MLM Final Project Part 2ab

12 2020

Team Members and division of work:

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Question 1

Refit the model in Part 1 that has all fixed effects as well as random intercepts (in schools and classrooms). Recall that `math1st = mathkind + mathgain` is the outcome. The model is `math1st ~ housepov + yearstea + mathprep + mathknow + ses + sex + minority + (1|schoolid/classid), REML = T)`

```
# Insert code to fit model and print summary
dat <- read.csv("./data/classroom.csv")
dat$math1st = dat$mathkind + dat$mathgain
fit1 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathprep + mathknow +
    ses + sex + minority + (1 | schoolid/classid), data = dat, REML = T)
summary(fit1)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathprep + mathknow + ses + sex +
##     minority + (1 | schoolid/classid)
##    Data: dat
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8581 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   93.89   9.689
##  schoolid         (Intercept)  169.45  13.017
##  Residual                     1064.96  32.634
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63041    5.31209  275.39010 101.585  < 2e-16 ***
## housepov     -17.64850   13.21755  113.87814  -1.335    0.184
## yearstea       0.01129    0.14141  226.80861   0.080    0.936
## mathprep      -0.27705    1.37583  205.27111  -0.201    0.841
## mathknow       1.35004    1.39168  234.49768   0.970    0.333
## ses           10.05076    1.54485 1066.56211   6.506 1.18e-10 ***
## sex           -1.21419    2.09483 1022.42110  -0.580    0.562
## minority     -16.18676    3.02605  704.47787  -5.349 1.20e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthprp mthknw ses    sex
## housepov -0.451
## yearstea -0.259  0.071
## mathprep -0.631  0.038 -0.172
## mathknow -0.083  0.058  0.029  0.004
## ses      -0.121  0.082 -0.028  0.053 -0.007
## sex      -0.190 -0.007  0.016 -0.006  0.007  0.020
## minority -0.320 -0.178  0.024  0.001  0.115  0.162 -0.011
```

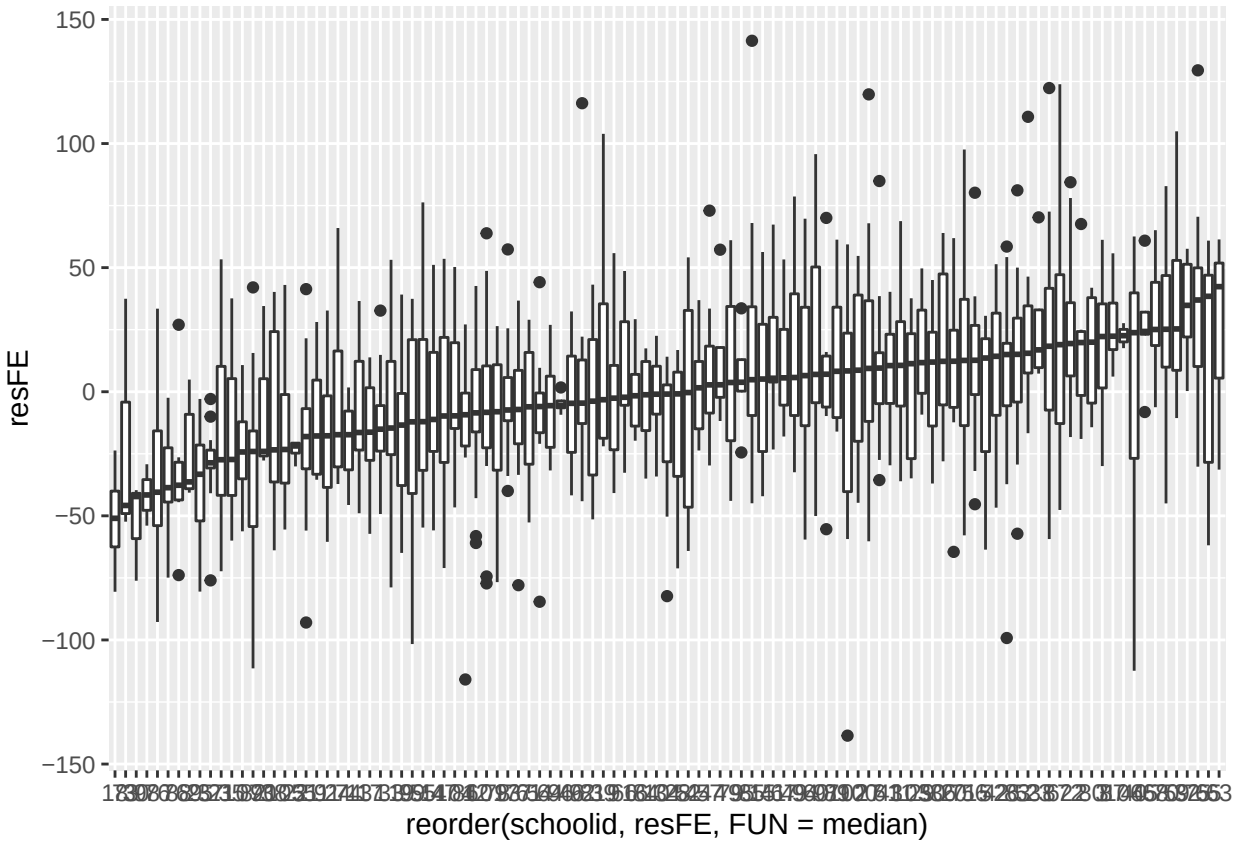
a. Construct the residual that removes only the 'fixed effects' then subtract it from the outcome; call this residual resFE
   i. R hint 1: `predict` has an option to generate the prediction based on the fixed effects only.
   ii. R hint 2: If you decide to add a column to your data frame with resFE, note that predict only generates predictions for cases uses in the model *after listwise deletion.*

```
math1st.pred <- predict(fit1, re.form = ~0)
dat$resFE[!is.na(dat$mathknow)] <- dat$math1st[!is.na(dat$mathknow)] - math1st.pred


# Check with lm function lm2<-lm(math1st ~ housepov + yearstea + mathprep +
# mathknow + ses + sex + minority , data = dat) lm.predct<-predict(lm2)
```

Question 2

Show that the residual is not indepedent within schools in some manner.

```
# Insert code to show that the residual, resFE, is not indepedent within
# schools

# stacked boxplot
require(ggplot2)
ggplot(dat[!is.na(dat$resFE), ], aes(x = reorder(schoolid, resFE, FUN = median),
    y = resFE)) + geom_boxplot()
```



Question 3

a. Construct the residual that utilizes the BLUPs for the random effects using the R command `residuals`.

