Effects of the Distribution of Synaptic Strength of Neuronal Network on its Spiking Dynamics

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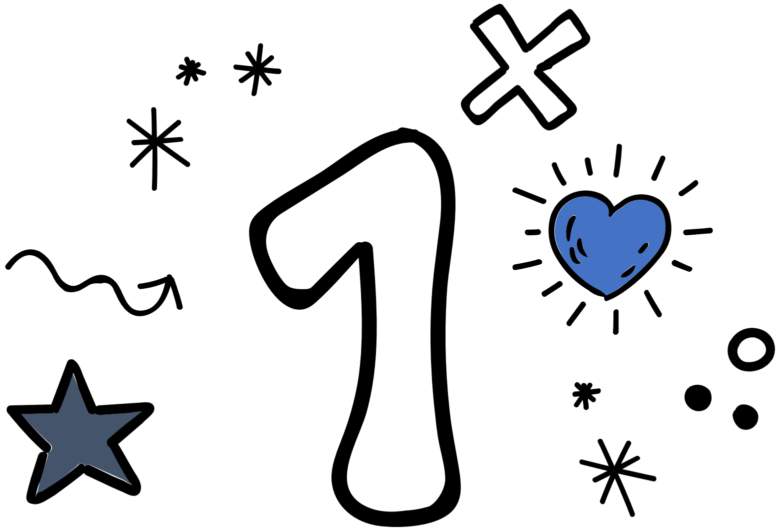


What is a Network?

The Basics

Components of a Network

Measures of a Network

Dynamics of a Network

Components of a Network

Measures of a Network

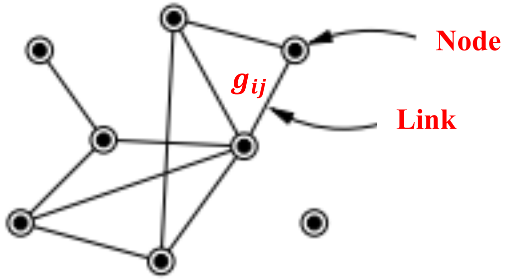
Dynamics of a Network

1. What is a Network?

- A network models a system with interacting individual components, and consists of **nodes**, **links** and **coupling strengths**

	Representation	**Notation**
Node	Individual component *e.g.* Neuron	$\boldsymbol{i}$, where $1 \leq i \leq N$ with N = total no. of nodes
Directed Link	Mutual interaction *e.g.* Linkage btw. neurons	$\boldsymbol{A_{ij}} = 1$ if node j links to i and 0 otherwise
Coupling Strength	Strength of interaction *e.g.* Synaptic weight	$\boldsymbol{g_{ij}} \neq 0$ if node j links to i and 0 otherwise

1. What is a Network?



- Coupling strength matrix **G** contains all g_{ij}
- **G** is the most crucial piece of information about a network

  ∵ it *fully* specifies the network structure, *i.e.*, **connectivity** & **interaction**

Features of G	
0-Diagonal	Nodes not self-connecting ⇒ Diagonal entries ≡ 0 and $N(N-1)$ possible directed links
Sparsity	Ratio btw. no. of directed links & $N(N-1)$ *Note*: Most real networks are sparse
Non-Symmetry	Directed network ⇒ $g_{ij} \neq g_{ji}$ in general

- **G** has high theoretical importance but, in practice, is difficult to extract

  ⇒ **Requires network reconstruction techniques**

Components of a Network

Measures of a Network

Dynamics of a Network

1. What is a Network?

- Network measures *summarize* the network structure
- This project focuses on **degree** and **strength**

Network Measure	Mathematical	Computational
In/Out-Degree	$k_{\text{in}}(i) = \sum_j \mathbb{I}(g_{ij} \neq 0)$	Count the non-zero entries in the ith *row* of **G**
	$k_{\text{out}}(i) = \sum_j \mathbb{I}(g_{ji} \neq 0)$	Count the non-zero entries in the ith *column* of **G**
In/Out-Strength	$s_{\text{in}}(i) = \sum_j g_{ij} / k_{\text{in}}(i)$	Average the non-zero entries in the ith *row* of **G**
	$s_{\text{out}}(i) = \sum_j g_{ji} / k_{\text{out}}(i)$	Average the non-zero entries in the ith *column* of **G**

- Other finer measures: $k_{\text{in}}^{+}(i) = \sum_j \mathbb{I}(g_{ij} > 0)$ and $s_{\text{in}}^{+}(i) = \sum_j g_{ij}\mathbb{I}(g_{ij} > 0) / k_{\text{in}}^{+}(i)$

