2042 MLM Final Group Project (Spring 2020)

Part 1

Group 1

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Team members and division of work

Group 1 Team Members:

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Division of Work:

Frank Jiang: Group project part 1

Lisa Song: Group project part 2

Yuyue Hua: Group project part 2

Seeun Jang: Group project part 1

Tong Jin: The mini project

All team members: Review all submissions

Question 0

We will use the `classroom.csv` data for this project.

a. `math1st` will be the outcome of interest for this first part.

   i. Recall that `math1st` = `mathkind` + `mathgain`

b. Read in the data (R: store as `dat`)

c. Fit all models using **REML** (not the default in R)

d. It's best if you use `lmerTest::lmer` rather than `lme4::lmer` to call the MLM function. The former provides *p*-values for fixed effects in the summary.

e. There are 2 common error messages one can get from lmer calls: failed to converge (problem with hessian: negative eigenvalue; max|grad| = …); and singularity. They may both be problematic in a real problem, but the latter suggests that a variance component is on the boundary of the parameter space.

   - In your discussion/writeup, consider the latter to be a "convergence problem" and ignore the former.

Solution

Load `classroom.csv` and create `math1st` (fit all models using REML)

```
# Load data and create math1st variable
dat <- read.csv(
  "data/classroom.csv",
  header = TRUE
)
# Create a variable and named as math1st
dat$math1st <- dat$mathkind + dat$mathgain

# School-level predictors:
#   housepov - average household poverty
# Classroom-level predictors:
#   yearstea - years teaching
#   mathknow - math knowledge
#   mathprep - math preparation (number of courses)
# Student-level predictors:
#   sex
#   minority
#   ses - socioeconomic status
#   mathkind - math score in spring of kindergarten
#   mathgain - increase in math score from kindergarten to spring of first grade
```

Question 1

Estimate an Unconditional Means Model (UMM) with random intercepts for *both* schools and classrooms (nested in schools).

a. Report the ICC for schools and the ICC for classrooms.

b. **Write out this model** using your preferred notation, but use the same choice of notation for the remainder of your project.

i. Be mindful and explicit about any assumptions made.

Solution

```
# Fit the unconditional model
unconditional_model <- lmer(math1st ~ (1|schoolid/classid),
                            data = dat)
# Report the model fit
print(summary(unconditional_model))
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ (1 | schoolid/classid)
##    Data: dat
##
## REML criterion at convergence: 11944.6
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -5.1872 -0.6174 -0.0204  0.5821  3.8339
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   85.47   9.245
##  schoolid         (Intercept)  280.69  16.754
##  Residual                      1146.79  33.864
## Number of obs: 1190, groups:  classid:schoolid, 312; schoolid, 107
##
## Fixed effects:
##             Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  522.540      2.037 104.403   256.6   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
# Calculate ICC for schools and classrooms
var_schoolid <- round(as.numeric(VarCorr(unconditional_model)$'schoolid'),
                      digits = 3)
var_classid <- round(as.numeric(VarCorr(unconditional_model)$'classid:schoolid'),
                     digits = 3)
var_res <- round(attr(VarCorr(unconditional_model), "sc")^2,
                 digits = 3)
```

```
var_total <- var_schoolid + var_classid + var_res

ICC_class <- round(var_classid / var_total, digits = 3)
ICC_school <- round(var_schoolid / var_total, digits = 3)
```

a. The ICC for schools is 0.186. Here is the equation:

$$ICC_{\text{school}} = \frac{\sigma^2_\zeta}{\sigma^2_{total} + \sigma^2_\varepsilon} = \frac{280.69}{85.47 + 1146.79 + 280.69} = 0.056$$

, where σ^2_ζ is the variance of school-level random effects, σ^2_{total} is the total variance of all random effects, and σ^2_ε is the variance of residual.

The ICC for classrooms is 0.056. Here is the equation:

$$ICC_{\text{class}} = \frac{85.47}{85.47 + 1146.79 + 280.69} = 0.186$$

, where σ^2_η is the variance of classroom-level random effects, σ^2_{total} is the total variance of all random effects, and σ^2_ε is the variance of residual.

b. The unconditional model fitting on `math1st` with random intercepts for `schoolid` and `classid` is:

$$MATH1ST_{ijk} = b_0 + \eta_{jk} + \zeta_k + \varepsilon_{ijk} \ ,$$

where $\eta_{jk} \sim \mathcal{N}(0, \sigma^2_\eta)$, $\zeta_k \sim \mathcal{N}(0, \sigma^2_\zeta)$, independently of each other, and $\varepsilon_{ijk} \sim \mathcal{N}(0, \sigma^2_\varepsilon)$, i represents individuals, j represents classrooms, and k represents schools.

Question 2

Add **all** school level predictors.

a. Report if adding the predictors **as a block** is justified.

b. Report change in σ^2_ζ.

Solution

```
# Add school-level predictor: housepov
model_all_school <- lmer(math1st ~ housepov + (1|schoolid/classid),
                         data = dat)
# Report the model fit
print(summary(model_all_school))
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ housepov + (1 | schoolid/classid)
##    Data: dat
##
## REML criterion at convergence: 11927.4
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -5.1142 -0.6011 -0.0350  0.5600  3.8154
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   82.36   9.075
##  schoolid         (Intercept)  250.93  15.841
##  Residual                      1146.96  33.867
## Number of obs: 1190, groups:  classid:schoolid, 312; schoolid, 107
##
## Fixed effects:
##             Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  531.294      3.341 102.807 159.023   <2e-16 ***
## housepov     -45.783     14.236 111.060  -3.216   0.0017 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr)
## housepov -0.810
```

```
# Store the variances of random effects
var_schoolid_q2 <- round(as.numeric(VarCorr(model_all_school)$'schoolid'),
                         digits = 3)
var_classid_q2 <- round(as.numeric(VarCorr(model_all_school)$'classid'),
                        digits = 3)
var_res_q2 <- round(attr(VarCorr(model_all_school), "sc")^2,
```

```
                    digits = 3)

# Examine justification of adding the predictor
fit_test_q2 <- anova(model_all_school,
                     unconditional_model,
                     refit = FALSE)
fit_test_q2
```

```
## Data: dat
## Models:
## unconditional_model: math1st ~ (1 | schoolid/classid)
## model_all_school: math1st ~ housepov + (1 | schoolid/classid)
##                      Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## unconditional_model  4 11953 11973 -5972.3    11945
## model_all_school     5 11937 11963 -5963.7    11927 17.186      1   3.39e-05
##
## unconditional_model
## model_all_school    ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
p_q2 <- round(
  fit_test_q2$'Pr(>Chisq)'[2],
  digits = 3
)
p_housepov_q2 <- round(
  summary(model_all_school)$coefficients['housepov', 'Pr(>|t|)'],
  digits = 3
)
```

a. The p-value of adding the housepov variable to the model is 0, which is less than 0.05. Therefore, adding the housepov is significant to the model. Additionally, the p-value for the coefficient on the housepov variable is 0.002, which is significant (smaller than 0.05). This also justifies the significance of adding this variable to the model.

b. After adding the school-level variable (housepov), the σ^2_ζ dropped from 280.691 to 250.933.

Question 3

Add **all** classroom level predictors.

a. Report if adding the predictors **as a block** is justified.

b. Report change in σ^2_η and change in σ^2_ϵ.

c. Give a potential reason as to why σ^2_ϵ is reduced, but not σ^2_η?

Solution

```
# Add classroom-level predictor: yearstea, mathknow, mathprep
model_all_class <- lmer(
  math1st ~ yearstea + mathknow + mathprep + housepov + (1|schoolid/classid),
  data = dat
)
# Report the model fit
print(summary(model_all_class))
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ yearstea + mathknow + mathprep + housepov + (1 | schoolid/classid)
##    Data: dat
##
## REML criterion at convergence: 10821
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.5552 -0.6118 -0.0311  0.5863  3.8315
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   94.36   9.714
##  schoolid         (Intercept)  223.31  14.943
##  Residual                      1136.43  33.711
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error        df t value Pr(>|t|)
## (Intercept) 532.29852    5.20495 228.85767 102.268  < 2e-16 ***
## yearstea      0.06193    0.14717 223.76570   0.421  0.67432
## mathknow      2.55143    1.44530 231.06560   1.765  0.07883 .
## mathprep     -0.75440    1.42809 203.20755  -0.528  0.59790
## housepov    -41.62117   14.08834 109.83230  -2.954  0.00383 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) yearst mthknw mthprp
## yearstea -0.264
```

```
## mathknow -0.052  0.030
## mathprep -0.666 -0.175  0.004
## housepov -0.568  0.077  0.082  0.032
```

```r
# Store the variances of random effects
var_schoolid_q3 <- round(as.numeric(VarCorr(model_all_class)$'schoolid'),
                         digits = 3)
var_classid_q3 <- round(as.numeric(VarCorr(model_all_class)$'classid'),
                        digits = 3)
var_res_q3 <- round(attr(VarCorr(model_all_class), "sc")^2,
                    digits = 3)

# Examine the fit of predictors
fit_test_q3 <- linearHypothesis(model_all_class,
                                c('yearstea','mathknow','mathprep'))
fit_test_q3
```

```
## Linear hypothesis test
##
## Hypothesis:
## yearstea = 0
## mathknow = 0
## mathprep = 0
##
## Model 1: restricted model
## Model 2: math1st ~ yearstea + mathknow + mathprep + housepov + (1 | schoolid/classid)
##
##   Df  Chisq Pr(>Chisq)
## 1
## 2  3 3.4804     0.3233
```

```r
p_q3 <- round(fit_test_q3$`Pr(>Chisq)`[2], digits = 3)
```

a. Based the Wald test, we can conclude that the p-value for adding classroom-level variable is 0.323, which is not significant at the level of 0.05.

b. The σ^2_η increased from 82.357 (the previous model) to 94.363, while the σ^2_ε dropped from 1146.955 (the previous model) to 1136.431.

c. The residuals of the model are decreased because some of the variance are explained by adding classroom-level variable. However, to the classroom-level effect, some added variable might not be significant or there might exists some correlation between the added classroom-level variables.

Question 4

Add (nearly) **all** student level predictors (but not `mathgain` or `mathkind`, as these are outomes in this context).

a. Report if justified statistically **as a block** of predictors.

b. Report change in variance components for all levels.

c. Give a potential reason as to why the school level variance component drops from prior model.

d. **Write out this model** using your chosen notation.

