Izhikevich's spiking neuron model:

$$\dot{v}_i = 0.04 v_i^2 + 5 v_i + 140 - u_i + I_i + \underbrace{\xi_i}_{\text{noise}}$$

$$\dot{u}_i = a(bv - u)$$

$a = 0.02$

$b = 0.2$

$c = -65$

$d = 8$

if $v_i \geq 30$

$v_i \to c$

$u_i \to u_i + d$

$$I_i(t) = G_i^{Exc}(t)\left(V_E - V_i(t)\right) - G_i^{Inh}(t)\left(V_i(t) - V_I\right)$$

$V_E = 0$

$V_I = -80$

$$\frac{dG_i^{exc}}{dt} = -\frac{G_i^{exc}}{\tau_{exc}} + \beta \sum_{j=1, g_{ij}>0}^{N} g_{ij} \sum_k \delta\left(t - t_{j,k}\right)$$

$\tau_{exc} = 5$

$$\frac{dG_i^{inh}}{dt} = -\frac{G_i}{\tau_{inh}} + \beta \sum_{j=1, g_{ij}<0}^{N} |g_{ij}| \sum_k \delta\left(t - t_{j,k}\right)$$

$\tau_{inh} = 6$

$$\Rightarrow \quad G_i^{exc} = \beta \sum_{j=1, g_{ij}>0} g_{ij} \sum_k e^{-\frac{(t - t_{j,k})}{\tau_{exc}}} \theta\left(t - t_{j,k}\right)$$

θ: Heaviside step function

$$G_i^{inh} = \beta \sum_{j=1, g_{ij}>0} |g_{ij}| \sum_k e^{-\frac{(t - t_{j,k})}{\tau_{inh}}} \theta\left(t - t_{j,k}\right)$$