and very simple, yet the results are surprisingly rich. From a theoretical point of view, our model contributes an explanation of the observed composition of an individual portfolio, usually biased towards liquidity; from a policy standpoint, it addresses some important questions regarding the best instruments to reduce tax evasion. The paper could be extended in several directions – one of the most interesting would allow for stochastic interest rates and tax audits with memory.

A Optimization

If $J(t,R)$ is the value function, then the Hamilton-Jacobi-Bellman (HJB) equation is

$$
\begin{aligned}
0 &= \frac{\partial J(t,R)}{\partial t} - \rho J(t,R) \\
&+ \max_{w(t),w_0(t),c(t)} \left[\begin{array}{c}
\frac{c(t)^{1-\delta}}{1-\delta} + \frac{\partial J(t,R)}{\partial R} R(t)(1-\tau_G) r \\
+\frac{\partial J(t,R)}{\partial R} w(t) S(t)((1-\tau)\mu - (1-\tau_G) r) \\
+\frac{\partial J(t,R)}{\partial R} (w_0(t) S(t)(\mu - (1-\tau_G) r) - c(t)) \\
+\frac{1}{2}\frac{\partial^2 J(t,R)}{\partial R^2} (w(t)(1-\tau) S(t) + w_0(t) S(t))^2 \sigma^2 \\
+(J(t, R - w_0(t) S(t)\theta) - J(t,R))\lambda
\end{array} \right],
\end{aligned}
$$

whose boundary (final) condition is

$$
J(T,R) = \chi \frac{R(T)^{1-\delta}}{1-\delta}.
$$

The first order conditions on consumption, declared assets, and undeclared assets are

$$
\begin{aligned}
c^*(t) &= \left(\frac{\partial J(t,R)}{\partial R}\right)^{-\frac{1}{\delta}}, \\
w^*(t)(1-\tau) S(t) + w_0^*(t) S(t) &= -\frac{\frac{\partial J(t,R)}{\partial R}}{\frac{\partial^2 J(t,R)}{\partial R^2}} \frac{(1-\tau)\mu - (1-\tau_G) r}{(1-\tau)\sigma^2}, \\
w^*(t)(1-\tau) S(t) + w_0^*(t) S(t) &= -\frac{\frac{\partial J(t,R)}{\partial R}}{\frac{\partial^2 J(t,R)}{\partial R^2}} \frac{\mu - (1-\tau_G) r}{\sigma^2} \\
&\quad + \frac{1}{\frac{\partial^2 J(t,R)}{\partial R^2}} \frac{\partial J(t, R - w_0^*(t) S(t)\theta)}{\partial (R - w_0^*(t) S(t)\theta)} \frac{\theta\lambda}{\sigma^2}.
\end{aligned}
$$

Note that if either $\lambda = 0$ or $\theta = 0$ (that is, there are either no jumps or evasion is not punished), the *last two* conditions are incompatible. In fact, they become

$$
\begin{aligned}
w^*(t)(1-\tau) S(t) + w_0^*(t) S(t) &= -\frac{\frac{\partial J(t,R)}{\partial R}}{\frac{\partial^2 J(t,R)}{\partial R^2}} \frac{(1-\tau)\mu - (1-\tau_G) r}{(1-\tau)\sigma^2}, \\
w^*(t)(1-\tau) S(t) + w_0^*(t) S(t) &= -\frac{\frac{\partial J(t,R)}{\partial R}}{\frac{\partial^2 J(t,R)}{\partial R^2}} \frac{\mu - (1-\tau_G) r}{\sigma^2},
\end{aligned}
$$

and the optimization problem does not have any feasible solution (in this case, a solution exists if and only if $\tau = 0$).

