# Effects of the Distribution of Synaptic Strength on the Spiking Dynamics in a Network of Spiking Neurons

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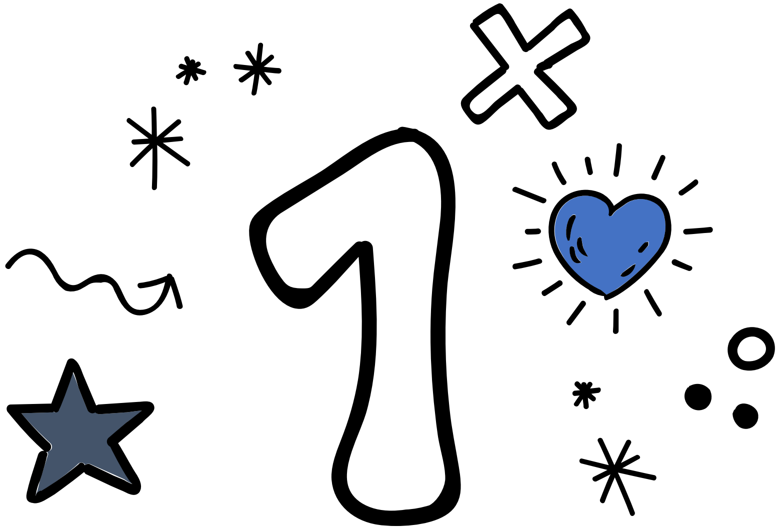


# What is a Network?

## *The Basics*

***Components of a Network***

***Measures of a Network***

***Dynamics of a Network***

***Components of a Network***

***Measures of a Network***

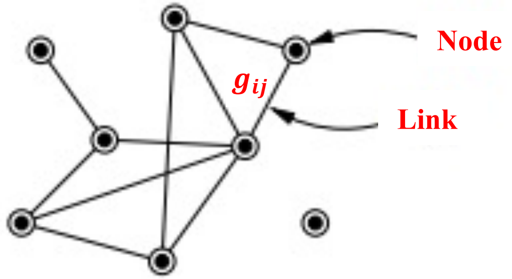
***Dynamics of a Network***

# 1. What is a Network?

- A network models a system with interacting individual components, and consists of **nodes**, **links** and **coupling strengths**

| | Representation | Notation |
|---|---|---|
| **Node** | Individual component<br>*e.g.* Neuron | $\boldsymbol{i}$, where $1 \le i \le N$ with $N$ = total no. of nodes |
| **Directed Link** | Mutual interaction<br>*e.g.* Linkage btw. neurons | $\boldsymbol{A_{ij}} = 1$ if node $j$ links to $i$ and 0 otherwise |
| **Coupling Strength** | Strength of interaction<br>*e.g.* Synaptic weight | $\boldsymbol{g_{ij}} \neq 0$ if node $j$ links to $i$ and 0 otherwise |

# 1. What is a Network?




- Coupling strength matrix **G** contains all $g_{ij}$
- **G** is the most crucial piece of information about a network

  ∵ it *fully* specifies the network structure, *i.e.*, **connectivity** & **interaction**

| Features of G | |
|---|---|
| **0-Diagonal** | Nodes not self-connecting ⇒ Diagonal entries ≡ 0 and $N(N-1)$ possible directed links |
| **Sparsity** | Ratio btw. no. of directed links & $N(N-1)$<br>*Note*: Most real networks are sparse |
| **Non-Symmetry** | Directed network ⇒ $g_{ij} \neq g_{ji}$ in general |

- **G** has high theoretical importance but, in practice, is difficult to extract

  ⇒ **Requires network reconstruction techniques**

*Components of a Network*

***Measures of a Network***

*Dynamics of a Network*

# 1. What is a Network?

- Network measures *summarize* the network structure
- This project focuses on **degree** and **strength**

| Network Measure | Mathematical | Computational |
|---|---|---|
| **In/Out-Degree** | $k_{\text{in}}(i) = \sum_j \mathbb{I}(g_{ij} \neq 0)$ | Count the non-zero entries in the $i$th *row* of **G** |
| | $k_{\text{out}}(i) = \sum_j \mathbb{I}(g_{ji} \neq 0)$ | Count the non-zero entries in the $i$th *column* of **G** |
| **In/Out-Strength** | $s_{\text{in}}(i) = \sum_j g_{ij} / k_{\text{in}}(i)$ | Average the non-zero entries in the $i$th *row* of **G** |
| | $s_{\text{out}}(i) = \sum_j g_{ji} / k_{\text{out}}(i)$ | Average the non-zero entries in the $i$th *column* of **G** |

