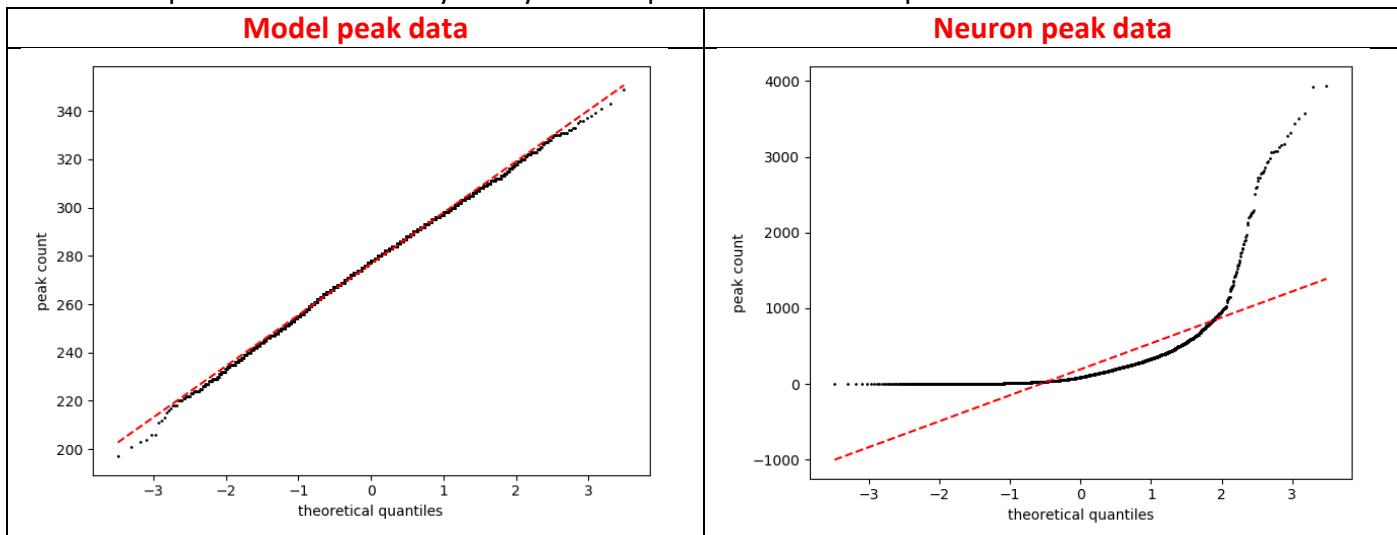


Table of content

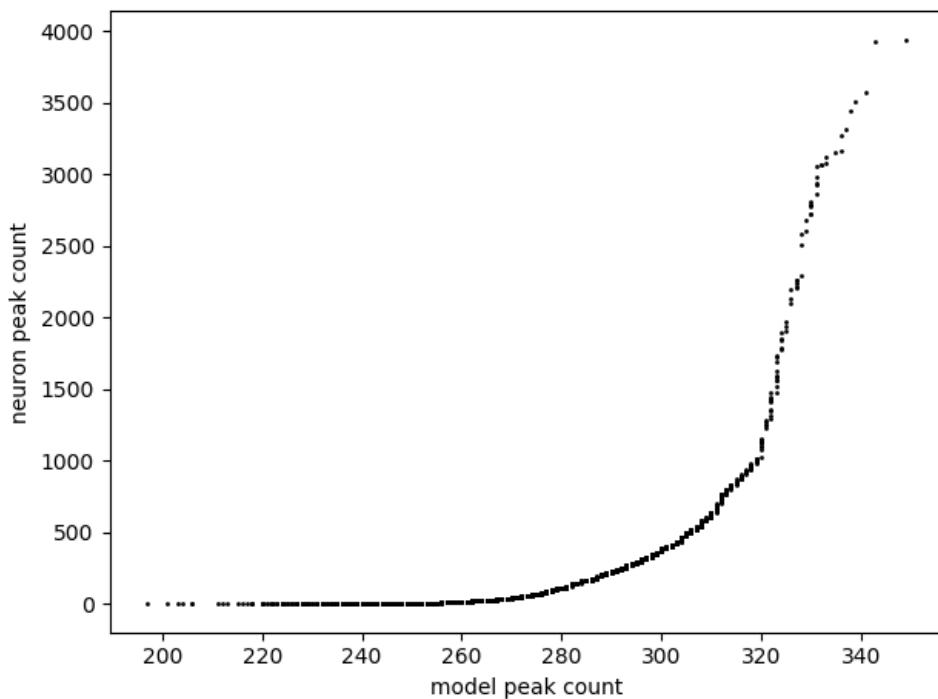
- Neuron spike analysis (DIV25_spks)
- Dynamics (case 4: directed weighted random network)
- Dynamics (DIV25_PREmethod, synaptic coupling function)
- Dynamics (DIV25_PREmethod, diffusive coupling function)
- Spike analysis (peak histogram, raster plot)
- Spike analysis (peak distribution, peak vs strength plot)
- Autocorrelation analysis
- Spike analysis (Gaussian reference network)
- Reference network analysis

Neuron spike analysis (DIV25_spks)

- Analyze how neuron spikes distribute
- Here all peak count data are experimental
- (Continuing from last time) **With-noise case:** params
 $(r_0, g_{ij} \times \text{multiplier}, \sigma_i) = (r_0 = 100, g_{ij} \times 10, \sigma_i = 1.5)$
- Question to address: how peaks distribute?
- How peak data contrast with **normal quantiles**
Quantile-quantile plot (QQ plot): plot percentiles of data against percentiles of another set of data (maybe from a specified distribution)
Model peak data almost perfectly align with normal quantiles
Neuron peak data have a very heavy tail compared with normal quantiles

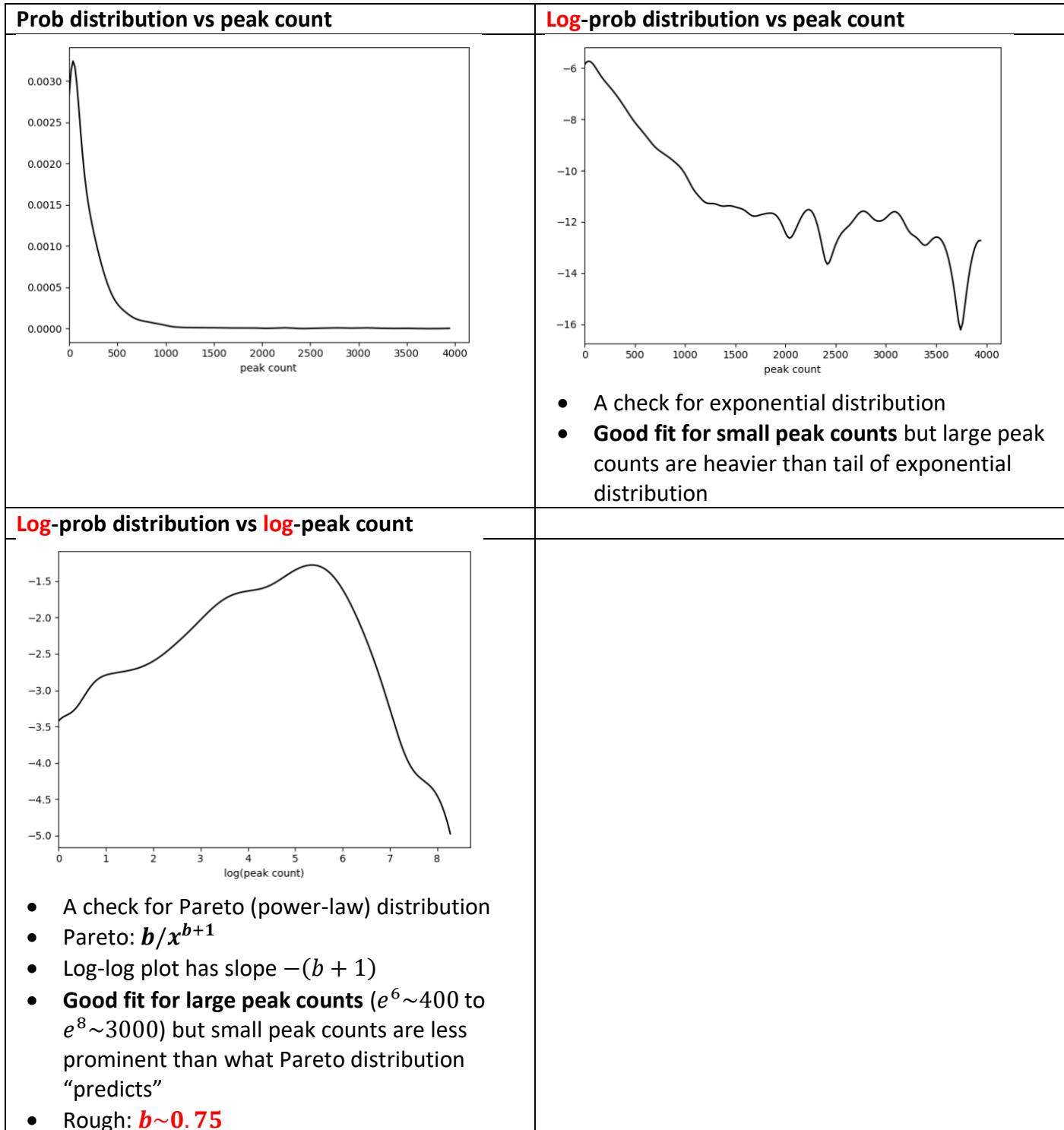


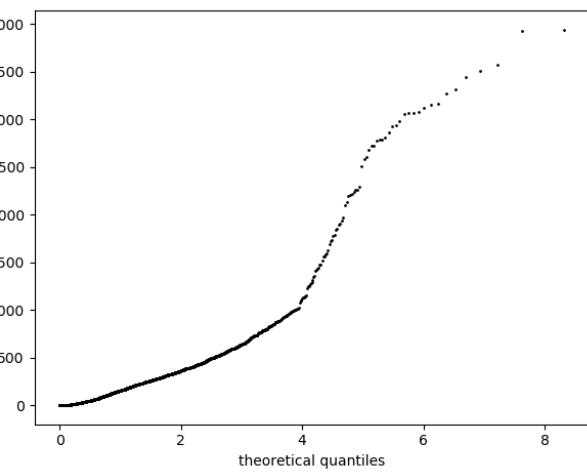
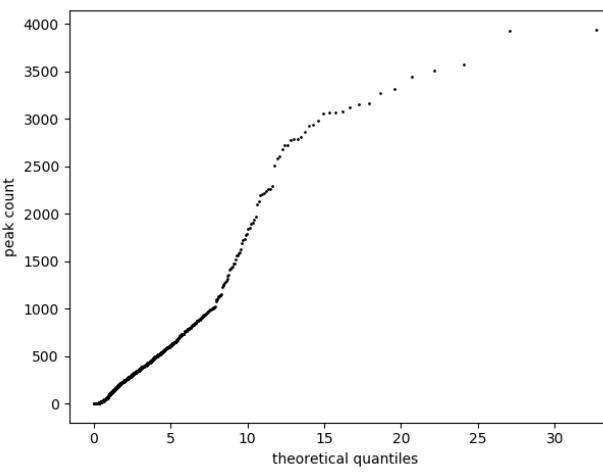
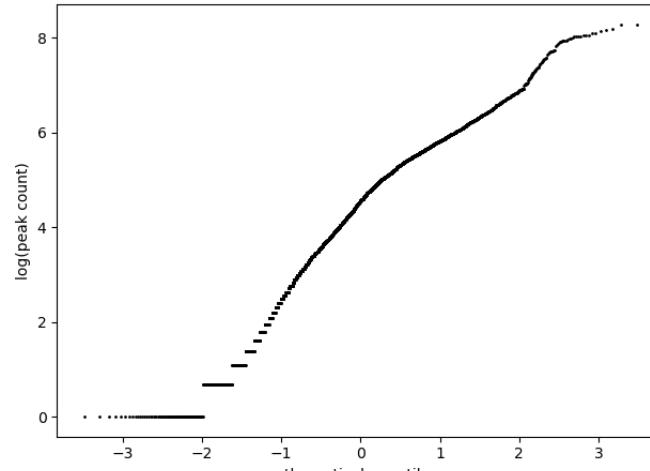
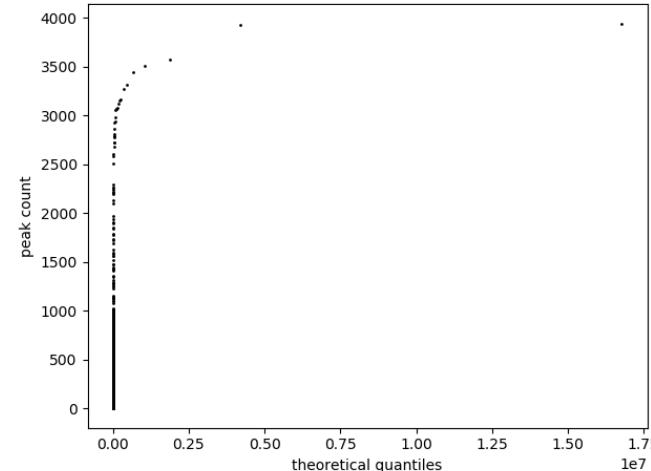
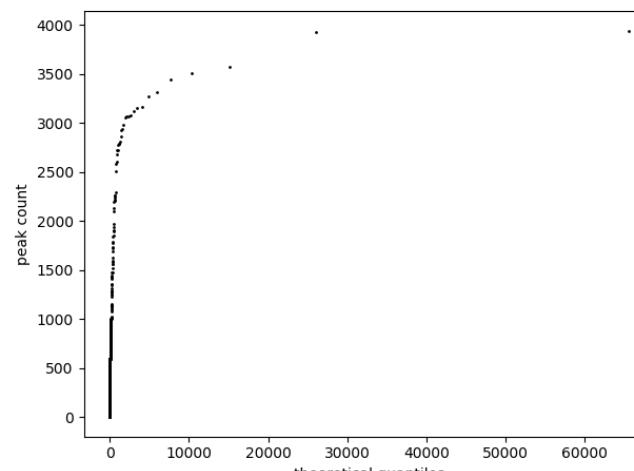
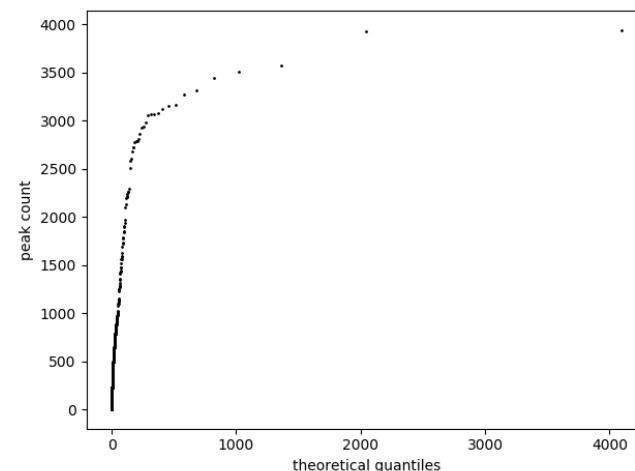
- How peak data contrast with each other
Expected as model peak data are near normal



