

MATH40011 Calculus for JMC  
Revision Note

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# Chapter 1

## Differentiation

### 1.1 Basic Differentiation

#### 1.1.1 Parametric Graph Sketching

We are going to sketch a parametric curve  $x = f(t)$ ,  $y = g(t)$ .

Some basic rules:

- Try to find the function  $y = h(x)$  between  $y$  and  $x$   
and try to determine the symmetry of the curve:  
i.e. determine whether  $h(x) = h(-x)$   
(even function, curve is symmetrical about  $y$  axis)  
or  $h(x) + h(-x) = 0$   
(odd function, curve is symmetrical about the origin)  
or if we have  $y^2 = t(x) \implies y = \pm\sqrt{t(x)}$   
(not a function, but curve is symmetrical about the  $x$  axis  
since a value of  $x$  points to two values with opposite signs)
- Find the zero points of the curve:  
Calculate the value of  $t$  such that  $y = g(t) = 0$ , and calculate the corresponding value of  $x$  to get the zero point  $(x, 0)$

Some special rules for the parametric curves:

- Determine when the tangent line to the curve is *horizontal* and *vertical*:

*Horizontal:* find the value of  $t$  such that

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = 0$$

Hence, the tangent is horizontal if

$$\frac{dy}{dt} = g'(t) = 0 \text{ and } \frac{dx}{dt} = f'(t) \neq 0$$

*Vertical:* find the value of  $t$  such that

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \pm\infty$$

Hence, the tangent is horizontal if

$$\frac{dy}{dt} = g'(t) \neq 0 \text{ and } \frac{dx}{dt} = f'(t) = 0$$

- Determine at which points is the tangent line to the curve parallel to  $y = x$ :  
Since for the line  $y = x$ ,  $\frac{dy}{dx} = 1$ ,

Therefore, if the tangent line is parallel to  $y = x$ ,

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = 1$$

Hence, we have

$$g'(t) = \frac{dy}{dt} = \frac{dx}{dt} = f'(t) \text{ where } g'(t) = f'(t) \neq 0$$

- Find the limit of the gradient when  $t$  is close to 0:

$$\lim_{t \rightarrow 0^+} \frac{dy}{dx} = \lim_{t \rightarrow 0^+} \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \text{ and } \lim_{t \rightarrow 0^+} \frac{dy}{dx} = \lim_{t \rightarrow 0^+} \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

- Similarly, find the limit of the gradient when  $t$  tends to  $\pm\infty$ :

$$\lim_{t \rightarrow \pm\infty} \frac{dy}{dx} = \lim_{t \rightarrow \pm\infty} \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

In conclusion, we can combine the elements above and find the "direction" of  $t$  to sketch the parametric curve.

*Example: Sketch the curve*

$$\begin{cases} x = t^2 \\ y = t^3 - t \end{cases}$$

in the cartesian plane.

1. Firstly we know that  $y = t^3 - t = t(t^2 - 1) = t(x - 1)$ .  
And we squared y to get:

$$y^2 = t^2(x - 1)^2 = x(x - 1)^2$$

Therefore,

$$y = \pm \sqrt{x(x - 1)^2}$$

Clearly this curve is symmetrical about x axis. (symmetry ✓)

- 2.

## 1.2 Differentiability & Continuity of a function