- Other finer measures: $k_{\text{in}}^{+}(i) = \sum_j \mathbb{I}(g_{ij} > 0)$ and $s_{\text{in}}^{+}(i) = \sum_j g_{ij} \mathbb{I}(g_{ij} > 0) / k_{\text{in}}^{+}(i)$

  ⇒ **Constructed by replacing the argument in the indicator function with the desired condition**

*Components of a Network*

*Measures of a Network*

***Dynamics of a Network***

# 1. What is a Network?

- Network **dynamics** associates with network **structure** through a set of $N$ dynamical equations, with the most *generic* form

$$\frac{\mathrm{d}x_i}{\mathrm{d}t} = F(\{x_i(t)\}_{i=1:N}, \mathbf{G}, \eta_i, \dots),$$

where $x_i$ is the state of node $i$, and

$\eta_i$ is the noise term with covariance matrix **D**

  - ➢ Numerically solved given suitable initial conditions $\{x_i(0)\}_{i=1:N}$
  - ➢ **Specifying a Network Model.** We require $\underline{F, \mathbf{G}, \mathbf{D}}$
- Time series $\{x_i(t)\}_{i=1:N}$ form the **network dynamics**


# Long-tailed Neuronal Spikes

## *The Motivation*

# 2. Long-tailed Neuronal Spikes

- A network reconstruction method is applied to the *empirical neuronal time series* of cultures of rat embryonic cortices (25 days in vitro) to estimate the *coupling strengths* $g_{ij}$, leading to the following dataset

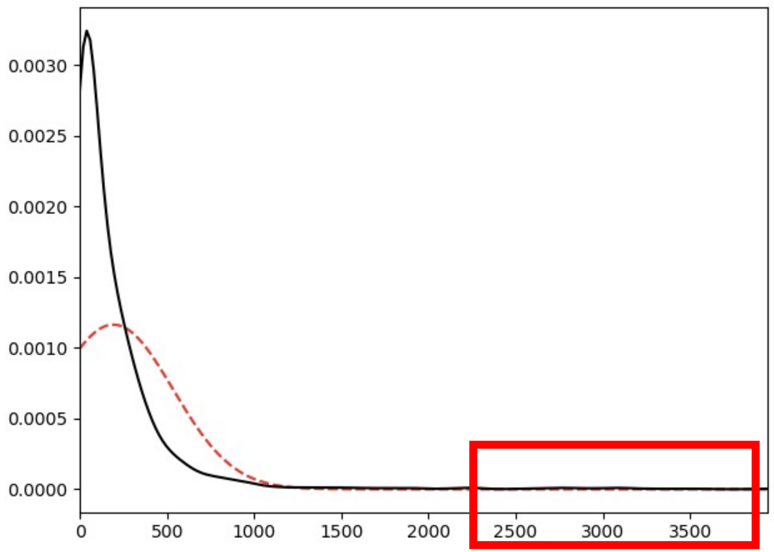
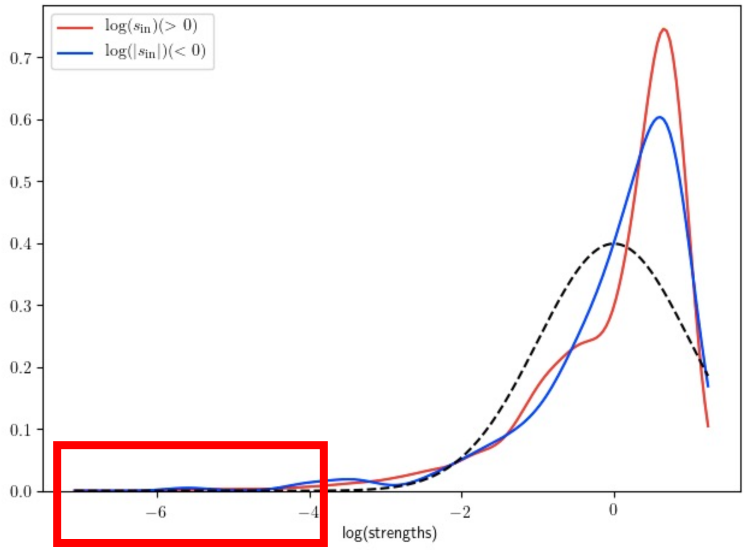
| Node $i$ | Node $j$ | $g_{ij}$ |
|---|---|---|
| 1 | 196 | 0.0208720006 |
| 1 | 266 | 0.0156720001 |
| 1 | 267 | 0.0218959991 |
| $\vdots$ | $\vdots$ | $\vdots$ |
| 2 | 1 | −0.0234200004 |
| 2 | 21 | −0.00388139999 |
| 2 | 23 | −0.00472760014 |
| $\vdots$ | $\vdots$ | $\vdots$ |
| 4095 | 4094 | 0.0089673996 |