If we equate the left-hand sides of the second and third equation, $w_0^*(t)$ must solve

$$
\frac{\partial J(t,R)}{\partial R} \frac{(1-\tau_G) r\tau}{(1-\tau)\theta\lambda} = \frac{\partial J(t, R - w_0^*(t) S(t)\theta)}{\partial (R - w_0^*(t) S(t)\theta)}.
$$

Thus, we can compute $w_0^*(t)$, $w^*(t)$, and $c^*(t)$ as functions of $J(t, R)$ that must solve the HJB differential equation. One of the most common methods of determining $J\left(t, \hat{R}\right)$ is to try a guess function. Here, we use

$$J(t, R) = F(t)^\delta \frac{R(t)^{1-\delta}}{1-\delta},$$

where $F(t)$ must be determined in order to solve the HJB differential equation with the boundary condition

$$F(T) = \chi^{\frac{1}{\delta}}.$$

Accordingly, the optimal values of the decision variables are

$$\begin{aligned}
c^*(t) &= \frac{R(t)}{F(t)}, \\
\frac{w_0^*(t) S(t)}{R(t)} &= \frac{1}{\theta}\left(1 - \left(\frac{(1-\tau)\theta\lambda}{(1-\tau_G) r\tau}\right)^{\frac{1}{\delta}}\right), \\
\frac{w^*(t) S(t)}{R(t)} &= \frac{1}{\delta}\frac{(1-\tau)\mu - (1-\tau_G) r}{(1-\tau)^2 \sigma^2} - \frac{1}{1-\tau}\frac{w_0^*(t) S(t)}{R(t)},
\end{aligned}$$

and the value of $F(t)$ must solve

$$0 = \frac{\partial F(t)}{\partial t} - \phi F(t) + 1$$

where

$$\begin{aligned}
\phi \equiv{}& \frac{\rho+\lambda}{\delta} - \frac{1-\delta}{\delta}(1-\tau_G) r\left(1 + \frac{\tau}{(1-\tau)\theta}\right) \\
&- \frac{1}{2}\frac{1-\delta}{\delta^2}\left(\frac{(1-\tau)\mu - (1-\tau_G) r}{(1-\tau)\sigma}\right)^2 - \lambda\left(\frac{(1-\tau)\theta\lambda}{(1-\tau_G) r\tau}\right)^{\frac{1-\delta}{\delta}}.
\end{aligned}$$

Given the boundary condition, the unique solution for this equation is

$$F(t) = \frac{1}{\phi} + \left(\chi^{\frac{1}{\delta}} - \frac{1}{\phi}\right) e^{-\phi(T-t)}.$$

B Government revenue

If $\Theta(t)$ is the total government revenue, then in the differential terms we have

$$d\Theta(t) = \tau_G w_G(t)\, dG(t) + \tau w(t)\, dS(t) + w_0(t) S(t)\, \theta d\Pi(t)$$

whose expected value is

$$\mathbb{E}_t[d\Theta(t)] = (\tau_G w_G(t) G(t) r + \tau w(t) S(t) \mu + w_0(t) S(t) \theta\lambda)\, dt.$$

Investor's wealth is

$$R(t) = w_G(t)\, G(t) + w(t)\, S(t) + w_0(t)\, S(t)\,;$$

hence, the expected revenue from capital gains income tax will be (substituting for $w_G(t)\, G(t)$):

$$\mathbb{E}_t\left[d\Theta(t)\right] = R(t)\left(\tau_G r + \frac{w(t)\, S(t)}{R(t)}\left(\tau\mu - \tau_G r\right) + \frac{w_0(t)\, S(t)}{R(t)}\left(\theta\lambda - \tau_G r\right)\right) dt.$$

If we substitute the optimal values of both $\frac{w(t)S(t)}{R(t)}$ and $\frac{w_0(t)S(t)}{R(t)}$, we obtain:

$$\mathbb{E}_t\left[d\Theta(t)\right] = R(t)\begin{pmatrix} \tau_G r + \frac{1}{\delta}\frac{(1-\tau)\mu-(1-\tau_G)r}{(1-\tau)^2\sigma^2}\left(\tau\mu - \tau_G r\right) \\ +\frac{1}{\theta}\left(1-\left(\frac{(1-\tau)\theta\lambda}{(1-\tau_G)r\tau}\right)^{\frac{1}{\delta}}\right)\left(\theta\lambda - \tau\frac{\mu-\tau_G r}{1-\tau}\right)\end{pmatrix} dt. \tag{20}$$

As expected, government revenue depends on all market and fiscal variables. The first two terms of the equation represent the expected revenue in the absence of tax evasion. We now concentrate on the third term, which also depends on the tax audit parameters λ and θ:

$$F(\theta,\lambda) \equiv \frac{1}{\theta}\left(1-\left(\frac{(1-\tau)\,\theta\lambda}{(1-\tau_G)\, r\tau}\right)^{\frac{1}{\delta}}\right)\left(\theta\lambda - \tau\frac{\mu-\tau_G r}{1-\tau}\right).$$

If evasion is expedient (that is, if (18) holds), then this function is always negative.