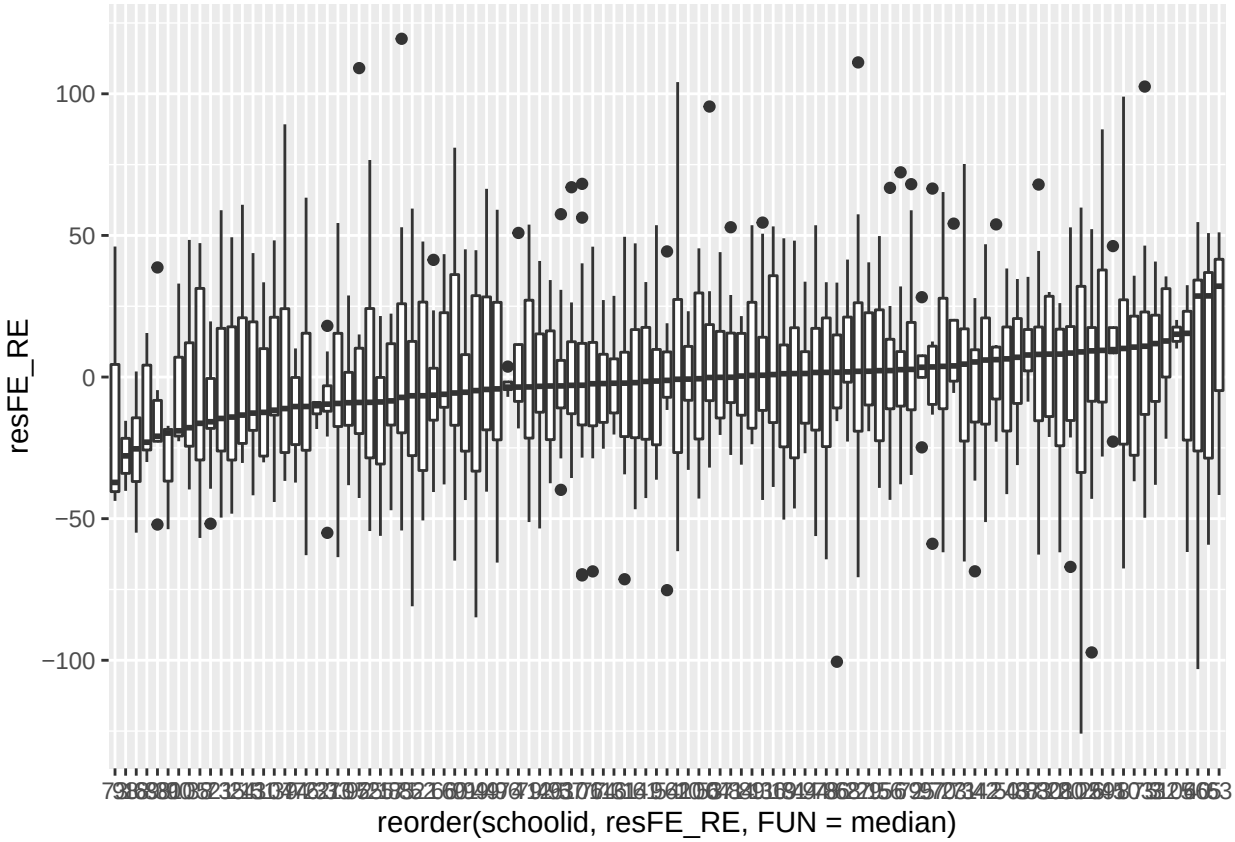
i. Call the new residual resFE_RE

```
# Insert code to construct the residual
dat$resFE_RE[!is.na(dat$mathknow)] <- residuals(fit1)
```

Question 4

a. Show that these new residuals, resFE_RE are MUCH LESS (if not completely un-) correlated within schools, using the same method as before (boxplot?) (you should comment)

```
# Insert code to show that new residuals, resFE_RE, is much less correlated
# within schools

# stacked boxplot
ggplot(dat[!is.na(dat$resFE_RE), ], aes(x = reorder(schoolid, resFE_RE, FUN = median),
    y = resFE_RE)) + geom_boxplot()
```



Response: The new residuals, resFE_RE, are much less correlated within school. This is because the boxplots are more centered around zero (median for each school would be much closer to zero if the errors are independent). The range of medians for the new residuals is also smaller.