- ➢ 4095 electrodes for data collection ⇒ 4095 nodes in the giant network
- ➢ (Order) $g_{ij} \sim 10^{-3}$ to $10^{-2}$
- ➢ (Sign) $g_{ij}$ can be +ve or –ve
- ➢ Sparse network with 1.4% sparsity
- ➢ Forms the foundation of *all* later simulations

# 2. Long-tailed Neuronal Spikes

- Experimental neuronal *spike counts* are **highly skewed** and **long-tailed** in distribution, and so are the *synaptic strengths*

***Can the very large spike counts be explained by the very large synaptic strengths?***

| Spike Distribution | Strength Distribution |
| --- | --- |
|  |  |





**Any causal relation?**

# <u>Synaptic Spiking Model</u>

## *Model Implementation*

**_Model Formulation_**

**_Simulation Results_**

***Model Formulation***

***Simulation Results***

# 3. Synaptic Spiking Model

- Every neuron is described by

  (i) **membrane potential** $\boldsymbol{v}(t)$, and

  (ii) **recovery variable** $\boldsymbol{u}(t)$, following

$$\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = 0.04v^2 + v + 140 - u + I(t)$$

$$\frac{\mathrm{d}\boldsymbol{u}}{\mathrm{d}t} = a(bv - u),$$

  where $I(t)$ is the **synaptic current** term,

  with a ***fire and reset*** rule:

  When $v$ assumes $v_{\text{peak}} = 30\text{eV}$,

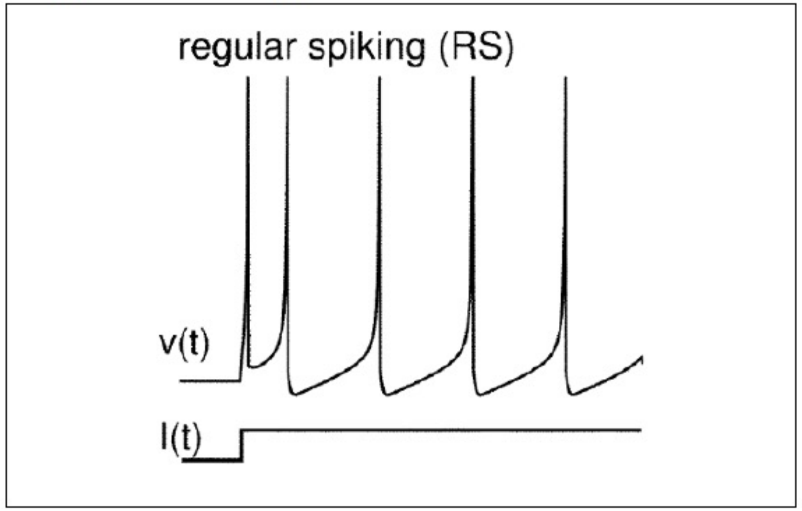
  $v(t) \leftarrow c$ and $u(t) \leftarrow u(t) + d$

# 3. Synaptic Spiking Model

$$\begin{cases} \dfrac{\mathrm{d}v}{\mathrm{d}t} = 0.04v^2 + v + 140 - u + I(t) \\ \dfrac{\mathrm{d}u}{\mathrm{d}t} = a(bv - u), \\ v(t) \leftarrow c \text{ and } u(t) \leftarrow u(t) + d \end{cases}$$

- Parameters $a, b, c, d$ are specific to a neuron type (require calibration)
- *e.g.* Regular spiking neuron: $a = 0.02, b = 0.2, c = -65, d = 8$

Typical Voltage Series of an RS Neuron



# 3. Synaptic Spiking Model

$$\begin{cases} \frac{\mathrm{d}v}{\mathrm{d}t} = 0.04v^2 + v + 140 - u + I(t) \\ \frac{\mathrm{d}u}{\mathrm{d}t} = a(bv - u), \\ v(t) \leftarrow c \text{ and } u(t) \leftarrow u(t) + d \end{cases}$$

- For neuron $i$,
  **conductance** is incorporated in $I_i(t)$:

$$I_i(t) = \boldsymbol{G}^{\mathbf{exc}}(t)\big(V_E - v_i(t)\big) + \boldsymbol{G}^{\mathbf{inh}}(t)\big(V_I - v_i(t)\big) + \eta_i$$

