# Intro To Regression

James Woods

This is something like a notebook. You can work with data and text at the same time. Things chunks are for code

```
print("I am in a chunk.")
## [1] "I am in a chunk."
```

By default they echo the code and then produce the result.

You can turn off the echo and just see the result.

## [1] "No Echo"

### **Assignment and Access**

1. Lets start with a few R conventions. Lets start with scalars. '<-' is the assignment operator in R. '=' also works but '==' is used for comparisons.

```
1 + 1

## [1] 2

A <- 2 + 3

A + 2

## [1] 7

A <- A+2

B <- A + 4

A + B

## [1] 18

C = A + B

C

## [1] 18

A == B
```

## [1] FALSE

Note that when you save values to variables you can see them in the Environment tab.

Play with this in the chunk below and run to see what happens.

```
ThisIsAVariable <-10
this_is_a_variable <-23
```

2. R has many ways of representing values

```
A <- "Shale"

## [1] "Shale"

C <- c("Tom", "Dick", "Harry")

C
```

```
## [1] "Tom"
                  "Dick"
                            "Harry"
C[[1]]
## [1] "Tom"
C[1]
## [1] "Tom"
1:1000
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                                                                     999 1000
D <- 1:10
D
             2 3 4 5
                           6 7
                                8
                                    9 10
    [1] 1
as.character(D)
    [1] "1" "2"
                    "3"
                          "4"
                               "5" "6" "7" "8"
                                                      "9" "10"
as.factor(D)
    [1] 1 2 3 4 5 6 7
                                8
## Levels: 1 2 3 4 5 6 7 8 9 10
summary(D)
                                                    Max.
##
      Min. 1st Qu.
                      Median
                                  Mean 3rd Qu.
##
      1.00
               3.25
                         5.50
                                  5.50
                                           7.75
                                                   10.00
summary(as.character(D))
##
      Length
                   Class
                               Mode
           10 character character
summary(as.factor(D))
```

```
2 3 4 5 6 7 8 9 10
1 1 1 1 1 1
            1
              1 1
```

There are many others, "list" being the most important not shown.

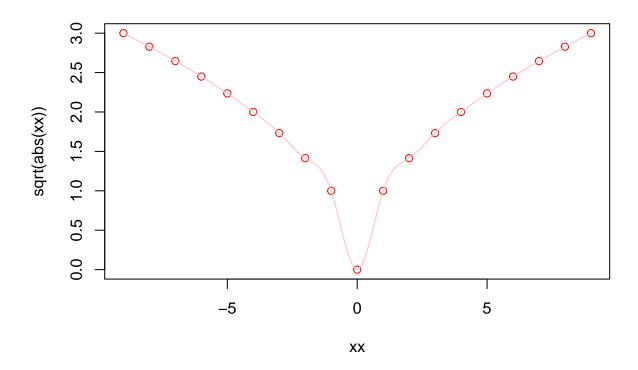
3. There are also functions. Most, but not all, work on many things at once.

```
sqrt(D)
    [1] 1.000000 1.414214 1.732051 2.000000 2.236068 2.449490 2.645751
##
   [8] 2.828427 3.000000 3.162278
# sqrt(A)
sqrt(D[6])
## [1] 2.44949
sqrt(D[1:3])
## [1] 1.000000 1.414214 1.732051
  4. If you need help, use the help pane or ask for help. Everything has a help file and many functions come
     with examples of how to use.
```

help(sqrt)

```
## starting httpd help server ... done
example(sqrt)
##
## sqrt> require(stats) # for spline
## sqrt> require(graphics)
##
## sqrt> xx <- -9:9
```

## sqrt> plot(xx, sqrt(abs(xx)), col = "red")



```
##
## sqrt> lines(spline(xx, sqrt(abs(xx)), n=101), col = "pink")
```

4. You can work with vectors and matrix but you will most frequently deal with dataframes, which is a matrix with extra attributes. Dataframes are objects that have variables inside them. You can access those variables with specific functions or with a '\$'.

```
X <- runif(20, 2, 40)
MyData <- as.data.frame(X)
MyData$Y <- 100 - 2 * MyData$X + rnorm(20, mean = 0, sd = 1)

names(MyData)
## [1] "X" "Y"
names(MyData) <- c("Price", "Quantity")
names(MyData)
## [1] "Price" "Quantity"
summary(MyData)</pre>
```

```
##
        Price
                         Quantity
           : 2.365
                              :30.81
##
    Min.
                      Min.
    1st Qu.: 8.964
                      1st Qu.:42.22
##
   Median :18.036
                      Median :63.24
##
##
   Mean
           :18.345
                      Mean
                              :63.38
##
    3rd Qu.:28.680
                      3rd Qu.:81.46
   Max.
           :34.156
                             :96.15
                      Max.
```

