What Is Dark Matter?

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1 Introduction

In this paper, I will be discussing the theoretical origin of Dark Matter, and demonstrating why we think the things that we think about it. More specifically, by defining why Galactic Rotation Curves, and Gravitational Lensing describe the Missing Mass Problem, whose solution is Dark Matter, I will bring about the conclusion that Dark Matter is necessary when considering structures - large and small - in the Universe. Similarly, I will introduce differing ideas about possible solutions to the Missing Mass Problem, leading to the conclusion that our Missing Mass is not an aberration of Newtonian Gravity being applied to situations beyond its domain, but rather that the Missing Mass is indeed Matter. Finally, I will discuss the leading ideas in Particle Physics which seek to describe and find suitable candidate particles, and then conclude with some closing thoughts.

2 Why Dark Matter?

2.1 Galactic Rotation Curves

One of the Big Ideas and reasons that we believe in the existence of Dark Matter is buried in the nature of Galactic Rotation Curves. We know from classical mechanics that the force of Gravity follows the inverse-square law, thus falling off at a rate proportional to $\frac{1}{r^2}$. This makes sense when we analyze small-scale systems - like our solar system. However, when we move to larger-scale systems - like the Milky-Way - or other Galaxies, we find that the orbital velocities of stars around the respective galactic centers do not make sense in the context of Newtonian Gravity. Let us dive into the particulars for a moment to see if we can make this make any sense at all.

2.1.1 Derivation

In order to understand why these Rotation Curves are able to tell us about the masses and mass distributions of Galaxies, let us take a look at the math.

Presuming that Newtonian Mechanics hold, we know that for a particle in circular orbit in a spherically symmetric mass distribution, the radial(centripetal) force F_r on that particle is equal to $\frac{m_p V_p^2}{R}$, where m_p, V_p, R are the mass of the particle, the velocity of the particle, and the Radius of the particle's orbit respectively. If we assume that gravitational force is the dominant force on that particle, ($F_g \approx F_{total}$), we get that

$$F_r = F_g \tag{1}$$

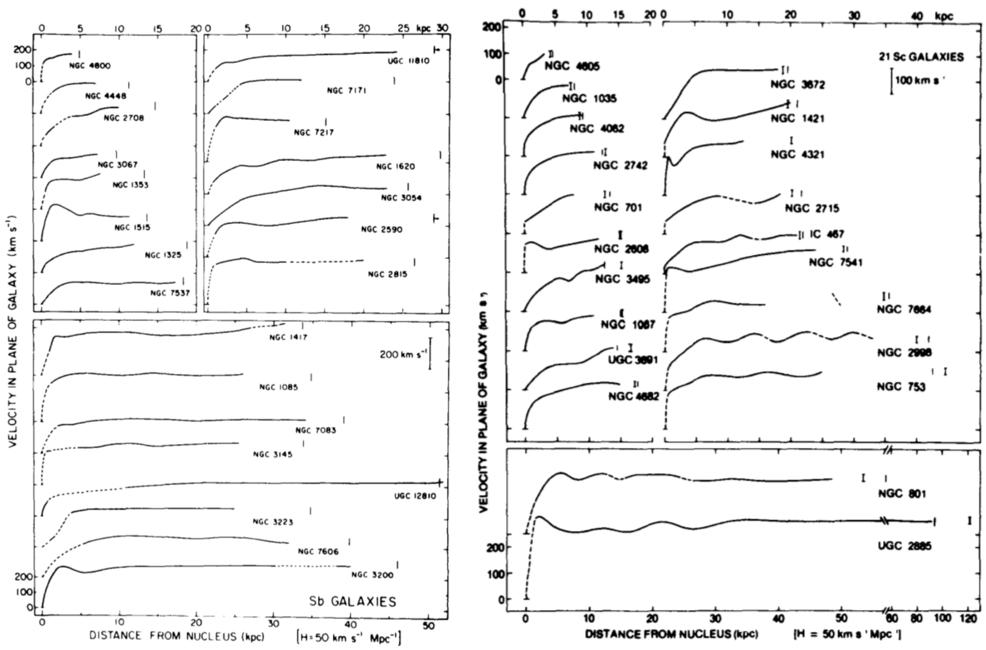

Figure 1: Rotation Curves (Rubin, 1983)

