

$$11. y = \frac{1}{\sqrt{3x-2}}.$$

$$12. y = \frac{7}{\sqrt{5-2x^2}}.$$

$$13. y = \sqrt{(x^2+4)}.$$

$$14. y = \sqrt{(x+1)}.$$

$$15. y = \frac{7}{\sqrt[3]{x^2+1}}.$$

$$16. y = \frac{5}{(x^2-1)^2}.$$

$$17. y = \frac{2}{(x^2+4)^2}.$$

$$18. y = \frac{3}{(5-x^3)^2}.$$

**33. The Derivative of  $u^n$ ,  $n$  a Positive Fraction.** We are now in a position to prove that the rule for the derivative of  $u^n$  holds when  $n$  is a positive fraction of the form  $\frac{p}{q}$ , where  $p$  and  $q$  are integers. Let

$$y = u^{\frac{p}{q}}.$$

Raise each member to the power  $q$ :

$$y^q = u^p.$$

Since  $u$  is a function of  $x$ ,  $y$  is a function of  $x$ . Hence each member is a function of  $x$  raised to a positive integral power. Then each member can be differentiated by the rule of §32 which was proved for positive integral exponents. We find

$$qy^{q-1} \frac{dy}{dx} = pu^{p-1} \frac{du}{dx}.$$

From which

$$\frac{dy}{dx} = \frac{p}{q} \frac{u^{p-1}}{y^{q-1}} \frac{du}{dx}.$$

Substitute for  $y$  in the second member and obtain

$$\frac{dy}{dx} = \frac{p}{q} \frac{u^{p-1}}{\left[u^{\frac{p}{q}}\right]^{q-1}} \frac{du}{dx} = \frac{p}{q} \frac{u^{p-1}}{u^{p-\frac{p}{q}}} \frac{du}{dx}.$$

Then

$$\frac{dy}{dx} = \frac{p}{q} u^{\frac{p}{q}-1} \frac{du}{dx},$$