5. You can get at columns and rows in other ways.

```
MyData[1:2]
```

```
##
         Price Quantity
## 1
      7.133680 85.99715
## 2
     33.653215 34.64993
## 3 16.936706 64.72794
## 4 28.767068 42.70684
## 5 34.155720 30.80695
     13.742116 73.44657
## 6
## 7
     16.878220 68.35650
## 8 21.242790 57.53247
## 9
      8.261871 83.66602
## 10 3.252648 93.63589
## 11 33.699309 33.02094
## 12 19.974595 61.64471
## 13 32.879849 33.16642
## 14 21.032628 58.56979
## 15 4.485552 91.28612
## 16 9.198394 80.71940
## 17 11.454320 75.11109
## 18 2.365383 96.14816
## 19 19.135119 61.75415
## 20 28.651570 40.74370
```

#### MyData[1]

```
##
          Price
## 1
      7.133680
## 2 33.653215
## 3
     16.936706
## 4
     28.767068
## 5 34.155720
## 6 13.742116
## 7
     16.878220
## 8
     21.242790
## 9
      8.261871
## 10 3.252648
## 11 33.699309
## 12 19.974595
## 13 32.879849
## 14 21.032628
## 15 4.485552
## 16 9.198394
## 17 11.454320
## 18 2.365383
## 19 19.135119
## 20 28.651570
```

#### MyData[1:3,]

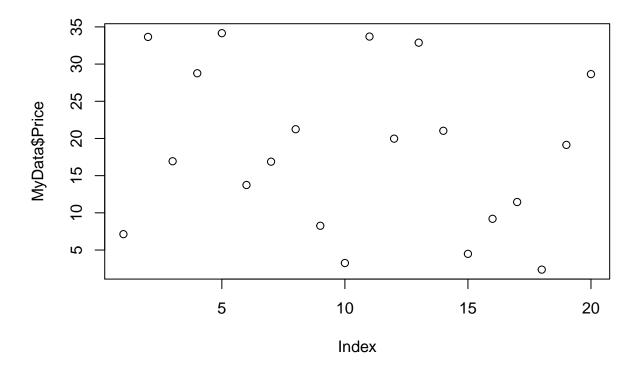
```
##
       Price Quantity
## 1 7.13368 85.99715
## 2 33.65322 34.64993
## 3 16.93671 64.72794
```

```
MyData[1:3,"Price"]
## [1] 7.13368 33.65322 16.93671
MyData["Price"]
##
          Price
## 1
       7.133680
## 2 33.653215
## 3 16.936706
## 4 28.767068
## 5 34.155720
## 6 13.742116
## 7 16.878220
## 8 21.242790
## 9
      8.261871
## 10 3.252648
## 11 33.699309
## 12 19.974595
## 13 32.879849
## 14 21.032628
## 15 4.485552
## 16 9.198394
## 17 11.454320
## 18 2.365383
## 19 19.135119
## 20 28.651570
MyData[c(1,5,8),]
##
       Price Quantity
## 1 7.13368 85.99715
## 5 34.15572 30.80695
## 8 21.24279 57.53247
```

### **Pictures**

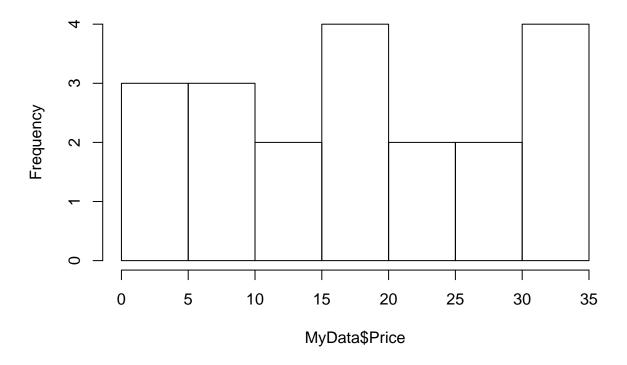
Now lets make pictures.

plot(MyData\$Price)



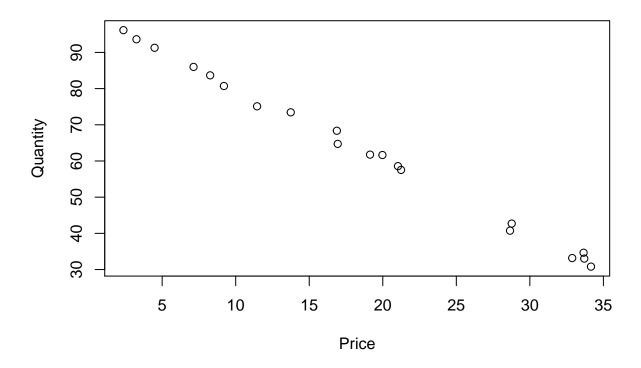
If you don't want to see the code and just the result, make sure to add echo=false to the chunk. Here is a Histogram without the code.