- More analysis for **neuron** peak data

Distributions are smoothed



QQ plot (vs exponential distribution)	QQ plot (vs lognormal distribution)
 <ul style="list-style-type: none"> Exponential distribution may be a good fit for small peak counts But QQ plot has some initial curvature 	 <ul style="list-style-type: none"> Lognormal distribution may be a good fit for small peak counts QQ plot is almost straight for small peak counts
QQ plot (log peak count vs normal distribution)	QQ plot (vs Pareto distribution, $b = 0.5$)
 <ul style="list-style-type: none"> To further check lognormal distribution Quite good fit for small peak counts 	
QQ plot (vs Pareto distribution, $b = 0.75$)	QQ plot (vs Pareto distribution, $b = 1$)
 <ul style="list-style-type: none"> Good fit for large peak counts (400-3000), as what previous result suggests But outliers (>3000) are not as heavy 	

- **Guess**

Peak counts consist of 3 regimes:

- (1) **Less than 400**: lognormal distribution
- (2) **Between 400 and 3000**: Pareto $b \sim 0.75$
- (3) **Larger than 3000** (only 14 nodes fall in this extreme outlier range): another Pareto with larger b (so that tail is thinner), but may be meaningless to fit distribution \because small number of extreme outliers

Dynamics (case 4: directed weighted random network)

- Examine linear in Q vs M plot

- Parameters

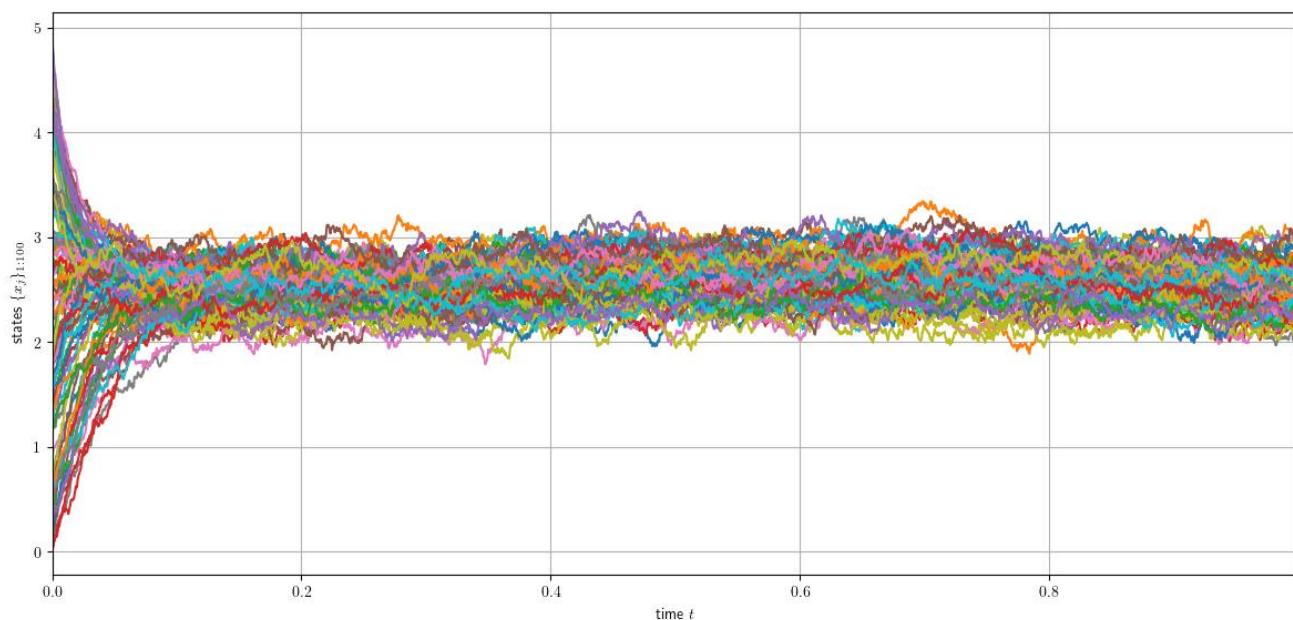
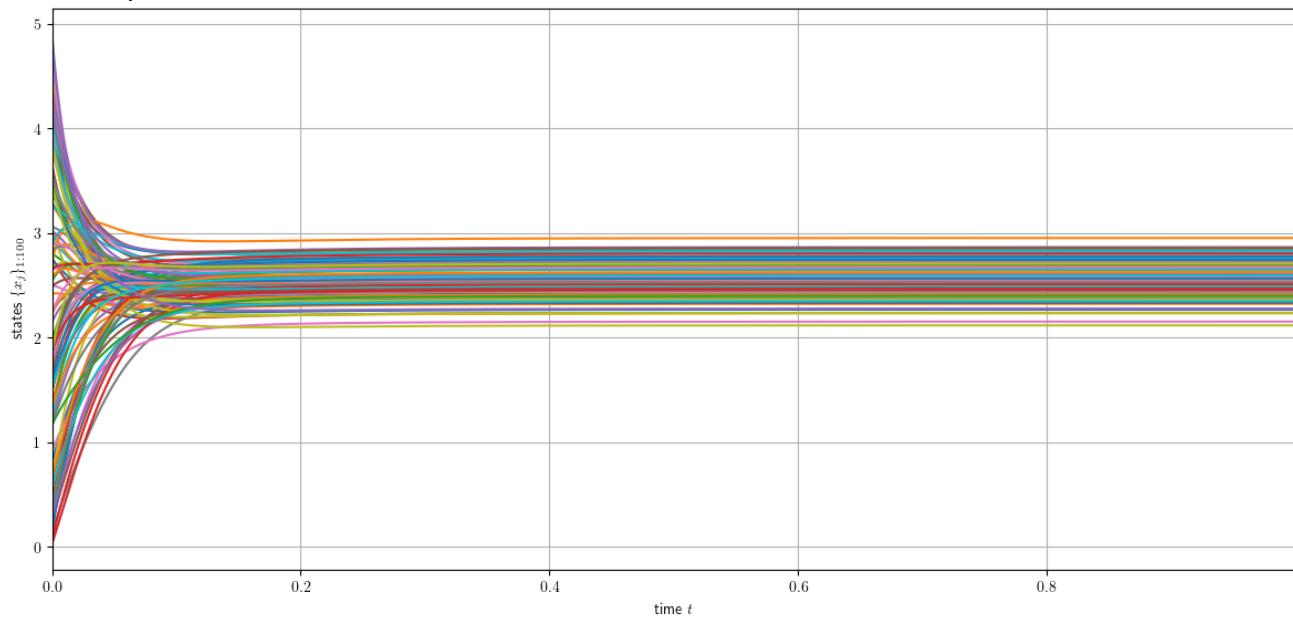
- (a) Random directed weighted network
- (b) Size = 100, connection probability = 0.2, weights $\sim N(10, 2)$, $\sigma_i = 1$
- (c) Intrinsic dynamics coefficient $r_i = 10$
- (d) Synaptic coupling function $(\beta_1, \beta_2, y_0) = (2, 0.5, 4)$
- (e) Initial conditions uniform[0,5]
- (f) Step size $\delta t = 5 \times 10^{-4}$, time steps $N_{\text{data}} = 2 \times 10^6$
- (g) Computation time: (run dynamics) 7 min for $N_{\text{data}} = 2 \times 10^6$

- Time series: noise-free & with-noise

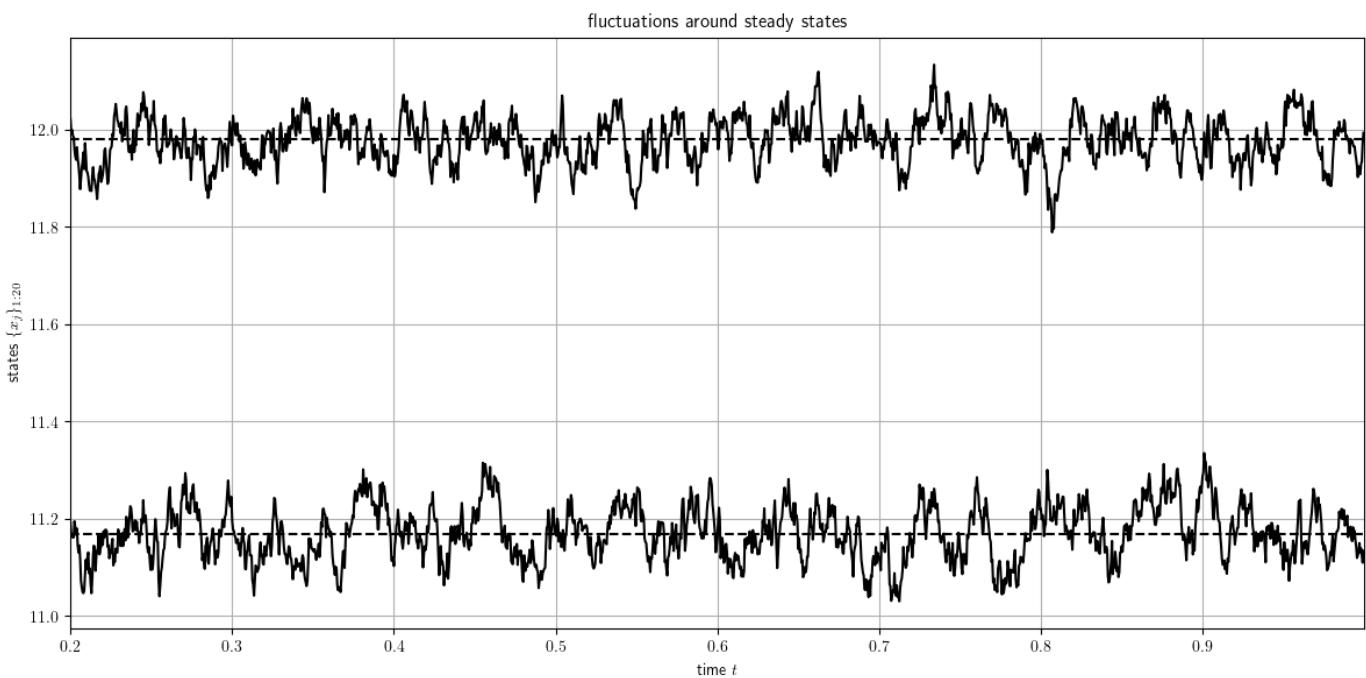
$$N_{\text{data}} = 2 \times 10^3$$

Transient behavior before $t = 0.2$

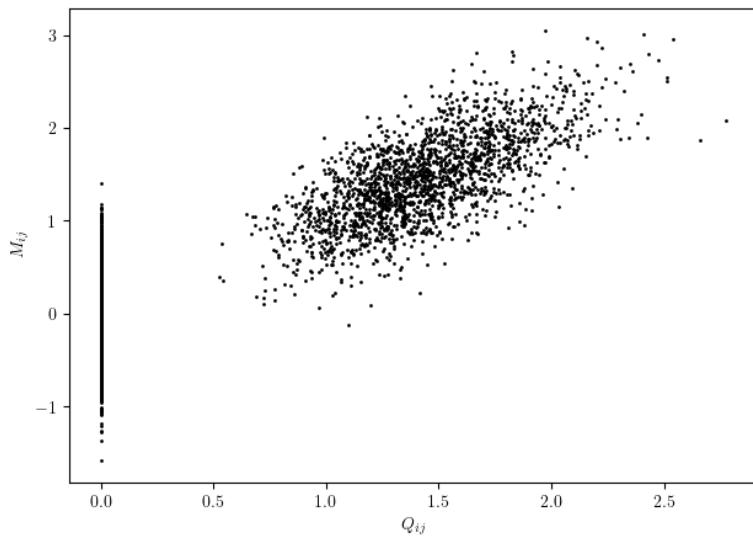
Steady values btw. 2 and 3



- (Same params but 20 nodes) fluctuations about steady values

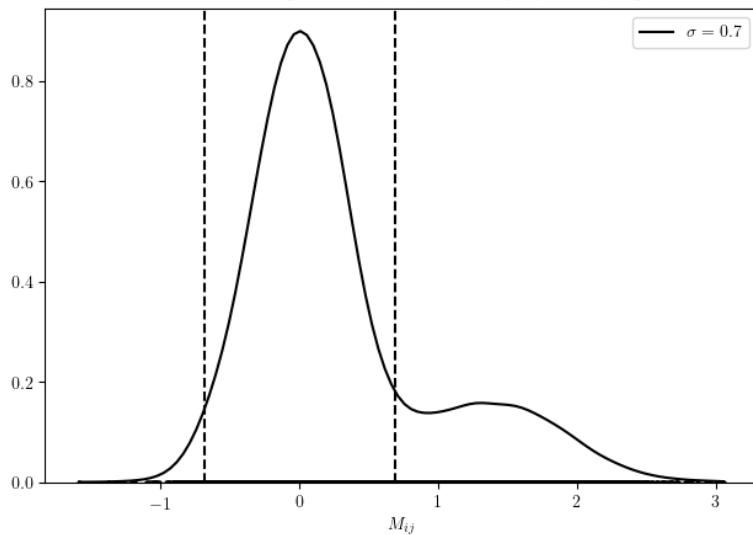


- Q vs M plot



Two peaks: 0 & non-0

size = 100, weight $\sim N(10, 2)$, $r_i = 10$, synaptic coupling



Dynamics (DIV25_PREmethod, synaptic coupling function)