Proposition 8. *The government's expected revenue (20) declines when taxes are evaded; that is,*

$$\mathbb{E}_t\left[d\Theta(t)\right] < \mathbb{E}_t\left[d\Theta(t)|_{\frac{w_0(t)S(t)}{R(t)}=0}\right].$$

Proof. Let us assume that condition (18) holds. Then,

$$\frac{1}{\theta}\left(1-\left(\frac{(1-\tau)\,\theta\lambda}{(1-\tau_G)\, r\tau}\right)^{\frac{1}{\delta}}\right)\left(\theta\lambda - \tau\frac{\mu-\tau_G r}{1-\tau}\right) < 0$$

implies

$$\theta\lambda - \tau\frac{\mu-\tau_G r}{1-\tau} < 0,$$

which can be written as

$$1 - \frac{\tau(\mu - r) + (1-\tau_G)\,\tau r}{(1-\tau)\,\theta\lambda} < 0$$

or

$$1 - \frac{(1-\tau_G)\,\tau r}{(1-\tau)\,\theta\lambda} - \frac{\tau}{1-\tau}\frac{\mu - r}{\theta\lambda} < 0.$$

Since the sum of the first two terms is negative because 18, and the last term is negative because $\mu > r$, then the inequality holds. □

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Figure 1: Six simulations of $5,000$ daily values (i.e. $T = 20$ years with $dt = \frac{1}{250}$) of the optimal wealth with evasion (in bold) and without evasion (dotted line), with initial wealth 100. Risky asset parameters are $\mu = 0.08$ and $\sigma = 0.2$. The fiscal parameters are: $\lambda = 0.1$, $\theta(\tau) = 0.08$, $\tau = 0.27$, $\tau_G = 0.235$. The riskless interest rate is $r = 0.04$ and the preference parameters are $\rho = 0.04$, $\chi = 1$, and $\delta = 2.5$. The bottom-right graph shows the average behaviour of wealth over 1000 simulations.