# **Histogram of MyData\$Price**



Show the relationship between two variables with "plot" or more than two with "pairs".

plot(Quantity ~ Price, data= MyData)



## Regression with Pictures

5. A regression line can be thought of as just putting a line through a cloud of data in a well defined way. Remember the data we created? Here is what the regression gives you. You may want to make some notes on how to write formulas for regression in this file.

```
FirstRegression<- lm(Quantity ~ Price, data=MyData)
summary(FirstRegression)</pre>
```

```
##
## Call:
  lm(formula = Quantity ~ Price, data = MyData)
##
##
##
  Residuals:
##
        Min
                  1Q
                       Median
                                    3Q
                                            Max
##
  -2.10044 -0.89349
                      0.06973
                              0.60832
                                        2.02863
##
##
   Coefficients:
##
                Estimate Std. Error t value Pr(>|t|)
## (Intercept) 100.19590
                                     187.63
                            0.53401
                                              <2e-16 ***
## Price
                -2.00661
                            0.02524
                                     -79.51
                                              <2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
```

```
## Residual standard error: 1.19 on 18 degrees of freedom
## Multiple R-squared: 0.9972, Adjusted R-squared: 0.997
## F-statistic: 6322 on 1 and 18 DF, p-value: < 2.2e-16</pre>
```

Note that the estimates are the parameter, or very close to the parameters we used to create the data. Note also that the residual standard errors are the same as what we used to create the data. Nice when you know what is true.

Some other things you should note:

- MORE NOTES FOR YOU TO ADD

You should probably make a lot of notes in this document so you can remember what we are talking about. Just add things. By the way, you can do equations too,  $\frac{1}{2}x$  or even

$$\lim_{x \to +\infty} \frac{3x^2 + 7x^3}{x^2 + 5x^4} = 3$$

If you take 400 level math classes, your homework will be in latex like this.

Regression can also show you things that are not true. You always have to assume a functional form and if you get that functional form wrong you get garbage. Sometimes the garbage makes some kind of sense and sometimes it leads you to believe that you have to make some kind of statistical fix. Students that have taken econometrics will be aware of autocorrelation, heterskedasticity and endogenaity as common problems. Having the wrong functional form will often present as one of these problems.

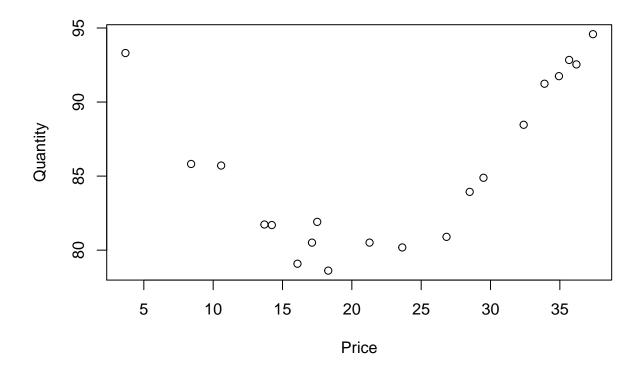
Lets wake new data and show how getting the regression model wrong can cause problems.

```
MyData2 <- data.frame(Price = runif(20, 2, 40))
MyData2$Quantity <- 100 - 2 * MyData2$Price + .05 * MyData2$Price^2 + rnorm(20, mean =0, sd = 1)
summary(MyData2)</pre>
```

```
##
        Price
                          Quantity
##
    Min.
           : 3.672
                      Min.
                              :78.62
##
    1st Qu.:15.617
                      1st Qu.:80.80
    Median :22.456
                      Median :84.41
##
    Mean
            :22.989
                      Mean
                              :85.51
##
    3rd Qu.:32.766
                      3rd Qu.:91.37
##
    Max.
            :37.388
                      Max.
                              :94.58
```

Not a line

```
plot(Quantity ~ Price, data = MyData2)
```

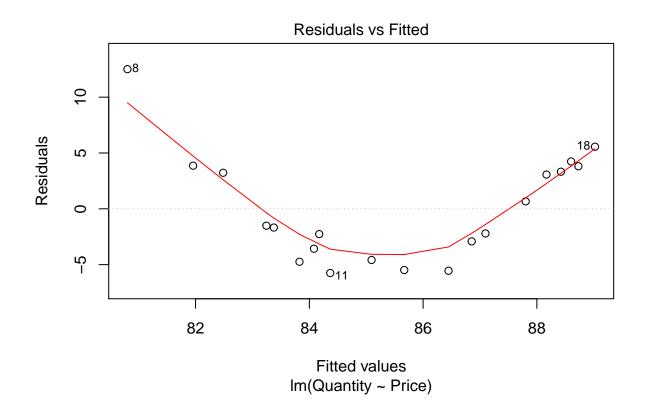


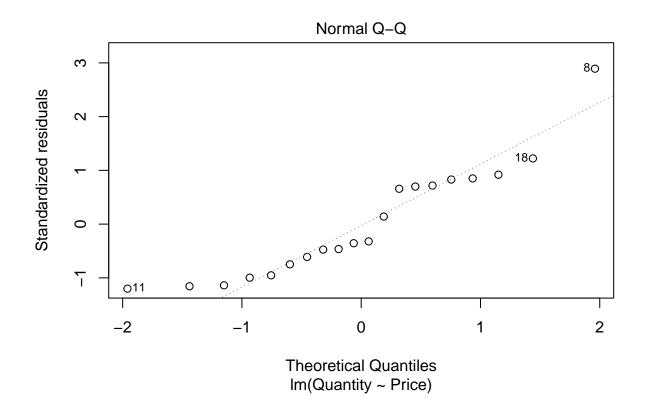
Give me a line anyway.