- **Noise-free case: parameters**

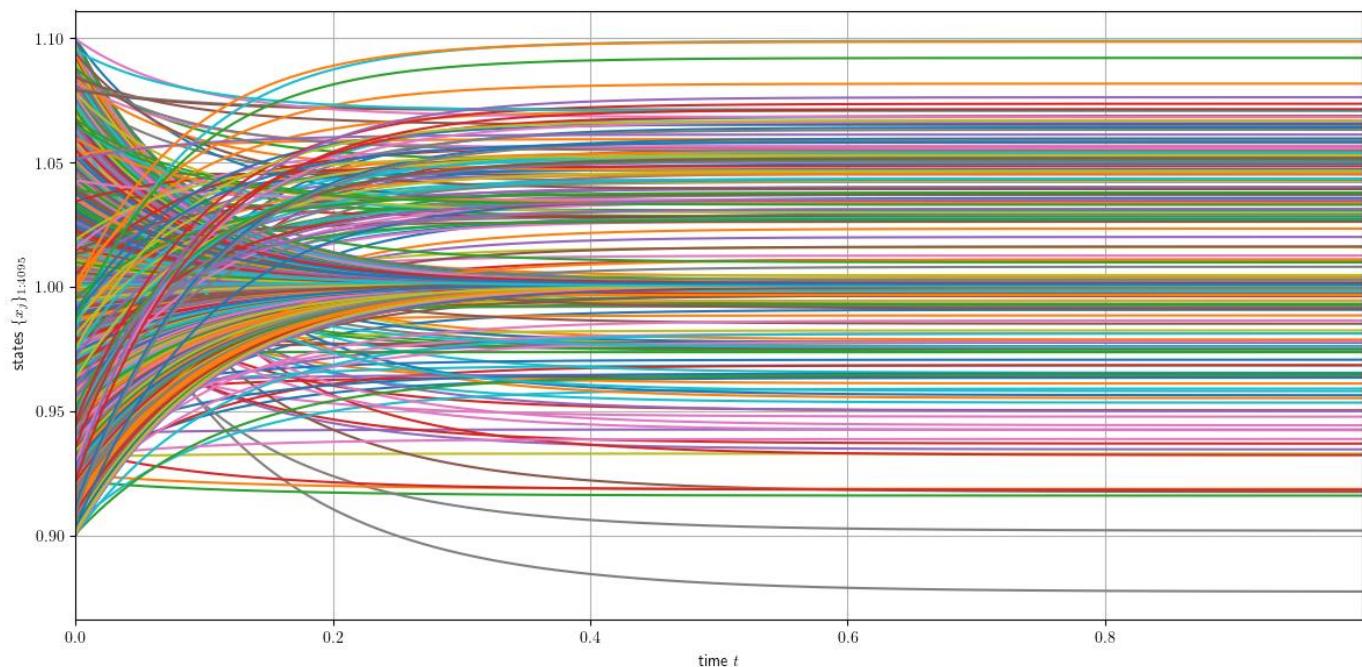
- (model-dependent) effective neuronal network
- Size = 4095, from file "DIV25_PREmethod" (contains g_{ij})
- Synaptic coupling h^{syn} , ($r_0 = 10, \beta_1 = 2, \beta_2 = 0.5, y_0 = 4, \sigma_i = 0.5$)**
- Initial conditions uniform[0.9,1.1]
- Step size $\delta t = 2 \times 10^{-4}$, time steps $N_{\text{data}} = 1 \times 10^6$ (hence time series ends at $t = 200$)
- Computation time: (run dynamics) 1 h 50 min for $N_{\text{data}} = 1 \times 10^6$

- **Noise-free case: time series**

$$N_{\text{data}} = 2 \times 10^3$$

Transient behavior before $t = 0.8$

Steady values btw. 0.9 and 1.1 → initial conditions uniform[0.9,1.1]



- **With-noise case**

$(h^{\text{syn}}) y_0 \sim \text{steady values } X_i$ (X_i obtained under $y_0 = 4$)

- finding the right set of params

$(r_0, \beta_1, \beta_2, y_0 = 1, \sigma_i)$

Fail (divergent dynamics)

1. (10,2,0.5,1,0.5)
2. (10,10,0.5,1,0.5)
3. (10,50,0.5,1,0.5) [stable up to $N_{\text{data}} = 2 \times 10^5$]
4. (10,2,0.1,1,0.5)
5. (50,2,0.5,1,0.5)
6. (10,20,1,1,0.5) [case 9ii]

OK (stable dynamics, up to $N_{\text{data}} = 1 \times 10^6$)

1. (100,2,0.5,1,0.5) [Rationale: stronger tendency to revert to 1]
2. (10,20,1,1,0.25) [case 9ii, from Chris] [Rationale: weaker couplings + smaller noise]

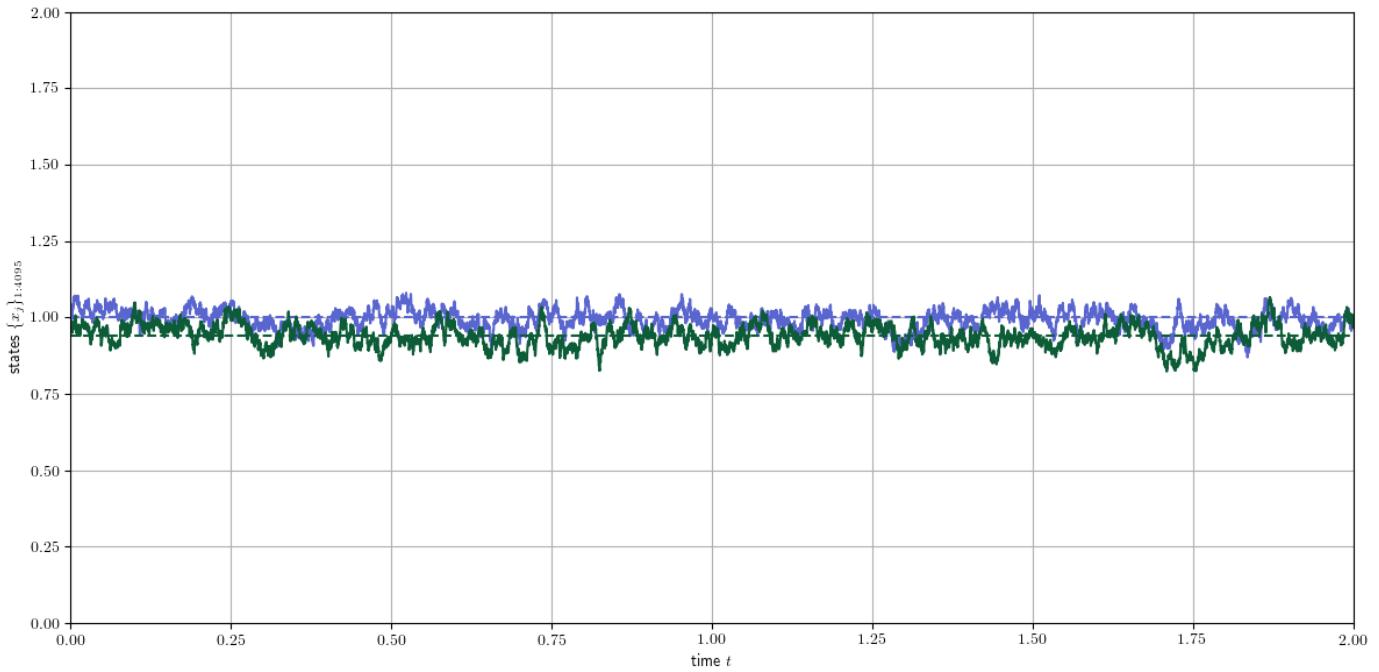
Used $(100,2,0.5,1,0.5)$ & $(10,20,1,1,0.25)$ for the following analysis.

(a) With-noise case: (100,2,0.5,1,0.5)

- Fluctuations around steady values (note: node 0 & 4)

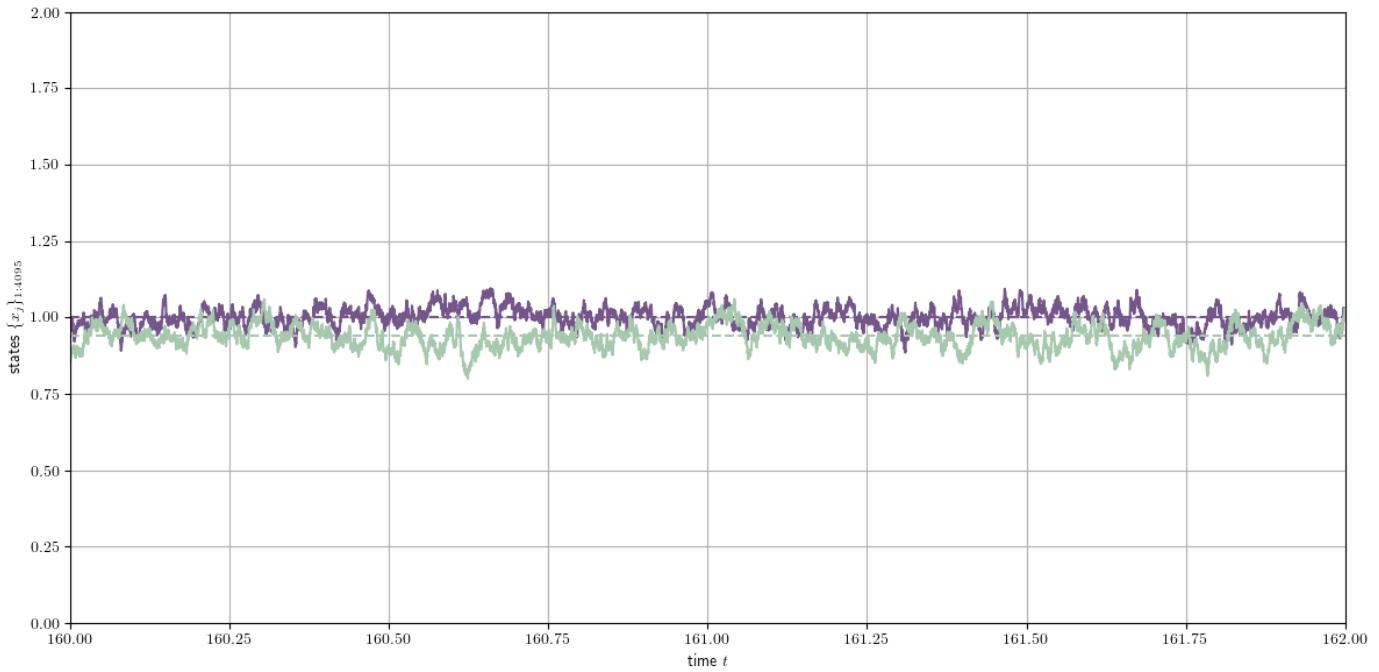
Time: 0 to 2

Recall: transient behavior before $t = 0.8$



Time: 160 to 162

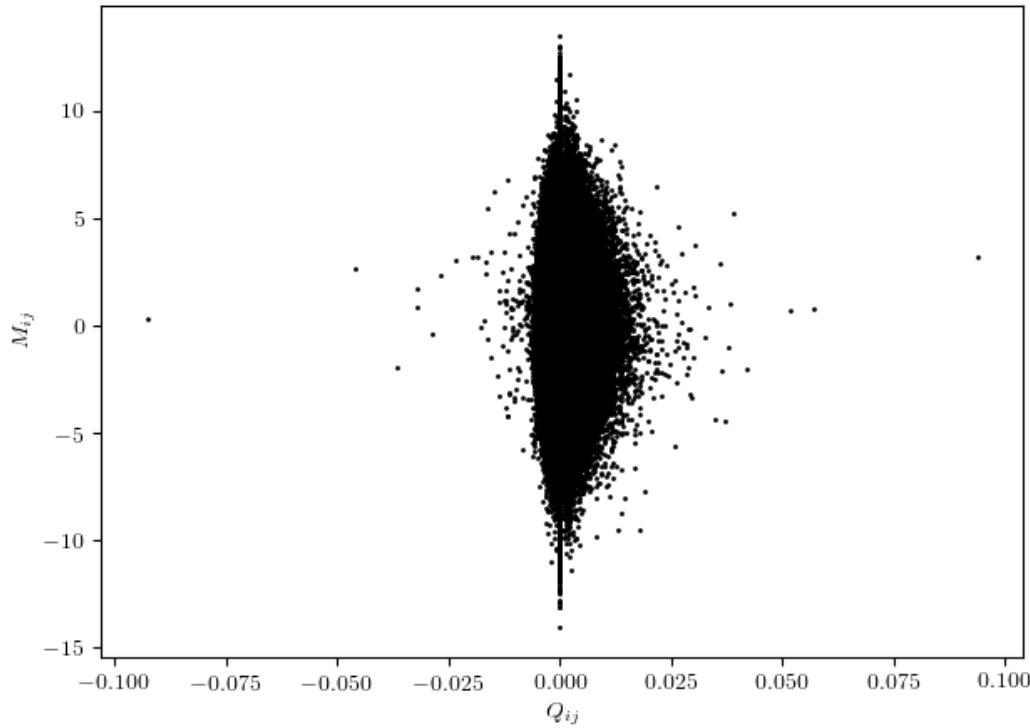
Still stable, as expected



- Condition $\tau \max_i |\operatorname{Im} \lambda_i| < \pi$ is true $\rightarrow \log \exp Q = Q$, i.e., OK to take log
- **Q vs M plot ($N_{\text{data}} = 4 \times 10^5$)**

Linear pattern not obvious, need longer time series

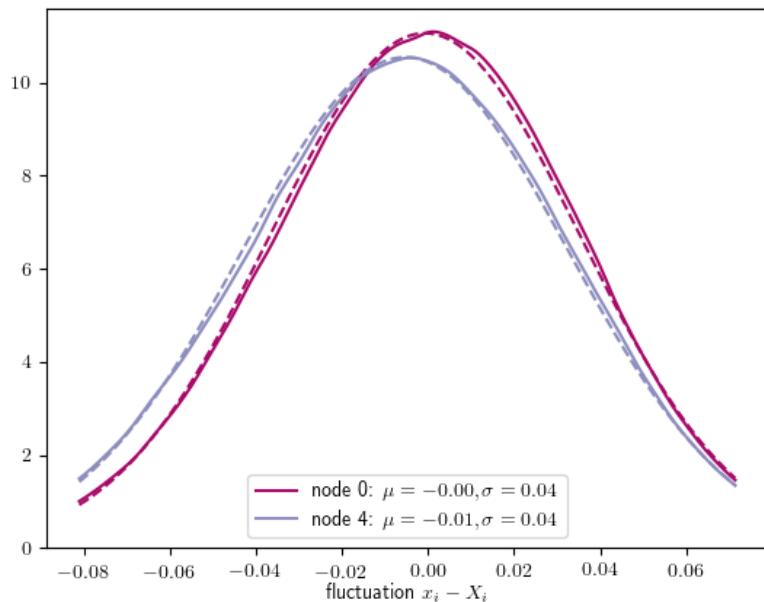
size = 4095, effective weight, $r_i = 100$, synaptic coupling



- **Distribution of fluctuations ($N_{\text{data}} = 1 \times 10^6$)**

Gaussian, s. d. $< \sigma_i = 0.5$

Network connectivity modifies s.d. of fluctuation but not shape (Gaussian noise \rightarrow Gaussian fluc.)