Solution

```
# Add student-level predictor: ses, minority, ses
model_all_student <- lmer(
  math1st ~ sex + minority + ses + yearstea + mathknow + mathprep + housepov + (1|schoolid/classid),
  data = dat
)

# Report the model fit
print(summary(model_all_student))
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ sex + minority + ses + yearstea + mathknow + mathprep +
##     housepov + (1 | schoolid/classid)
##    Data: dat
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8581 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   93.89   9.689
##  schoolid         (Intercept)  169.45  13.017
##  Residual                      1064.96  32.634
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63041    5.31209  275.39010 101.585  < 2e-16 ***
## sex           -1.21419    2.09483 1022.42110  -0.580    0.562
## minority     -16.18676    3.02605  704.47787  -5.349 1.20e-07 ***
## ses           10.05076    1.54485 1066.56211   6.506 1.18e-10 ***
## yearstea       0.01129    0.14141  226.80861   0.080    0.936
## mathknow       1.35004    1.39168  234.49768   0.970    0.333
## mathprep      -0.27705    1.37583  205.27111  -0.201    0.841
## housepov     -17.64850   13.21755  113.87814  -1.335    0.184
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) sex    minrty ses    yearst mthknw mthprp
## sex      -0.190
## minority -0.320 -0.011
## ses      -0.121  0.020  0.162
## yearstea -0.259  0.016  0.024 -0.028
## mathknow -0.083  0.007  0.115 -0.007  0.029
## mathprep -0.631 -0.006  0.001  0.053 -0.172  0.004
## housepov -0.451 -0.007 -0.178  0.082  0.071  0.058  0.038
```

```
# Store the variances of random effects
var_schoolid_q4 <- round(as.numeric(VarCorr(model_all_student)$'schoolid'),
                         digits = 3)
var_classid_q4 <- round(as.numeric(VarCorr(model_all_student)$'classid'),
                        digits = 3)
var_res_q4 <- round(attr(VarCorr(model_all_student), "sc")^2,
                    digits = 3)

# Examine the fit of predictors
fit_test_q4_aov <- anova(model_all_class,
                     model_all_student,
                     refit = FALSE)
fit_test_q4_aov
```

```
## Data: dat
## Models:
## model_all_class: math1st ~ yearstea + mathknow + mathprep + housepov + (1 | schoolid/classid)
## model_all_student: math1st ~ sex + minority + ses + yearstea + mathknow + mathprep +
## model_all_student:     housepov + (1 | schoolid/classid)
##                   Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## model_all_class    8 10837 10877 -5410.5    10821
## model_all_student 11 10752 10806 -5364.8    10730 91.446      3  < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
fit_test_q4_hypo <- linearHypothesis(model_all_student,
                                     c('sex','minority','ses'))
fit_test_q4_hypo
```

```
## Linear hypothesis test
##
## Hypothesis:
## sex = 0
## minority = 0
## ses = 0
##
## Model 1: restricted model
## Model 2: math1st ~ sex + minority + ses + yearstea + mathknow + mathprep +
##     housepov + (1 | schoolid/classid)
##
##   Df  Chisq Pr(>Chisq)
```

```
## 1
## 2  3 85.055  < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
p_q4 <- round(fit_test_q4_hypo$`Pr(>Chisq)`[2],
              digits = 3)
```

a. The p-value of adding the student-level predictors ((`sex`, `minority`, `ses`)) to the model is 0, which is less than 0.05. This implies that adding the student-level predictors to the model is significant at the 0.05 level.

b. Comparing to the previous model before adding the student-level predictors, the σ^2_η dropped from 94.363 to 93.885.

The σ^2_ε dropped from 1136.431 to 1064.956.

The σ^2_ζ dropped from 223.306 to 169.448.

c. The school level variance dropped might due to adding the student-level predictors that may be associated with group (school) effects in the aggregation. Thus, adding the student-level predictors caused the decrease of the school-level variance.

d. The model after adding the student-level predictors is:

$$\begin{aligned} MATH1ST_{ijk} = b_0 + b_1 SES_{ijk} + b_2 SEX_{ijk} + b_3 MINORITY_{ijk} + \\ b_4 YEARSTEA_{jk} + b_5 MATHKNOW_{jk} + b_6 MATHPREP_{jk} + \\ b_7 HOUSEPOV_k + \eta_{jk} + \zeta_k + \varepsilon_{ijk} \end{aligned}$$

, where $\eta_{jk} \sim \mathcal{N}(0, \sigma^2_\eta)$, $\zeta_k \sim \mathcal{N}(0, \sigma^2_\zeta)$, independently of each other, and $\varepsilon_{ijk} \sim \mathcal{N}(0, \sigma^2_\varepsilon)$, i represents individuals, j represents classrooms, and k represents schools.

Question 5

a. Try to add a random slope for each **teacher level** predictor (varying at the **school level**; one by one separately - not all together).

b. Report the model fit or lack of fit.

c. Why is it a bad idea to include a random slope on the `housepov` effect?

d. Retry the above, allowing the slopes to be correlated with the random intercepts (still one by one).

e. Report anything unusual about the variance components (changes that are in a direction you didn't expect) and any potential explanation for why those changes occurred (*hint: what did you add to the model?*).

Solution

a.

```
# Add random slope for teacher-level predictors
# mathknow
model_mathknow <- lmer(
  math1st ~ sex + minority + ses + yearstea + mathknow + mathprep + housepov +
    (0 + mathknow | schoolid) + (1 | schoolid) + (1 | classid),
  data = dat
)
```

```
## boundary (singular) fit: see ?isSingular
```

```
# Report the model fit
print(summary(model_mathknow))
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ sex + minority + ses + yearstea + mathknow + mathprep +
##     housepov + (0 + mathknow | schoolid) + (1 | schoolid) + (1 |      classid)
##    Data: dat
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups     Name        Variance  Std.Dev.
##  classid    (Intercept) 9.389e+01  9.689914
##  schoolid   (Intercept) 1.694e+02 13.016328
##  schoolid.1 mathknow    1.700e-06  0.001304
##  Residual               1.065e+03 32.633705
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
```

```
## (Intercept)  539.63047    5.31204  275.40357 101.586  < 2e-16 ***
## sex           -1.21417    2.09483 1022.42010  -0.580    0.562
## minority     -16.18681    3.02603  704.47306  -5.349 1.20e-07 ***
## ses           10.05075    1.54485 1066.56262   6.506 1.18e-10 ***
## yearstea       0.01129    0.14141  226.81110   0.080    0.936
## mathknow       1.34993    1.39168  234.50060   0.970    0.333
## mathprep      -0.27708    1.37583  205.27196  -0.201    0.841
## housepov     -17.64821   13.21718  113.88792  -1.335    0.184
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) sex    minrty ses    yearst mthknw mthprp
## sex      -0.190
## minority -0.320 -0.011
## ses      -0.121  0.020  0.162
## yearstea -0.259  0.016  0.024 -0.028
## mathknow -0.083  0.007  0.115 -0.007  0.029
## mathprep -0.631 -0.006  0.001  0.053 -0.172  0.004
## housepov -0.451 -0.007 -0.178  0.082  0.071  0.058  0.038
## convergence code: 0
## boundary (singular) fit: see ?isSingular
```

```r
# Store the variances of random effects
var_schoolid_q5_mathknow <- round(
  as.numeric(VarCorr(model_mathknow)$'schoolid'),
  digits = 3)
var_classid_q5_mathknow <- round(
  as.numeric(VarCorr(model_mathknow)$'classid'),
  digits = 3)
var_res_q5_mathknow <- round(
  attr(VarCorr(model_mathknow), "sc")^2,
  digits = 3)

# Examine the fit of random slope
fit_test_q5_mathknow <- anova(model_mathknow,
                              model_all_student,
                              refit = FALSE)
fit_test_q5_mathknow
```

```
## Data: dat
## Models:
## model_all_student: math1st ~ sex + minority + ses + yearstea + mathknow + mathprep +
## model_all_student:     housepov + (1 | schoolid/classid)
## model_mathknow: math1st ~ sex + minority + ses + yearstea + mathknow + mathprep +
## model_mathknow:     housepov + (0 + mathknow | schoolid) + (1 | schoolid) + (1 |
## model_mathknow:     classid)
##                   Df   AIC   BIC  logLik deviance Chisq Chi Df Pr(>Chisq)
## model_all_student 11 10752 10806 -5364.8    10730
## model_mathknow    12 10754 10813 -5364.8    10730     0      1          1
```

```r
# mathprep
model_mathprep <- lmer(
```

```
  math1st ~ sex + minority + ses + yearstea + mathknow + mathprep + housepov +
    (0 + mathprep | schoolid) + (1 | schoolid) + (1 | classid),
  data = dat
)
```

```
## boundary (singular) fit: see ?isSingular
```

```
# Report the model fit
print(summary(model_mathprep))
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ sex + minority + ses + yearstea + mathknow + mathprep +
##     housepov + (0 + mathprep | schoolid) + (1 | schoolid) + (1 |      classid)
##    Data: dat
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8581 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups     Name        Variance  Std.Dev.
##  classid    (Intercept) 9.388e+01 9.689e+00
##  schoolid   (Intercept) 1.694e+02 1.302e+01
##  schoolid.1 mathprep    2.171e-07 4.659e-04
##  Residual               1.065e+03 3.263e+01
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63039    5.31207  275.39223 101.586  < 2e-16 ***
## sex           -1.21419    2.09483 1022.42070  -0.580    0.562
## minority     -16.18676    3.02605  704.47629  -5.349 1.20e-07 ***
## ses           10.05076    1.54485 1066.56201   6.506 1.18e-10 ***
## yearstea       0.01129    0.14141  226.80838   0.080    0.936
## mathknow       1.35003    1.39167  234.49786   0.970    0.333
## mathprep      -0.27705    1.37582  205.27063  -0.201    0.841
## housepov     -17.64851   13.21749  113.87941  -1.335    0.184
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) sex    minrty ses    yearst mthknw mthprp
## sex      -0.190
## minority -0.320 -0.011
## ses      -0.121  0.020  0.162
## yearstea -0.259  0.016  0.024 -0.028
## mathknow -0.083  0.007  0.115 -0.007  0.029
## mathprep -0.631 -0.006  0.001  0.053 -0.172  0.004
## housepov -0.451 -0.007 -0.178  0.082  0.071  0.058  0.038
## convergence code: 0
```

```
## boundary (singular) fit: see ?isSingular
```

```
# Store the variances of random effects
var_schoolid_q5_mathprep <- round(
  as.numeric(VarCorr(model_mathprep)$'schoolid'),
  digits = 3)
var_classid_q5_mathprep <- round(
  as.numeric(VarCorr(model_mathprep)$'classid'),
  digits = 3)
var_res_q5_mathprep <- round(
  attr(VarCorr(model_mathprep), "sc")^2,
  digits = 3)

# Examine the fit of random slope
fit_test_q5_mathprep <- anova(model_mathprep,
                              model_all_student,
                              refit = FALSE)
fit_test_q5_mathprep
```

```
## Data: dat
## Models:
## model_all_student: math1st ~ sex + minority + ses + yearstea + mathknow + mathprep +
## model_all_student:     housepov + (1 | schoolid/classid)
## model_mathprep: math1st ~ sex + minority + ses + yearstea + mathknow + mathprep +
## model_mathprep:     housepov + (0 + mathprep | schoolid) + (1 | schoolid) + (1 |
## model_mathprep:     classid)
##                    Df   AIC   BIC  logLik deviance Chisq Chi Df Pr(>Chisq)
## model_all_student 11 10752 10806 -5364.8    10730
## model_mathprep    12 10754 10813 -5364.8    10730     0      1          1
```

```
# yearstea
model_yearstea <- lmer(
  math1st ~ sex + minority + ses + yearstea + mathknow + mathprep + housepov +
    (0 + yearstea | schoolid) + (1 | schoolid) + (1 | classid),
  data = dat
)
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl = control$checkConv, :
## Model failed to converge with max|grad| = 0.00805454 (tol = 0.002, component 1)
```

```
# Report the model fit
print(summary(model_yearstea))
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ sex + minority + ses + yearstea + mathknow + mathprep +
##     housepov + (0 + yearstea | schoolid) + (1 | schoolid) + (1 |      classid)
##    Data: dat
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
```

```
##     Min      1Q  Median      3Q     Max
## -3.8482 -0.6147 -0.0322  0.5979  3.6603
##
## Random effects:
##  Groups     Name        Variance  Std.Dev.
##  classid    (Intercept) 9.247e+01  9.6159
##  schoolid   (Intercept) 1.684e+02 12.9758
##  schoolid.1 yearstea    1.008e-02  0.1004
##  Residual               1.065e+03 32.6361
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.59885    5.30780  266.47953 101.662  < 2e-16 ***
## sex           -1.21060    2.09480 1022.21558  -0.578    0.563
## minority     -16.16715    3.02635  702.61831  -5.342 1.24e-07 ***
## ses           10.04528    1.54492 1066.09816   6.502 1.21e-10 ***
## yearstea       0.01128    0.14192  122.87740   0.079    0.937
## mathknow       1.33106    1.39155  234.33195   0.957    0.340
## mathprep      -0.26584    1.37588  204.90504  -0.193    0.847
## housepov     -17.72082   13.21686  113.58577  -1.341    0.183
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) sex    minrty ses    yearst mthknw mthprp
## sex      -0.191
## minority -0.320 -0.010
## ses      -0.121  0.020  0.162
## yearstea -0.258  0.015  0.023 -0.027
## mathknow -0.082  0.006  0.115 -0.007  0.028
## mathprep -0.632 -0.006  0.001  0.053 -0.172  0.003
## housepov -0.450 -0.007 -0.179  0.082  0.070  0.057  0.037
## convergence code: 0
## Model failed to converge with max|grad| = 0.00805454 (tol = 0.002, component 1)
```

```r
# Store the variances of random effects
var_schoolid_q5_yearstea <- round(
  as.numeric(VarCorr(model_yearstea)$'schoolid'),
  digits = 3)
var_classid_q5_yearstea <- round(
  as.numeric(VarCorr(model_yearstea)$'classid'),
  digits = 3)
var_res_q5_yearstea <- round(
  attr(VarCorr(model_yearstea), "sc")^2,
  digits = 3)

# Examine the fit of random slope
fit_test_q5_yearstea <- anova(model_yearstea,
                              model_all_student,
                              refit = FALSE)

# Store the p-values
p_q5_mathknow <- round(fit_test_q5_mathknow$`Pr(>Chisq)`[2], digits = 3)
```

```r
p_q5_mathprep <- round(fit_test_q5_mathprep$`Pr(>Chisq)`[2], digits = 3)
p_q5_yearstea <- round(fit_test_q5_yearstea$`Pr(>Chisq)`[2], digits = 3)
```