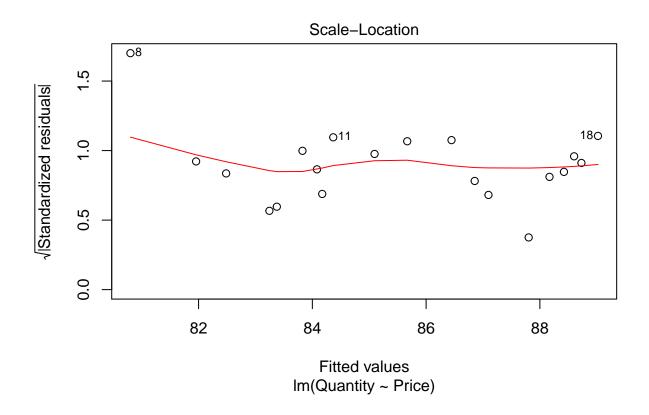
```
NotALine<- lm(Quantity ~ Price, data=MyData2)
summary(NotALine)
##
## Call:</pre>
```

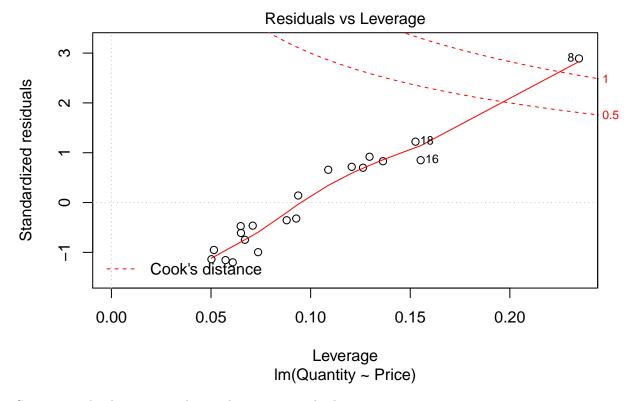
```
lm(formula = Quantity ~ Price, data = MyData2)
##
##
## Residuals:
##
      Min
              1Q Median
                            3Q
                                  Max
  -5.749 -3.826 -1.596 3.446 12.507
##
##
## Coefficients:
##
              Estimate Std. Error t value Pr(>|t|)
                                   28.955
## (Intercept) 79.9065
                            2.7596
                                             <2e-16 ***
                                            0.0398 *
## Price
                 0.2438
                            0.1100
                                     2.217
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 4.944 on 18 degrees of freedom
## Multiple R-squared: 0.2144, Adjusted R-squared: 0.1708
## F-statistic: 4.913 on 1 and 18 DF, p-value: 0.03978
```