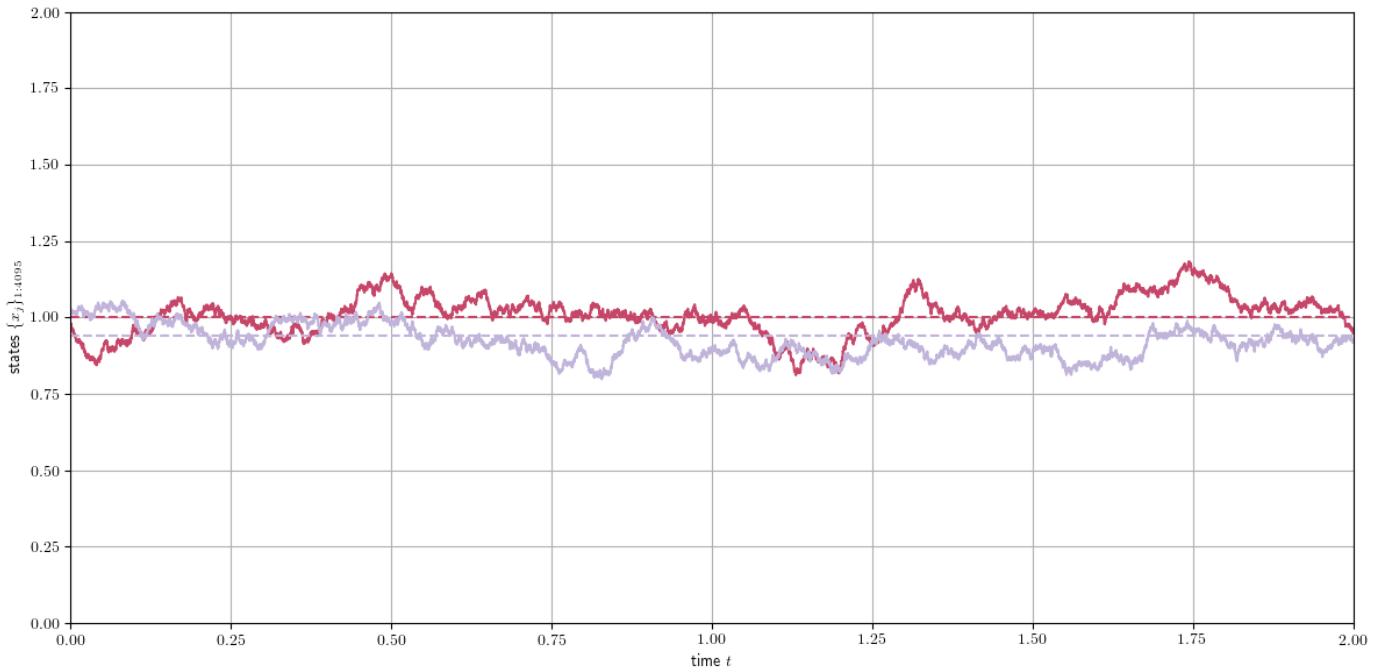


(b) With-noise case: (10,20,1,1,0.25)

- Fluctuations around steady values (note: node 0 & 4)

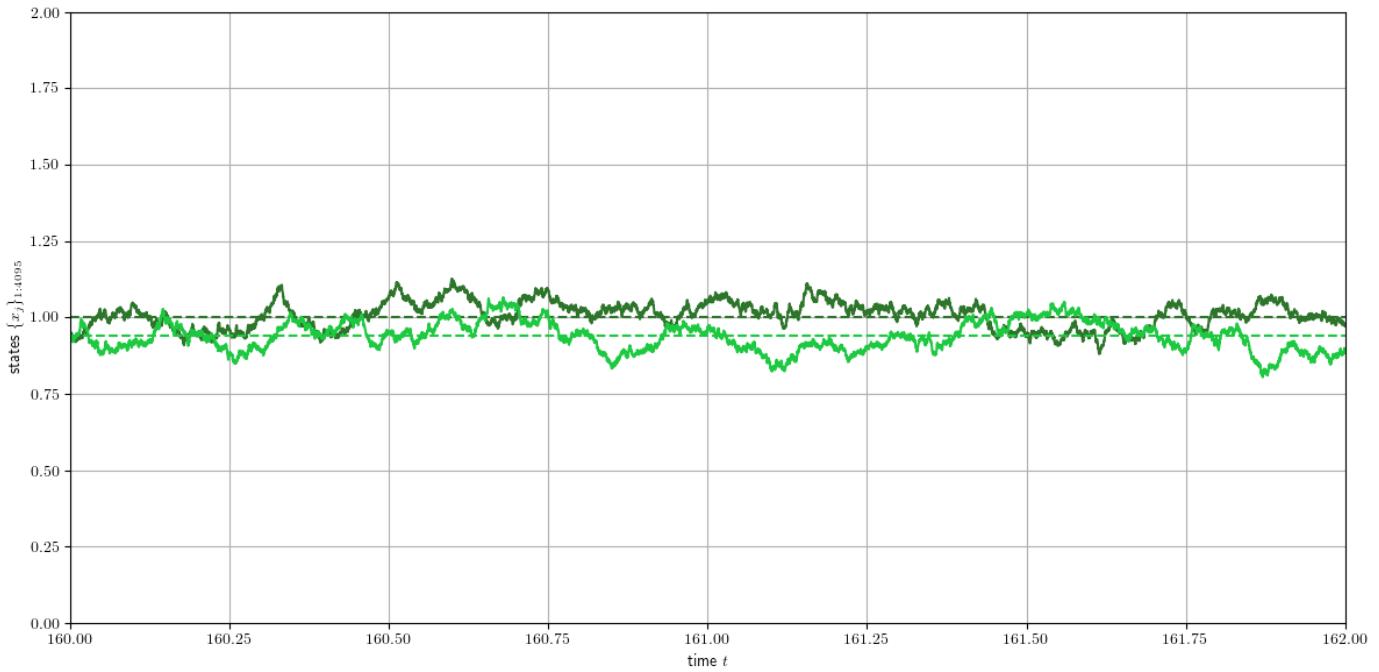
Time: 0 to 2

Recall: transient behavior before $t = 0.8$



Time: 160 to 162

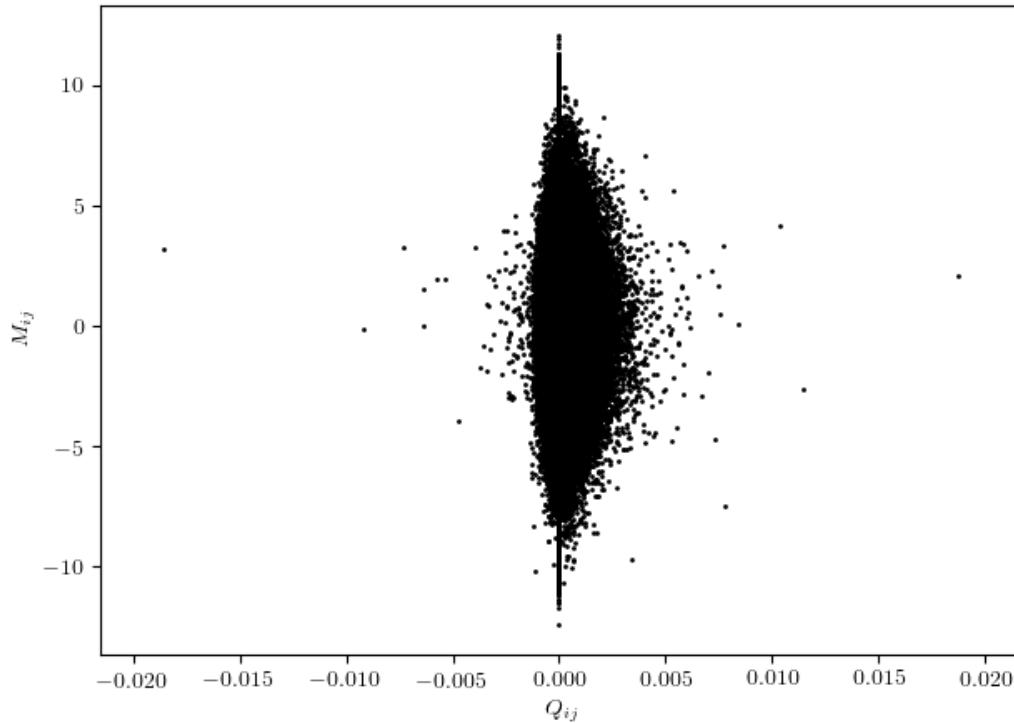
Still stable, as expected



- Condition $\tau \max_i |\operatorname{Im} \lambda_i| < \pi$ is true $\rightarrow \log \exp Q = Q$, i.e., OK to take log
- **Q vs M plot ($N_{\text{data}} = 4 \times 10^5$)**

Linear pattern not obvious, need longer time series

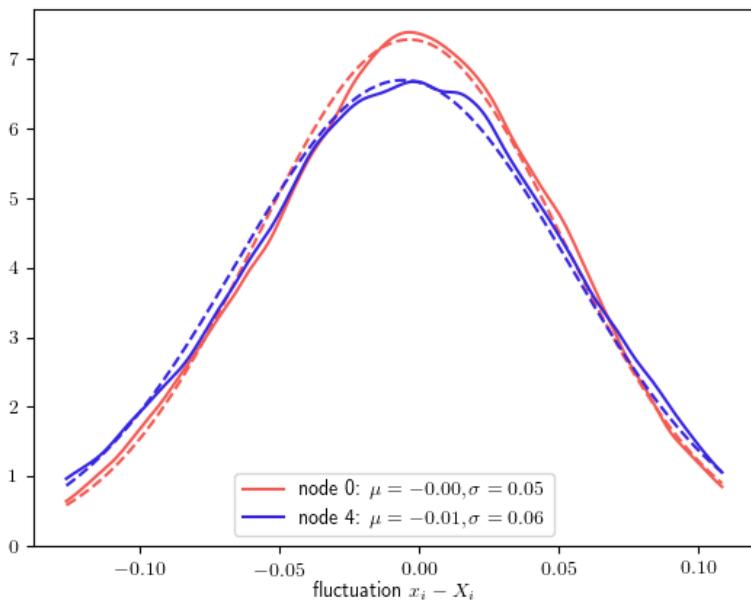
size = 4095, effective weight, $r_i = 10$, synaptic coupling



- **Distribution of fluctuations ($N_{\text{data}} = 1 \times 10^6$)**

Gaussian, s. d. $< \sigma_i = 0.25$

Network connectivity modifies s.d. of fluctuation but not shape (Gaussian noise \rightarrow Gaussian fluc.)



Dynamics (DIV25_PREmethod, diffusive coupling function)

- Model params: diffusive, ($r_0 = 10, g_{ij} \times 10, \sigma_i = 0.25$)
- Noise-free case: parameters
 - (model-dependent) effective neuronal network
 - Size = 4095, from file "DIV25_PREmethod" (contains g_{ij})
 - Diffusive coupling h^{diff} , ($r_0 = 10, g_{ij} \times 10$)**
 - Initial conditions uniform[0.9,1.1]
 - Step size $\delta t = 5 \times 10^{-4}$, time steps $N_{\text{data}} = 2 \times 10^6$ (hence time series ends at $t = 1000$)
 - Computation time: (run dynamics) 3 h 30 min for $N_{\text{data}} = 2 \times 10^6$

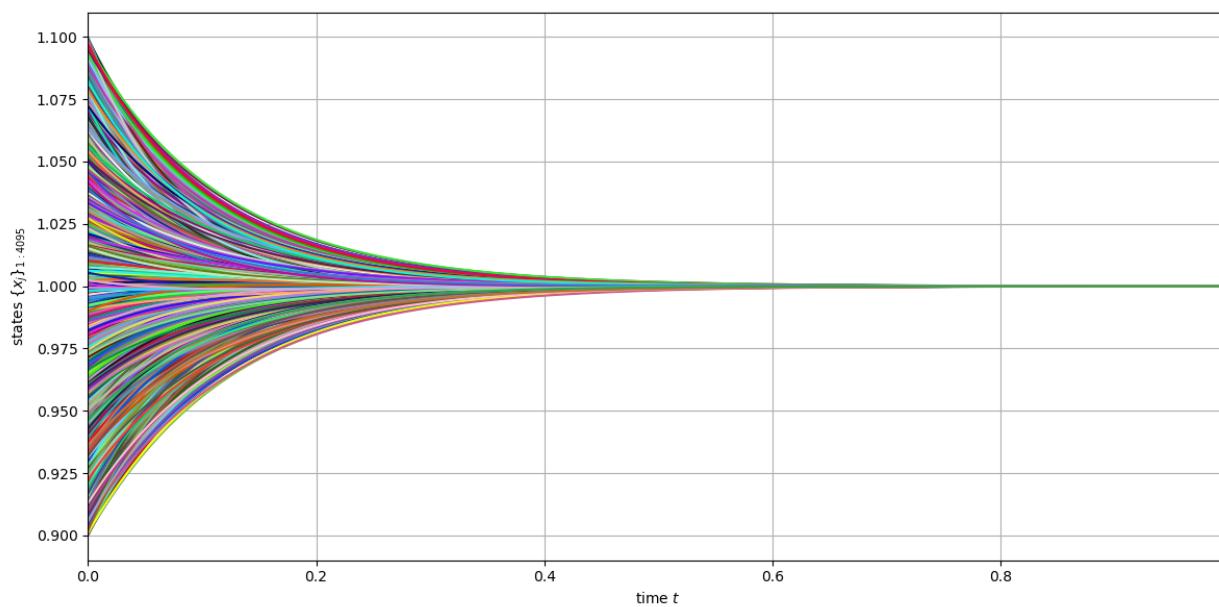
- Noise-free case: time series

1. ($r_0 = 10, g_{ij} \times 10$): $g_{ij} \times 10 \Rightarrow \max g_{ij} \sim 0.3, \min g_{ij} \sim -0.3$

$$N_{\text{data}} = 2 \times 10^3$$

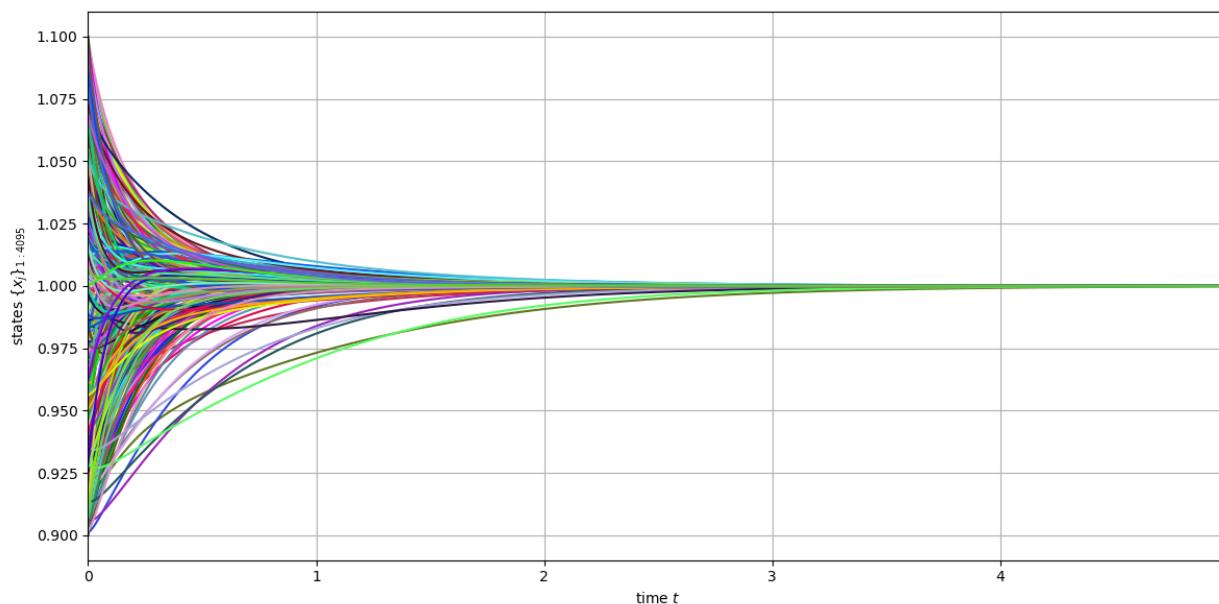
Transient behavior before $t = 0.8$

Steady values all 1 → initial conditions uniform[0.9,1.1]