  $\Rightarrow$ **Constructed by replacing the argument in the indicator function with the desired condition**

Components of a Network

Measures of a Network

Dynamics of a Network

1. What is a Network?

- Network **dynamics** associates with network **structure** through a set of N dynamical equations, with each consisting of
  - ➢ intrinsic dynamics $f_i(x)$
  - ➢ nodal interaction $h(x, y)$
  - ➢ noise η_i with noise covariance matrix **D**
- The time-evolution is governed by $\frac{\mathrm{d}x_i}{\mathrm{d}t} = f_i(x_i) + \sum_j g_{ij} h(x_i, x_j) + \eta_i$, where x_i is the state of node i
  - ➢ Numerically solved given suitable initial conditions $\{x_i(0)\}_{i=1:N}$
  - ➢ **Specifying a Network Model.** We require <u>$\mathbf{G}, f_i(x), h(x,y), \mathbf{D}$</u>
- Time series $\{x_i(t)\}_{i=1:N}$ form the **network dynamics**

Network Reconstruction

A Literature Review

The Core Idea & its Application

Motivations of the Project - Neuronal Spikes

The Core Idea & its Application

Motivations of the Project - Neuronal Spikes

2. Network Reconstruction

- A network reconstruction method proposed by Ching and Tam
  - Reconstructs links & coupling strengths *using empirical time series data*
  - Targets systems that have stationary fluctuation around (const.) **noise-free steady states** X_i
- The dynamics is *linearized* around X_i to give $\frac{\mathrm{d}}{\mathrm{d}t}\delta\boldsymbol{x} = \mathbf{Q}\delta\boldsymbol{x} + \boldsymbol{\eta}$, where $\delta\boldsymbol{x}_i = x_i - X_i$ and $\boldsymbol{\eta}_i = \eta_i$. Non-diagonal entries of $\mathbf{Q}$ is shown to be

$$Q_{ij} = g_{ij}\frac{\partial h}{\partial y}\bigg|_{(X_i,X_j)}$$

- Define the **lagged covariance matrix**, computed entirely based on *time series data*

$$\mathbf{K}_\tau \equiv \langle[\boldsymbol{x}(t+\tau) - \langle\boldsymbol{x}(t+\tau)\rangle][\boldsymbol{x}(t) - \langle\boldsymbol{x}(t)\rangle]^T\rangle$$

2. Network Reconstruction

- The following relation is shown to hold

$$\mathbf{Q} \approx \mathbf{M} \equiv \frac{1}{\tau}\log(\mathbf{K}_\tau \mathbf{K}_0^{-1})$$

  - **Q** reflects the *underlying* model-based network structure
  - **M** is computed from the *empirical time series data* of the nodes
- Recall $Q_{ij} = g_{ij}\frac{\partial h}{\partial y}(X_i, X_j) \sim g_{ij}$. Divide M_{ij} into two groups:
  - $M_{ij} \approx 0 \Rightarrow g_{ij} \approx 0 \Rightarrow$ Node i and j are unlinked
  - $|M_{ij}| \gg 0 \Rightarrow |g_{ij}| \gg 0 \Rightarrow$ Node j links to i
- Grouping is done using *Gaussian mixture model* by calculating the probability of M_{ij} belonging to the two groups
- Through clustering analysis, *coupling strengths* g_{ij} can be inferred too

Thus knowing the network links & coupling strengths

2. Network Reconstruction

- The method is applied to the *empirical neuronal time series* of cultures of rat embryonic cortices (25 days in vitro) to estimate the *coupling strengths* g_{ij}, leading to the following dataset

Node i	**Node** j	g_{ij}
1	196	0.0208720006
1	266	0.0156720001
1	267	0.0218959991
$\vdots$	$\vdots$	$\vdots$
2	1	−0.0234200004
2	21	−0.00388139999
2	23	−0.00472760014
$\vdots$	$\vdots$	$\vdots$
4095	4094	0.0089673996

- ➢ 4095 electrodes for data collection ⇒ 4095 nodes in the giant network
- ➢ (Order) $g_{ij} \sim 10^{-3}$ to 10^{-2}
- ➢ (Sign) g_{ij} can be +ve or –ve
- ➢ Sparse network with 1.4% sparsity
- ➢ Forms the foundation of *all* later simulations