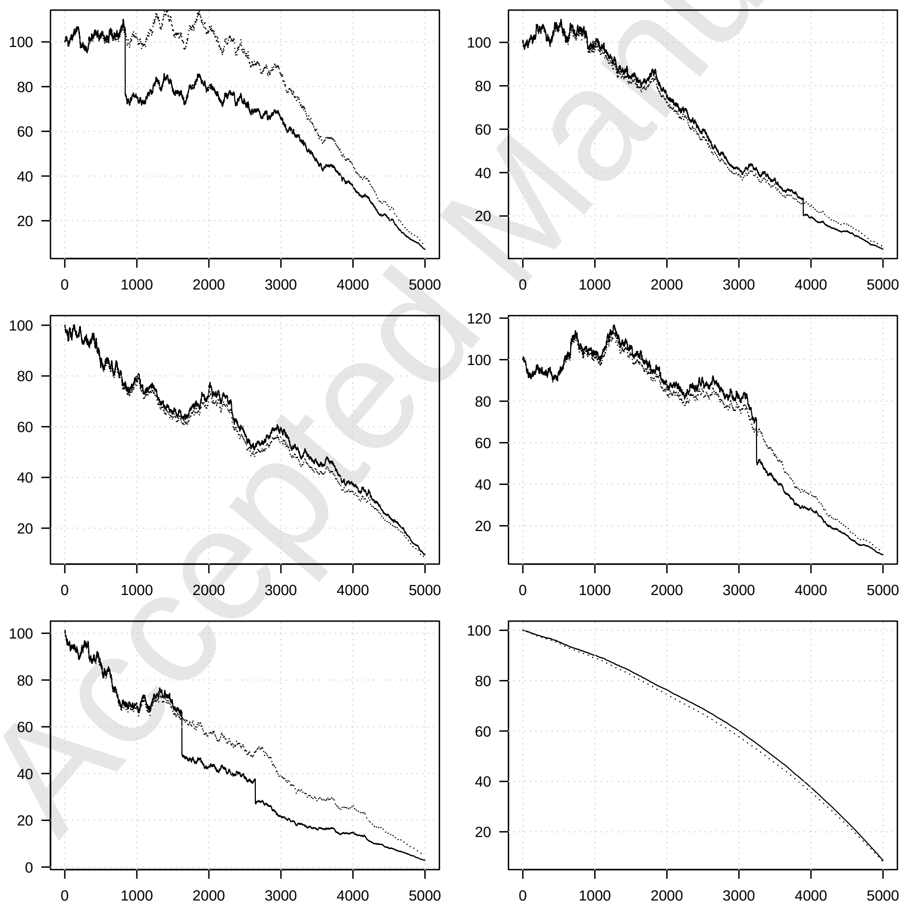
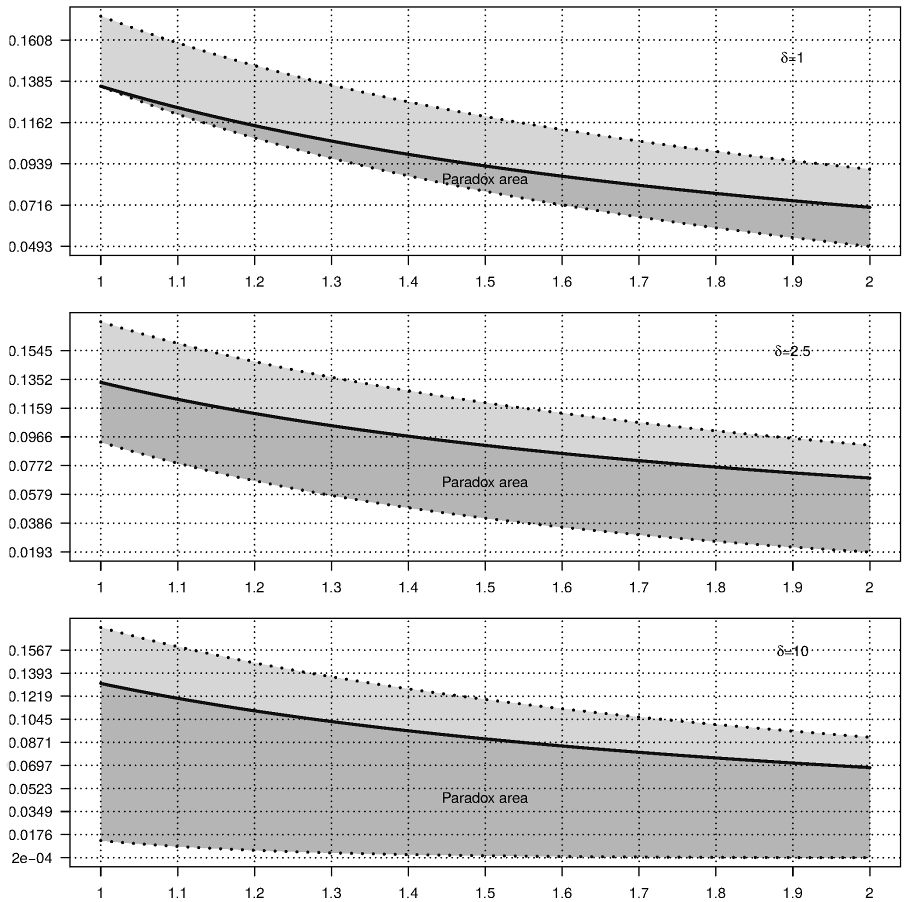