How to tell you were wrong thinking it was a line. Make a pictures



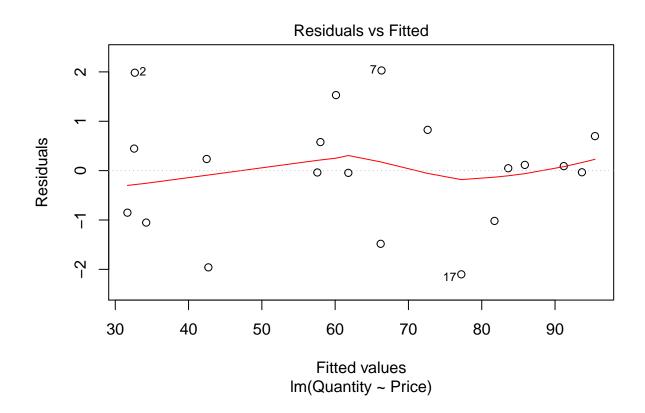


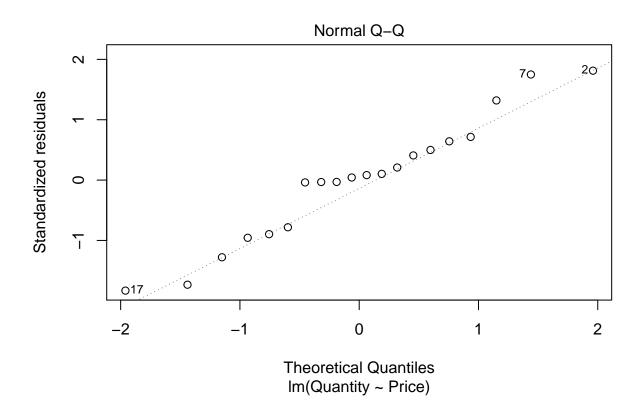


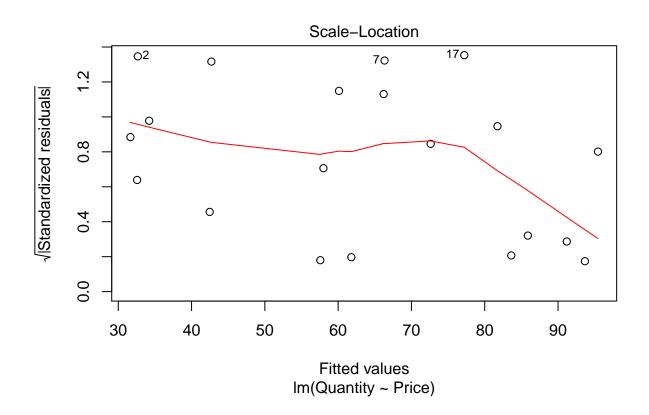


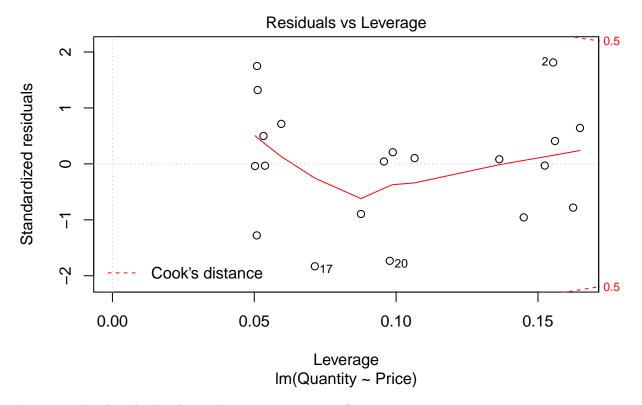
Compare with when it was a line and you estimated a line

plot(FirstRegression)



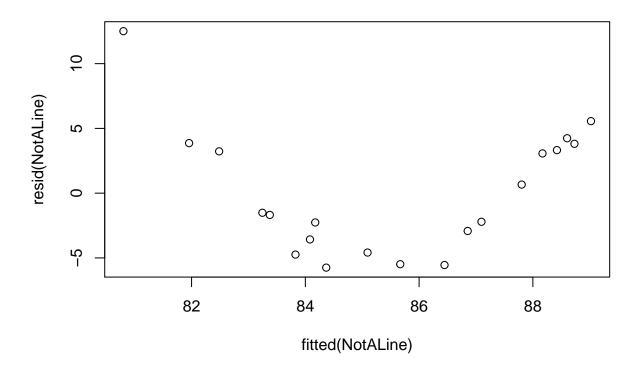






You can make these by hand too. Here are two accessor functions.

plot(resid(NotALine)~fitted(NotALine))



## **Dummy Variables**

Not every variable is real valued, some are true false or categorical like night and day. R deals with categorical variables, called factors, very well and has fewer of the problems you see in other languages.

Lets make new fake data with dummy variables. I will reuse the first artificial dataset that was truly linear.

```
MyData$Time <- as.factor(ifelse(runif(20) < .5, "Day", "Night"))

MyData$Quantity[MyData$Time == "Day"] <- MyData$Quantity[MyData$Time == "Day"] + 12</pre>
```

I just made it so that if it is Day the quantity is 12 higher.

```
WTime <- lm(Quantity ~ Price + Time, data=MyData)
summary(WTime)</pre>
```

```
##
## Call:
## lm(formula = Quantity ~ Price + Time, data = MyData)
##
##
  Residuals:
##
                1Q
                    Median
                                 3Q
                                         Max
   -2.2816 -0.5447
                    0.1296
                             0.5854
                                     1.7162
##
##
## Coefficients:
```

```
##
               Estimate Std. Error t value Pr(>|t|)
## (Intercept) 111.96260
                           0.52931 211.52 < 2e-16 ***
## Price
               -2.01764
                           0.02501
                                   -80.66 < 2e-16 ***
## TimeNight
              -11.12888
                           0.52758
                                   -21.09 1.25e-13 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 1.137 on 17 degrees of freedom
## Multiple R-squared: 0.998, Adjusted R-squared: 0.9978
## F-statistic: 4231 on 2 and 17 DF, p-value: < 2.2e-16
```

See something odd. Interpreting this can be tricky. Best take some notes.

You can also make it so that the reaction to price is different depending on if it is night or day.

```
WTimeInteract <- lm(Quantity ~ Price*Time, data=MyData)
summary(WTimeInteract)</pre>
```

```
##
## Call:
## lm(formula = Quantity ~ Price * Time, data = MyData)
##
## Residuals:
##
       Min
                  1Q
                      Median
                                    3Q
                                           Max
## -2.16963 -0.62535 0.09932 0.60599
                                       1.90283
##
## Coefficients:
##
                   Estimate Std. Error t value Pr(>|t|)
                               0.65321 171.101 < 2e-16 ***
## (Intercept)
                   111.76468
## Price
                   -2.00489
                               0.03478 -57.639 < 2e-16 ***
                               1.09391 -9.704 4.17e-08 ***
## TimeNight
                   -10.61482
## Price:TimeNight -0.02768
                               0.05127 -0.540
                                                  0.597
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.161 on 16 degrees of freedom
## Multiple R-squared: 0.998, Adjusted R-squared: 0.9977
## F-statistic: 2703 on 3 and 16 DF, p-value: < 2.2e-16
```

You will need a lot of notes on this. Interpreting interactive effects is tricky.