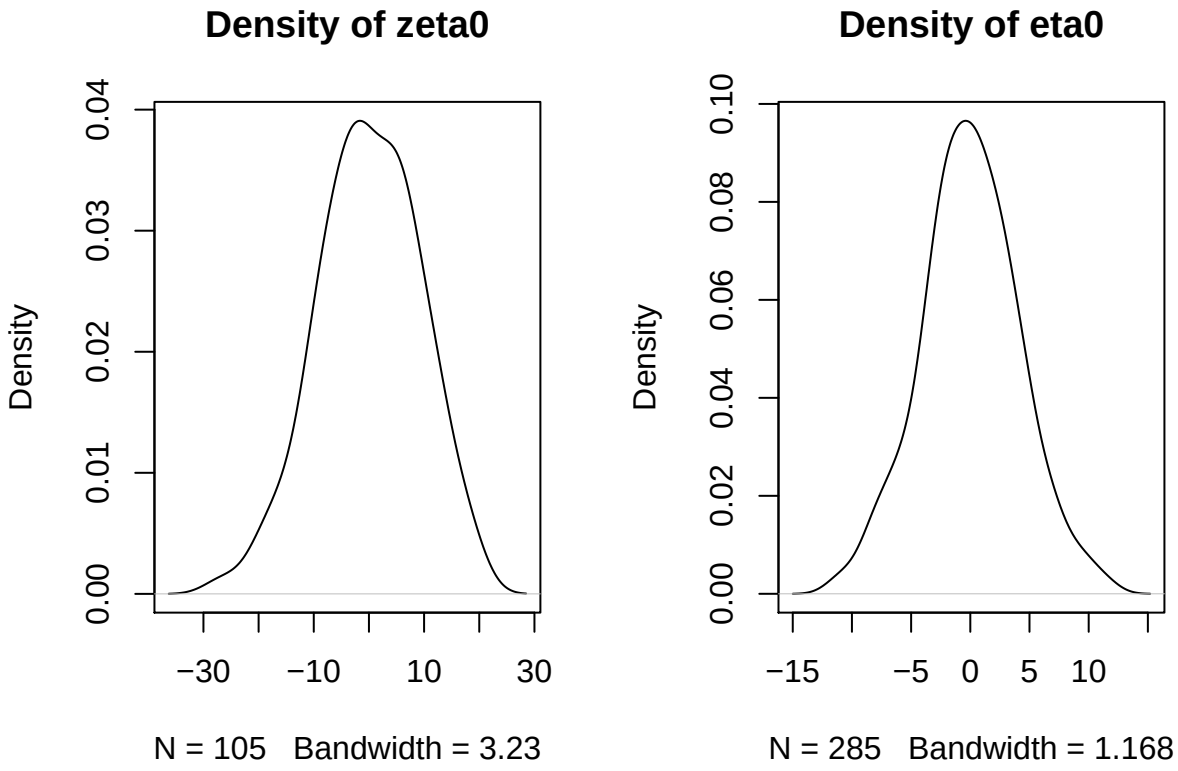
Question 5

a. Generate the two sets of BLUPs (for random effects zeta0 and eta0)

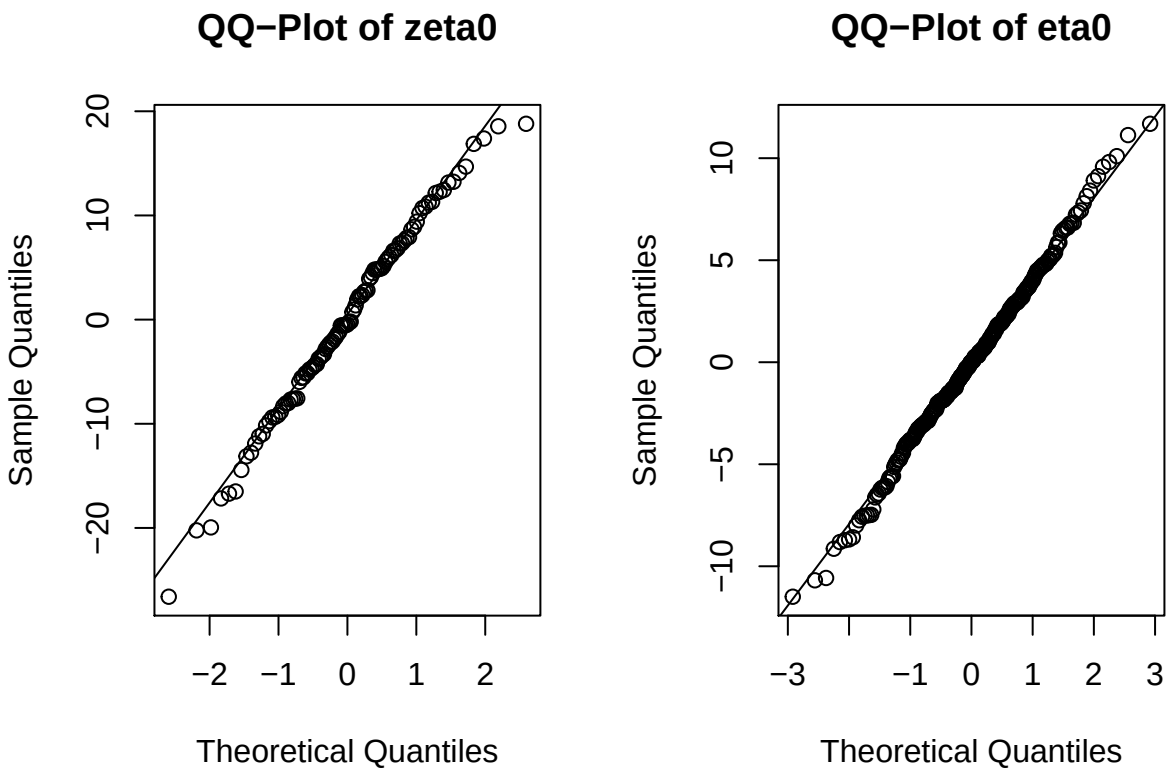
```
# Insert code to generate the two sets of BLUPS (zeta0 and eta0)
ranefs1 <- lme4::ranef(fit1)
zeta0 <- ranefs1$schoolid[, 1]
eta0 <- ranefs1$classid[, 1]
```

b. Examine these for normality (include evidence), and comment.

```
# Insert code to examine BLUPs for normality par(mfrow=c(1,2)) produces
# palette for one row of plots with two columns

# Create density plot
par(mfrow = c(1, 2))
plot(density(zeta0), main = "Density of zeta0")
plot(density(eta0), main = "Density of eta0")
```




```r
# QQplot
par(mfrow = c(1, 2))
qqnorm(zeta0, main = "QQ-Plot of zeta0")
qqline(zeta0)
qqnorm(eta0, main = "QQ-Plot of eta0")
qqline(eta0)
```



Response: It appears that the BLUPs for classroom effects are fairly normal. Their density plot has a bell-shaped, symmetric distribution and their Q-Q plot has most of the points falling about the straight line. The BLUPs for school effects appear slightly less normal. Their density plot has a less symmetric distribution and the points on their Q-Q plot deviate a bit more from forming a straight line.

Question 6

a. Fit a slightly more complicated model with the same fixed effects, but now add a random slope for minority, correlated with the random intercept, at the school level (keep the classroom level random intercept).

```
# Insert code to fit the slightly more complicated model and print the
# summary
fit2 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathprep + mathknow +
    ses + sex + minority + (minority | schoolid) + (1 | schoolid:classid), data = dat,
    REML = T)

summary(fit2)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathprep + mathknow + ses + sex +
##     minority + (minority | schoolid) + (1 | schoolid:classid)
##    Data: dat
##
## REML criterion at convergence: 10717.5
```

```
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8952 -0.6358 -0.0345  0.6129  3.6444
##
## Random effects:
##  Groups           Name        Variance Std.Dev. Corr
##  schoolid:classid (Intercept)   86.7    9.311
##  schoolid         (Intercept)  381.2   19.524
##                   minority     343.2   18.525   -0.83
##  Residual                      1039.4   32.240
## Number of obs: 1081, groups:  schoolid:classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.49369    5.65513  173.09178  95.399  < 2e-16 ***
## housepov     -16.06251   12.57477   99.99134  -1.277    0.204
## yearstea      -0.00437    0.13765  217.17884  -0.032    0.975
## mathprep      -0.29178    1.33537  198.06922  -0.218    0.827
## mathknow       1.63216    1.35929  224.78144   1.201    0.231
## ses            9.43095    1.54335 1063.13485   6.111 1.39e-09 ***
## sex           -0.86278    2.08382 1021.81437  -0.414    0.679
## minority     -16.37547    3.89604   58.24604  -4.203 9.17e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthprp mthknw ses    sex
## housepov -0.394
## yearstea -0.253  0.091
## mathprep -0.576  0.037 -0.167
## mathknow -0.078  0.061  0.024 -0.002
## ses      -0.105  0.089 -0.021  0.052 -0.005
## sex      -0.172 -0.013  0.014 -0.005  0.010  0.024
## minority -0.494 -0.157  0.027 -0.002  0.099  0.113 -0.014
```