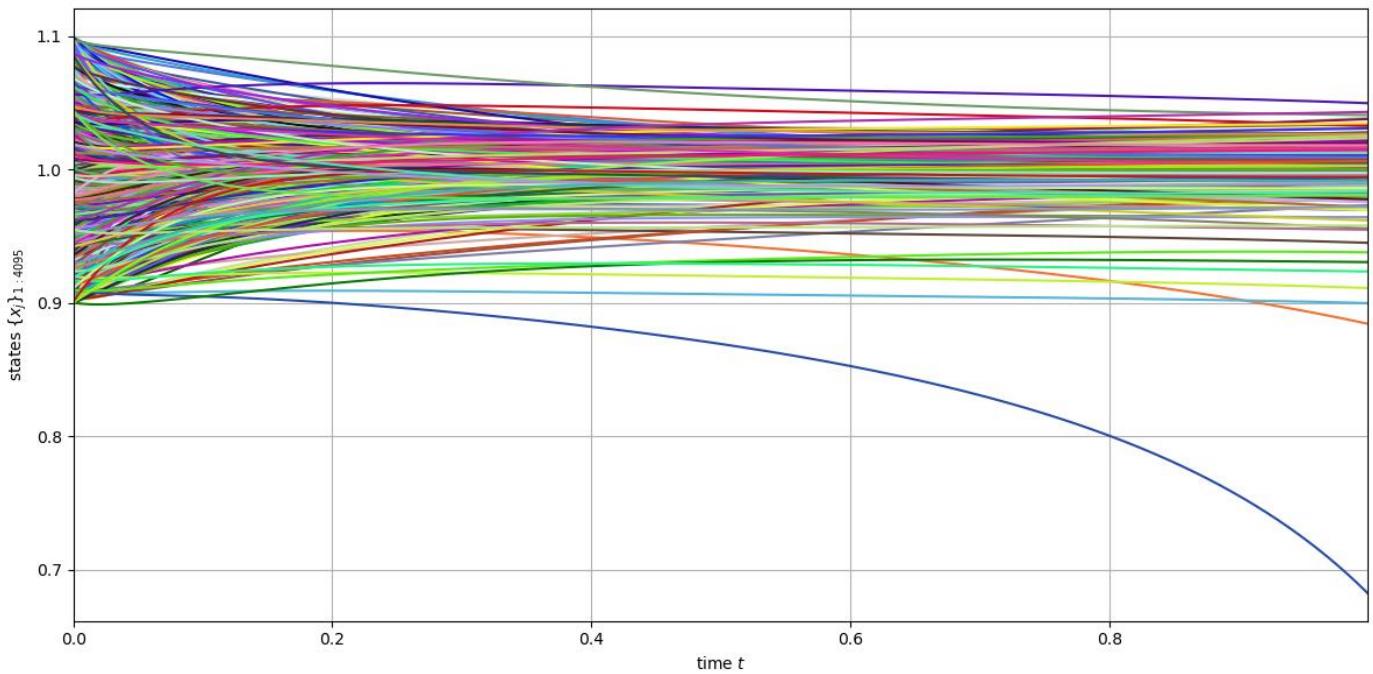
2. ($r_0 = 10, g_{ij} \times 40$): similar behaviour but longer transient behaviour

Transient behavior before $t = 4$



3. $(r_0 = 10, g_{ij} \times 50)$: divergent

Tend to be less stable when $g_{ij} \sim r_0$



- **With-noise case**

Finding the right set of params

$(r_0, g_{ij} \times \text{multiplier}, \sigma_i)$

Fail (divergent dynamics)

1. (10,10,0.5)

OK (stable dynamics, up to $N_{\text{data}} = 2 \times 10^6$)

1. (10,10,0.25)

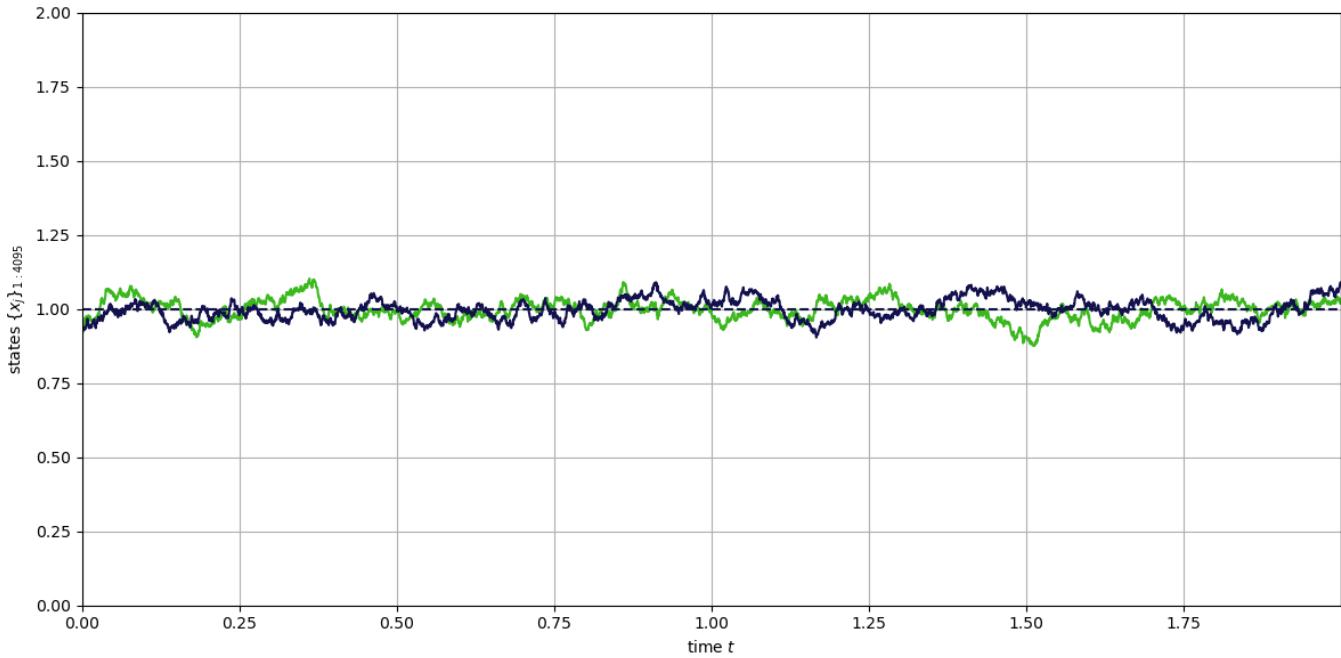
Used (10,10,0.25) for the following analysis.

With-noise case: (10,10,0.25)

- Fluctuations around steady values (note: node 0 & 1)