b. According to the LRT test conducted above, we can conclude that the p-value for adding random slope for teacher level variable `mathknow`, `mathprep` and `yearstea` are 1, 1, and 0.934. This implies that there is no significant variation for adding those random slopes. Thus, the model for adding all three variables **does not fit**.

c. Because `housepov` are school-level predictor and, therefore, it only varies at the school level. Including `housepov` as a random slope on teacher-level only create an redundant school-level random effects.

d.

```r
# Add random slope for teacher-level predictors with correlation
# mathknow
model_mathknow_cor <- lmer(
  math1st ~ sex + minority + ses + yearstea + mathknow + mathprep + housepov +
    (mathknow|schoolid) + (1|classid),
  data = dat
)
# Report the model fit
print(summary(model_mathknow_cor))
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ sex + minority + ses + yearstea + mathknow + mathprep +
##     housepov + (mathknow | schoolid) + (1 | classid)
##    Data: dat
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8581 -0.6131 -0.0324  0.5969  3.6603
##
## Random effects:
##  Groups   Name        Variance  Std.Dev. Corr
##  classid  (Intercept) 9.393e+01  9.6915
##  schoolid (Intercept) 1.693e+02 13.0118
##           mathknow    9.182e-04  0.0303  0.97
##  Residual             1.065e+03 32.6341
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.64041    5.31203  275.38950 101.588  < 2e-16 ***
## sex           -1.21328    2.09485 1021.79809  -0.579    0.563
## minority     -16.19378    3.02608  703.80362  -5.351 1.18e-07 ***
## ses           10.04788    1.54488 1062.12259   6.504 1.20e-10 ***
## yearstea       0.01114    0.14141  226.85275   0.079    0.937
## mathknow       1.35458    1.39201  214.62535   0.973    0.332
## mathprep      -0.27754    1.37599  201.27744  -0.202    0.840
## housepov     -17.64141   13.21242  103.98189  -1.335    0.185
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) sex    minrty ses    yearst mthknw mthprp
## sex      -0.190
## minority -0.320 -0.011
## ses      -0.121  0.020  0.162
## yearstea -0.259  0.016  0.024 -0.028
## mathknow -0.082  0.007  0.115 -0.007  0.029
## mathprep -0.631 -0.006  0.001  0.053 -0.173  0.004
## housepov -0.451 -0.007 -0.178  0.082  0.071  0.057  0.038
```

```
# Store the variances of random effects
var_schoolid_q5_mathknow_cor <- round(
  as.numeric(VarCorr(model_mathknow_cor)$'schoolid'[1]),
  digits = 3)
var_classid_q5_mathknow_cor <- round(
  as.numeric(VarCorr(model_mathknow_cor)$'classid'[1]),
  digits = 3)
var_res_q5_mathknow_cor <- round(
  attr(VarCorr(model_mathknow_cor), "sc")^2,
  digits = 3)

# Examine the fit of random slopes
fit_test_q5_mathknow_cor <- anova(model_mathknow_cor,
                                  model_all_student,
                                  refit = FALSE)
fit_test_q5_mathknow_cor
```

```
## Data: dat
## Models:
## model_all_student: math1st ~ sex + minority + ses + yearstea + mathknow + mathprep +
## model_all_student:     housepov + (1 | schoolid/classid)
## model_mathknow_cor: math1st ~ sex + minority + ses + yearstea + mathknow + mathprep +
## model_mathknow_cor:     housepov + (mathknow | schoolid) + (1 | classid)
##                     Df   AIC   BIC  logLik deviance Chisq Chi Df Pr(>Chisq)
## model_all_student   11 10752 10806 -5364.8    10730
## model_mathknow_cor  13 10756 10820 -5364.8    10730 3e-04      2     0.9998
```

```
# mathprep
model_mathprep_cor <- lmer(
  math1st ~ sex + minority + ses + yearstea + mathknow + mathprep + housepov +
    (mathprep|schoolid) + (1|classid),
  data = dat
)
```

```
## boundary (singular) fit: see ?isSingular
```

```
# Report the model fit
print(summary(model_mathprep_cor))
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
```

```
## lmerModLmerTest]
## Formula: math1st ~ sex + minority + ses + yearstea + mathknow + mathprep +
##     housepov + (mathprep | schoolid) + (1 | classid)
##    Data: dat
##
## REML criterion at convergence: 10724.7
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8542 -0.6034 -0.0221  0.5915  3.6475
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  classid  (Intercept)   78.46   8.858
##  schoolid (Intercept)  552.68  23.509
##           mathprep      15.88   3.985   -1.00
##  Residual             1064.27  32.623
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  538.60872    5.60800  159.90159  96.043  < 2e-16 ***
## sex           -1.16756    2.08698 1023.14887  -0.559    0.576
## minority     -16.46421    2.99522  663.67376  -5.497 5.52e-08 ***
## ses           10.14167    1.53961 1060.93433   6.587 7.04e-11 ***
## yearstea      -0.02586    0.13948  223.50197  -0.185    0.853
## mathknow       1.29874    1.37192  229.68410   0.947    0.345
## mathprep       0.04067    1.34844  139.04795   0.030    0.976
## housepov     -14.01327   12.88649  116.07068  -1.087    0.279
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) sex    minrty ses    yearst mthknw mthprp
## sex      -0.183
## minority -0.275 -0.013
## ses      -0.121  0.024  0.161
## yearstea -0.260  0.023  0.025 -0.033
## mathknow -0.071  0.002  0.107 -0.001  0.048
## mathprep -0.692 -0.008 -0.035  0.061 -0.155  0.012
## housepov -0.461  0.003 -0.187  0.095  0.089  0.027  0.107
## convergence code: 0
## boundary (singular) fit: see ?isSingular
```

```r
# Store the variances of random effects
var_schoolid_q5_mathprep_cor <- round(
  as.numeric(VarCorr(model_mathprep_cor)$'schoolid'[1]),
  digits = 3)
var_classid_q5_mathprep_cor <- round(
  as.numeric(VarCorr(model_mathprep_cor)$'classid'[1]),
  digits = 3)
var_res_q5_mathprep_cor <- round(
  attr(VarCorr(model_mathprep_cor), "sc")^2,
  digits = 3)
```

```r
# Examine the fit of random slopes
fit_test_q5_mathprep_cor <- anova(model_mathprep_cor,
                                  model_all_student,
                                  refit = FALSE)
fit_test_q5_mathprep_cor
```

```
## Data: dat
## Models:
## model_all_student: math1st ~ sex + minority + ses + yearstea + mathknow + mathprep +
## model_all_student:     housepov + (1 | schoolid/classid)
## model_mathprep_cor: math1st ~ sex + minority + ses + yearstea + mathknow + mathprep +
## model_mathprep_cor:     housepov + (mathprep | schoolid) + (1 | classid)
##                     Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## model_all_student   11 10752 10806 -5364.8    10730
## model_mathprep_cor 13 10751 10816 -5362.3    10725 4.8144      2    0.09007 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
# yearstea
model_yearstea_cor <- lmer(
  math1st ~ sex + minority + ses + yearstea + mathknow + mathprep + housepov +
    (yearstea|schoolid) + (1|classid),
  data = dat
)
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl = control$checkConv, :
## Model failed to converge with max|grad| = 0.0406922 (tol = 0.002, component 1)
```

```r
# Report the model fit
print(summary(model_yearstea_cor))
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ sex + minority + ses + yearstea + mathknow + mathprep +
##     housepov + (yearstea | schoolid) + (1 | classid)
##    Data: dat
##
## REML criterion at convergence: 10723.7
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.7469 -0.6028 -0.0286  0.6038  3.8423
##
## Random effects:
##  Groups   Name        Variance  Std.Dev. Corr
##  classid  (Intercept)   38.8149  6.2302
##  schoolid (Intercept)  363.7664 19.0727
##           yearstea       0.5471  0.7396  -0.78
##  Residual             1066.2528 32.6535
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
```

```
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  538.96287    5.48722  223.18317  98.221  < 2e-16 ***
## sex           -1.33373    2.08780 1024.43110  -0.639    0.523
## minority     -16.44519    2.99647  669.80484  -5.488 5.77e-08 ***
## ses           10.14914    1.53875 1062.65856   6.596 6.66e-11 ***
## yearstea       0.02194    0.15756   75.91038   0.139    0.890
## mathknow       1.04741    1.34469  210.43084   0.779    0.437
## mathprep       0.04601    1.34639  191.56330   0.034    0.973
## housepov     -17.14168   13.45317  119.73031  -1.274    0.205
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) sex    minrty ses    yearst mthknw mthprp
## sex      -0.185
## minority -0.305 -0.012
## ses      -0.119  0.022  0.168
## yearstea -0.369  0.009  0.032 -0.019
## mathknow -0.085  0.008  0.122 -0.001  0.012
## mathprep -0.606 -0.004 -0.007  0.049 -0.139  0.014
## housepov -0.455 -0.004 -0.170  0.079  0.084  0.049  0.050
## convergence code: 0
## Model failed to converge with max|grad| = 0.0406922 (tol = 0.002, component 1)
```

```r
# Store the variances of random effects
var_schoolid_q5_yearstea_cor <- round(
  as.numeric(VarCorr(model_yearstea_cor)$'schoolid'[1]),
  digits = 3)
var_classid_q5_yearstea_cor <- round(
  as.numeric(VarCorr(model_yearstea_cor)$'classid'[1]),
  digits = 3)
var_res_q5_yearstea_cor <- round(
  attr(VarCorr(model_yearstea_cor), "sc")^2,
  digits = 3)

# Examine the fit of random slopes
fit_test_q5_yearstea_cor <- anova(model_yearstea_cor,
                                  model_all_student,
                                  refit = FALSE)
fit_test_q5_yearstea_cor
```

```
## Data: dat
## Models:
## model_all_student: math1st ~ sex + minority + ses + yearstea + mathknow + mathprep +
## model_all_student:     housepov + (1 | schoolid/classid)
## model_yearstea_cor: math1st ~ sex + minority + ses + yearstea + mathknow + mathprep +
## model_yearstea_cor:     housepov + (yearstea | schoolid) + (1 | classid)
##                    Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## model_all_student  11 10752 10806 -5364.8    10730
## model_yearstea_cor 13 10750 10814 -5361.8    10724 5.8248      2    0.05434 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

e. There are a couple things I noticed interesting after allowing for correlation between random slopes and intercepts.

For `mathknow`, the variance component seems not to have a huge influence on the variance component as the variance for class-level effect and residual only slightly varies.

For `mathprep`, the class-level variance dropped by 15.422. However, the school-level intercept variance is 552.682, which is a huge increase from 169.446. This might due to the correlation added to the model between `mathprep` random slope and school-level intercept.

For `yearstea`, the class-level effect variance dropped from 92.466 to 38.815. This variance are explained by the school-level random intercept that are added to the model. Also, the random slope on teacher-level variance is close to 0.54, which is around 0. This might due to the added `yeastea` that aggregate to have an effect and influence on the variation between school-level predictors.

Question 6

a. Try to add a random slope for each **student level** predictor (varying at the **classroom level**; one by one separately - not all together).

b. Why is it a bad idea to include a classroom-level variable with random slopes at the classroom level?

c. Retry the above, allowing the slopes to be correlated with the random intercepts. Report findings.

Solution

a.