Expanding and using Newton's formulation of gravity gives us

$$\frac{m_p V_p^2}{R} = \frac{G M m_p}{R^2} \tag{2}$$

Where M is the mass enclosed within the orbit of our particle. Simplifying this expression and solving for the enclosed mass we get

$$M(< R) = \frac{R V_p^2}{G} \tag{3}$$

Which is useful because now we have enclosed mass as a function of the particle's velocity and its radius.
From this relation, we can easily see that if we take V_p to be a constant, we get that

$$M(< R) = R \cdot (constant) \tag{4}$$

Combining this with the observations in Figure 1, which show the dynamics of spiral galaxies with largely constant V_p's, we get that the Mass distributions of these galaxies must, by Newtonian Dynamics, increase linearly with R.

2.2 Gravitational Lensing

Another reason why we believe that Dark Matter is necessary for an accurate description of the universe is Gravitational Lensing. Similarly to rotation curves, Gravitational Lensing gives us the ability to assess the mass contained within a certain bound, but this time we are able to ascertain this information by viewing the effect of this phenomena which is bent light(which leads to the light of galaxies or other objects being presented to us as streaks, or sometimes magnified multiple different times like SN Zwicky and other supernovae).

2.3 Conclusion

We are able to accurately model the enclosed mass of a system by observing phenomena of both the lensing of light and the circular velocities of orbiting bodies. However, when we estimate the visible mass of these systems, we find that there is a significant lack of it, if we want to be able to accurately describe observations. For example, the visible mass in spiral galaxies decreases in density dramatically outside of the bulge, which contradicts our assertion that the enclosed mass(in order to sustain a constant circular velocity), must increase linearly with R. This is the epitome of the Missing Mass Problem, to which Dark Matter is the leading explanation.

3 What could Dark Matter be?

3.1 Modified Newtonian Dynamics(MOND)

Modified Newtonian Dynamics or MOND is an attempt to solve the Missing Mass Problem first published in 1983 by Israeli Physicist Mordehai Milgrom.
MOND proposes a solution to the Missing Mass Problem which does not include the inclusion of Dark Matter at all. MOND asserts that instead of introducing invisible, and altogether very hard to sense particles in order to explain the Missing Mass Problem, the issue could instead lie with the Physics out of which it was derived, namely, Newtonian Dynamics. This actually makes a lot of logical sense. This is because this assertion parallels General Relativity in the sense that General Relativity amends Newtonian Gravity in the limit of very large masses, MOND attempts to amend Newtonian Gravity in the limit of very small accelerations. In more math-y terms,

$$F = m_g a \rightarrow F = (m_g \mu(\frac{a}{a_0}))a \tag{5}$$

with the limits

$$\mu(x >> 1) \approx 1, \mu(x << 1) \approx x \tag{6}$$

This function μ, which modifies the force of gravity for accelerations $a <= a_0$, also serves another purpose. As long as we have $a <= a_0$, we can rewrite our equations from the Galactic Rotation Curves

$$\frac{m_p V_p^2}{R} = \frac{GMm_p}{R^{\alpha+1}} \tag{7}$$

Which gives us

$$M(< R) \approx R^{\alpha} \cdot (constant), a <= a_0 \tag{8}$$

$$M(< R) \approx (constant), a << a_0 \tag{9}$$

Where α is dependent on different μ functions. Some μ functions that have been used include

$$\mu(x) = \frac{x}{\sqrt{1+x^2}} \tag{10}$$

$$\mu(x) = \frac{x}{1+x} \tag{11}$$

To make things fit into perspective, $a_0 \approx 1.27^{-10} m/s^2$, which means that MOND does not take effect until we have very small accelerations. This means that MOND does not have an appreciable effect on Newtonian Gravity until we get to a regime of the gravitational force of attraction comparable to the gravity exerted between myself and my girlfriend, say, if we were about 48 meters apart. This being negligible compared to the tangible attraction that we feel(to the earth).