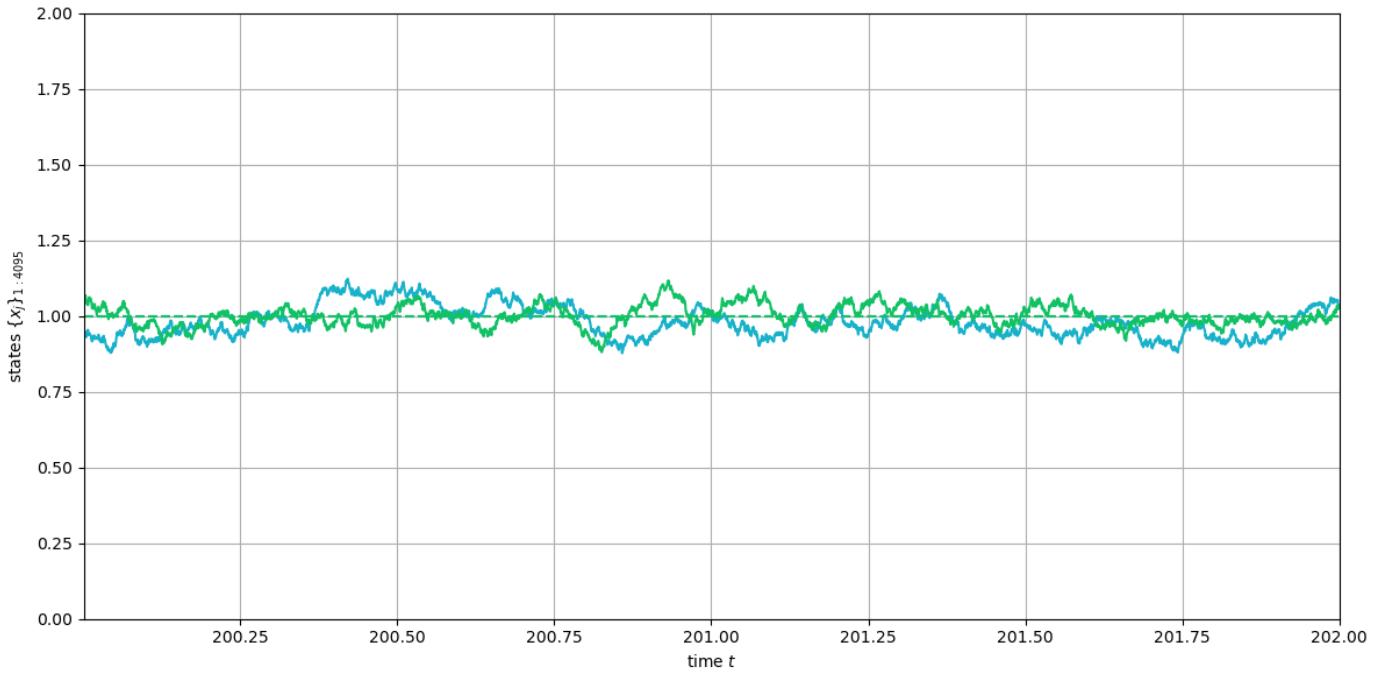
Time: 0 to 2

Recall: transient behavior before $t = 0.8$, steady values all 1



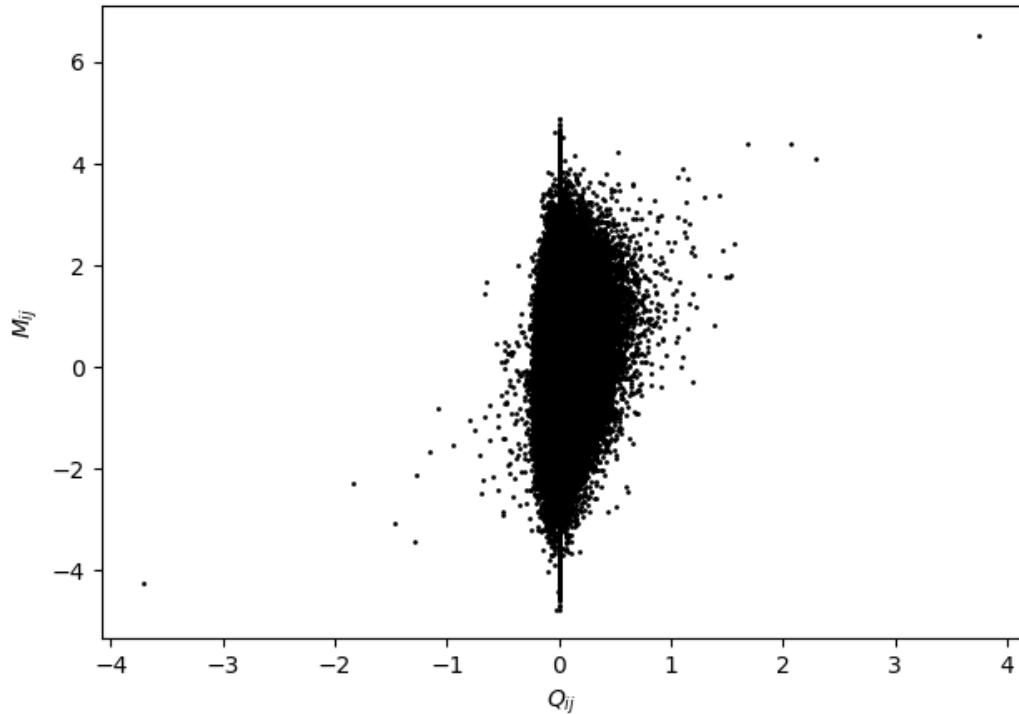
Time: 200 to 202

Still stable, as expected



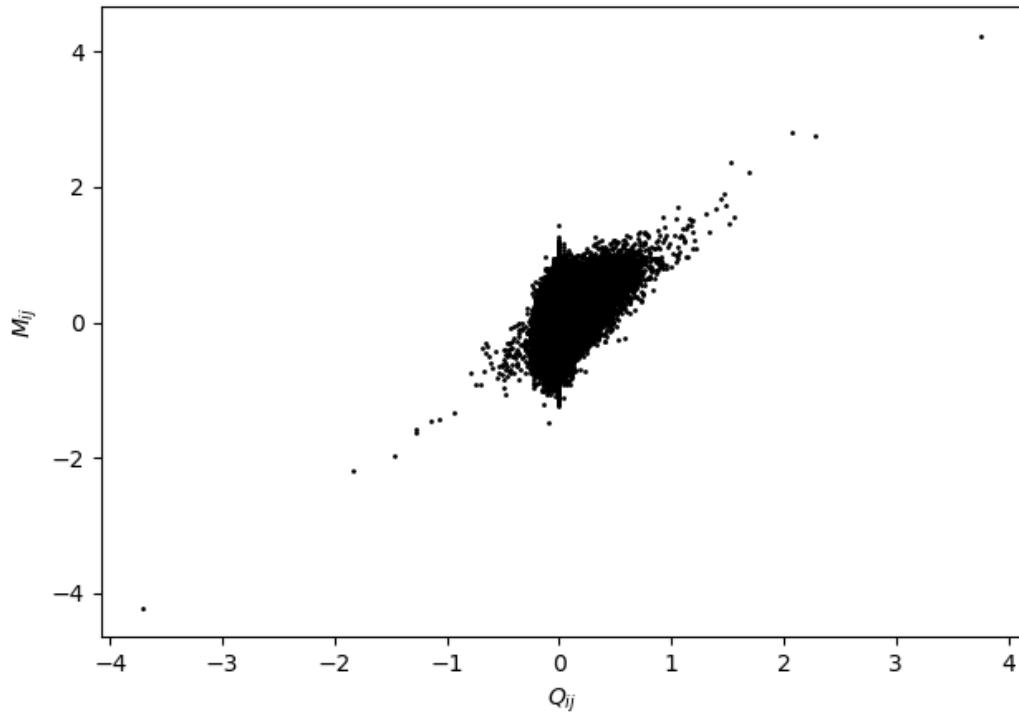
- Condition $\tau \max_i |\operatorname{Im} \lambda_i| < \pi$ is true $\rightarrow \log \exp Q = Q$, i.e., OK to take log
- Q vs M plot ($N_{\text{data}} = 4 \times 10^5$)
 - Linear pattern starting to become obvious, M_{ij} & Q_{ij} have same order
 - M converges faster to Q with diffusive coupling function

size = 4095, effective weight, $r_i = 10$, diffusive coupling



- Q vs M plot ($N_{\text{data}} = 2 \times 10^6$)

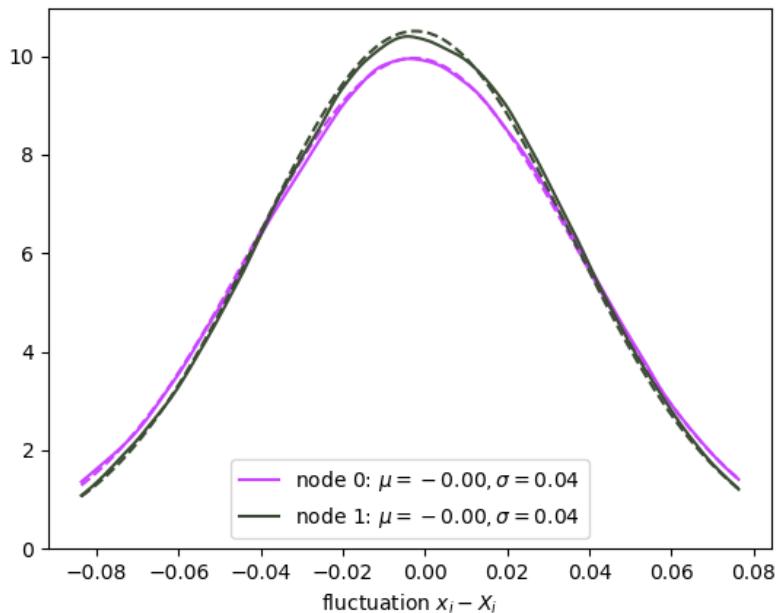
size = 4095, effective weight, $r_i = 10$, diffusive coupling



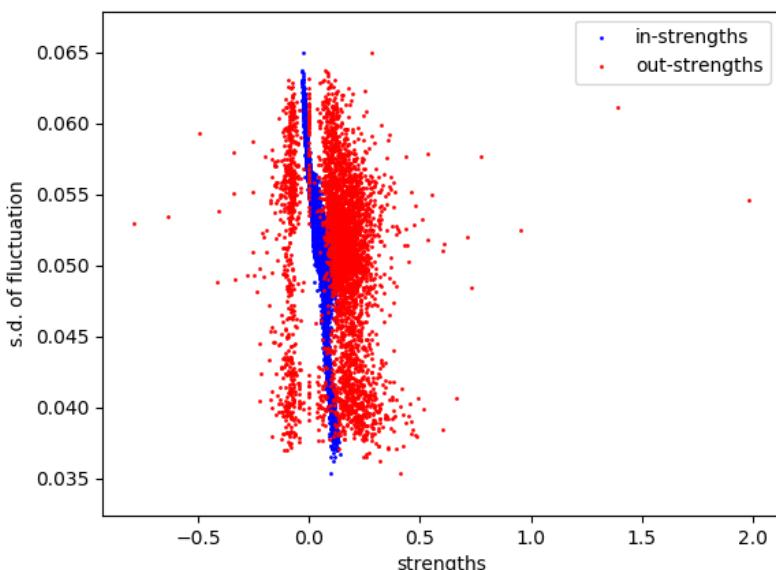
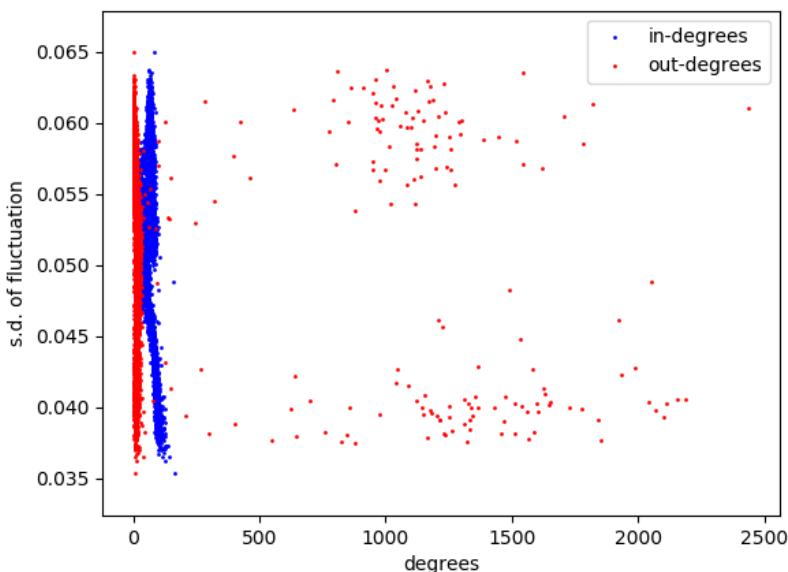
- **Distribution of fluctuations ($N_{\text{data}} = 1 \times 10^6$)**

Gaussian, s. d. $< \sigma_i = 0.25$

Network connectivity modifies s.d. of fluctuation but not shape (Gaussian noise \rightarrow Gaussian fluc.)

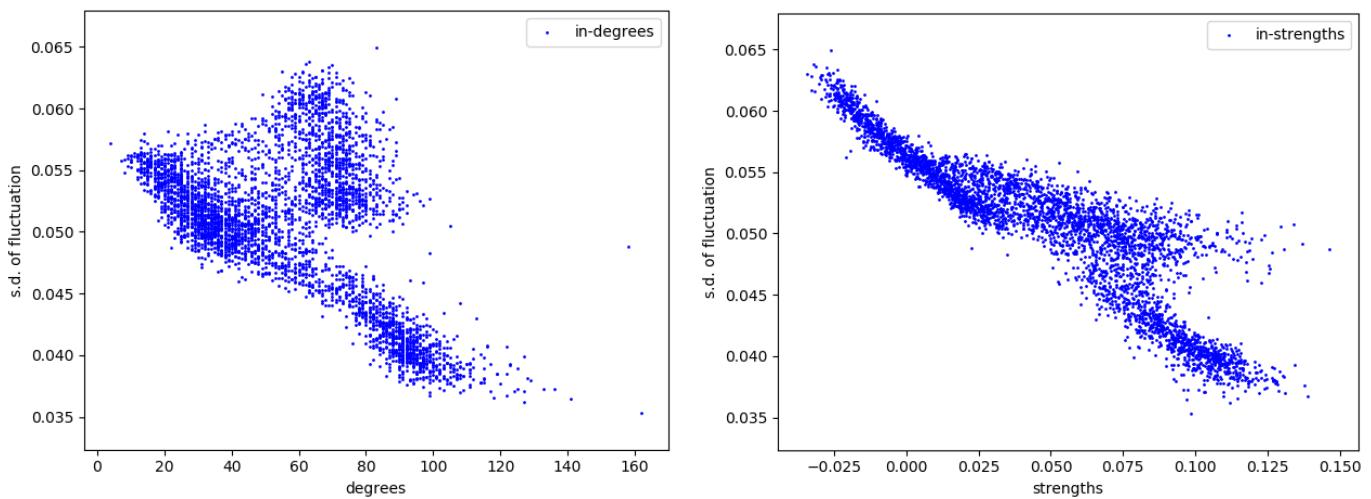


- **Dependence of s.d. of fluctuations on degrees and strengths ($N_{\text{data}} = 1 \times 10^6$)**

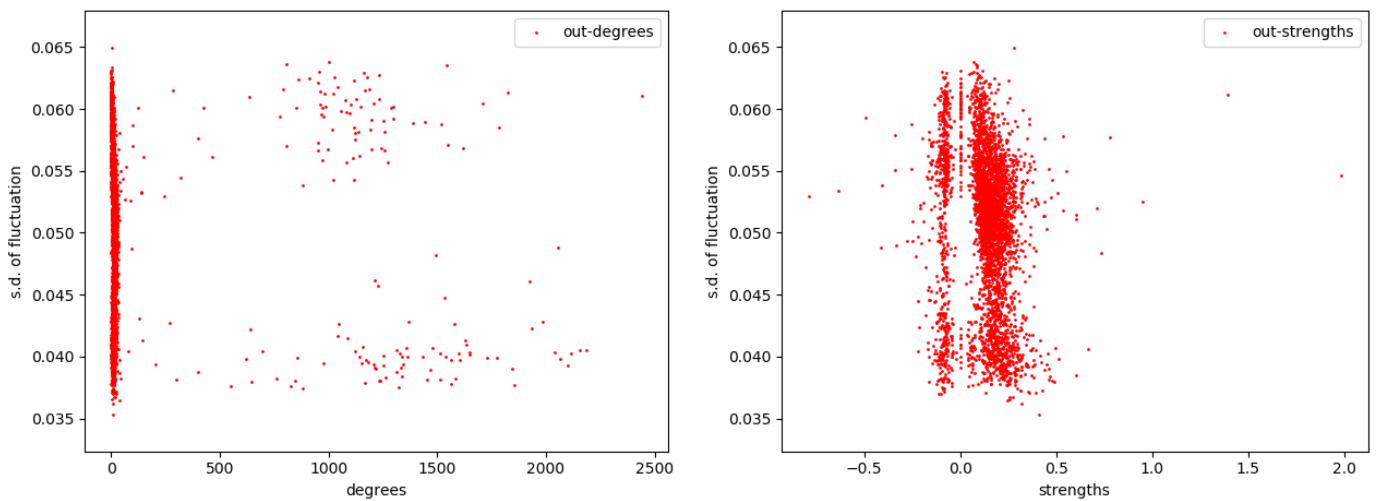


Linear pattern for in-strengths

- Dependence of s.d. of fluctuations on in-degrees and in-strengths ($N_{\text{data}} = 1 \times 10^6$)



- Dependence of s.d. of fluctuations on out-degrees and out-strengths ($N_{\text{data}} = 1 \times 10^6$)



- **Cross-sectional distribution (distribution of all nodes at some time): time-evolution**

Time: 0 to 1

Evolution: uniform → normal

<https://drive.google.com/file/d/1LrP6pSeSgs6OIdGqB56v5eANH0IWatan/view?usp=sharing>

- **Spiking detection**

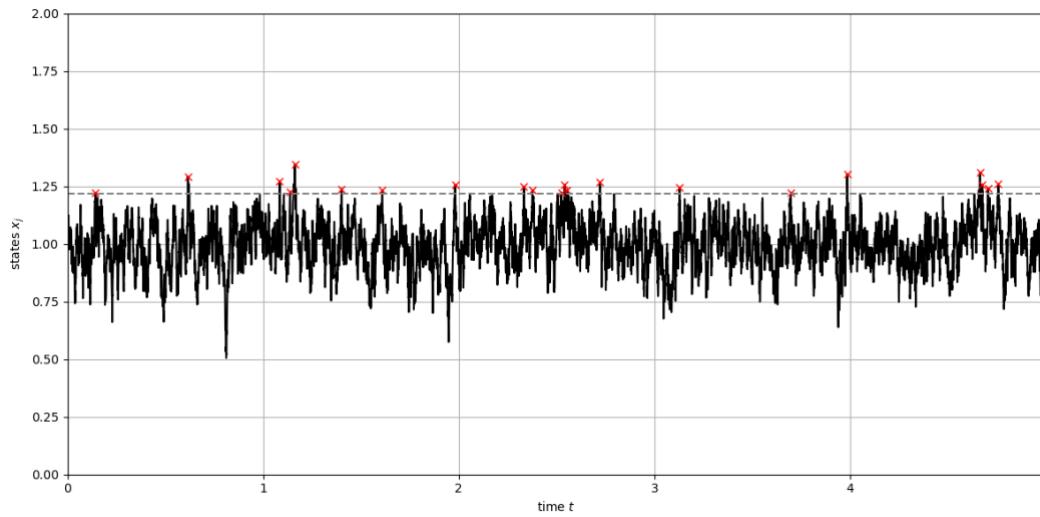
In first 1e5 steps, node 0 reaches its max at around $t = 35.55$

Time series of node 0 & its adjacent nodes in neighborhood of $t = 35.55$:

https://drive.google.com/file/d/1fQD313ho7DmrQsBmWhyej_O4P7Nm8QED/view?usp=sharing