```
# Add random slope for each student level predictor: varying at classrooms
# ses
model_ses <- lmer(
  math1st ~ sex + minority + ses + yearstea + mathknow + mathprep + housepov +
    (0 + ses|classid) + (1|schoolid) + (1|classid),
  data = dat
)
# Report the model fit
print(summary(model_ses))
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ sex + minority + ses + yearstea + mathknow + mathprep +
##     housepov + (0 + ses | classid) + (1 | schoolid) + (1 | classid)
##    Data: dat
##
## REML criterion at convergence: 10727.9
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.7163 -0.6032 -0.0331  0.5855  3.6840
##
## Random effects:
##  Groups    Name        Variance Std.Dev.
##  classid   (Intercept)   87.11   9.333
##  classid.1 ses           49.60   7.043
##  schoolid  (Intercept)  171.02  13.077
##  Residual              1043.44  32.302
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.71226    5.30641  274.46506 101.710  < 2e-16 ***
## sex           -1.37733    2.09334 1022.81814  -0.658    0.511
## minority     -16.29362    3.02464  703.33746  -5.387 9.78e-08 ***
## ses           10.14363    1.64248  176.39731   6.176 4.41e-09 ***
## yearstea       0.01103    0.14117  226.97682   0.078    0.938
## mathknow       1.36796    1.38563  229.40643   0.987    0.325
## mathprep      -0.27938    1.37171  204.89332  -0.204    0.839
## housepov     -17.50879   13.21775  113.44882  -1.325    0.188
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) sex    minrty ses    yearst mthknw mthprp
## sex      -0.190
## minority -0.321 -0.011
## ses      -0.108  0.020  0.145
## yearstea -0.259  0.014  0.025 -0.026
## mathknow -0.082  0.006  0.111  0.002  0.029
## mathprep -0.631 -0.005  0.002  0.050 -0.172  0.005
## housepov -0.451 -0.007 -0.180  0.081  0.070  0.058  0.040
```

```
# Store the variances of random effects
var_schoolid_q6_ses <- round(
  as.numeric(VarCorr(model_ses)$'schoolid'),
  digits = 3)
var_classid_q6_ses <- round(
  as.numeric(VarCorr(model_ses)$'classid'),
  digits = 3)
var_res_q6_ses <- round(
  attr(VarCorr(model_ses), "sc")^2,
  digits = 3)

# Examine the fit of random slopes
fit_test_q6_ses <- anova(model_all_student,
                         model_ses,
                         refit = FALSE)
fit_test_q6_ses
```

```
## Data: dat
## Models:
## model_all_student: math1st ~ sex + minority + ses + yearstea + mathknow + mathprep +
## model_all_student:     housepov + (1 | schoolid/classid)
## model_ses: math1st ~ sex + minority + ses + yearstea + mathknow + mathprep +
## model_ses:     housepov + (0 + ses | classid) + (1 | schoolid) + (1 | classid)
##                    Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## model_all_student 11 10752 10806 -5364.8    10730
## model_ses         12 10752 10812 -5364.0    10728 1.5969      1     0.2063
```

```
# sex
model_sex <- lmer(
  math1st ~ sex + minority + ses + yearstea + mathknow + mathprep + housepov +
    (0 + sex|classid) + (1|schoolid) + (1|classid),
  data = dat
)
# Report the model fit
print(summary(model_sex))
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ sex + minority + ses + yearstea + mathknow + mathprep +
##     housepov + (0 + sex | classid) + (1 | schoolid) + (1 | classid)
```

```
##    Data: dat
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8581 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups    Name        Variance  Std.Dev.
##  classid   (Intercept) 9.387e+01  9.688824
##  classid.1 sex         3.310e-05  0.005753
##  schoolid  (Intercept) 1.695e+02 13.017987
##  Residual              1.065e+03 32.633681
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63033    5.31211  275.37966 101.585  < 2e-16 ***
## sex           -1.21421    2.09483 1022.41575  -0.580    0.562
## minority     -16.18672    3.02607  704.48078  -5.349 1.20e-07 ***
## ses           10.05076    1.54485 1066.56152   6.506 1.18e-10 ***
## yearstea       0.01129    0.14141  226.80606   0.080    0.936
## mathknow       1.35013    1.39167  234.49478   0.970    0.333
## mathprep      -0.27702    1.37582  205.26984  -0.201    0.841
## housepov     -17.64878   13.21784  113.87028  -1.335    0.184
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) sex    minrty ses    yearst mthknw mthprp
## sex      -0.190
## minority -0.320 -0.011
## ses      -0.121  0.020  0.162
## yearstea -0.259  0.016  0.024 -0.028
## mathknow -0.083  0.007  0.115 -0.007  0.029
## mathprep -0.631 -0.006  0.001  0.053 -0.172  0.004
## housepov -0.451 -0.007 -0.178  0.082  0.071  0.058  0.038
```

```r
# Store the variances of random effects
var_schoolid_q6_sex <- round(
  as.numeric(VarCorr(model_sex)$'schoolid'[1]),
  digits = 3)
var_classid_q6_sex <- round(
  as.numeric(VarCorr(model_sex)$'classid'[1]),
  digits = 3)
var_res_q6_sex <- round(
  attr(VarCorr(model_sex), "sc")^2,
  digits = 3)

# Examine the fit of random slopes
fit_test_q6_sex <- anova(model_all_student,
                         model_sex,
                         refit = FALSE)
```

```
fit_test_q6_sex
```

```
## Data: dat
## Models:
## model_all_student: math1st ~ sex + minority + ses + yearstea + mathknow + mathprep +
## model_all_student:     housepov + (1 | schoolid/classid)
## model_sex: math1st ~ sex + minority + ses + yearstea + mathknow + mathprep +
## model_sex:     housepov + (0 + sex | classid) + (1 | schoolid) + (1 | classid)
##                    Df   AIC   BIC  logLik deviance Chisq Chi Df Pr(>Chisq)
## model_all_student 11 10752 10806 -5364.8    10730
## model_sex         12 10754 10813 -5364.8    10730     0      1          1
```

```
# minority
model_minority <- lmer(
  math1st ~ sex + minority + ses + yearstea + mathknow + mathprep + housepov +
    (0 + minority|classid) + (1|schoolid) + (1|classid),
  data = dat
)
```

```
## boundary (singular) fit: see ?isSingular
```

```
# Report the model fit
print(summary(model_minority))
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ sex + minority + ses + yearstea + mathknow + mathprep +
##     housepov + (0 + minority | classid) + (1 | schoolid) + (1 |      classid)
##    Data: dat
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups    Name        Variance Std.Dev.
##  classid   (Intercept)   93.89   9.69
##  classid.1 minority       0.00   0.00
##  schoolid  (Intercept)  169.45  13.02
##  Residual              1064.95  32.63
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63042    5.31210  275.38909 101.585  < 2e-16 ***
## sex           -1.21419    2.09483 1022.42137  -0.580    0.562
## minority     -16.18678    3.02605  704.47894  -5.349 1.20e-07 ***
## ses           10.05075    1.54484 1066.56222   6.506 1.18e-10 ***
## yearstea       0.01129    0.14141  226.80896   0.080    0.936
## mathknow       1.35004    1.39168  234.49773   0.970    0.333
```

```
## mathprep      -0.27705    1.37583  205.27155  -0.201    0.841
## housepov     -17.64848   13.21758  113.87764  -1.335    0.184
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) sex    minrty ses    yearst mthknw mthprp
## sex      -0.190
## minority -0.320 -0.011
## ses      -0.121  0.020  0.162
## yearstea -0.259  0.016  0.024 -0.028
## mathknow -0.083  0.007  0.115 -0.007  0.029
## mathprep -0.631 -0.006  0.001  0.053 -0.172  0.004
## housepov -0.451 -0.007 -0.178  0.082  0.071  0.058  0.038
## convergence code: 0
## boundary (singular) fit: see ?isSingular
```

```
# Store the variances of random effects
var_schoolid_q6_minority <- round(
  as.numeric(VarCorr(model_minority)$'schoolid'[1]),
  digits = 3)
var_classid_q6_minority <- round(
  as.numeric(VarCorr(model_minority)$'classid'[1]),
  digits = 3)
var_res_q6_minority <- round(
  attr(VarCorr(model_minority), "sc")^2,
  digits = 3)

# Examine the fit of random slopes
fit_test_q6_minority <- anova(model_all_student,
                              model_sex,
                              refit = FALSE)
fit_test_q6_minority
```

```
## Data: dat
## Models:
## model_all_student: math1st ~ sex + minority + ses + yearstea + mathknow + mathprep +
## model_all_student:     housepov + (1 | schoolid/classid)
## model_sex: math1st ~ sex + minority + ses + yearstea + mathknow + mathprep +
## model_sex:     housepov + (0 + sex | classid) + (1 | schoolid) + (1 | classid)
##                    Df   AIC   BIC  logLik deviance Chisq Chi Df Pr(>Chisq)
## model_all_student 11 10752 10806 -5364.8    10730
## model_sex         12 10754 10813 -5364.8    10730     0      1          1
```

b. Including classroom-level variable with random slopes at the classroom-level will lead to the same effect for each group, which is each classroom. Thus, adding a random slope for the same level does not show the relationship between the outcome variable on different group.

c.