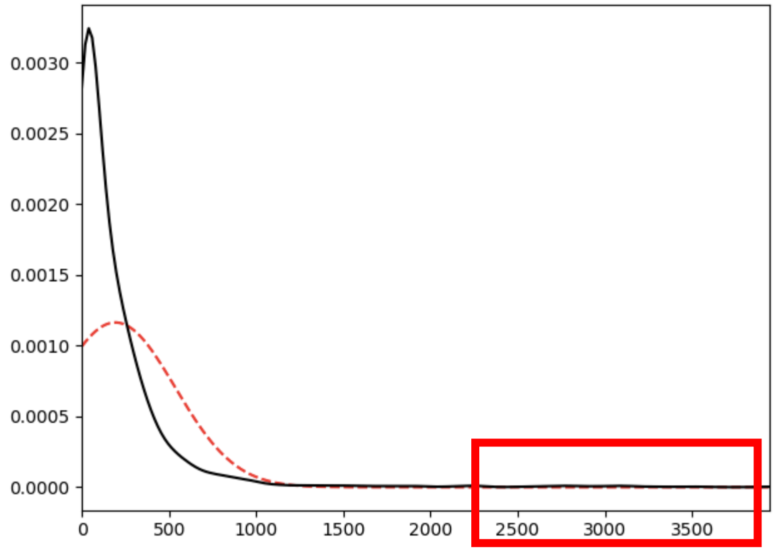
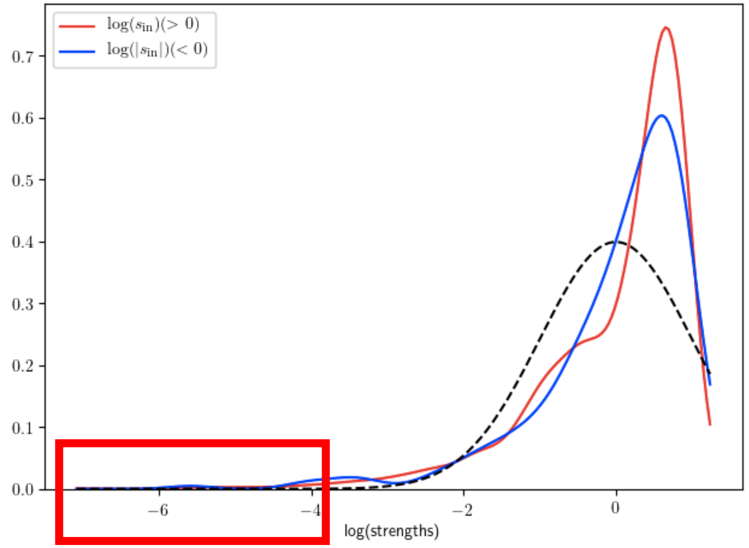
The Core Idea & its Application

Motivations of the Project - Neuronal Spikes

2. Network Reconstruction

- Experimental neuronal *spike counts* are **highly skewed** and **long-tailed** in distribution, and so are the *synaptic strengths*

Can the very large spike counts be explained by the very large synaptic strengths?

Spike Distribution	Strength Distribution
	

Any *statistical* or *causal* relation?

Trials of Different Dynamics

Model Testing

The Logistic Network Model

The FitzHugh-Nagumo Network Model

The Logistic Network Model

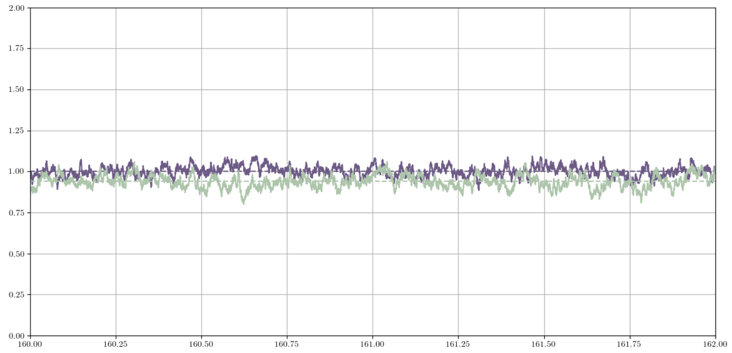
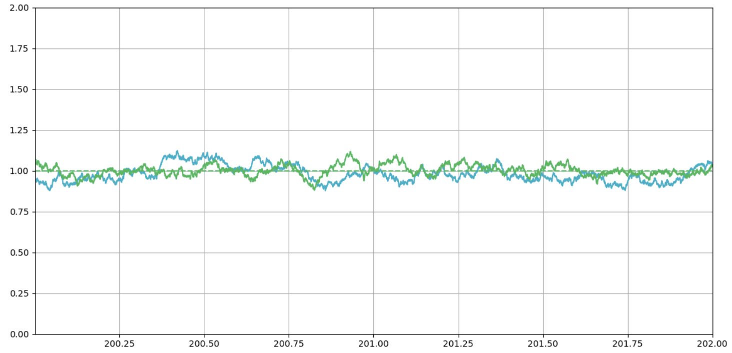
The FitzHugh-Nagumo Network Model