- Time evolution of $\boldsymbol{G}^{\mathbf{exc/inh}}$ is analytically given by:

$$\boldsymbol{G}_{\boldsymbol{i}}^{\mathbf{exc}} = \beta \sum_{\{j:g_{ij}>0\}} \left(g_{ij} \sum_k \exp\left(-\frac{t-t_{j,k}}{\tau_{\mathrm{exc}}}\right) \Theta\big(t - t_{j,k}\big)\right)$$

$$\boldsymbol{G}_{\boldsymbol{i}}^{\mathbf{inh}} = \beta \sum_{\{j:g_{ij}<0\}} \left(g_{ij} \sum_k \exp\left(-\frac{t-t_{j,k}}{\tau_{\mathrm{inh}}}\right) \Theta\big(t - t_{j,k}\big)\right)$$

- where $V_E, V_I, \tau_{\mathrm{exc}}, \tau_{\mathrm{inh}}, \beta$ are parameters
- Conductance $\boldsymbol{G}^{\mathbf{exc/inh}}$ summarize the *historical* spikes of the *connecting* exc./inh. neurons with impacts scaled by *coupling strength* $g_{ij}$
- In turn, conductance affects the current *I* neuron *i* experiences

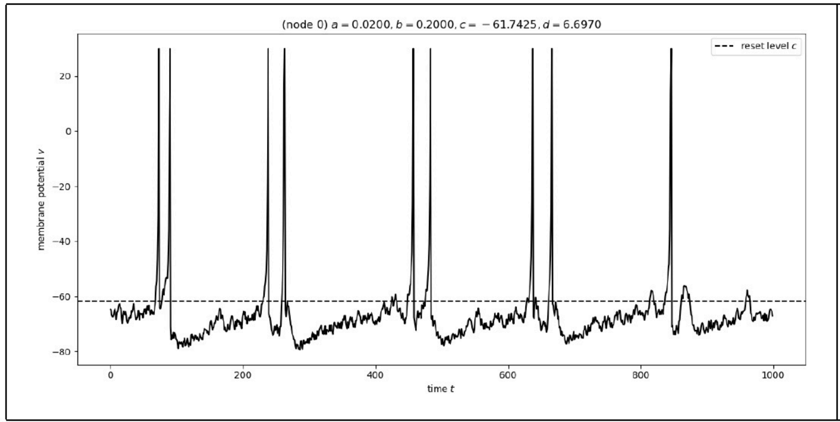
***Model Formulation***

***Simulation Results***

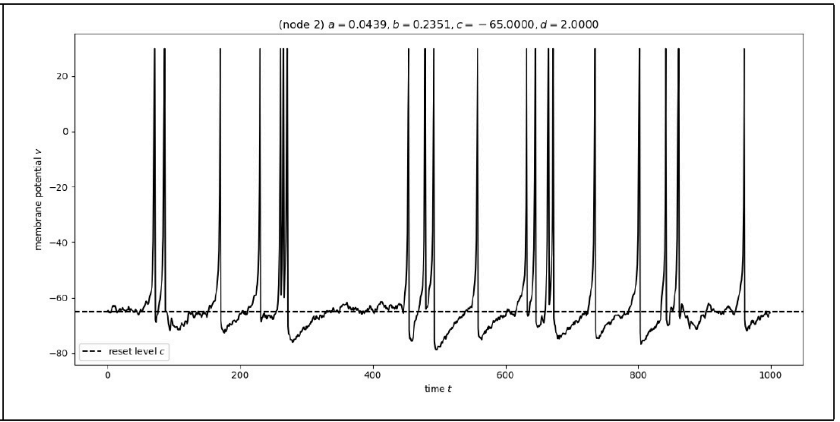
# 3. Synaptic Spiking Model

- We drive the dynamics using uniformly distributed $\eta_i \sim \mathcal{U}[0, \delta]$
- Within the model, there are two neuron types: *Regular* & *Fast* Spiking