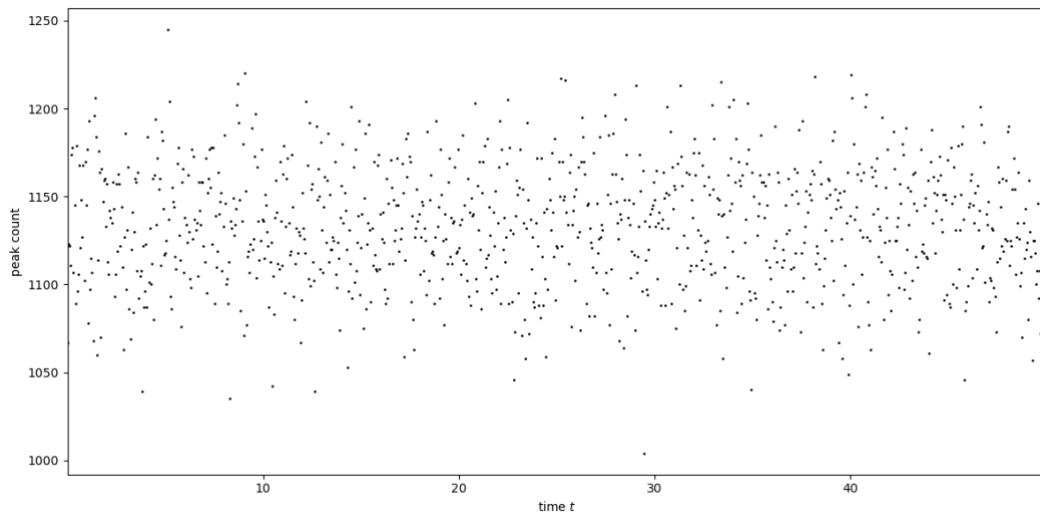
Spike analysis (peak histogram, raster plot)

- Model params: diffusive, ($r_0 = 100, g_{ij} \times 10, \sigma_i = 1.5$)
- $\sigma_i = 1.5$ to magnify spiking behavior
- With-noise case: params
 $(r_0, g_{ij} \times \text{multiplier}, \sigma_i) = (r_0 = 100, g_{ij} \times 10, \sigma_i = 1.5)$
⇒ larger noise to look for more apparent spiking
- [last time] (Visual) spiking detection
In first 1e5 steps, node 0 reaches its max at around $t = 9.9$
Time series of node 0 & its adjacent nodes in neighborhood of $t = 9.9$:
https://drive.google.com/file/d/18MMoLGlHhWUJZ3vGw1XtBM-c7N_4vqJ/view?usp=sharing
(Visual) correlation for node 14,162,223,329,406,4041
- (Numerical) spiking detection
Criteria:
 - Peak value > steady value + ($\alpha = 2$ or 2.5) * s.d. of fluctuations (of all nodes) over $[0, T]$,
s.d. of fluctuations ~ 0.1
 - Time steps btw. peak values $> d = 20$ (avoid multiple-counting for very close peaks)
- $\alpha = 2, d = 20$, node 0 dynamics & peaks over $[0, 5]$

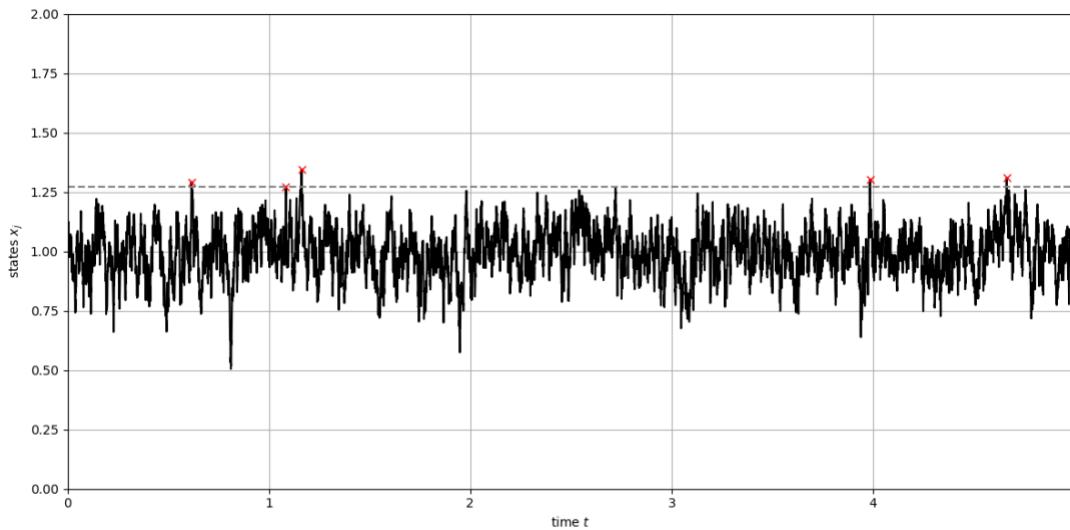


Peak histogram over $[0, 50]$ (i.e. 1e5 steps), 1000 bins

To look for concentrations of peaks but distribution appears to be uniform

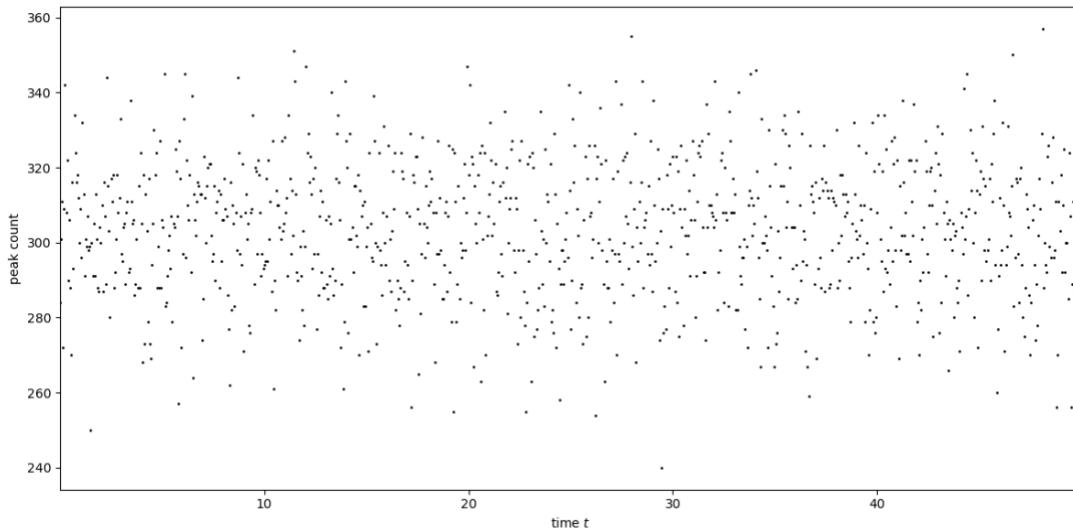


- $\alpha = 2.5, d = 20$, node 0 dynamics & peaks over [0, 5]



Peak histogram over [0, 50] (i.e. 1e5 steps), 1000 bins

To look for concentrations of peaks but distribution appears to be uniform

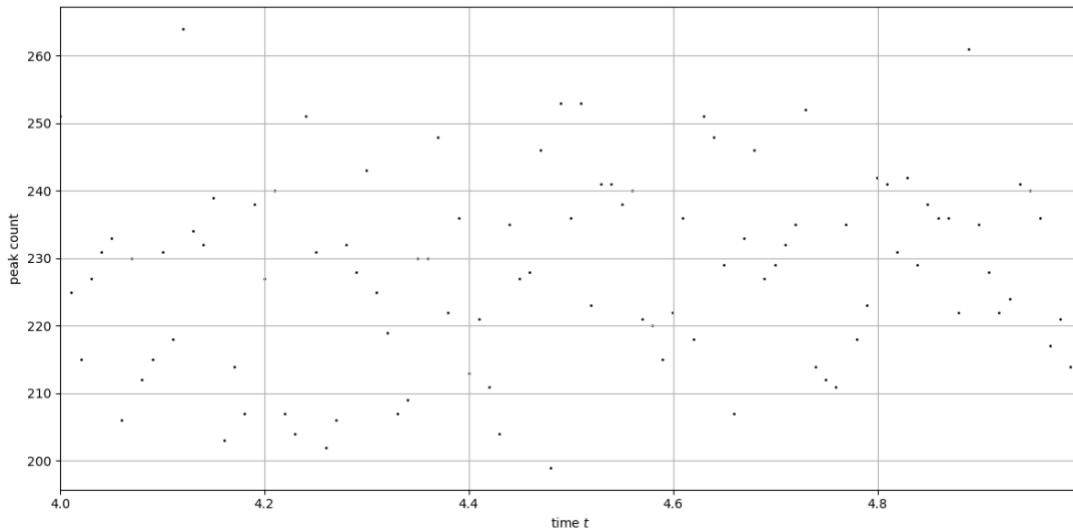


- $\alpha = 2, d = 20$, node 0 to 100 over [4, 5]

<https://drive.google.com/file/d/1W20hD-8YxLiXhi6n7G5I08lmtS4nomNM/view?usp=sharing>

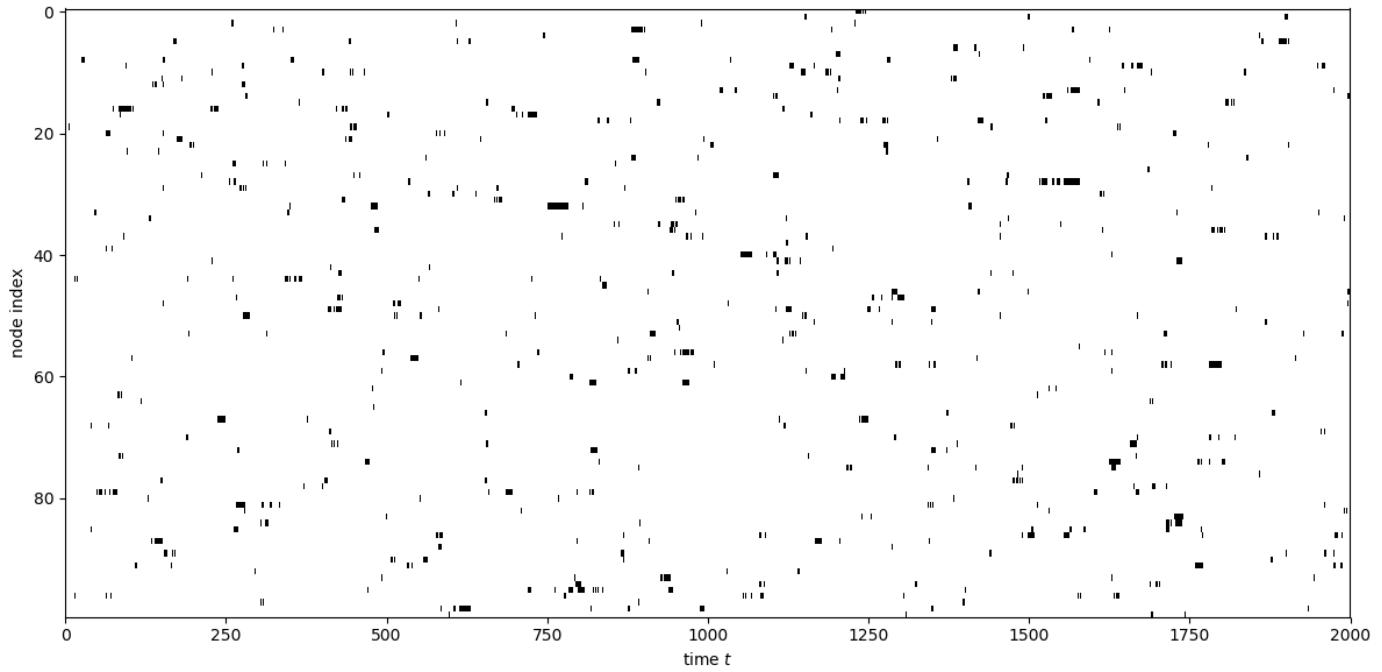
To visually inspect correlated spiking activities

Peak histogram over [4, 5] (i.e. 2e3 steps), 100 bins

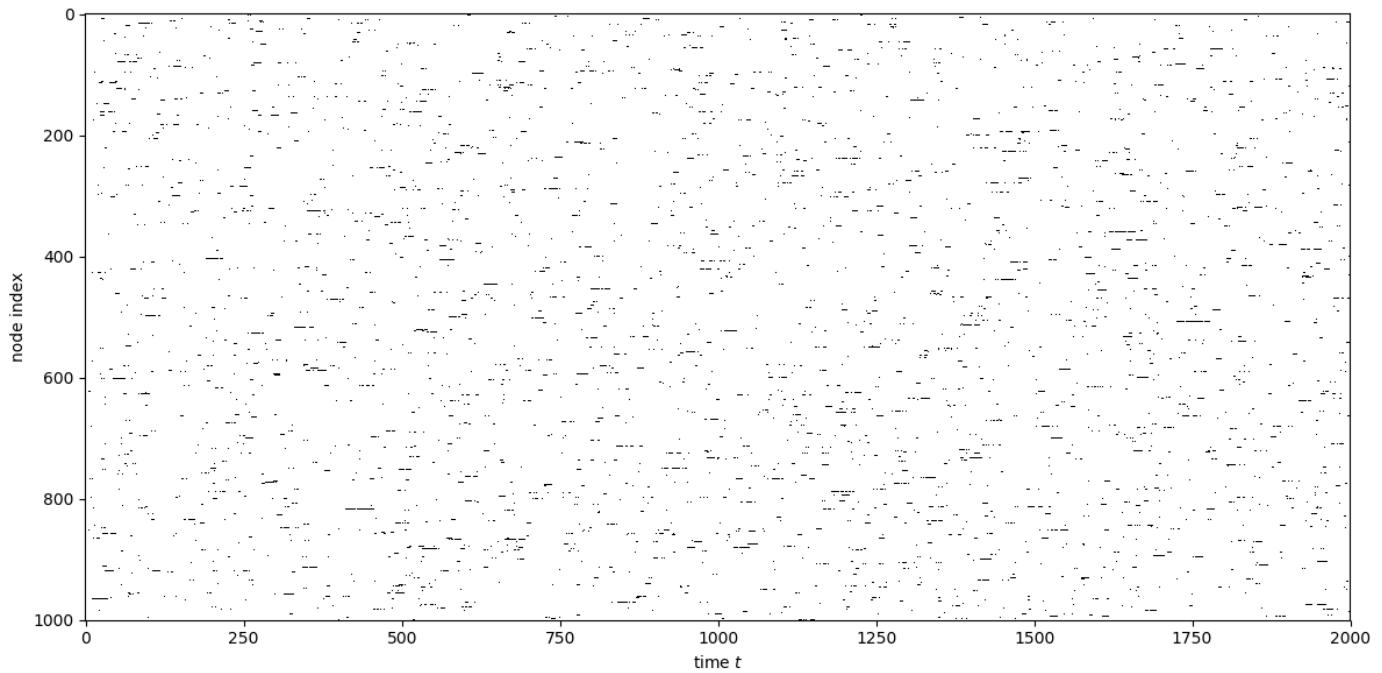


- Raster plot: $\alpha = 2, d = 0$ (allow “continuous” peaks)