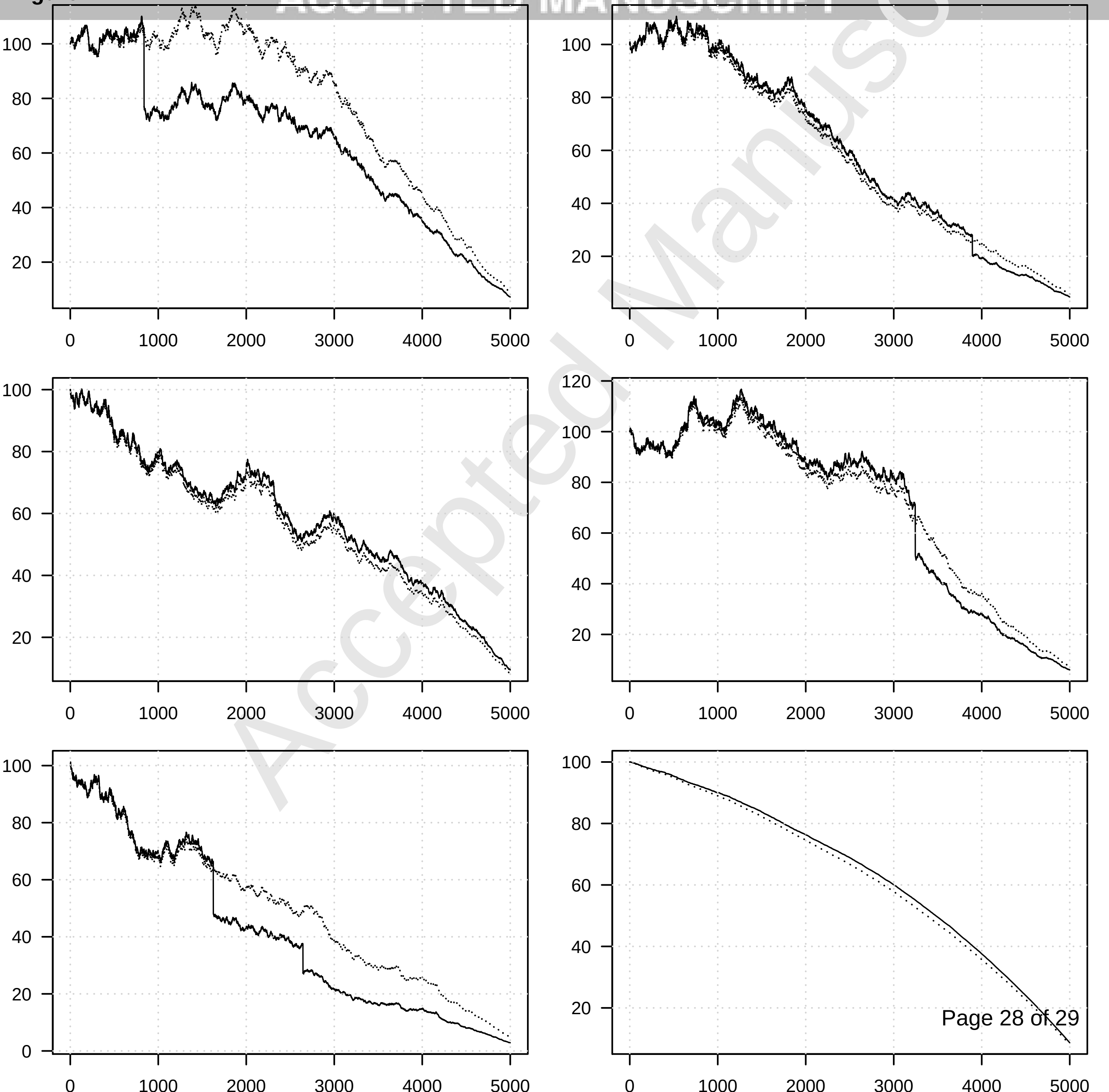