```
# Add random slope for student-level predictors with correlation
# ses with correlation
model_ses_cor <- lmer(
```

```r
  math1st ~ ses + minority + sex + yearstea + mathknow + mathprep + housepov +
    (1|schoolid) + (ses||classid),
  data = dat
)

# Report the model fit
print(summary(model_ses_cor))
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ ses + minority + sex + yearstea + mathknow + mathprep +
##     housepov + (1 | schoolid) + (ses || classid)
##    Data: dat
##
## REML criterion at convergence: 10727.9
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.7163 -0.6032 -0.0331  0.5855  3.6840
##
## Random effects:
##  Groups    Name        Variance Std.Dev.
##  classid   ses           49.60   7.043
##  classid.1 (Intercept)   87.11   9.333
##  schoolid  (Intercept)  171.02  13.077
##  Residual              1043.44  32.302
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.71226    5.30641  274.46506 101.710  < 2e-16 ***
## ses           10.14363    1.64248  176.39731   6.176 4.41e-09 ***
## minority     -16.29362    3.02464  703.33746  -5.387 9.78e-08 ***
## sex           -1.37733    2.09334 1022.81814  -0.658    0.511
## yearstea       0.01103    0.14117  226.97682   0.078    0.938
## mathknow       1.36796    1.38563  229.40643   0.987    0.325
## mathprep      -0.27938    1.37171  204.89332  -0.204    0.839
## housepov     -17.50879   13.21775  113.44882  -1.325    0.188
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) ses    minrty sex    yearst mthknw mthprp
## ses      -0.108
## minority -0.321  0.145
## sex      -0.190  0.020 -0.011
## yearstea -0.259 -0.026  0.025  0.014
## mathknow -0.082  0.002  0.111  0.006  0.029
## mathprep -0.631  0.050  0.002 -0.005 -0.172  0.005
## housepov -0.451  0.081 -0.180 -0.007  0.070  0.058  0.040
```

```r
# Store the variances of random effects
var_schoolid_q6_ses_cor <- round(
  as.numeric(VarCorr(model_ses_cor)$'schoolid'[1]),
  digits = 3)
var_classid_q6_ses_cor <- round(
  as.numeric(VarCorr(model_ses_cor)$'classid'[1]),
  digits = 3)
var_res_q6_ses_cor <- round(
  attr(VarCorr(model_ses_cor), "sc")^2,
  digits = 3)

# Examine the fit of random slopes
fit_test_q6_ses_cor <- anova(model_all_student,
                             model_ses_cor,
                             refit = FALSE)
fit_test_q6_ses_cor
```

```
## Data: dat
## Models:
## model_all_student: math1st ~ sex + minority + ses + yearstea + mathknow + mathprep +
## model_all_student:     housepov + (1 | schoolid/classid)
## model_ses_cor: math1st ~ ses + minority + sex + yearstea + mathknow + mathprep +
## model_ses_cor:     housepov + (1 | schoolid) + (ses || classid)
##                    Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## model_all_student 11 10752 10806 -5364.8    10730
## model_ses_cor     12 10752 10812 -5364.0    10728 1.5969      1     0.2063
```

```r
# sex with correlation
model_sex_cor <- lmer(
  math1st ~ sex + minority + ses +  yearstea + mathknow + mathprep + housepov +
    (1|schoolid) + (sex|classid),
  data = dat
)

# Report the model fit
print(summary(model_sex_cor))
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ sex + minority + ses + yearstea + mathknow + mathprep +
##     housepov + (1 | schoolid) + (sex | classid)
##    Data: dat
##
## REML criterion at convergence: 10729
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.7562 -0.6134 -0.0307  0.5916  3.7116
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  classid  (Intercept)  130.2   11.411
##           sex           31.5    5.612   -0.67
```

```
##  schoolid (Intercept)  169.9   13.035
##  Residual             1056.3   32.502
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  5.400e+02  5.332e+00  2.723e+02 101.279  < 2e-16 ***
## sex         -1.197e+00  2.123e+00  2.158e+02  -0.564    0.573
## minority    -1.619e+01  3.028e+00  7.042e+02  -5.347 1.21e-07 ***
## ses          1.010e+01  1.544e+00  1.065e+03   6.539 9.61e-11 ***
## yearstea     3.053e-03  1.416e-01  2.270e+02   0.022    0.983
## mathknow     1.306e+00  1.391e+00  2.315e+02   0.939    0.349
## mathprep    -3.460e-01  1.374e+00  2.014e+02  -0.252    0.801
## housepov    -1.829e+01  1.323e+01  1.145e+02  -1.382    0.170
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) sex    minrty ses    yearst mthknw mthprp
## sex      -0.203
## minority -0.321 -0.009
## ses      -0.123  0.020  0.164
## yearstea -0.258  0.015  0.024 -0.027
## mathknow -0.085  0.003  0.116 -0.005  0.029
## mathprep -0.628 -0.008  0.003  0.054 -0.174  0.005
## housepov -0.452 -0.005 -0.178  0.083  0.072  0.060  0.040
```

```
# Store the variances of random effects
var_schoolid_q6_sex_cor <- round(
  as.numeric(VarCorr(model_sex_cor)$'schoolid'[1]),
  digits = 3)
var_classid_q6_sex_cor <- round(
  as.numeric(VarCorr(model_sex_cor)$'classid'[2, 2]),
  digits = 3)
var_res_q6_sex_cor <- round(
  attr(VarCorr(model_sex_cor), "sc")^2,
  digits = 3)

# Examine the fit of random slopes
fit_test_q6_sex_cor <- anova(model_all_student,
                             model_sex_cor,
                             refit = FALSE)
fit_test_q6_sex_cor
```

```
## Data: dat
## Models:
## model_all_student: math1st ~ sex + minority + ses + yearstea + mathknow + mathprep +
## model_all_student:     housepov + (1 | schoolid/classid)
## model_sex_cor: math1st ~ sex + minority + ses + yearstea + mathknow + mathprep +
## model_sex_cor:     housepov + (1 | schoolid) + (sex | classid)
##                   Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## model_all_student 11 10752 10806 -5364.8    10730
## model_sex_cor     13 10755 10820 -5364.5    10729 0.5003      2     0.7787
```

```r
# minority with correlation
model_minority_cor <- lmer(
  math1st ~ sex + minority + ses + yearstea + mathknow + mathprep + housepov +
    (1|schoolid) + (minority|classid),
  data = dat
)
# Report the model fit
print(summary(model_minority_cor))
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ sex + minority + ses + yearstea + mathknow + mathprep +
##     housepov + (1 | schoolid) + (minority | classid)
##    Data: dat
##
## REML criterion at convergence: 10726.3
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.9036 -0.6221 -0.0295  0.6033  3.4574
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  classid  (Intercept)  225.4   15.01
##           minority     171.3   13.09    -0.82
##  schoolid (Intercept)  157.4   12.55
##  Residual             1045.3   32.33
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.73593    5.38021  270.70839 100.319  < 2e-16 ***
## sex           -1.01014    2.08966 1015.73459  -0.483    0.629
## minority     -16.48615    3.21756  183.24221  -5.124 7.55e-07 ***
## ses            9.89350    1.54595 1062.82952   6.400 2.33e-10 ***
## yearstea      -0.01636    0.14285  234.25604  -0.115    0.909
## mathknow       1.45697    1.39354  234.05425   1.046    0.297
## mathprep      -0.13522    1.37018  203.97781  -0.099    0.921
## housepov     -17.34685   12.91273  103.34823  -1.343    0.182
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) sex    minrty ses    yearst mthknw mthprp
## sex      -0.188
## minority -0.368 -0.009
## ses      -0.117  0.021  0.149
## yearstea -0.265  0.015  0.025 -0.023
## mathknow -0.079  0.009  0.108  0.001  0.038
## mathprep -0.618 -0.005 -0.004  0.051 -0.171 -0.006
## housepov -0.435 -0.009 -0.171  0.085  0.080  0.061  0.037
```

```r
# Store the variances of random effects
var_schoolid_q6_minority_cor <- round(
  as.numeric(VarCorr(model_minority_cor)$'schoolid'[1]),
  digits = 3)
var_classid_q6_minority_cor <- round(
  as.numeric(VarCorr(model_minority_cor)$'classid'[1]),
  digits = 3)
var_res_q6_minority_cor <- round(
  attr(VarCorr(model_minority_cor), "sc")^2,
  digits = 3)

# Examine the fit of random slopes
fit_test_q6_minority_cor <- anova(model_all_student,
                                  model_minority_cor,
                                  refit = FALSE)
fit_test_q6_minority_cor
```

```
## Data: dat
## Models:
## model_all_student: math1st ~ sex + minority + ses + yearstea + mathknow + mathprep +
## model_all_student:     housepov + (1 | schoolid/classid)
## model_minority_cor: math1st ~ sex + minority + ses + yearstea + mathknow + mathprep +
## model_minority_cor:     housepov + (1 | schoolid) + (minority | classid)
##                     Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## model_all_student   11 10752 10806 -5364.8    10730
## model_minority_cor 13 10752 10817 -5363.2    10726 3.1967      2     0.2022
```

For allowing correlation on `ses` variable, the classroom-level variance drops from 87.114 to 49.597. The school-level variance does not change (171.015). The residual also remains the same. The p-value is larger than 0.05, which is not significant.

For allowing correlation on `sex` variable, the classroom-level variance drops from 93.873 to 31.5. The school-level variance does not change. The residual changes slightly. The p-value implies that it might not be a good fit to allow correlation on `sex` variable with random slope.

For allowing correlation on `minority` variable, the classroom-level variance significantly increased from 93.889 to 225.389, while the interaction effect variance between student-level variable minority and classroom-level are around 171 with a correlation of -0.82. The p-value is not significant. This significant increase might due to the aggregate effect of student-level predictor `minority` on the classroom-level effect.

Question 7

a. Try to add a random slope for each **student level** predictor (varying at the **school level**; one by one separately - not all together).

b. Retry the above, allowing the slopes to be correlated with the random intercepts.

c. Report anything unusual about the variance components (changes that are unexpected).

Solution

a.

```
# Add random slope for student-level predictors at school-level
# ses varying at school-level
model_ses_school <- lmer(
  math1st ~ sex + minority + ses + yearstea + mathknow + mathprep + housepov +
    (0 + ses|schoolid) + (1|schoolid) + (1|classid),
  data = dat
)
# Report the model fit
print(summary(model_ses_school))
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ sex + minority + ses + yearstea + mathknow + mathprep +
##     housepov + (0 + ses | schoolid) + (1 | schoolid) + (1 | classid)
##    Data: dat
##
## REML criterion at convergence: 10724.8
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.6138 -0.6185 -0.0289  0.5798  3.7130
##
## Random effects:
##  Groups     Name        Variance Std.Dev.
##  classid    (Intercept)   88.56   9.411
##  schoolid   (Intercept)  168.00  12.961
##  schoolid.1 ses           72.50   8.515
##  Residual               1035.11  32.173
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.13752    5.27926  270.52802 102.124  < 2e-16 ***
## sex           -1.40187    2.08169 1011.29089  -0.673    0.501
## minority     -16.52526    3.02191  700.07600  -5.468 6.32e-08 ***
## ses            9.78982    1.82216   79.01650   5.373 7.61e-07 ***
## yearstea       0.03079    0.14052  223.94252   0.219    0.827
## mathknow       1.35586    1.38461  232.19737   0.979    0.328
## mathprep      -0.19799    1.35995  198.59551  -0.146    0.884
## housepov     -16.94575   13.21161  112.81447  -1.283    0.202
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) sex    minrty ses    yearst mthknw mthprp
## sex      -0.190
## minority -0.323 -0.010
## ses      -0.091  0.017  0.124
## yearstea -0.260  0.018  0.024 -0.019
## mathknow -0.079  0.006  0.110  0.006  0.028
## mathprep -0.628 -0.007  0.001  0.042 -0.172  0.002
## housepov -0.451 -0.007 -0.180  0.076  0.070  0.056  0.041
```

```
# Store the variances of random effects
var_schoolid_q7_ses <- round(
  as.numeric(VarCorr(model_ses_school)$'schoolid'),
  digits = 3)
var_classid_q7_ses <- round(
  as.numeric(VarCorr(model_ses_school)$'classid'),
  digits = 3)
var_res_q7_ses <- round(
  attr(VarCorr(model_ses_school), "sc")^2,
  digits = 3)

# Examine the fit of random slopes
fit_test_q7_ses <- anova(model_all_student,
                         model_ses_school,
                         refit = FALSE)
fit_test_q7_ses
```

```
## Data: dat
## Models:
## model_all_student: math1st ~ sex + minority + ses + yearstea + mathknow + mathprep +
## model_all_student:     housepov + (1 | schoolid/classid)
## model_ses_school: math1st ~ sex + minority + ses + yearstea + mathknow + mathprep +
## model_ses_school:     housepov + (0 + ses | schoolid) + (1 | schoolid) + (1 | classid)
##                    Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## model_all_student 11 10752 10806 -5364.8    10730
## model_ses_school  12 10749 10809 -5362.4    10725 4.6972      1    0.03021 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
# sex varying at school-level
model_sex_school <- lmer(
  math1st ~ sex + minority + ses + yearstea + mathknow + mathprep + housepov +
    (0 + sex|schoolid) + (1|schoolid) + (1|classid),
  data = dat
)
# Report the model fit
print(summary(model_sex_school))
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
```

```
## Formula: math1st ~ sex + minority + ses + yearstea + mathknow + mathprep +
##     housepov + (0 + sex | schoolid) + (1 | schoolid) + (1 | classid)
##    Data: dat
##
## REML criterion at convergence: 10728.9
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8578 -0.6110 -0.0259  0.5922  3.5556
##
## Random effects:
##  Groups     Name        Variance Std.Dev.
##  classid    (Intercept)   96.08   9.802
##  schoolid   (Intercept)  161.63  12.713
##  schoolid.1 sex           35.85   5.987
##  Residual               1054.36  32.471
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.43513    5.30741  272.54817 101.638  < 2e-16 ***
## sex           -1.33538    2.18749  138.10018  -0.610    0.543
## minority     -16.16537    3.02862  704.25875  -5.338 1.27e-07 ***
## ses            9.98475    1.54243 1058.28030   6.473 1.46e-10 ***
## yearstea       0.01448    0.14163  226.44545   0.102    0.919
## mathknow       1.40068    1.39464  234.45910   1.004    0.316
## mathprep      -0.27193    1.38011  205.78600  -0.197    0.844
## housepov     -16.77631   13.22883  112.39531  -1.268    0.207
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) sex    minrty ses    yearst mthknw mthprp
## sex      -0.179
## minority -0.320 -0.015
## ses      -0.120  0.020  0.161
## yearstea -0.259  0.013  0.024 -0.029
## mathknow -0.081  0.007  0.114 -0.007  0.028
## mathprep -0.633 -0.004  0.001  0.052 -0.172  0.004
## housepov -0.449 -0.010 -0.178  0.081  0.070  0.055  0.036
```

```r
# Store the variances of random effects
var_schoolid_q7_sex <- round(
  as.numeric(VarCorr(model_sex_school)$'schoolid'[1]),
  digits = 3)
var_classid_q7_sex <- round(
  as.numeric(VarCorr(model_sex_school)$'classid'[1]),
  digits = 3)
var_res_q7_sex <- round(
  attr(VarCorr(model_sex_school), "sc")^2,
  digits = 3)

# Examine the fit of random slopes
fit_test_q7_sex <- anova(model_all_student,
```

```
                        model_sex_school,
                        refit = FALSE)
fit_test_q7_sex
```

```
## Data: dat
## Models:
## model_all_student: math1st ~ sex + minority + ses + yearstea + mathknow + mathprep +
## model_all_student:     housepov + (1 | schoolid/classid)
## model_sex_school: math1st ~ sex + minority + ses + yearstea + mathknow + mathprep +
## model_sex_school:     housepov + (0 + sex | schoolid) + (1 | schoolid) + (1 | classid)
##                   Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## model_all_student 11 10752 10806 -5364.8    10730
## model_sex_school  12 10753 10813 -5364.4    10729 0.6137      1     0.4334
```

```
# minority varying at school-level
model_minority_school <- lmer(
  math1st ~ sex + minority + ses + yearstea + mathknow + mathprep + housepov + (0 + minority|schoolid) +
  data = dat
)
```

```
## boundary (singular) fit: see ?isSingular
```

```
# Report the model fit
print(summary(model_minority_school))
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ sex + minority + ses + yearstea + mathknow + mathprep +
##     housepov + (0 + minority | schoolid) + (1 | schoolid) + (1 |      classid)
##    Data: dat
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8581 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups     Name        Variance  Std.Dev.
##  classid    (Intercept) 9.388e+01  9.689369
##  schoolid   (Intercept) 1.694e+02 13.017176
##  schoolid.1 minority    1.777e-06  0.001333
##  Residual               1.065e+03 32.633690
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63040    5.31208  275.39129 101.586  < 2e-16 ***
## sex           -1.21419    2.09483 1022.42090  -0.580    0.562
## minority     -16.18676    3.02605  704.47671  -5.349 1.20e-07 ***
## ses           10.05076    1.54485 1066.56207   6.506 1.18e-10 ***
## yearstea       0.01129    0.14141  226.80855   0.080    0.936
```

```
## mathknow       1.35003    1.39168  234.49782   0.970    0.333
## mathprep      -0.27705    1.37582  205.27091  -0.201    0.841
## housepov     -17.64850   13.21752  113.87887  -1.335    0.184
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) sex    minrty ses    yearst mthknw mthprp
## sex      -0.190
## minority -0.320 -0.011
## ses      -0.121  0.020  0.162
## yearstea -0.259  0.016  0.024 -0.028
## mathknow -0.083  0.007  0.115 -0.007  0.029
## mathprep -0.631 -0.006  0.001  0.053 -0.172  0.004
## housepov -0.451 -0.007 -0.178  0.082  0.071  0.058  0.038
## convergence code: 0
## boundary (singular) fit: see ?isSingular
```

```
# Store the variances of random effects
var_schoolid_q7_minority <- round(
  as.numeric(VarCorr(model_minority_school)$'schoolid'[1]),
  digits = 3)
var_classid_q7_minority <- round(
  as.numeric(VarCorr(model_minority_school)$'classid'[1]),
  digits = 3)
var_res_q7_minority <- round(
  attr(VarCorr(model_minority_school), "sc")^2,
  digits = 3)

# Examine the fit of random slopes
fit_test_q7_minority <- anova(model_all_student,
                              model_minority_school,
                              refit = FALSE)
fit_test_q7_minority
```

```
## Data: dat
## Models:
## model_all_student: math1st ~ sex + minority + ses + yearstea + mathknow + mathprep +
## model_all_student:     housepov + (1 | schoolid/classid)
## model_minority_school: math1st ~ sex + minority + ses + yearstea + mathknow + mathprep +
## model_minority_school:     housepov + (0 + minority | schoolid) + (1 | schoolid) + (1 |
## model_minority_school:     classid)
##                        Df   AIC   BIC  logLik deviance Chisq Chi Df Pr(>Chisq)
## model_all_student      11 10752 10806 -5364.8    10730
## model_minority_school 12 10754 10813 -5364.8    10730     0      1          1
```

b.

```
# Add random slope for student-level predictors with correlation at school-level
# ses with correlation
model_ses_school_cor <- lmer(
  math1st ~ sex + minority + ses + yearstea + mathknow + mathprep + housepov +
    (ses||schoolid) + (1|classid),
```

```
  data = dat
)
# Report the model fit
print(summary(model_ses_school_cor))
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ sex + minority + ses + yearstea + mathknow + mathprep +
##     housepov + (ses || schoolid) + (1 | classid)
##    Data: dat
##
## REML criterion at convergence: 10724.8
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.6138 -0.6185 -0.0289  0.5798  3.7130
##
## Random effects:
##  Groups     Name        Variance Std.Dev.
##  classid    (Intercept)   88.56   9.411
##  schoolid   ses           72.50   8.515
##  schoolid.1 (Intercept)  168.00  12.961
##  Residual               1035.11  32.173
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.13752    5.27926  270.52802 102.124  < 2e-16 ***
## sex           -1.40187    2.08169 1011.29089  -0.673    0.501
## minority     -16.52526    3.02191  700.07600  -5.468 6.32e-08 ***
## ses            9.78982    1.82216   79.01650   5.373 7.61e-07 ***
## yearstea       0.03079    0.14052  223.94252   0.219    0.827
## mathknow       1.35586    1.38461  232.19737   0.979    0.328
## mathprep      -0.19799    1.35995  198.59551  -0.146    0.884
## housepov     -16.94575   13.21161  112.81447  -1.283    0.202
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) sex    minrty ses    yearst mthknw mthprp
## sex      -0.190
## minority -0.323 -0.010
## ses      -0.091  0.017  0.124
## yearstea -0.260  0.018  0.024 -0.019
## mathknow -0.079  0.006  0.110  0.006  0.028
## mathprep -0.628 -0.007  0.001  0.042 -0.172  0.002
## housepov -0.451 -0.007 -0.180  0.076  0.070  0.056  0.041
```

```
# Store the variances of random effects
var_schoolid_q7_ses_cor <- round(
  as.numeric(VarCorr(model_ses_school_cor)$'schoolid'[1]),
  digits = 3)
var_classid_q7_ses_cor <- round(
```