#### Useful facts

## Residuals:

You can get elasticites directly out of a regression model if both quantity and price are logged. In every other case you have to calculate elasticities by hand and they are different at every point.

```
ElastForm <- lm(log(Quantity) ~ log(Price) + Time, data=MyData)
summary(ElastForm)

##
## Call:
## lm(formula = log(Quantity) ~ log(Price) + Time, data = MyData)
##</pre>
```

```
##
                       Median
                  1Q
## -0.25916 -0.15725 0.06056 0.14053 0.21103
##
## Coefficients:
##
               Estimate Std. Error t value Pr(>|t|)
                           0.13225 40.004 < 2e-16 ***
## (Intercept) 5.29041
## log(Price)
              -0.37942
                           0.04882
                                    -7.771 5.41e-07 ***
## TimeNight
               -0.20447
                           0.07643
                                    -2.675
                                               0.016 *
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 0.1662 on 17 degrees of freedom
## Multiple R-squared: 0.8278, Adjusted R-squared: 0.8075
## F-statistic: 40.86 on 2 and 17 DF, p-value: 3.208e-07
I did it, but is it right?
If only the right hand side variable is logged, you interpret the parameter as the effect of a percent change.
PercentForm <- lm(Quantity ~ log(Price) + Time, data=MyData)</pre>
summary(PercentForm)
##
## Call:
## lm(formula = Quantity ~ log(Price) + Time, data = MyData)
## Residuals:
##
       Min
                1Q
                    Median
                                30
                                       Max
## -12.332 -7.017
                     3.762
                             5.551
                                     9.574
##
## Coefficients:
##
               Estimate Std. Error t value Pr(>|t|)
                             6.172 23.247 2.53e-14 ***
## (Intercept)
              143.493
                -25.291
                             2.279 -11.098 3.29e-09 ***
## log(Price)
## TimeNight
                -13.239
                             3.567 -3.711 0.00173 **
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 7.755 on 17 degrees of freedom
## Multiple R-squared: 0.9067, Adjusted R-squared: 0.8957
## F-statistic: 82.61 on 2 and 17 DF, p-value: 1.753e-09
```

## Common statistical problems

This is the basics. You now know enough to be insanely overconfident. Don't destroy the world.

Beyond getting the specification wrong, which you usually support with several similar models that show similar results, there are common statistical problems that you learn about in econometrics

- Autocorrelation, which makes your regression look better than it actually is.
- Heteroskadasticity, which makes it worse.
- Endogenaity, which biases your parameters in unknown directions.
- Errors in variables (RHS), which bias parameters towards zero.
- ...

Don't get cocky.

What we are going to do next is read in some real data that looks like either supply or demand but will be neither. Why, because supply and demand determine price and quantity simultaneously, that means that something either price or quantity is endogenous. At this point, you are at about 1910 as far as standard econometric practice.

### Real Data

- I will walk you through a few steps on reading in the data. The biggest hurdle to doing stats on the data is reading it in. There is actually an R library for working with EIA data directly, EIAdata, but we will use the more general tools to read files.
- Download into R data on coal prices and quantities. Again, the assignment operator in R is the "<-" symbol.

```
Coal <- read.csv("https://www.eia.gov/totalenergy/data/browser/csv.cfm?tbl=T06.01")
```

There are many ways of loading data into R (http://www.r-tutor.com/r-introduction/data-frame/data-import). Some work some of the time. In most cases Comma Separated Values (CSV) is the safest format to work with.

• Take a look at the summary of the data

#### summary(Coal)

##	MSN	MMYYYY	Value	Э	Column_Order
##	CLEXPUS: 603	Min. :194913	Not Available:	244	Min. :1.00
##	CLIMPUS: 603	1st Qu.:198210	2 :	3	1st Qu.:2.75
##	CLLUPUS: 603	Median :199405	3 :	3	Median:4.50
##	CLNIPUS: 603	Mean :199348	-4657 :	2	Mean :4.50
##	CLPRPUS: 603	3rd Qu.:200513	114 :	2	3rd Qu.:6.25
##	CLSCPUS: 603	Max. :201707	129 :	2	Max. :8.00
##	(Other):1206		(Other) :	4568	
##		Desc	ription		Unit
##	Coal Consumpti	on	: 603 Thousand	d Shor	t Tons:4824
##	Coal Exports		: 603		
##	Coal Imports		: 603		
##	Coal Losses and Unaccounted for: 603				
##	Coal Net Impor	ts	: 603		
##	Coal Production	n	: 603		
##	(Other)		:1206		

You will notice that for some variables they give counts, e.g., MSN, and for others you get numerical summaries, e.g., Column\_Order. The difference has to do with the data types (http://www.statmethods.net/input/datatypes.html).