3. Trials of Different Dynamics

- Recall that the network dynamics is governed by

$$\frac{\mathrm{d}x_i}{\mathrm{d}t} = f_i(x_i) + \sum_j g_{ij} h(x_i, x_j) + \eta_i$$

- The **logistic** network model assumes $f_i(x) = r_i x(1 - x)$
- With both the **synaptic** and **diffusive** nodal interaction term h, the model *fails* at generating realistic spikes but rather, gives only *fluctuations*

Synaptic h	Diffusive h
	

➢ **Synaptic *h***

$h(x, y) = 1/\beta_1[1 + \tanh \beta_2(y - y_0)]$

➢ **Diffusive *h***

$h(x, y) = y - x$

The Logistic Network Model

The FitzHugh-Nagumo Network Model

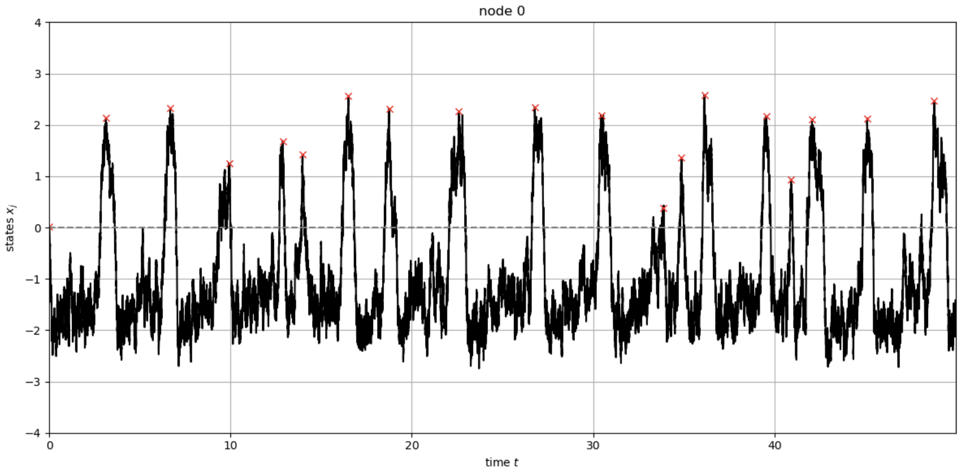
3. Trials of Different Dynamics

- The **FHN** network model assumes two-dimensional states (x_i, y_i) with dynamics governed by

$$\frac{\mathrm{d}x_i}{\mathrm{d}t} = \frac{1}{\epsilon}\left(x_i - \frac{x_i^3}{3} - y_i\right) + \sum_j g_{ij} h(x_i, x_j) + \eta_i$$

$$\frac{\mathrm{d}y_i}{\mathrm{d}t} = x_i + \alpha$$

- With the **diffusive** nodal interaction term $h(x, y) = y - x$, the model succeeds in generating realistic spikes with parameters $(\epsilon, \alpha) = (0.1, 0.95)$



- ➢ Active spiking is seen
- ➢ Spikes are selected based on numerical criteria and counted for each node
- ➢ *The subsequent analysis is based on this FHN network model*

What Explain the Spikes?