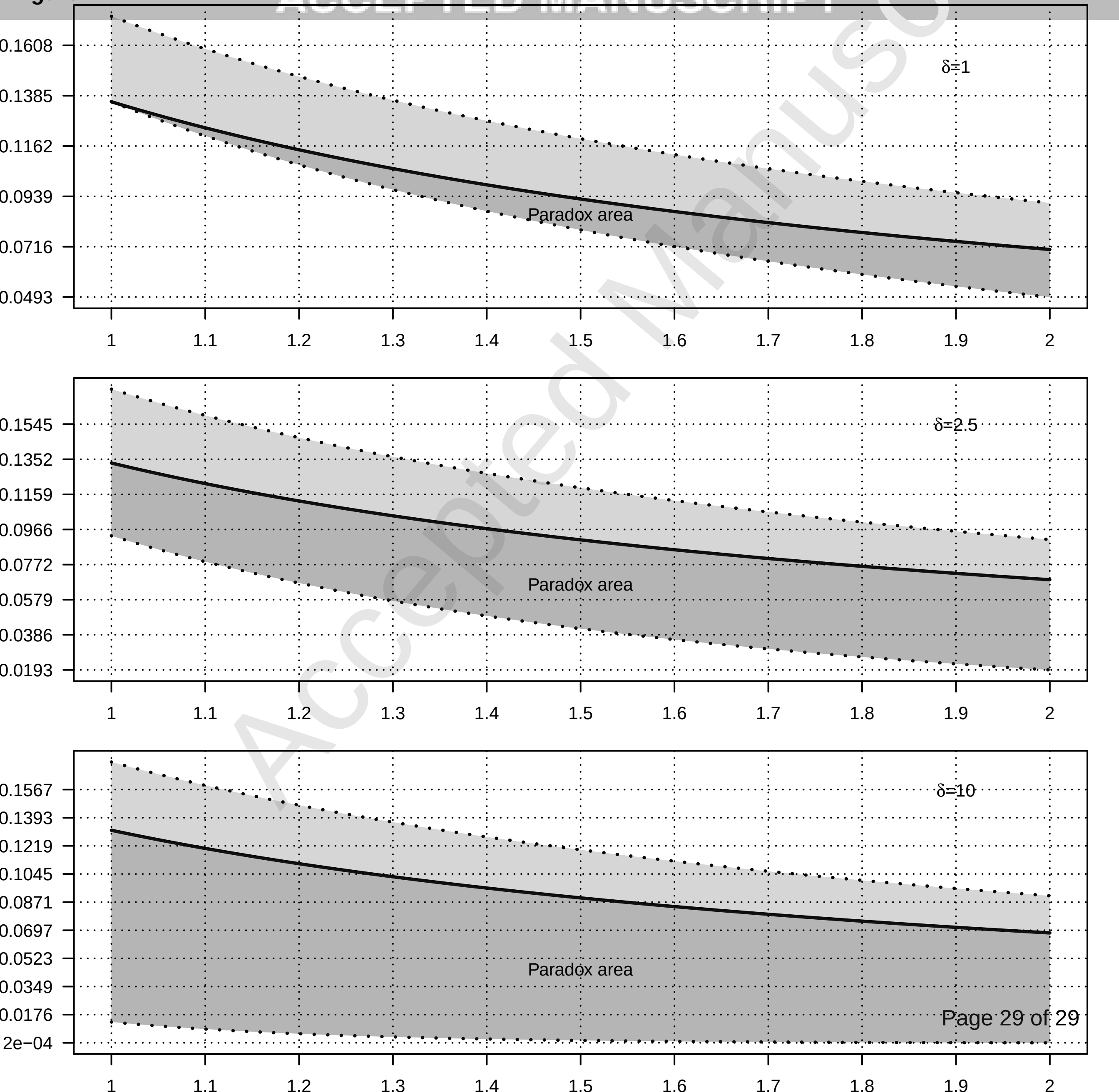
Figure 2: α appears on the horizontal axis and the probability of getting caught for five periods $(1 - e^{-\lambda 5})$ appears on the vertical axis. The shaded area shows the feasible area (evasion$\in [0, 1]$), while the dark grey area shows the locus where Yitzhaki [1974]'s paradox arises. We have chosen $\tau_G = 0.235$, $\tau = 0.27$, and $r = 0.04$, and three values for risk aversion.



Figure



Figure


δ=1
Paradox area
0.1608
0.1385
0.1162
0.0939
0.0716
0.0493
δ=2.5
Paradox area
0.1545
0.1352
0.1159
0.0966
0.0772
0.0579
0.0386
0.0193
δ=10
Paradox area
0.1567
0.1393
0.1219
0.1045
0.0871
0.0697
0.0523
0.0349
0.0176
2e−04
1 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2