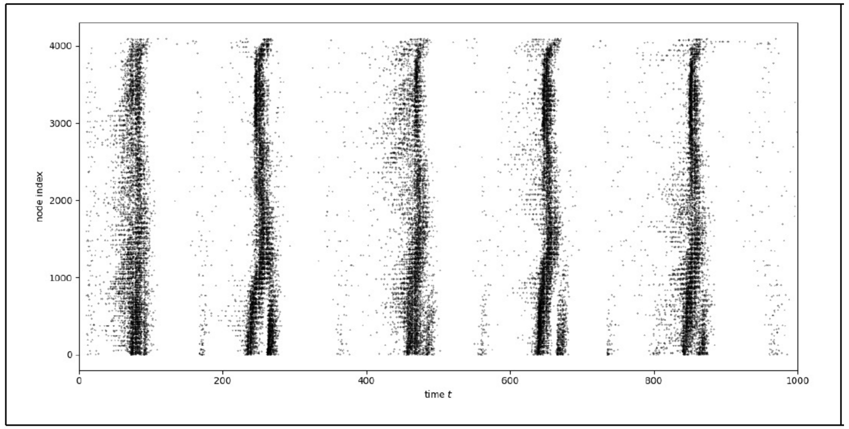
(Node 0) RS-Type Neuron




(Node 2) FS-Type Neuron



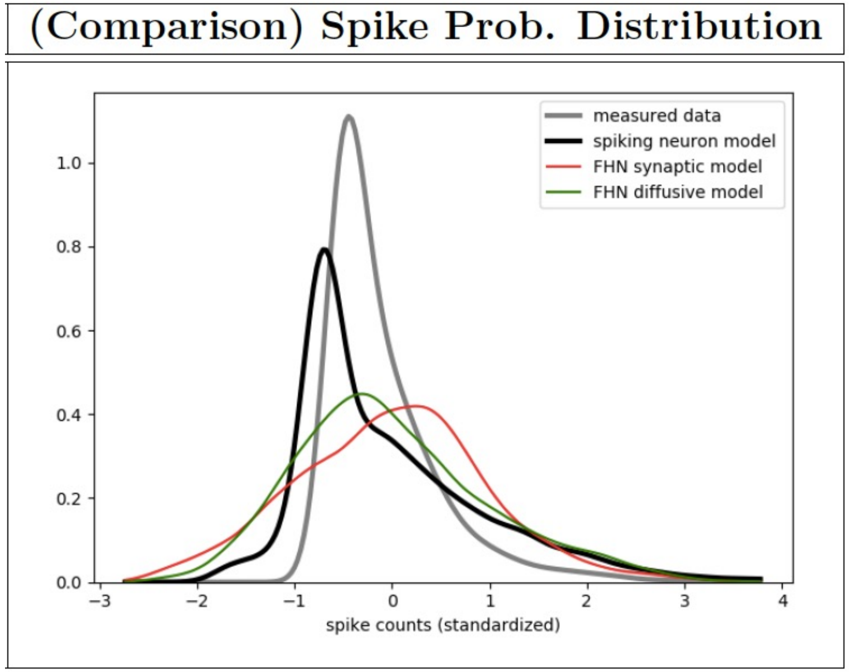

Model Raster Plot




- Large-scale periodic, simultaneous spiking
- Sub-period spiking
- Non-uniform spike train

# 3. Synaptic Spiking Model

- The model succeeds in producing a *long right tail*, with spike counts spreading over a wide range [0,409]
- Synaptic spiking model exhibits more realistic dynamics than previous models
- Spike count distribution of different models:




# What Explain the Spikes?

***Analysis & Discussion***

# 4. What Explain the Spikes?

- **Reference networks** *vary* network features of interest while *preserving* the rest, and are constructed through artificial manipulation of the reconstructed **G**
- Effects of the varied features on the dynamics are studied
- Main results from **reference network analysis** – *causal* relation
  - ➢ The reference network that has *rows of* **G** *shuffled*, thus (i) keeping the *long-tailed distribution of* $k_{\mathrm{in}}, s_{\mathrm{in}}$ and (ii) making *distribution of* $k_{\mathrm{out}}, s_{\mathrm{out}}$ *bell-shaped*, preserves the distribution of spike counts
  - ➢ Other reference networks with *bell-shaped* $k_{\mathrm{in}}, s_{\mathrm{in}}$ or *long-tailed* $k_{\mathrm{in}}$ distort the spike count distribution
  - ➢ Only $\boldsymbol{s_{\mathrm{in}}}$ remains as the important driving factor of the spiking dynamics
  - ➢ **Conclusion.** Long-tailed $s_{\mathrm{in}}$ leads to long-tailed spike counts

# Conclusion

- The synaptic spiking model successfully generates realistic neuronal spikes with a long tail, with a better performance than previous models
- **Reference network analysis.** The long-tailed incoming synaptic strength $s_{\text{in}}$ leads to the long-tailed spike counts
- This echoes with the conclusion in my FYP part I

***Can the very large spike counts be explained by the very large synaptic strengths?***

***- Yes, in the context of the synaptic spiking model.***

***In particular, it's the very large incoming synaptic strengths***
***that have an important effect on spike counts.***

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