2042 MLM Mini Project (Spring 2020)

Group 1

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Team members and division of work

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```
set.seed(2042001)
```

Question 1

You will generate simulated data for a single school with 100 classrooms, each of which has 200 students.

a. Outcome for student i in classroom j: Y_{ij}.
b. There is a single predictor, $X_{ij} \sim U(0,1)$ (uniform on [0,1])
c. There is a classroom random effect, $\eta_j \sim N(0, \sigma^2_\eta)$, where $\sigma^2_\eta = 2$.
d. Subject level error, $\varepsilon_{ij} \sim N(0, \sigma^2_\varepsilon)$, where $\sigma^2_\varepsilon = 2$.
e. `set.seed(2042001)` once at the beginning of your code.
f. Generate the random quantities in this order to ensure the same solution for everyone: X, η_j, ε_{ij}
g. The outcome has the following form (DGP, given the modeling parameters above):

$$Y_{ij} = 0 + 1X_{ij} + \eta_j + \varepsilon_{ij};\ \eta_j \sim N(0, \sigma^2_\eta), \varepsilon_{ij} \sim N(0, \sigma^2_\varepsilon), \text{indep.}$$

h. Generate a single simulated dataset (you will need a "classid" variable to track classrooms); you can optionally assign a "studentid") i. **Important:** construct classid such that classrooms appear consecutively within the dataframe. As per: `rep(1:J,each=n_j)`

```
# Compute variables related to the equation
N_j <- 100 # Total number of classrooms
n_i <- 200 # Number of students in each classroom
N_i <- N_j * 200 # Total number of students

X_ij <- runif( # Single predictor: Uniform on [0, 1]
  N_i,
  min = 0,
  max = 1
)
eta_j <- rnorm( # Classroom random effect: Normal on (0, 2)
```

```
  N_j,
  mean = 0,
  sd = sqrt(2)
)
epsilon_ij <- rnorm( # Subject level error: Normal on (0, 2)
  N_i,
  mean = 0,
  sd = sqrt(2)
)

# Create equation elements
eta_j <- rep( # Assign classroom random effect to students in each classroom
  eta_j,
  each = n_i
)
Y_ij <- 0 + 1 * X_ij + eta_j + epsilon_ij

classid <- rep(
  1:N_j,
  each = n_i
)
studentid <- seq(
  1:N_i
)

# Create a dataframe to store all elements
classroom_sim <- data.frame(
  outcome = Y_ij,
  predictor = X_ij,
  cls_raneff = eta_j,
  subject_error = epsilon_ij,
  classid = classid,
  studentid = studentid,
  row.names = studentid
)
```

Question 2

Fit the model corresponding to the DGP on your simulated data.

```
# Fit the model in Q1
fit_q2 <- lmer(outcome ~ predictor + (1 | classid), data = classroom_sim)
summary(fit_q2)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: outcome ~ predictor + (1 | classid)
##    Data: classroom_sim
##
## REML criterion at convergence: 71227.3
##
## Scaled residuals:
```

```
##      Min       1Q   Median       3Q      Max
## -4.0143 -0.6761  0.0024  0.6711  3.7584
##
## Random effects:
##  Groups   Name        Variance Std.Dev.
##  classid  (Intercept) 1.893    1.376
##  Residual             2.008    1.417
## Number of obs: 20000, groups:  classid, 100
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept) -7.493e-03  1.391e-01  1.022e+02  -0.054    0.957
## predictor    9.864e-01  3.496e-02  1.990e+04  28.216   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##           (Intr)
## predictor -0.126
```

```
slope_X_q2 <- round(summary(fit_q2)$coefficients["predictor", "Estimate"], digits = 4)
```

a. Report coefficient estimate for slope on X.

   Response: 0.9864.

b. Does a 95% confidence band for this coefficient estimate cover the "truth" that you used to generate the data?

```
CI_Q2_b <- confint(fit_q2, parm = "predictor", level = 0.95)
```

```
## Computing profile confidence intervals ...
```

```
lower <- round(CI_Q2_b[1], digits = 4)
upper <- round(CI_Q2_b[2], digits = 4)
```

```
**Response:** Yes. The confidence band (95% of confidence) for this coefficient estimate is between 0.9
```

Question 3

3. Next, we simulate missing data in several ways. This is the first:

a. Make a copy of the data, then modify the copy following these instructions:

```
classroom_miss <- classroom_sim
```

b. Generate $Z_{ij} \sim \text{Bernoulli}(p)$, with $p = 0.5$.

c. Set Y to NA when $Z_{ij} == 1$. This should look a lot like "MCAR" missingness.