- Since we are trying to make a simple supply model, i.e., trying to explain coal production, lets select just the production part of the data set and also get only the annual production values. This is a little primer on changing data types and taking part of data.
- Load the dplyr package. This is the normal way to load libraries of functions that you need.

```
library(dplyr)
```

```
##
## Attaching package: 'dplyr'
```

```
## The following objects are masked from 'package:stats':
##
## filter, lag
## The following objects are masked from 'package:base':
##
## intersect, setdiff, setequal, union
```

• Select only the Coal Production Figures and save it as CoalProduction. There is a cheat sheet for dplyr built into R. Look under the help menu.

```
CoalProduction <- Coal %>% filter(MSN == "CLPRPUS")
```

• Note that you have monthly data but that the annual data is shown as month 13. Grab all of the observations with month not equal to 13.

```
library(stringr)

CoalProduction <- CoalProduction %>% filter(str_sub(as.character(YYYYMM),5) != "13")
```

• At this point you will have noticed that we don't need all the columns so lets keep just the ones we need and then give the two columns we saved new names.

```
CoalProduction <- CoalProduction %>% select(YYYYMM, Value)

names(CoalProduction) <- c("RawMonth", "ProductionKShortTon")
summary(CoalProduction)</pre>
```

```
##
       RawMonth
                      ProductionKShortTon
##
    Min.
           :197301
                      100000.27 : 1
##
    1st Qu.:198403
                      100427.638:
   Median:199504
                      100627.455:
##
  Mean
           :199486
                      100653.897:
##
    3rd Qu.:200606
                      101144.833:
##
   Max.
           :201707
                      101490.13:
##
                      (Other)
                                :529
```

summary(CoalProduction)

• Notice that the ProductionKShortTon variable shows a count rather than a numerical summary. This means that R thinks it is a factor rather than a number. Lets fix that. It requires to first convert the factor, which is an integer, to the real value as a character and then convert that to numeric.

```
CoalProduction$ProductionKShortTon <- as.numeric(as.character(CoalProduction$ProductionKShortTon))</pre>
```

Play around with this doing one function at a time to see what each does and what each does alone.

• Next lets create a column for the year and make it a numeric value.

```
CoalProduction <- CoalProduction %>% mutate(Year = as.numeric(str_sub(as.character(RawMonth),0,4))) %
```

```
##
       RawMonth
                      ProductionKShortTon
                                                               Month
                                                Year
##
   Min.
           :197301
                             : 23664
                                           Min.
                                                  :1973
                                                           Min.
                                                                  : 1.000
   1st Qu.:198403
                      1st Qu.: 69716
                                                           1st Qu.: 3.000
##
                                           1st Qu.:1984
## Median :199504
                      Median: 82470
                                           Median:1995
                                                           Median : 6.000
                             : 79110
                                                                  : 6.467
## Mean
           :199486
                      Mean
                                           Mean
                                                   :1995
                                                           Mean
    3rd Qu.:200606
                      3rd Qu.: 90540
                                           3rd Qu.:2006
                                                           3rd Qu.: 9.000
## Max.
           :201707
                             :104390
                                           Max.
                                                  :2017
                                                           Max.
                                                                  :12.000
                      Max.
```

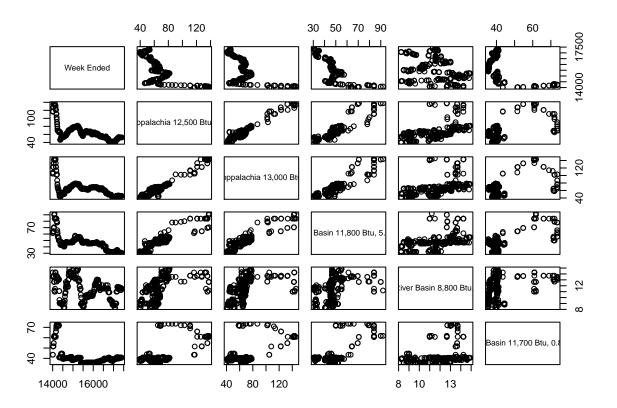
• Now grab some price data. We are going to use Quandl (https://www.quandl.com/), which has a bunch of data on energy and a lot of other things (https://www.quandl.com/collections/markets/coal). Make sure you have the Quandl library installed. If you don't install.packages("Quandl") should do it.