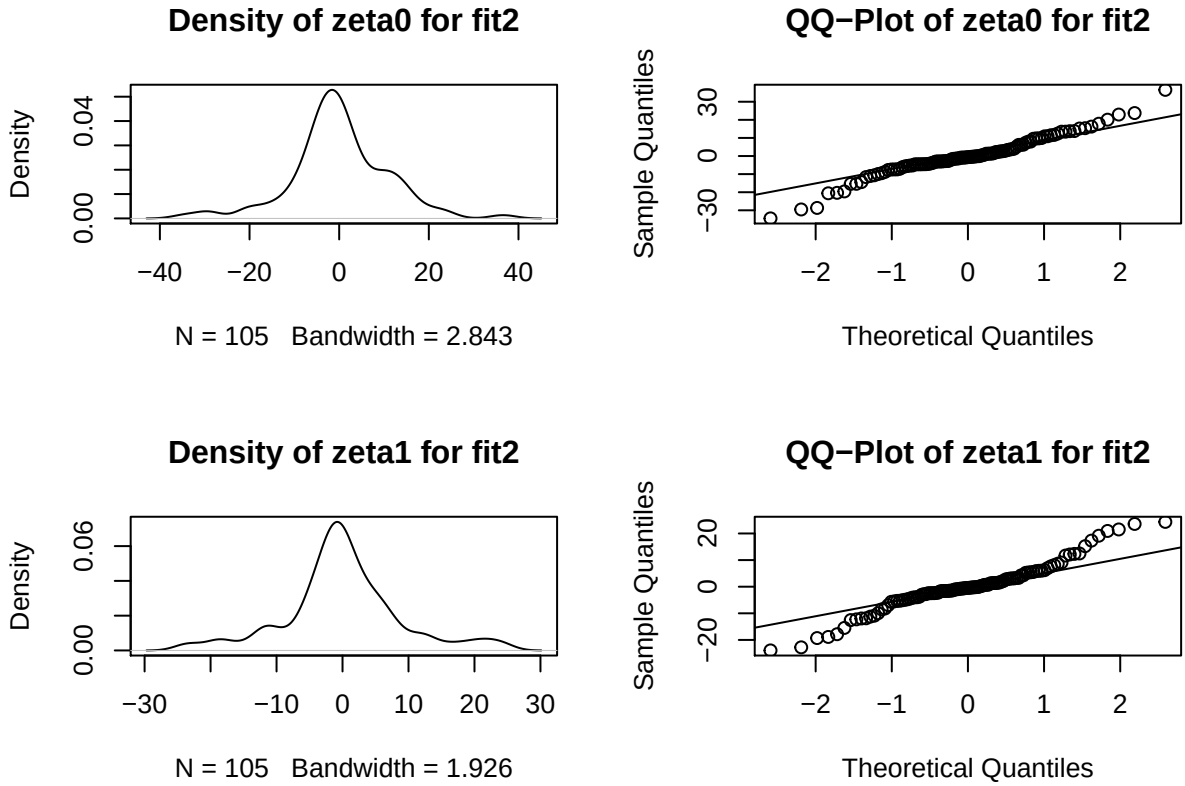
b. Construct the residual (individual, level 1) and the BLUPs for the remaining random effects. Call the new residual resFE_RE as before.

```r
# Insert code to construct residual and BLUPs

resFE_RE2 <- residuals(fit2)
ranefs2 <- lme4::ranef(fit2)
zeta0_2 <- ranefs2$schoolid[, 1]
zeta1_2 <- ranefs2$schoolid[, 2]
eta0_2 <- ranefs2$`schoolid:classid`[, 1]
```

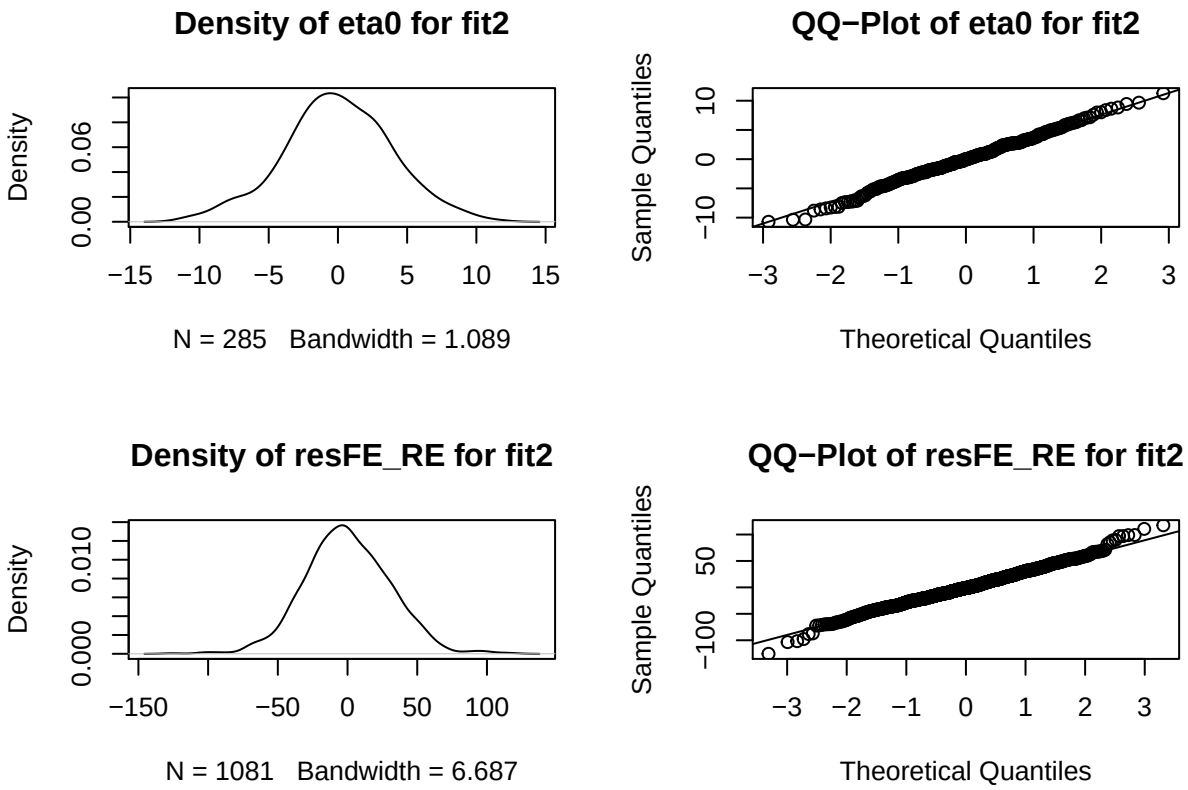
c. Examine all error estimates (individual level residuals, BLUPs (school and classroom level) for normality (and comment)).

```r
# Insert code to examine error estimates.

par(mfrow = c(2, 2))
```

```r
plot(density(zeta0_2), main = "Density of zeta0 for fit2")
qqnorm(zeta0_2, main = "QQ-Plot of zeta0 for fit2")
qqline(zeta0_2)

plot(density(zeta1_2), main = "Density of zeta1 for fit2")
qqnorm(zeta1_2, main = "QQ-Plot of zeta1 for fit2")
qqline(zeta1_2)
```




```r
par(mfrow = c(2, 2))
plot(density(eta0_2), main = "Density of eta0 for fit2")
qqnorm(eta0_2, main = "QQ-Plot of eta0 for fit2")
qqline(eta0_2)

plot(density(resFE_RE2), main = "Density of resFE_RE for fit2")
qqnorm(resFE_RE2, main = "QQ-Plot of resFE_RE for fit2")
qqline(resFE_RE2)
```