3.1.1 Successes

MOND has been found to accurately model spiral, and elliptical galaxy dynamics. Similarly, MOND was able to predict the behavior of Low Surface Brightness Galaxies before and better than models based on Dark Matter.

3.1.2 Failures

While MOND has some successes, its failures are more decisive. For example, its failures in accurately describing galaxy clusters mean that it still does not remove the need for a Dark Matter. Similarly, some iterations of MOND predict things wrong, like in 2017, when we measured the speed of gravitational waves to be equal to the speed of light, some MOND versions predicted that the speed of gravity would have been different.

3.2 Why is Dark Matter, Matter?

The idea that Dark Matter is Matter seems like circular logic, but saying that Dark Matter(i.e. the solution to the Missing Mass Problem) is Matter means that we are accepting the notion of mass that is completely invisible to us making up the vast majority of mass in the Universe. The common idiom *there's more here than meets the eye* is really true in the Universe.

3.2.1 Bullet Cluster

The most significant evidence that the solution to the Missing Mass Problem must be Dark Matter is the existence of phenomena like the Bullet Cluster. In the Bullet Cluster, there is a distinct separation between the visible matter and matter that is mapped via gravitational lensing. This means that the Bullet Cluster supremely points to the existence of Dark Matter because there is evidence of mass in places where no visible mass should be, thus eliminating attempts at explaining the result with kinds of Modified Gravity, and pointing the solution directly at a form of Dark **Matter**.

3.3 WIMPS

WIMPs, or Weakly Interacting Massive Particles, are the bucket nature for particle Dark Matter. Weakly Interacting(only interacting via the Weak force[and gravity]) Massive(having a somewhat significant mass) Particle(self-explanatory) is the explanation for the name WIMP. These categorizations are all necessary for particulate Dark Matter candidates. Amongst this category of WIMPs, there are many particles that *fit the bill*, or are suitable for consideration as potential resolutions to the Particle Physics question. These suitors to the title of Dark Matter hail from the Standard Model and Beyond, to fields like Quantum Chromodynamics. However, experiments in direct detection of WIMPs have come back empty-handed, succeeding at making increasingly smaller upper bounds on these WIMPs.

3.3.1 Structure Formation and Galaxies

Because of its cohesive gravitational effects, Dark Matter is integral to understanding many structures in the Universe. The most obvious of these is the Galaxy, of which there exist many types, but in most, there exists the need for Dark Matter in order for them to not tear themselves apart. In fact, there exists a dual sort of highway between Structures in the Universe and the Dark Matter that exists within them. Since we know that since Dark Matter is integral to the formation of many of these structures, the nature and pattern of these structures(which we are able to observe) are able to tell us something about their creators, i.e. their Dark Matter. Different hypotheses about Dark Matter can be differentiated by this method. For example, the differences between Cold, Warm/Fuzzy, and Hot Dark Matter are characterized by the velocities of the average Dark Matter particle, with Cold being the Slowest up to Hot which reaches near relativistic speeds. These different kinds of Dark Matter actually lead to different paradigms of structure formation throughout the universe, with Cold on up to Hot having increasingly less defined, and more scattered structure.

4 Conclusion

The search for Dark Matter, which began as the Missing Mass Problem, has become a conjoined effort between Astro-Physics and Particle Physics in order to better understand the minutiae and macroscopic functions, behaviors, and structures of the Universe. It is regrettable that there has not yet been any experimental verification of the potential particle(s) responsible, but it is a worthwhile project nonetheless.

5 References

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