```
  as.numeric(VarCorr(model_ses_school_cor)$'classid'[1]),
  digits = 3)
var_res_q7_ses_cor <- round(
  attr(VarCorr(model_ses_school_cor), "sc")^2,
  digits = 3)

# Examine the fit of random slopes
fit_test_q7_ses_cor <- anova(model_all_student,
                             model_ses_school_cor,
                             refit = FALSE)

# sex with correlation
model_sex_school_cor <- lmer(
  math1st ~ sex + minority + ses + yearstea + mathknow + mathprep + housepov +
    (sex|schoolid) + (1|classid),
  data = dat
)
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl = control$checkConv, :
## Model failed to converge with max|grad| = 0.00526733 (tol = 0.002, component 1)
```

```
# Report the model fit
print(summary(model_sex_school_cor))
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ sex + minority + ses + yearstea + mathknow + mathprep +
##     housepov + (sex | schoolid) + (1 | classid)
##    Data: dat
##
## REML criterion at convergence: 10727.6
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8050 -0.6094 -0.0223  0.5970  3.5528
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  classid  (Intercept)   97.29   9.863
##  schoolid (Intercept)  206.02  14.353
##           sex           83.73   9.151   -0.43
##  Residual             1041.89  32.278
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  5.399e+02  5.363e+00  2.626e+02 100.675  < 2e-16 ***
## sex         -1.340e+00  2.300e+00  8.740e+01  -0.583    0.562
## minority    -1.642e+01  3.027e+00  7.076e+02  -5.425 7.97e-08 ***
## ses          9.929e+00  1.540e+00  1.055e+03   6.448 1.72e-10 ***
## yearstea     6.921e-03  1.418e-01  2.277e+02   0.049    0.961
## mathknow     1.379e+00  1.396e+00  2.364e+02   0.988    0.324
## mathprep    -2.796e-01  1.378e+00  2.061e+02  -0.203    0.839
```

```
## housepov    -1.742e+01  1.325e+01  1.136e+02  -1.314    0.191
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) sex    minrty ses    yearst mthknw mthprp
## sex      -0.222
## minority -0.319 -0.011
## ses      -0.121  0.018  0.163
## yearstea -0.258  0.014  0.024 -0.028
## mathknow -0.082  0.006  0.114 -0.006  0.027
## mathprep -0.627 -0.005  0.004  0.053 -0.172  0.004
## housepov -0.449 -0.003 -0.178  0.083  0.072  0.060  0.038
## convergence code: 0
## Model failed to converge with max|grad| = 0.00526733 (tol = 0.002, component 1)
```

```r
# Store the variances of random effects
var_schoolid_q7_sex_cor <- round(
  as.numeric(VarCorr(model_sex_school_cor)$'schoolid'[1]),
  digits = 3)
var_classid_q7_sex_cor <- round(
  as.numeric(VarCorr(model_sex_school_cor)$'classid'[1]),
  digits = 3)
var_res_q7_sex_cor <- round(
  attr(VarCorr(model_sex_school_cor), "sc")^2,
  digits = 3)

# Examine the fit of random slopes
fit_test_q7_sex_cor <- anova(model_all_student,
                             model_sex_school_cor,
                             refit = FALSE)

# minority with correlation
model_minority_school_cor <- lmer(
  math1st ~ sex + minority + ses + yearstea + mathknow + mathprep + housepov +
    (minority|schoolid) + (1|classid),
  data = dat
)
# Report the model fit
print(summary(model_minority_school_cor))
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ sex + minority + ses + yearstea + mathknow + mathprep +
##     housepov + (minority | schoolid) + (1 | classid)
##    Data: dat
##
## REML criterion at convergence: 10717.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8952 -0.6358 -0.0345  0.6129  3.6444
##
## Random effects:
```

```
##  Groups   Name        Variance Std.Dev. Corr
##  classid  (Intercept)   86.7    9.311
##  schoolid (Intercept)  381.2   19.524
##           minority     343.2   18.525   -0.83
##  Residual             1039.4   32.240
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.49369    5.65513  173.09178  95.399  < 2e-16 ***
## sex           -0.86278    2.08382 1021.81437  -0.414    0.679
## minority     -16.37547    3.89604   58.24604  -4.203 9.17e-05 ***
## ses            9.43095    1.54335 1063.13485   6.111 1.39e-09 ***
## yearstea      -0.00437    0.13765  217.17884  -0.032    0.975
## mathknow       1.63216    1.35929  224.78144   1.201    0.231
## mathprep      -0.29178    1.33537  198.06922  -0.218    0.827
## housepov     -16.06251   12.57477   99.99134  -1.277    0.204
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) sex    minrty ses    yearst mthknw mthprp
## sex      -0.172
## minority -0.494 -0.014
## ses      -0.105  0.024  0.113
## yearstea -0.253  0.014  0.027 -0.021
## mathknow -0.078  0.010  0.099 -0.005  0.024
## mathprep -0.576 -0.005 -0.002  0.052 -0.167 -0.002
## housepov -0.394 -0.013 -0.157  0.089  0.091  0.061  0.037
```

```r
# Store the variances of random effects
var_schoolid_q7_minority_cor <- round(
  as.numeric(VarCorr(model_minority_school_cor)$'schoolid'[1]),
  digits = 3)
var_classid_q7_minority_cor <- round(
  as.numeric(VarCorr(model_minority_school_cor)$'classid'[1]),
  digits = 3)
var_res_q7_minority_cor <- round(
  attr(VarCorr(model_minority_school_cor), "sc")^2,
  digits = 3)

# Examine the fit of random slopes
fit_test_q7_minority_cor <- anova(model_all_student,
                                  model_minority_school_cor,
                                  refit = FALSE)
```

c. One thing that is unusual is that, when allowing correlation between `minority` and `sex` (varying at school-level) random effect, the variance of school-level random intercept and minority random slope has a significant increase from 169.447 to 381.197 for `minority` and from 161.629 to 206.019. This might due to the high negative correlation between the random intercept and the random slope.

Another thing that is unusual is that, when allowing correlation between `ses` (varying at school-level) random effect, the variance has a decrease from 167.998 to 72.499.

Question 8

a. Take the two predictors that had significant (at 0.05 level) random slopes, in the forms in which they worked (independent or correlated) and add both to the model, and test for need of one conditional on needing the other.

b. Is the more complex model (with both random slopes in it) justified?

c. **Write out this model** in your preferred notation.

Solution

a.

```
# Add minority and ses to the model
model_complex <- lmer(
  math1st ~ ses + minority + sex + yearstea + mathknow + mathprep + housepov +
    (0 + ses|schoolid) + (minority||schoolid) + (1|schoolid) + (1|classid),
  data = dat
)
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl = control$checkConv, :
## unable to evaluate scaled gradient
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl = control$checkConv, :
## Model failed to converge: degenerate Hessian with 1 negative eigenvalues
```

```
## Warning: Model failed to converge with 1 negative eigenvalue: -9.1e-02
```

```
# Report the model fit
print(summary(model_complex))
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ ses + minority + sex + yearstea + mathknow + mathprep +
##     housepov + (0 + ses | schoolid) + (minority || schoolid) +
##     (1 | schoolid) + (1 | classid)
##    Data: dat
##
## REML criterion at convergence: 10724.8
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.6138 -0.6185 -0.0290  0.5800  3.7131
##
## Random effects:
##  Groups     Name        Variance  Std.Dev.
##  classid    (Intercept) 8.851e+01  9.408
##  schoolid   (Intercept) 1.679e+02 12.958
##  schoolid.1 minority    1.600e-05  0.004
##  schoolid.2 (Intercept) 3.422e-02  0.185
##  schoolid.3 ses         7.253e+01  8.517
##  Residual               1.035e+03 32.174
```

```
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.13716    5.27884  270.57293 102.132  < 2e-16 ***
## ses            9.78979    1.82229   79.01698   5.372 7.63e-07 ***
## minority     -16.52522    3.02184  700.03798  -5.469 6.32e-08 ***
## sex           -1.40190    2.08171 1011.28775  -0.673    0.501
## yearstea       0.03081    0.14051  223.93787   0.219    0.827
## mathknow       1.35563    1.38449  232.20069   0.979    0.329
## mathprep      -0.19797    1.35984  198.58482  -0.146    0.884
## housepov     -16.94566   13.21025  112.84396  -1.283    0.202
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) ses    minrty sex    yearst mthknw mthprp
## ses      -0.091
## minority -0.323  0.124
## sex      -0.190  0.017 -0.010
## yearstea -0.260 -0.019  0.024  0.018
## mathknow -0.079  0.006  0.110  0.006  0.028
## mathprep -0.628  0.042  0.001 -0.007 -0.172  0.002
## housepov -0.451  0.076 -0.180 -0.007  0.070  0.056  0.041
## convergence code: 0
## unable to evaluate scaled gradient
## Model failed to converge: degenerate  Hessian with 1 negative eigenvalues
```

```
# Examine the fit of new predictors
fit_test_q8 <- anova(model_complex,
                     model_ses_school,
                     refit = FALSE)
fit_test_q8
```

```
## Data: dat
## Models:
## model_ses_school: math1st ~ sex + minority + ses + yearstea + mathknow + mathprep +
## model_ses_school:     housepov + (0 + ses | schoolid) + (1 | schoolid) + (1 | classid)
## model_complex: math1st ~ ses + minority + sex + yearstea + mathknow + mathprep +
## model_complex:     housepov + (0 + ses | schoolid) + (minority || schoolid) +
## model_complex:     (1 | schoolid) + (1 | classid)
##                  Df   AIC   BIC  logLik deviance Chisq Chi Df Pr(>Chisq)
## model_ses_school 12 10749 10809 -5362.4    10725
## model_complex    14 10753 10823 -5362.4    10725     0      2          1
```

b. According to the LRT test, we can conclude that the p-value of adding the minority variable random slope (varying at school-level) while allowing correlation to the model conditioning on adding `ses` variable random slope (varying at school-level) is 0. This implies that there is a need for adding both variable to the model. This justified the more complex model.

c. The model for adding both `ses` variable and `minority` variable (varying at school-level while allowing correlation for minority) is:

$$
\begin{aligned}
MATH1ST_{ijk} = {} & b_0 + b_1 SES_{ijk} + b_2 MINORITY_{ijk} + b_3 SEX_{ijk} + \\
& b_4 YEARSTEA_{jk} + b_5 MATHKNOW_{jk} + b_6 MATHPREP_{jk} + \\
& b_7 HOUSEPOV_k + \\
& \zeta_{1k} SES_{ijk} + \zeta_{2k} MINORITY_{ijk} + \\
& \eta_{0jk} + \zeta_{0k} + \varepsilon_{ijk},
\end{aligned}
$$

where $\eta_{0jk} \sim \mathcal{N}(0, \sigma^2_{\eta_0})$, $\zeta_{0k} \sim \mathcal{N}(0, \sigma^2_{\zeta_0})$, $\zeta_{1k} \sim \mathcal{N}(0, \sigma^2_{\zeta_1})$, $\zeta_{2k} \sim \mathcal{N}(0, \sigma^2_{\zeta_2})$, $\varepsilon_{ijk} \sim \mathcal{N}(0, \sigma^2_{\varepsilon})$, i represents students, j represents classrooms and k represents schools.

$Corr(\zeta_{0k}, \zeta_{2k}) = \rho_{\zeta_0, \zeta_2}$, all other random terms independent of each other.

Question 9

a. For UMM, write down: `V_S`, `V_C`, `V_E` for the three variance components (simply the estimates).

b. For the most complicated (all fixed effects) random **intercepts only** model, what are: `V_S`, `V_C`, `V_E`?

c. By what fraction did these each decrease with the new predictors in the model?

Solution

