

Project outline:

- **Title:** Arbitrage-free Parametrization of Vol Surface
- **[Week 1] Options data:**
 - Gather historical weekly options data over 2010-2021 of indexes (SPX/NK); perform dataset cleaning/standardization

[1] and [2] focus on calibration to a **single-date** market snapshot (e.g. 20210617, a day before triple-witch) to conclude goodness of fits (ATM vol/skew/L2); [3] calibrates to **historical** snapshots to conclude params stability

[1] Explicit Parametrization

- **[Week 2] SVI:** simple/surface SVI
 - Overview of arbitrage constraints (butterfly/calendar/Lee bounds)
 - Overview of SVI: (1) large-time asymptotics of Heston (2) not arb-free -- can give negative density
 - Calibration routine: choice of loss functions (price/vol), weights (bid-ask/vega), arbitrage penalty (Gatheral "crossedness" approach)
 - Arb-free time-interpolation with monotonic spline in variance space
 - Overview of surface SVI: choice of arb-free skew kernel
 - Brief discussion of extended SVI (theory only): choice of arb-free corr kernel
 - Calibrate simple/surface SVI to **2021 SPX**
- **[Week 3] Ensemble Carr-Pelts:**
 - Introduce CP as arb-free extension of BS-formula
 - Overview of ensemble CP: interpretation of each CP surface
 - Calibrate ensemble CP to **2021 SPX**

[2] Parsimonious Model

Calibration framework: **Carr-Madan's FFT/Fang's COS** pricing formulas for **closed-form** characteristic functions; fit to **implied vols**; choice of **loss** function (bid-ask/vega/Gaussian-weighted); Black-Scholes vol/skew term-structure from **Lewis's formula** (all implemented...)

- **[Week 4] Heston-type:**
 - Formulate Heston under affine forward variance framework (Riccati eq.) and derive its characteristic function/ATM vol & skew
 - Visualize typical surface and effects of model params on skew
 - Exponential skew decay: small-time $\propto \sqrt{t}$; large-time $1/t$ (inconsistent with observations!)
 - Calibrate to **2021 SPX** and demonstrate deficiency fitting short-skew (too flat)
- **[Week 5] Jump-type: Merton/SVJ**
 - Introduce Merton and derive its characteristic function via Levy's thm
 - Visualize typical surface and effects of model params on skew
 - Long-skew flattening & moment decay
 - SVJ: decoupled Heston/Merton skew (skew adding)
 - Brief discussion of SVJJ (theory only)
 - Calibrate M/SVJ to **2021 SPX**
- **[Week 6] Levy-type: CGMY/BG/CGMY-SA**
 - Levy process: characteristic function; activity etc.
 - VG as time-changed Brownian (trading-time interpretation)
 - Jump measure: CGMY/BG (infinite activity)
 - Stochastic arrivals: CGMY-SA (random clock)
 - Visualize typical surface and effects of model params on skew
 - Calibrate CGMY/BG/CGMY-SA to **2021 SPX**

Calibration approach: (1) **slice-wise** & impose calendar-arb penalty (2) **whole surface**. (1) applies to Heston/Merton/CGMY/BG; (2) applies to SVJ/CGMY-SA

[3] Comparative Study

- **[Week 7] Comparison:** params interpretation & stability
 - Summarize typical surfaces and params interpretations
 - Discuss ease of enforcing no-arbitrage: (1) require explicit penalty in SVI/slice-models (2) automatic in ECP/surface-models
 - Params stability: fit to **historical** data (2010-2021)
 - Goodness of fits in crisis regime (e.g. 20200323, a day before rebound)
 - Speed considerations: which models usable in real-time fitting?
- **[Week 8] Discussion:** parametrized then what? (theory only)
 - Local vol: numerical computation or closed-form?
 - LSV calibration for exotics pricing
 - Breakdown/summary of surface into risk factors
 - Surface dynamics: skew-stickiness ratio etc.
 - Market-making off inter/extrapolated vols

- [Week 9&10] Report/presentation etc.