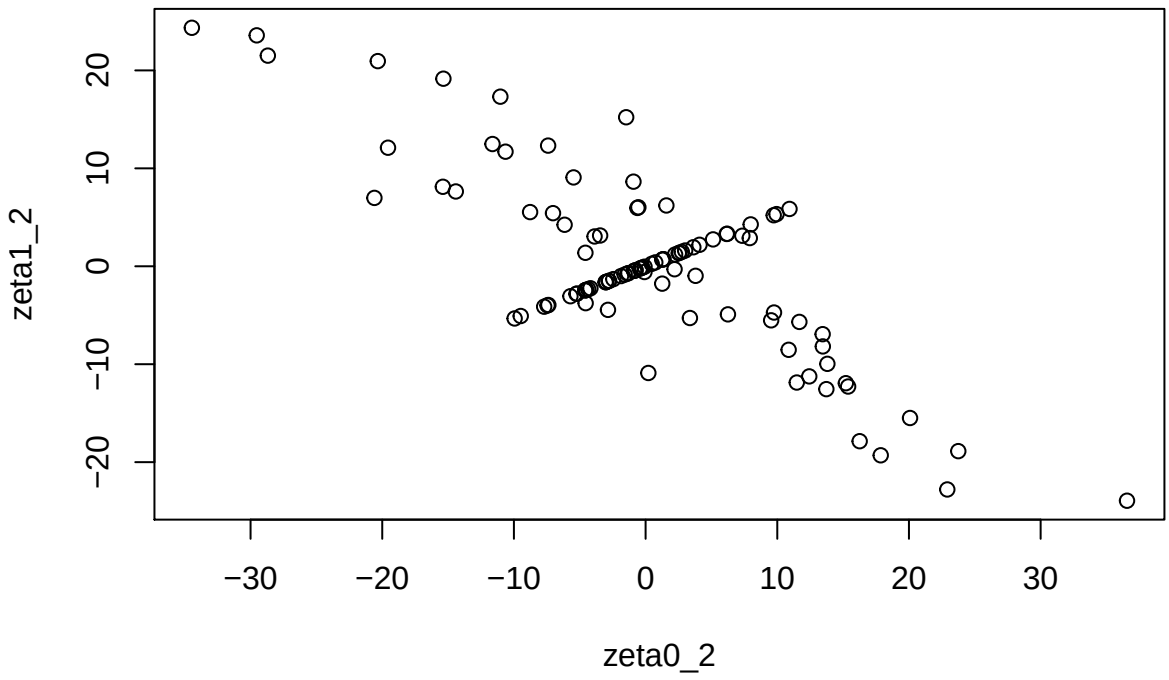
Response: For school level random effects, the deviation from theoretical quantiles at two tails of the line suggest some non-normality. The classroom effects and residuals are a bit more normal. Although there is still some deviation for residuals, it is probably tolerable.

d. Plot zeta0 vs. zeta1 to see whether the estimated correlation is consistent with the observed. Briefly comment.

```
# Insert code for plot and estimate correlation

cor(zeta0_2, zeta1_2)
```

```
## [1] -0.7852153
```

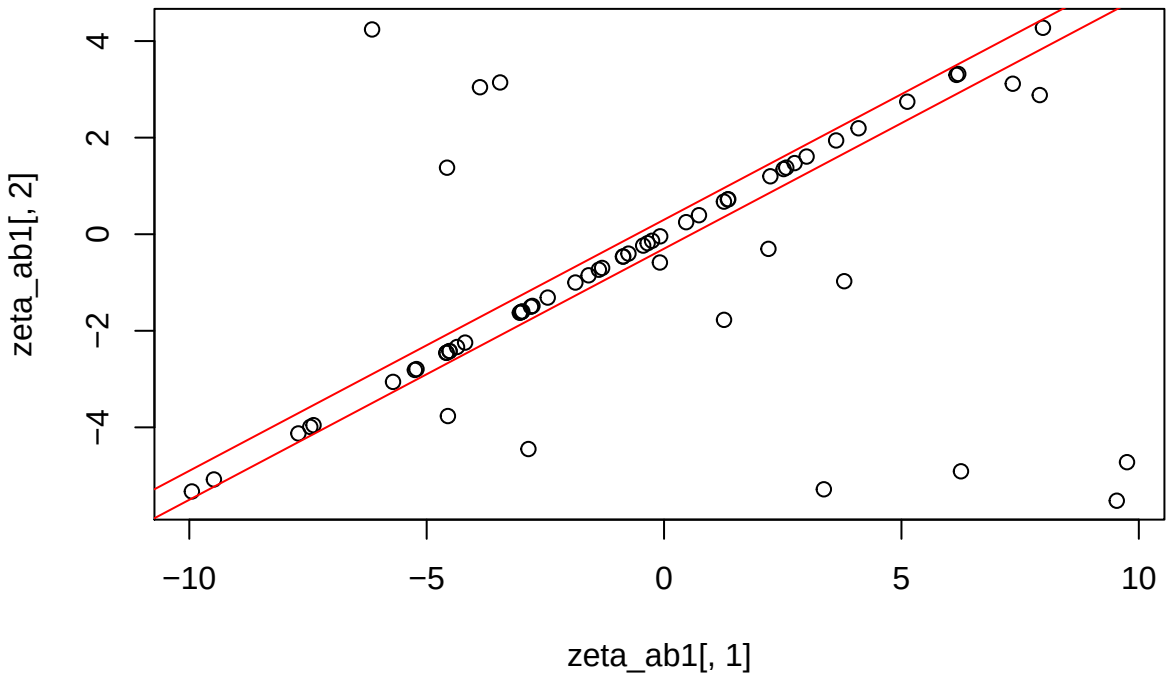
```
plot(zeta0_2, zeta1_2)
```



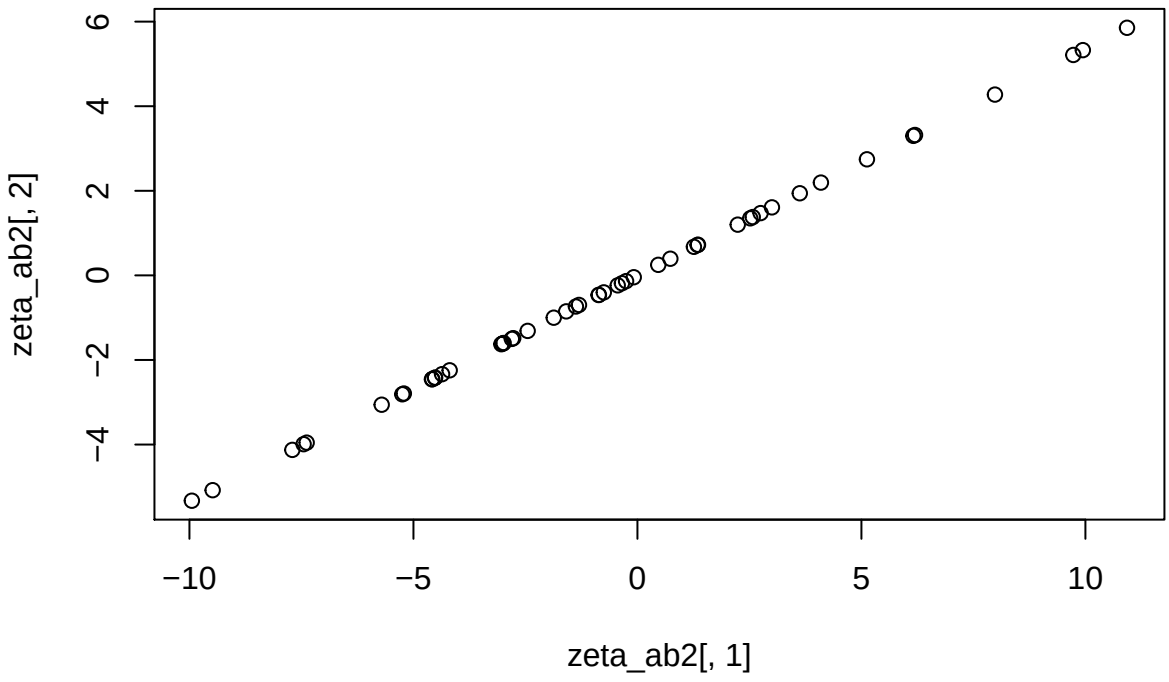
Response: In spite of the "weird" positively correlated points shown in the plot, the estimated correlation above is -0.7852 which is consistent with what we get from the model(-0.83). These are all large and negative correaltions.

e. Track down those odd points in the scatterplot. What schools are they? Do they have anything in common? (You should comment)

```
# Insert code if you want to examine odd points
zeta <- ranefs2$schoolid
zeta$schooid <- row.names(zeta)
zeta_ab1 <- zeta[zeta0_2 > -10 & zeta0_2 < 10 & zeta1_2 > -10 & zeta1_2 < 5,
    ]
plot(zeta_ab1[, 1], zeta_ab1[, 2])
abline(a = 0.3, b = 0.52, col = 2)
abline(a = -0.3, b = 0.52, col = 2)
```




```
zeta_ab2 <- zeta[0.3 + 0.52 * zeta0_2 > zeta1_2 & -0.3 + 0.52 * zeta0_2 < zeta1_2,
    ]
plot(zeta_ab2[, 1], zeta_ab2[, 2])
```




```
# Part of Schoolids for the weird points
head(zeta_ab2[, 3])
```

```
## [1] "1"  "4"  "5"  "9"  "10" "12"
```

```
dat2 <- dat[dat$schoolid %in% zeta_ab2[, 3], ]

library(dplyr)
```

```
##
## Attaching package: 'dplyr'
```

```
## The following object is masked from 'package:car':
##
##     recode
```

```
## The following objects are masked from 'package:stats':
##
##     filter, lag
```

```
## The following objects are masked from 'package:base':
##
##     intersect, setdiff, setequal, union
```

```
head(dat2 %>% group_by(schoolid) %>% summarize(mean(minority)))
```

```
## # A tibble: 6 x 2
##   schoolid `mean(minority)`
##      <int>            <dbl>
```

```
## 1        1                1
## 2        4                1
## 3        5                1
## 4        9                1
## 5       10                1
## 6       12                1
```

Response: We can see that the odd points are from schools where high proportion of students or even all of them are minorities.

Question 7

Make a *person-period* file with math score (Kindergarten and First grade). That is, `math0 <- mathkind; math1 <- mathkind + mathgain` (you have to make this work in the dataframe). Using `reshape` in R, you have to be careful to specify the name of the math variable (`math0` and `math1`) as *varying*.

```
# Insert code to create the variables math0 and math1 and to reshape data

dat$math0 <- dat$mathkind
dat$math1 <- dat$mathkind + dat$mathgain
class_pp <- reshape(dat, varying = c("math0", "math1"), v.names = "math", timevar = "year",
    times = c(0, 1), direction = "long")
```

Question 8

We ignore classrooms in this analysis, but keep it in the notation.

a. Fit a model with math as outcome, and fixed effect for time trend (year), and random intercepts for schools.

```
# Insert code to fit model and print summary
fit8a <- lmerTest::lmer(math ~ year + (1 | schoolid), data = class_pp)
summary(fit8a)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math ~ year + (1 | schoolid)
##    Data: class_pp
##
## REML criterion at convergence: 23951.7
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -5.2833 -0.6084  0.0037  0.6329  3.7761
##
## Random effects:
##  Groups   Name        Variance Std.Dev.
##  schoolid (Intercept)  348.7   18.67
##  Residual             1268.4   35.62
## Number of obs: 2380, groups:  schoolid, 107
##
## Fixed effects:
##             Estimate Std. Error       df t value Pr(>|t|)
## (Intercept)  464.932      2.116  132.154  219.73   <2e-16 ***
## year          57.566      1.460 2270.855   39.43   <2e-16 ***
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##      (Intr)
## year -0.345
```

b. Write down the model

Equation:

$MATH_{tijk} = b_0 + \zeta_{0k} + (b_1 + \zeta_{1k})TIME_{tijk} + \varepsilon_{tijk}$ and assume $\zeta_{0k} \sim N(0, \sigma^2_{\zeta 0})$, $\varepsilon_{tijk} \sim N(0, \sigma^2_{\varepsilon})$, all independent with each other.

c. Add random intercepts for child

```
# Insert code to fit new model and print summary output
fit8c <- lmer(math ~ year + (1 | schoolid) + (1 | childid), data = class_pp)
summary(fit8c)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math ~ year + (1 | schoolid) + (1 | childid)
##    Data: class_pp
##
## REML criterion at convergence: 23554.7
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -4.7492 -0.4811  0.0085  0.4881  3.4957
##
## Random effects:
##  Groups   Name        Variance Std.Dev.
##  childid  (Intercept) 702.0    26.50
##  schoolid (Intercept) 307.5    17.54
##  Residual             599.1    24.48
## Number of obs: 2380, groups:  childid, 1190; schoolid, 107
##
## Fixed effects:
##             Estimate Std. Error       df t value Pr(>|t|)
## (Intercept)  465.118      2.042  117.023  227.74   <2e-16 ***
## year          57.566      1.003 1189.000   57.37   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##      (Intr)
## year -0.246
```

d. Write down the model

Equation:

$MATH_{tijk} = b_0 + \delta_{0ijk} + \zeta_{0k} + (b_1 + \zeta_{1k})TIME_{tijk} + \varepsilon_{tijk}$ and assume $\delta_{0ijk} \sim N(0, \sigma^2_{\delta 0})$, $\zeta_{0k} \sim N(0, \sigma^2_{\zeta 0})$, $\varepsilon_{tijk} \sim N(0, \sigma^2_{\varepsilon})$, all independent with each other.

Question 9

Report original and new variance estimates of $\sigma^2_{\zeta_0}$ (between schools) and σ^2_ε (within schools):

$\sigma^2_{\zeta_0}$: 348.7(original), 307.5(new)

σ^2_ε : 1268.4(original), 599.1(new)

a. Compute a pseudo R^2 relating the between school variation and ignoring between students in the same school. In other words, what fraction of the between-school variance in the first model is 'explained' by the addition of a student random effect?

```
# Insert code to compute psuedo R^2 or do this inline
Rb <- (348.7 - 307.5)/348.7
```

b. Does the total variation stay about the same (adding between children within schools variance as well, to the second model results) (you should comment)?

Response: Between children variance is 702. Total Variance for the first model is 348.7+ 1268.4=1617.1 and total Variance for the second model is 307.5+ 599.1+702= 1608.6. They are roughly the same.

Question 10

Add a random slope (ζ_1) for the trend (year) within schools (uncorrelated with random intercept (ζ_0))

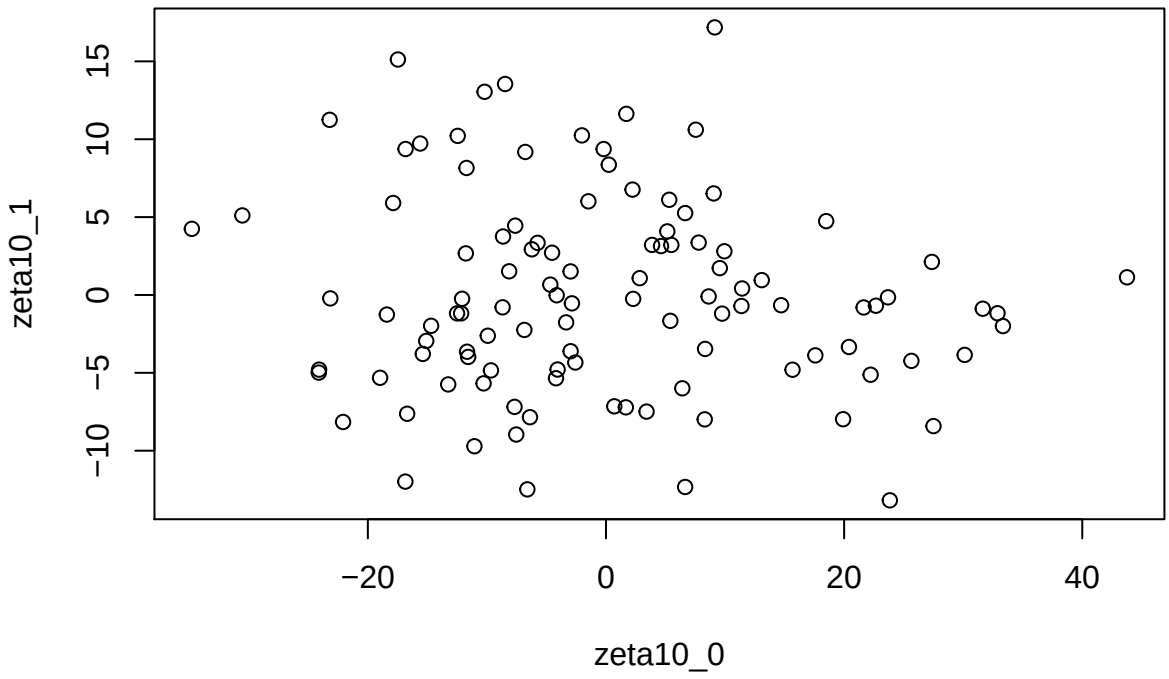
```
# Insert code to fit model and print out summary
fit10 <- lmer(math ~ year + (year || schoolid) + (1 | childid), data = class_pp)
summary(fit10)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math ~ year + (year || schoolid) + (1 | childid)
##    Data: class_pp
##
## REML criterion at convergence: 23529.1
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -4.7665 -0.4721  0.0139  0.4686  3.6080
##
## Random effects:
##  Groups     Name        Variance Std.Dev.
##  childid    (Intercept) 725.12   26.928
##  schoolid   year         88.67    9.417
##  schoolid.1 (Intercept) 324.81   18.023
##  Residual               552.20   23.499
## Number of obs: 2380, groups:  childid, 1190; schoolid, 107
##
## Fixed effects:
##             Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  465.087      2.081 109.946  223.44   <2e-16 ***
## year          57.499      1.370  99.916   41.97   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
```

```
##      (Intr)
## year -0.178
```