Analysis & Discussion

Approach 1:
Regression Analysis

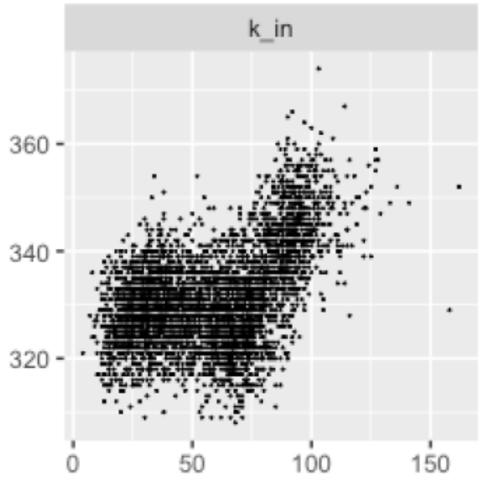
Approach 2:
Reference Networks

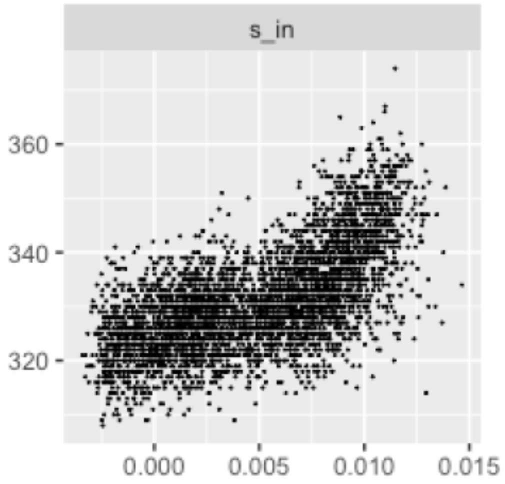
Approach 1:
Regression Analysis

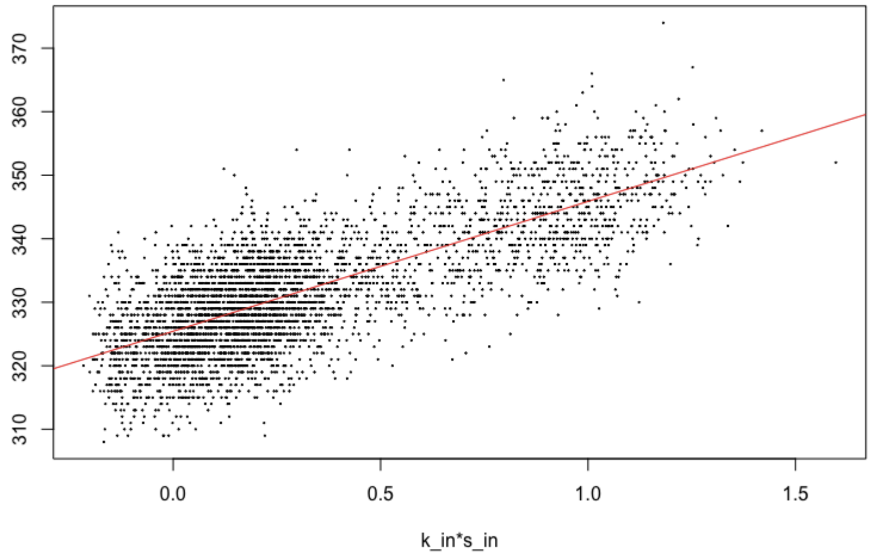
Approach 2:
Reference Networks

4. What Explain the Spikes?

- Main results from **regression analysis** – *statistical* relation
  - ➢ Non-linear relation btw. model spike counts and *in-measures*, including $k_{\text{in}}, s_{\text{in}}, k_{\text{in}}^{+}, s_{\text{in}}^{+}$, but less obvious relation with out-measures
  - ➢ Large spike counts are associated with large *in-degrees* and *in-strengths*
  - ➢ Strong *linear* correlation btw. model spike counts and $k_{\text{in}} \times s_{\text{in}}$, recognize that $(k_{\text{in}} \times s_{\text{in}})(i) = \sum_j g_{ij}$ (total incoming g_{ij})







Approach 1:
Regression Analysis

Approach 2:
Reference Networks

4. What Explain the Spikes?

- **Reference networks** *vary* network features of interest while *preserving* the rest, and are constructed through artificial manipulation of the reconstructed **G**
- Effects of the varied features on the dynamics are studied
- Main results from **reference network analysis** – *causal* relation
  - ➢ The reference network that has *rows of* **G** *shuffled*, thus (i) keeping the *long-tailed distribution of* $k_{\text{in}}, s_{\text{in}}$ and (ii) making *distribution of* $k_{\text{out}}, s_{\text{out}}$ *bell-shaped*, preserves the distribution of spike counts
  - ➢ Other reference networks with *bell-shaped* $k_{\text{in}}, s_{\text{in}}$ or *long-tailed* k_{in} have bell-shaped spike counts
  - ➢ Only $\boldsymbol{s_{\text{in}}}$ remains as the important driving factor of the spiking dynamics
  - ➢ **Conclusion.** Long-tailed s_{in} leads to long-tailed spike counts

Conclusion

- The FHN network model successfully generates realistic neuronal spikes
- **Regression analysis.** The model spike counts have stronger statistical relations with in-measures than out-measures
- **Reference network.** The long-tailed incoming synaptic strength s_{in} leads to the long-tailed spike counts

Can the very large spike counts be explained by the very large synaptic strengths?

- Yes, in the context of the FHN network model.

In particular, it's the very large incoming synaptic strengths

that have an important effect on spike counts.

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