CCAPP Research Summary: Modeling of EoR Ionization Front

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2 Introduction

Please refer to the introduction section in [1].

3 Analytic Analysis

We are considering interaction between five species: electron (e^-), hydrogen ion (H^+), helium ion (He^+), neutral hydrogen (H), and neutral helium (He). Some properties are listed in Table.1.

In the process we are discussing, the interactions are sorted in five groups: electrons and ions, ions and ions, electrons and neutrals, ions and neutrals, and between neutrals and neutrals. We

Table 1: Useful properties of species.

Parameters	e^-	H^+	He^+	H	He
Mass (kg)	9.109×10^{-31}	1.673×10^{-27}	6.647×10^{-27}	1.673×10^{-27}	6.647×10^{-27}
Mass (MeV/c^2)	0.511	938.272	3727.911	938.896	3728.422
$\ln \Lambda$					

aim to generate a 5 matrix of energy transferring rate between species. For all species we adopt Maxwellian distributions $f(v)$ for calculations of speeds.

3.1 Basic Principles of Plasma Physics

The average kinetic energy of a particle in three dimensions is

$$E_{av} = \frac{3}{2}kT = 3 \cdot \frac{1}{2}mv^2 \tag{1}$$

Accordingly, its average thermal speed is

$$v = \left(\frac{kT}{m}\right)^{1/2} \tag{2}$$

In addition, quasi-neutrality demands that

$$n_i(\text{total ion species}) \simeq n_e \equiv n \tag{3}$$

The strategy I adopt is illustrated as following:

relative velocity distribution $\times$ energy loss rate for single collision $\rightarrow$ distributional energy loss

3.2 Electron $\Longleftrightarrow$ Ions

In this section we aim to find the momentum and energy loss rate when electrons collide ions. To do so, we need two components: the thermal distribution of electrons and the collision rate as a function of velocity.

In the stationary reference frame of ions, the electrons obey the velocity distributions of [2]:

$$f_e(\boldsymbol{v}) = n_e\left(\frac{m_e}{2\pi kT_e}\right)^{3/2} \exp\left[-\frac{m_e(\boldsymbol{v}-\boldsymbol{v_d})^2}{2kT_e}\right] \tag{4}$$

where $\boldsymbol{v_d}$ is the drift velocity of the electrons ensemble, and $\boldsymbol{v}$ is the velocity of electrons **relative** to that of ions. We do not include the thermal motion of the ions because they are moving slowly compared with electrons.

Consider a single electron collides with an ion, the momentum collision frequency $\nu_{ei,p}$ (p denotes momentum) is [3]:

$$\nu_{ei,p} = n_i \frac{q_e^2 q_i^2}{(4\pi\epsilon_0)^2} \frac{4\pi(m_e+m_i)}{m_i m_e^2 v^3} \ln \Lambda_e \tag{5}$$

where $\ln\Lambda$ is the Coulomb Logarithm , and according to [4],

$$\ln\Lambda_e = 22.1 + \ln\Big[\Big(\frac{E_e}{kT_e}\Big)\Big(\frac{T_e}{10^4\mathrm{K}}\Big)\Big(\frac{\mathrm{cm}^{-3}}{n_e}\Big)\Big] \tag{6}$$

By introducing the Coulomb Logarithm factor, we constrain the lower and upper cutoffs when integrating over all relevant impact parameters b.

Therefore, combining Eq.4, 5, and 6, we obtain the total momentum loss rate per unit volume, assuming that $\ln\Lambda$ is a constant instead of function of v, and that $|\boldsymbol{v}_d| \ll v$:

$$\begin{aligned}
-\frac{d\boldsymbol{p}_e}{dt} &= \int d^3v\ f_e(\boldsymbol{v})\nu_{ei,p}(v)m_e\boldsymbol{v} \\
&= \int d^3v\ m_e\boldsymbol{v}n_e\Big(\frac{m_e}{2\pi kT_e}\Big)^{3/2}\exp\Big[-\frac{m_e(\boldsymbol{v}-\boldsymbol{v_d})^2}{2kT_e}\Big]\cdot n_i\frac{q_e^2q_i^2}{(4\pi\epsilon_0)^2}\frac{4\pi(m_e+m_i)}{m_im_e^2v^3}\ln\Lambda_e \\
&= m_en_e\Big(\frac{1}{2\pi v_e^2}\Big)^{3/2}\cdot n_i\frac{q_e^2q_i^2}{(4\pi\epsilon_0)^2}\frac{4\pi(m_e+m_i)}{m_im_e^2}\ln\Lambda_e\int d^3v\ \frac{\boldsymbol{v}}{v^3}\exp\Big[-\frac{m_e(\boldsymbol{v}-\boldsymbol{v_d})^2}{2kT_e}\Big]
\end{aligned} \tag{7}$$

where $v_e = (kT_e/m)^{1/2}$ is the average thermal speed at temperature T_e. In addition, we can define the the integral as