```r
Z_ij <- rbinom(N_i, size = c(0, 1), prob = 0.5)
classroom_miss$missing <- Z_ij
classroom_miss$outcome <- ifelse(classroom_miss$missing == 1, yes = NA, no = classroom_miss$outcome)
```

d. Refit the model on the new data and report the coefficient estimate for slope on X. Look at the other parameter estimates as well.

```r
fit_q3 <- lmer(outcome ~ predictor + (1 | classid), data = classroom_miss)
summary(fit_q3)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: outcome ~ predictor + (1 | classid)
##    Data: classroom_miss
##
## REML criterion at convergence: 53333.6
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -4.0171 -0.6742  0.0023  0.6631  3.7668
##
## Random effects:
##  Groups   Name        Variance Std.Dev.
##  classid  (Intercept) 1.878    1.371
##  Residual             2.000    1.414
## Number of obs: 14963, groups:  classid, 100
##
## Fixed effects:
##              Estimate Std. Error        df t value Pr(>|t|)
## (Intercept) 1.270e-02  1.390e-01 1.033e+02   0.091    0.927
## predictor   9.612e-01  4.029e-02 1.486e+04  23.856   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##           (Intr)
## predictor -0.146
```

```r
slope_X_q3 <- round(summary(fit_q3)$coefficients["predictor", "Estimate"], digits = 4)

# Calculate the conficience band of this model
CI_Q3_d <- confint(fit_q3, parm = "predictor", level = 0.95)
```

```
## Computing profile confidence intervals ...
```

```r
lower <- round(CI_Q3_d[1], digits = 4)
upper <- round(CI_Q3_d[2], digits = 4)
```

e. Do you see any real change in the β_X estimate?

Response: No. The β_X estimate of the model with missing data values is 0.9612, which is very close to the original estimate, 0.9864.

i. Does a 95% confidence band for this coefficient estimate cover the "truth" that you used to generate the data?

Response: Yes. The confidence band (95% of confidence) for this coefficient estimate is between 0.8822 and 1.0402. This confirms that a 95% confidence band covers the "truth" of the slope, which is 1.

f. What is the total sample size N used in the model fit?

Response: The total sample size used in this model fit is 14963.

Question 4:

Missing Data II: Make another copy of the original data, then modify the copy as follows: a. Generate $Z_{ij} \sim$ Bernoulli(X_{ij}), with X_{ij} your predictor generated previously. b. Set Y to NA when $Z_{ij} == 1$. This should look a lot like "MAR" missingness.

```
# Insert code the generate your data
```

c. Refit the model on the new data and report the coefficient estimate for slope on X. Look at the other parameter estimates as well. **comment**

```
# Insert code to fit model and compute confidence interval
```

`Response:`

d. Do you see any real change in the β_X estimate?

i. Does a 95% confidence band for this coefficient estimate cover the "truth" that you used to generate the data? **comment**

Response:

e. What is the total sample size N used in the model fit? **comment**

Response:

Question 5:

Missing Data III: Make another copy of the original data, then modify the copy as follows:

```
# Insert code to make a copy of the original data
```

a. First, define the expit function: `expit <- function(x) exp(x)/(1+exp(x))`

```
# Insert code to define expit function
```

b. Generate $Z_{ij} \sim$ Bernoulli($expit(Y_{ij})$), with Y_{ij} your *outcome* generated previously.

c. Set Y to NA when $Z_{ij} == 1$. This should look like a violation of "MAR" missingness (missingness depedents on outcome and cannot be *simply* predicted with the predictor set – Y should be correlated with X, though, so it might not be too bad a violation).

```
# Insert code the generate your data
```

d. Refit the model on the new data and report the coefficient estimate for slope on X. Look at the other parameter estimates as well. **comment**

```
# Insert code to fit model and compute confidence interval
```

`Response:`

e. Do you see any real change in the β_X estimate? **comment**

   i. Does a 95% confidence band for this coefficient estimate cover the "truth" that you used to generate the data? **comment**

   Response:

f. What is the total sample size N used in the model fit? **comment**

   Response: