Speaker: Frankie Higgs, University of Bath

Title: Connecting two points with random blobs

Abstract: The *random blob* model, or the Boolean model, places balls of radius r centred at the points of a homogeneous Poisson point process $\mathcal{P}$ in a set A with an intensity μ. It is closely related to the random geometric graph, in which two points of $\mathcal{P}$ are joined with an edge if the distance between them is less than $2r$. If the relationship between the number of balls and the radii is such that the average degree of a point, which is proportional to μr^d, converges to a constant then the model can be thought of as a form of percolation.

Indeed, if $A = \mathbb{R}^d$ or a half-space and $r = 1$, the random blob model is often called *continuum percolation*, parameterised by the intensity. Many well-known properties of discrete percolation are known to also hold for continuum percolation, such as existence of a non-trivial critical intensity and the existence of a unique unbounded component.

We will discuss recent work on the two-point connection function for the Boolean model. If A is a bounded subset of $\mathbb{R}^d$ with a sufficiently smooth boundary, then given distinct $x, y \in \partial A$, we prove a limit theorem for the probability that x and y are joined by a path in the Boolean model. In particular, if $\theta_A(\lambda)$ is the probability that the origin is contained in an unbounded component in continuum percolation with intensity λ in A, then we show that if $\lambda r^d \to \mu$ as $\lambda \to \infty$, then the probability that x and y lie in the same component converges to $\theta_{\mathbb{H}}(\lambda)^2$.

Our proof involves a renormalisation argument relating certain events for the Boolean model in A explicitly to corresponding events for continuum percolation in $\mathbb{H}$.

Based on joint work with Mathew Penrose.