$$\begin{aligned}
I &\equiv \int d^3v\ \frac{\boldsymbol{v}}{v^3}\exp\Big[-\frac{m_e(\boldsymbol{v}-\boldsymbol{v}_d)^2}{2kT_e}\Big] \\
&= \int d^3v\ \frac{\boldsymbol{v}}{v^3}\exp\Big[-\frac{m_e}{2kT_e}(\boldsymbol{v}-\boldsymbol{v}_d)^2\Big] \\
&= \int d^3v\ \frac{\boldsymbol{v}}{v^3}\exp\Big[-\frac{m_e}{2kT_e}(\boldsymbol{v}\cdot\boldsymbol{v}-2\boldsymbol{v}\cdot\boldsymbol{v}_d+\boldsymbol{v}_d\cdot\boldsymbol{v}_d)\Big] \\
&= \int d^3v\ \frac{\boldsymbol{v}}{v^3}\exp\Big[-\frac{m_e}{2kT_e}(\boldsymbol{v}\cdot\boldsymbol{v})\Big]\exp\Big[\frac{m_e}{T_e}(\boldsymbol{v}\cdot\boldsymbol{v}_d)\Big]\exp\Big[-\frac{m_e}{2kT_e}(\boldsymbol{v}_d\cdot\boldsymbol{v}_d)\Big] \\
&= \int d^3v\ \frac{\boldsymbol{v}}{v^3}\exp\Big[-\frac{m_e}{2m_ev_e^2}(\boldsymbol{v}\cdot\boldsymbol{v})\Big]\exp\Big[\frac{m_e}{m_ev_e^2}(\boldsymbol{v}\cdot\boldsymbol{v}_d)\Big]\exp\Big[-\frac{m_e}{2m_ev_e^2}(\boldsymbol{v}_d\cdot\boldsymbol{v}_d)\Big] \\
&= \int d^3v\ \frac{\boldsymbol{v}}{v^3}\exp\Big[-\frac{1}{2v_e^2}(\boldsymbol{v}\cdot\boldsymbol{v})\Big]\exp\Big[\frac{1}{v_e^2}(\boldsymbol{v}\cdot\boldsymbol{v}_d)\Big]\exp\Big[-\frac{1}{2v_e^2}(\boldsymbol{v}_d\cdot\boldsymbol{v}_d)\Big] \\
&= \exp\Big[-\frac{v_d^2}{2v_e^2}\Big]\int d^3v\ \frac{\boldsymbol{v}}{v^3}\exp\Big[-\frac{v^2}{2v_e^2}\Big]\exp\Big[\frac{1}{v_e^2}(\boldsymbol{v}\cdot\boldsymbol{v}_d)\Big]
\end{aligned} \tag{8}$$

For one component of the momentum vector $\boldsymbol{p}_e$, e.g. p_{ex}, we can align the corresponding axis to the drift velocity $\boldsymbol{v}_d$. Therefore, the integral I becomes

$$I = \exp\Big[-\frac{v_d^2}{2v_e^2}\Big]\int d^3v\ \frac{v_x}{v^3}\exp\Big[-\frac{v^2}{2v_e^2}\Big]\exp\Big[\frac{1}{v_e^2}(v_xv_d)\Big] \tag{9}$$

Assuming the Maxwellian distribution is isotropic, we can establish the relation such that $3v_x^2 = v^2$, or $v_x = v/\sqrt{3}$. Therefore,

$$
\begin{aligned}
I &= \exp\Big[-\frac{v_d^2}{2v_e^2}\Big]\int_0^\infty d^3v\ \frac{v}{\sqrt{3}v^3}\exp\Big[-\frac{1}{2v_e^2}v^2\Big]\exp\Big[\frac{v_d}{\sqrt{3}v_e^2}v\Big] \\
&= \exp\Big[-\frac{v_d^2}{2v_e^2}\Big]\int_0^\infty d^3v\ \frac{1}{\sqrt{3}v^2}\exp\Big[-\frac{1}{2v_e^2}v^2\Big]\exp\Big[\frac{v_d}{\sqrt{3}v_e^2}v\Big] \\
&= \exp\Big[-\frac{v_d^2}{2v_e^2}\Big]\int_0^\infty dv\ 4\pi v^2\frac{1}{\sqrt{3}v^2}\exp\Big[-\frac{1}{2v_e^2}v^2\Big]\exp\Big[\frac{v_d}{\sqrt{3}v_e^2}v\Big] \\
&= \frac{4\pi}{\sqrt{3}}\exp\Big[-\frac{v_d^2}{2v_e^2}\Big]\int_0^\infty dv\ \ \exp\Big[-\frac{1}{2v_e^2}v^2\Big]\exp\Big[\frac{v_d}{\sqrt{3}v_e^2}v\Big] \\
&= \frac{4\pi}{\sqrt{3}}\exp\Big[-\frac{v_d^2}{2v_e^2}\Big]\exp\Big[\frac{v_d^2}{6v_e^2}\Big]\sqrt{\frac{\pi}{2}}v_e\Big(\mathrm{Erf}(\frac{v_d}{\sqrt{6}v_e})+\lim_{a\to\infty}\mathrm{Erf}(\frac{3a-\sqrt{3}v_d}{3\sqrt{2}v_e})\Big) \\
&= \frac{4\pi}{\sqrt{3}}\exp\Big[-\frac{v_d^2}{2v_e^2}\Big]\exp\Big[\frac{v_d^2}{6v_e^2}\Big]\sqrt{\frac{\pi}{2}}v_e\Big(\mathrm{Erf}(\frac{v_d}{\sqrt{6}v_e})+1\Big)
\end{aligned}
\tag{10}
$$

1. look at the papers I downloaded by Dosledge 2. try to directly calculate dE/dt, assuming two maxwellian distributions

3.3 Ions $\Longleftrightarrow$ Ions

3.4 Electron $\Longleftrightarrow$ Neutrals

3.5 Ions $\Longleftrightarrow$ Neutrals

3.6 Neutrals $\Longleftrightarrow$ Neutrals

4 Numerical Calculation

References

[1] C. Hirata, MNRAS **474**, 2173 (2018)

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[5] M. McQuinn, Annu. Rev. Astron. Astrophys. **54**, 313 (2016)