```
print(summary(unconditional_model))
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ (1 | schoolid/classid)
##    Data: dat
##
## REML criterion at convergence: 11944.6
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -5.1872 -0.6174 -0.0204  0.5821  3.8339
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   85.47   9.245
##  schoolid         (Intercept)  280.69  16.754
##  Residual                      1146.79  33.864
## Number of obs: 1190, groups:  classid:schoolid, 312; schoolid, 107
##
## Fixed effects:
##             Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  522.540      2.037 104.403   256.6   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
print(summary(model_all_student))
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ sex + minority + ses + yearstea + mathknow + mathprep +
##     housepov + (1 | schoolid/classid)
##    Data: dat
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8581 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
```

```
##  classid:schoolid (Intercept)   93.89   9.689
##  schoolid         (Intercept)  169.45  13.017
##  Residual                     1064.96  32.634
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63041    5.31209  275.39010 101.585  < 2e-16 ***
## sex           -1.21419    2.09483 1022.42110  -0.580    0.562
## minority     -16.18676    3.02605  704.47787  -5.349 1.20e-07 ***
## ses           10.05076    1.54485 1066.56211   6.506 1.18e-10 ***
## yearstea       0.01129    0.14141  226.80861   0.080    0.936
## mathknow       1.35004    1.39168  234.49768   0.970    0.333
## mathprep      -0.27705    1.37583  205.27111  -0.201    0.841
## housepov     -17.64850   13.21755  113.87814  -1.335    0.184
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) sex    minrty ses    yearst mthknw mthprp
## sex      -0.190
## minority -0.320 -0.011
## ses      -0.121  0.020  0.162
## yearstea -0.259  0.016  0.024 -0.028
## mathknow -0.083  0.007  0.115 -0.007  0.029
## mathprep -0.631 -0.006  0.001  0.053 -0.172  0.004
## housepov -0.451 -0.007 -0.178  0.082  0.071  0.058  0.038
```

a. For Unconditional mean model, `V_C` is 85.47 , `V_S` is 280.69, and `V_E` is 1146.79.

b. For most complicated model (Random Intercept only), `V_C` is 93.89 , `V_S` is 169.45, and `V_E` is 1064.96.

c. For `V_C`, it increased by 9.85%. For `V_S`, it decreased by 39.63%. For `V_E`, it decreased by 7.14%.

Question 10

Now consider the model with a random slope in `ses`.

a. What are: `V_C`, `V_S` (`ses` = 0), `V_E`?

   - We need to list `ses = 0` here, or we don't know how to use the slope variance.

b. What are: `V_S` (`ses` = -0.50), `V_S` (`ses` = +0.50)?

Solution

```
print(summary(model_ses_school))
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ sex + minority + ses + yearstea + mathknow + mathprep +
##     housepov + (0 + ses | schoolid) + (1 | schoolid) + (1 | classid)
##    Data: dat
##
## REML criterion at convergence: 10724.8
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.6138 -0.6185 -0.0289  0.5798  3.7130
##
## Random effects:
##  Groups     Name        Variance Std.Dev.
##  classid    (Intercept)   88.56   9.411
##  schoolid   (Intercept)  168.00  12.961
##  schoolid.1 ses           72.50   8.515
##  Residual               1035.11  32.173
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.13752    5.27926  270.52802 102.124  < 2e-16 ***
## sex           -1.40187    2.08169 1011.29089  -0.673    0.501
## minority     -16.52526    3.02191  700.07600  -5.468 6.32e-08 ***
## ses            9.78982    1.82216   79.01650   5.373 7.61e-07 ***
## yearstea       0.03079    0.14052  223.94252   0.219    0.827
## mathknow       1.35586    1.38461  232.19737   0.979    0.328
## mathprep      -0.19799    1.35995  198.59551  -0.146    0.884
## housepov     -16.94575   13.21161  112.81447  -1.283    0.202
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) sex    minrty ses    yearst mthknw mthprp
## sex      -0.190
## minority -0.323 -0.010
## ses      -0.091  0.017  0.124
```

```
## yearstea -0.260  0.018  0.024 -0.019
## mathknow -0.079  0.006  0.110  0.006  0.028
## mathprep -0.628 -0.007  0.001  0.042 -0.172  0.002
## housepov -0.451 -0.007 -0.180  0.076  0.070  0.056  0.041
```

a. For model with a random slope in `ses`, the `V_C` is 88.56, `V_S`(`ses`=0) is 168, `V_E` is 1035.11

b. `V_S` (`ses` = -0.50) is $168 + 0.5 \times 72.5 = 204.5$

`V_S` (`ses` = +0.50) is $168 - 0.5 \times 72.5 = 131.75$

Question 11

Now consider the model with a random slope in `minority`.

a. What are: `V_C`, `V_S` (`minority` = 0), `V_E`?

   - We need to list `minority` = 0 here, or we don't know how to use the slope variance.

b. What are: `V_S` (`minority` = +0.25),`V_S` (`minority` = +0.50),`V_S` (`minority` = +0.75)?

Solution

```
print(summary(model_minority_school_cor))
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ sex + minority + ses + yearstea + mathknow + mathprep +
##     housepov + (minority | schoolid) + (1 | classid)
##    Data: dat
##
## REML criterion at convergence: 10717.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8952 -0.6358 -0.0345  0.6129  3.6444
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  classid  (Intercept)   86.7    9.311
##  schoolid (Intercept)  381.2   19.524
##           minority     343.2   18.525   -0.83
##  Residual             1039.4   32.240
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.49369    5.65513  173.09178  95.399  < 2e-16 ***
## sex           -0.86278    2.08382 1021.81437  -0.414    0.679
## minority     -16.37547    3.89604   58.24604  -4.203 9.17e-05 ***
## ses            9.43095    1.54335 1063.13485   6.111 1.39e-09 ***
## yearstea      -0.00437    0.13765  217.17884  -0.032    0.975
## mathknow       1.63216    1.35929  224.78144   1.201    0.231
## mathprep      -0.29178    1.33537  198.06922  -0.218    0.827
## housepov     -16.06251   12.57477   99.99134  -1.277    0.204
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) sex    minrty ses    yearst mthknw mthprp
## sex      -0.172
## minority -0.494 -0.014
## ses      -0.105  0.024  0.113
```

```
## yearstea -0.253  0.014  0.027 -0.021
## mathknow -0.078  0.010  0.099 -0.005  0.024
## mathprep -0.576 -0.005 -0.002  0.052 -0.167 -0.002
## housepov -0.394 -0.013 -0.157  0.089  0.091  0.061  0.037
```

a. For model with a random slope in `minority`(allowing correlation, varying at school-level random effect), the `V_C` is 86.7, `V_S` (`minority` = 0) is 381.2, `V_E` is 1039.4.

b. `V_S` (`minority` = +0.25) is $381.2+0.25^2 \times 343.2 = 402.65$ `V_S` (`minority` = +0.50) is $381.2+0.5^2 \times 343.2 = 467$ `V_S` (`minority` = +0.75) is $381.2 + 0.75^2 \times 343.2 = 574.25$

Question 12

Now consider the model with a random slope in `ses` and `minority`.

a. What are: `V_C`, `V_S` (`minority` = 0, `ses` = 0), `V_E`?

   - We need to list `minority` = 0 and `ses` = 0 here, or we don't know how to use the slope variance.

b. In the last model, what is a "likely" (+/- 1 s.d.) range for η_{0jk}.

c. Can we make a similar statement about ζ_{0k}?

d. If you had a large value for η_{0jk}, would you expect a large or small or "any" value for the two random slope terms, ζ_{1k} and ζ_{2k} for `ses` and `minority`?

e. If you had a large value for ζ_{0jk}, would you expect a large or small or "any" value for the two random slope terms, ζ_{1k} and ζ_{2k} for `ses` and `minority` (discuss each separately)?

solution

```
print(summary(model_complex))
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ ses + minority + sex + yearstea + mathknow + mathprep +
##     housepov + (0 + ses | schoolid) + (minority || schoolid) +
##     (1 | schoolid) + (1 | classid)
##    Data: dat
##
## REML criterion at convergence: 10724.8
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.6138 -0.6185 -0.0290  0.5800  3.7131
##
## Random effects:
##  Groups     Name        Variance  Std.Dev.
##  classid    (Intercept) 8.851e+01  9.408
##  schoolid   (Intercept) 1.679e+02 12.958
##  schoolid.1 minority    1.600e-05  0.004
##  schoolid.2 (Intercept) 3.422e-02  0.185
##  schoolid.3 ses         7.253e+01  8.517
##  Residual               1.035e+03 32.174
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.13716    5.27884  270.57293 102.132  < 2e-16 ***
## ses            9.78979    1.82229   79.01698   5.372 7.63e-07 ***
## minority     -16.52522    3.02184  700.03798  -5.469 6.32e-08 ***
## sex           -1.40190    2.08171 1011.28775  -0.673    0.501
## yearstea       0.03081    0.14051  223.93787   0.219    0.827
## mathknow       1.35563    1.38449  232.20069   0.979    0.329
```

```
## mathprep      -0.19797    1.35984  198.58482  -0.146    0.884
## housepov     -16.94566   13.21025  112.84396  -1.283    0.202
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) ses    minrty sex    yearst mthknw mthprp
## ses      -0.091
## minority -0.323  0.124
## sex      -0.190  0.017 -0.010
## yearstea -0.260 -0.019  0.024  0.018
## mathknow -0.079  0.006  0.110  0.006  0.028
## mathprep -0.628  0.042  0.001 -0.007 -0.172  0.002
## housepov -0.451  0.076 -0.180 -0.007  0.070  0.056  0.041
## convergence code: 0
## unable to evaluate scaled gradient
## Model failed to converge: degenerate  Hessian with 1 negative eigenvalues
```

a. Model with random slope in `SES` (varying at school-level effect, no correlation with random intercept), `Minority` (varying at school-level effect, with correlation), `V_C` = 80.62, `V_S` (`minority` = 0, `ses` = 0) = 815.17, `V_E`= 1009.73.

b. In the last model, η_{0jk} (+/- 1 s.d.) are likely to be around 89.599 and 71.641.

c. No. ζ_{0k} is the variance of school-level random effects.

d. If η_{0jk} is relatively large, then the two random slope ζ_{1k} and ζ_{2k} for `ses` and `minority` are not likely to be affected by this change, since ζ_{2k}, ζ_{1k} and η_{0jk} should be independent of each other. Therefore, we would expect any values.

e. If ζ_{0jk} are likely to be large, then for ζ_{1k} (`ses`) it would likely to not have an impact since those two coefficient are indepedent of each other. For ζ_{2k} (`minority`), it would likely to be relatively small since there is negative correlation of -0.85 between those two random effect coefficient.