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BMI 500 Week 11 BSS Lab
Question 1A

Let $C = \begin{bmatrix} 5 & 1 \\ 1 & 2 \end{bmatrix}$

$Cv_i = \lambda_i v_i$

The eigenvalues λ for C can be given by solving for λ in the following

$$\det(C - \lambda I) = 0$$

$$\det\left(\begin{bmatrix} 5 & 1 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda \end{bmatrix}\right) = 0$$

$$\det \begin{bmatrix} 5-\lambda & 1 \\ 1 & 2-\lambda \end{bmatrix} = 0$$

$$(5-\lambda)(2-\lambda) - (1)(1) = 0$$

$$10 - 5\lambda - 2\lambda + \lambda^2 - 1 = 0$$

$$\lambda^2 - 7\lambda + 9 = 0$$

Gives $\lambda = 1.69$ or $\lambda = 5.30$ — these are the eigenvalues

Let $\lambda = 1.69$

$Cv_1 = 1.69 v_1$ This is the definition of the eigenvector

$$(C - \lambda) v_1 = 0$$

$$\left(\begin{bmatrix} 5 & 1 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} 1.69 & 0 \\ 0 & 1.69 \end{bmatrix}\right) v_1 = 0$$

$$\begin{bmatrix} 5-1.69 & 1 \\ 1 & 2-1.69 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} 3.31 & 1 \\ 1 & 0.31 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0$$

Solving for this system of linear equations

$$v = \begin{bmatrix} 0.29 \\ -0.96 \end{bmatrix}$$

eigenvector corresponding to $\lambda = 1.69$

Let $\lambda = 5.30$

Again: $(C - \lambda) v = 0$

$$\begin{bmatrix} 5-5.30 & 1 \\ 1 & 2-5.30 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} -0.30 & 1 \\ 1 & -3.30 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0$$

Solving for this system of linear equations

$$v = \begin{bmatrix} -0.96 \\ -0.29 \end{bmatrix}$$

eigenvector corresponding to $\lambda = 5.30$