a. Generate the BLUPs for the random effects and examine whether the independence between zeta_0 and zeta_1 is reflected in a scatterplot of these two sets of effects. (you should comment)

```
# Insert code to generate BLUPs
ranefs10 <- ranef(fit10)
zeta10_1 <- ranefs10$schoolid[, 1]
zeta10_0 <- ranefs10$schoolid[, 2]
plot(zeta10_0, zeta10_1)
```




```
cor(zeta10_0, zeta10_1)
```

```
## [1] -0.11187
```

Response: The correlation between zeta_0 and zeta_1 is -0.11, which is pretty small and the scatterplot shows no obvious trend between zeta_0 and zeta_1. They are gernerally independent.

b. Compute V_S(year = 0) and V_S (year = 1). Since there are only two years, this is a form of heteroscedasticity in the random effects.

```
# Insert code to compute terms or do this inline
V_S_0 <- 324.81
V_S_1 <- 324.81 + 1 * 88.67
```

i. In which year is there more between school variation, net of all else, **(you should comment)**?

Response: Year 1 has more between school variation. Variances between schools are 324.81 for year 0 and 413.48 for year 1.

Question 11

If you ran the model BY YEAR, and removed the year trend from the model, would you get the same estimates for the variances between schools? **(you should comment)* *

```
# Insert code to fit the two models by year and print out the summary
fit11_0 <- lmer(math ~ 1 + (1 | schoolid), data = class_pp[class_pp$year ==
    0, ])
summary(fit11_0)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math ~ 1 + (1 | schoolid)
##    Data: class_pp[class_pp$year == 0, ]
##
## REML criterion at convergence: 12085.7
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -4.8223 -0.5749  0.0005  0.6454  3.6237
##
## Random effects:
##  Groups   Name        Variance Std.Dev.
##  schoolid (Intercept)  364.3   19.09
##  Residual             1344.5   36.67
## Number of obs: 1190, groups:  schoolid, 107
##
## Fixed effects:
##             Estimate Std. Error     df t value Pr(>|t|)
## (Intercept)   465.23       2.19 103.20   212.4   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
# fit110<-lmer((mathkind+mathgain)~(1|schoolid),data=dat)

fit11_1 <- lmer(math ~ 1 + (1 | schoolid), data = class_pp[class_pp$year ==
    1, ])
summary(fit11_1)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math ~ 1 + (1 | schoolid)
##    Data: class_pp[class_pp$year == 1, ]
##
## REML criterion at convergence: 11950.8
##
## Scaled residuals:
##    Min     1Q Median     3Q    Max
## -5.291 -0.612 -0.005  0.613  3.793
##
## Random effects:
```

```
##  Groups   Name        Variance Std.Dev.
##  schoolid (Intercept)  306.8   17.52
##  Residual             1205.0   34.71
## Number of obs: 1190, groups:  schoolid, 107
##
## Fixed effects:
##             Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  522.698      2.027 103.069   257.8   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Response: The results are different. When running the model separately by year,variances between schools are 364.3 for year 0 and 306.8 for year 1. The variation at the school level drops from kindergarten to first grade. However in question 10, year 1 has more between school variation, which might suggest a misspecification in the nested longitudinal model in question 10.

Question 12

Rerun the last nested longitudinal model, allowing correlation between intercept and slope.

a. Is the correlation significant? (you should comment)

```
# Insert code to fit model, print the summary output, and compare models
fit12 <- lmer(math ~ year + (year | schoolid) + (1 | childid), dat = class_pp)
summary(fit12)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math ~ year + (year | schoolid) + (1 | childid)
##    Data: class_pp
##
## REML criterion at convergence: 23520.3
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -4.7030 -0.4686  0.0066  0.4669  3.5142
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  childid  (Intercept) 728.0    26.98
##  schoolid (Intercept) 370.6    19.25
##           year        109.1    10.44    -0.45
##  Residual             547.0    23.39
## Number of obs: 2380, groups:  childid, 1190; schoolid, 107
##
## Fixed effects:
##             Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  465.099      2.188 102.918  212.60   <2e-16 ***
## year          57.668      1.440  94.572   40.04   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##      (Intr)
## year -0.439
```

```r
anova(fit10, fit12, refit = F)
```

```
## Data: class_pp
## Models:
## fit10: math ~ year + (year || schoolid) + (1 | childid)
## fit12: math ~ year + (year | schoolid) + (1 | childid)
##       Df   AIC   BIC logLik deviance  Chisq Chi Df Pr(>Chisq)
## fit10  6 23541 23576 -11764    23529
## fit12  7 23534 23575 -11760    23520 8.8241      1   0.002973 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Response: The correlation is -0.45 and is significant at 5% level, as shown by the LR test between fit10 and fit12.

b. Compute V_S (year = 0) and V_S(year = 1) for this new model (your formula should include covariance terms).

```r
# Insert code to compute terms or do this inline
V_S12_0 = 370.6 + 0 * 109.1
V_S12_1 = 370.6 + 1 * 109.1 - 2 * 0.45 * 19.25 * 10.44
```

i. Is this result (and thus model) more consistent with the separate grade analysis? You are implicity testing model fit here. **(you should comment)**

Response: Variance between school for year=0 is 370.6, which is now smaller than variance between school for year=1. This result from model that allows for correlation picks up the 'drop' in between-school variance in first grade and thus is more consistent with the previous separate grade analysis.