#### library(Quand1)

• Lets make a picture

```
## Loading required package: xts
## Loading required package: zoo
##
## Attaching package: 'zoo'
## The following objects are masked from 'package:base':
##
##
       as.Date, as.Date.numeric
##
## Attaching package: 'xts'
## The following objects are masked from 'package:dplyr':
##
##
       first, last
Prices <- Quandl("EIA/COAL")</pre>
summary(Prices)
##
      Week Ended
                         Central Appalachia 12,500 Btu, 1.2 SO2
   Min.
           :2008-05-06
                         Min.
                                 : 39.55
   1st Qu.:2010-09-11
                         1st Qu.: 52.75
##
                         Median: 60.90
##
  Median :2012-12-25
                                 : 64.07
##
  Mean
           :2013-01-01
                         Mean
##
  3rd Qu.:2015-04-27
                         3rd Qu.: 69.30
## Max.
           :2017-08-25
                         Max.
                                 :137.50
##
   Northern Apppalachia 13,000 Btu, <3.0 SO2
##
  \mathtt{Min}.
           : 40.75
   1st Qu.: 51.90
##
##
    Median : 64.10
##
  Mean
           : 64.62
##
   3rd Qu.: 68.65
## Max.
           :145.50
    Illinois Basin 11,800 Btu, 5.0 SO2 Powder River Basin 8,800 Btu, 0.8 SO2
##
##
  Min.
           :30.70
                                        Min.
                                               : 8.25
   1st Qu.:40.45
                                        1st Qu.:10.04
##
  Median :46.00
                                        Median :11.55
## Mean
           :45.87
                                        Mean
                                               :11.38
##
  3rd Qu.:47.90
                                        3rd Qu.:12.90
## Max.
           :92.00
                                        Max.
                                                :14.90
## Uinta Basin 11,700 Btu, 0.8 S02
## Min.
           :35.50
##
  1st Qu.:36.59
## Median :39.45
## Mean
           :41.04
##
   3rd Qu.:40.55
## Max.
           :73.50
```

# library(ggplot2) plot(Prices)



- Story time on the picture. What do you see? What stories come to mind?
- As before we will convert the year to a numeric value.

#### library(lubridate)

```
##
## Attaching package: 'lubridate'
## The following object is masked from 'package:base':
##
## date
# Prices$Year <- as.numeric(str_sub(Prices$Year,0,4))
Prices <- Prices %>% mutate(Month = month(`Week Ended`) ) %>% mutate(Year = year (`Week Ended`))
```

• Now we are going to get monthly averages

```
Prices <- Prices %>% group_by(Month, Year) %>% summarise_all(mean)
```

For illustrative purposes lets let stick with one price and then merge the price and volume data.

```
Prices <- Prices %>% select(Year, Month, Price = `Central Appalachia 12,500 Btu, 1.2 SO2` )
```

• Merge the two data frames

```
summary(CoalProduction)
      RawMonth
                   ProductionKShortTon
##
                                          Year
                                                        Month
                   Min. : 23664
##
  Min. :197301
                                                   Min. : 1.000
                                 Min. :1973
  1st Qu.:198403
                   1st Qu.: 69716
                                      1st Qu.:1984
                                                    1st Qu.: 3.000
## Median :199504
                   Median : 82470
                                      Median:1995
                                                    Median : 6.000
## Mean :199486
                   Mean : 79110
                                      Mean :1995
                                                    Mean : 6.467
## 3rd Qu.:200606
                   3rd Qu.: 90540
                                      3rd Qu.:2006
                                                    3rd Qu.: 9.000
## Max.
         :201707
                   Max.
                         :104390
                                      Max. :2017
                                                    Max. :12.000
summary(Prices)
##
        Year
                     Month
                                   Price
## Min.
         :2008
                 Min. : 1.0
                              Min. : 40.50
## 1st Qu.:2010
                 1st Qu.: 4.0
                              1st Qu.: 52.81
                 Median: 6.5
                               Median : 61.37
## Median :2012
                 Mean : 6.5
                               Mean : 64.17
## Mean
        :2012
                 3rd Qu.: 9.0
## 3rd Qu.:2015
                               3rd Qu.: 69.33
## Max.
          :2017
                 Max. :12.0
                               Max. :137.50
CoalMarket <- inner_join(Prices, CoalProduction, by =c("Year", "Month"))</pre>
summary(CoalMarket)
        Year
                                                    RawMonth
                     Month
                                     Price
## Min.
        :2008
                 Min. : 1.000
                                 Min. : 40.50
                                                      :200805
                                                Min.
   1st Qu.:2010
                 1st Qu.: 4.000
                                 1st Qu.: 52.87
                                                 1st Qu.:201009
##
                                 Median : 61.50
## Median :2012
                Median : 6.000
                                                Median :201212
                 Mean : 6.486
## Mean :2012
                                 Mean : 64.28
                                                 Mean :201252
## 3rd Qu.:2015
                 3rd Qu.: 9.000
                                 3rd Qu.: 69.55
                                                 3rd Qu.:201504
                                 Max. :137.50 Max. :201707
## Max.
         :2017
                 Max.
                      :12.000
## ProductionKShortTon
## Min. : 48115
## 1st Qu.: 76995
## Median: 84568
## Mean : 82171
## 3rd Qu.: 90313
## Max. :104390
Lets look at the data.
```

pairs(~ Year+ Price + ProductionKShortTon, data = CoalMarket)