Node 0 to 100, 2e3 steps

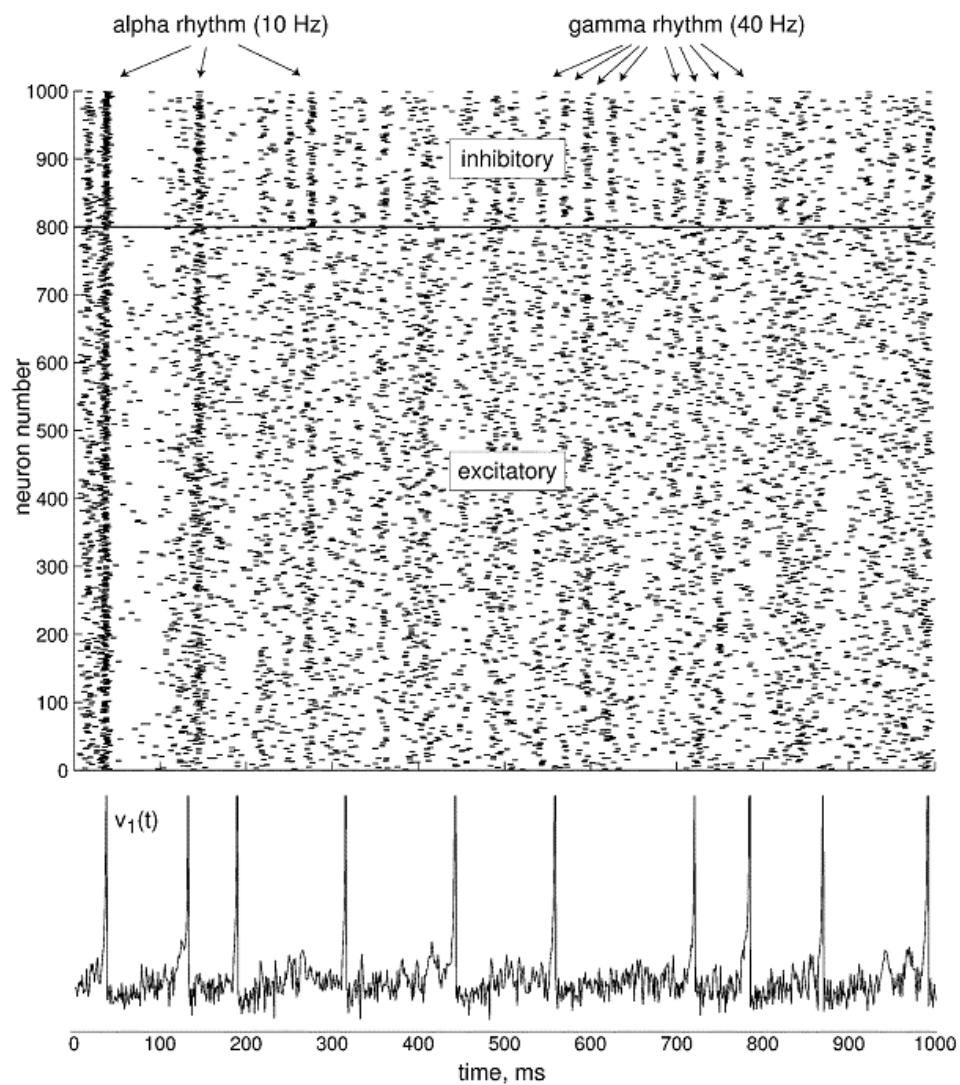


Node 0 to 1000, 2e3 steps



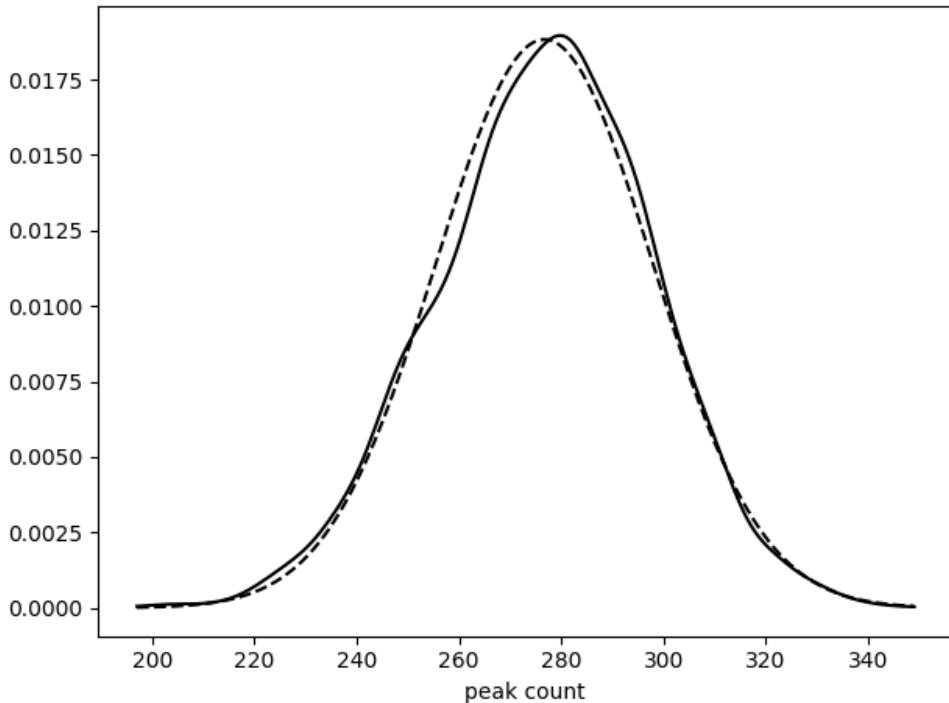
- **Neuron raster plot found on internet:**

Not similar ...



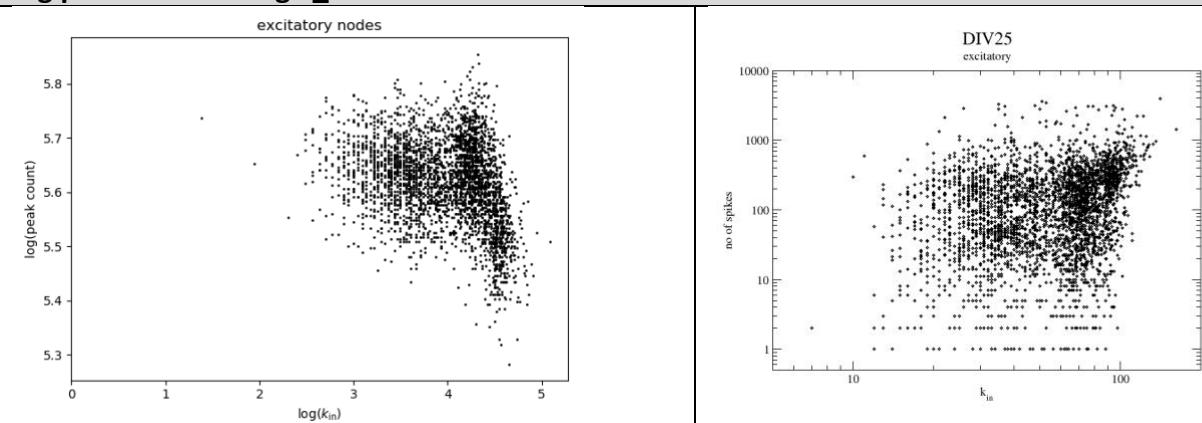
Spike analysis (peak distribution, peak vs strength plot)

- Model params: diffusive, ($r_0 = 100$, $g_{ij} \times 10$, $\sigma_i = 1.5$)
- Draw comparison btw. simulation data & experimental data
- (Continuing from last time) With-noise case: params
 $(r_0, g_{ij} \times \text{multiplier}, \sigma_i) = (r_0 = 100, g_{ij} \times 10, \sigma_i = 1.5)$
⇒ larger noise to look for more apparent spiking
- (Continuing from last time) (Numerical) spiking detection
Criteria:
 - Peak value > steady value + ($\alpha = 2$ or 2.5) * s.d. of fluctuations (of all nodes) over $[0, T]$,
s.d. of fluctuations ~ 0.1
 - Time steps btw. peak values $> d = 20$ (avoid multiple-counting for very close peaks)
- Note for the following plots:
 - Peak criteria: $\alpha = 2, d = 20$
 - Number of peaks range from 200 to 340 (1e5 steps)
- Distribution of number of peaks (near normal)

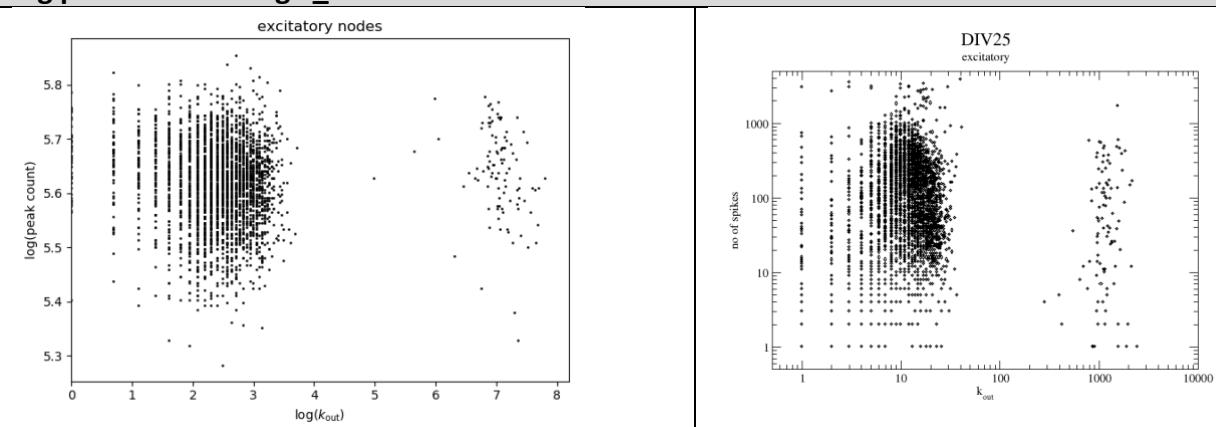


- **Excitatory nodes**

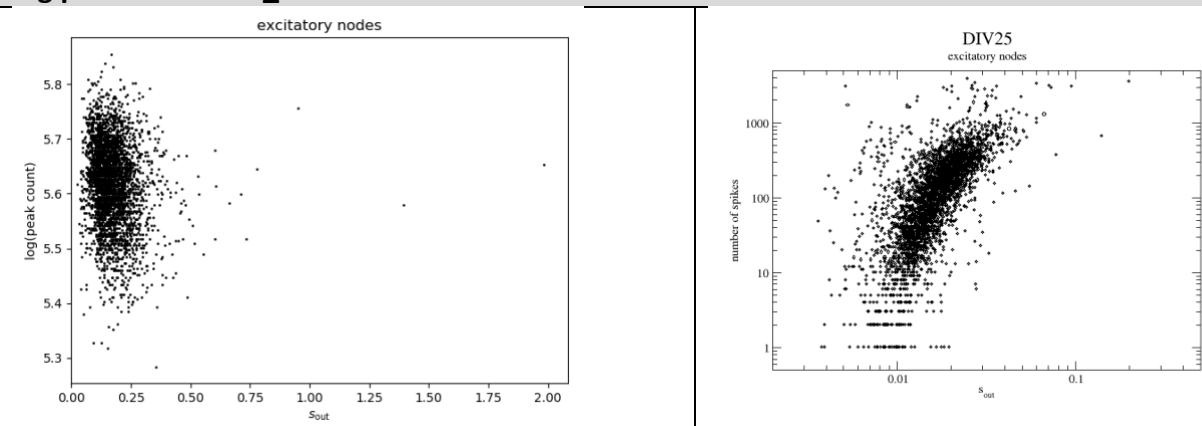
log peak count vs log k_in



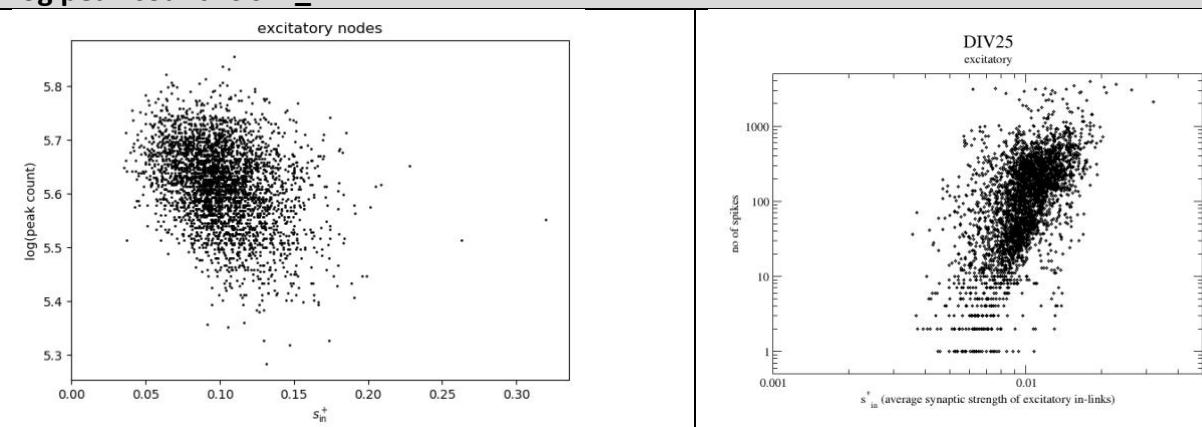
log peak count vs log k_out



log peak count vs s_out

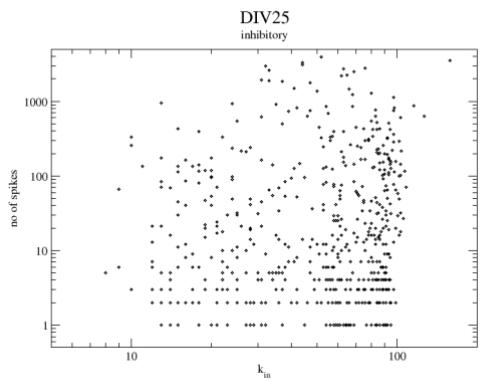
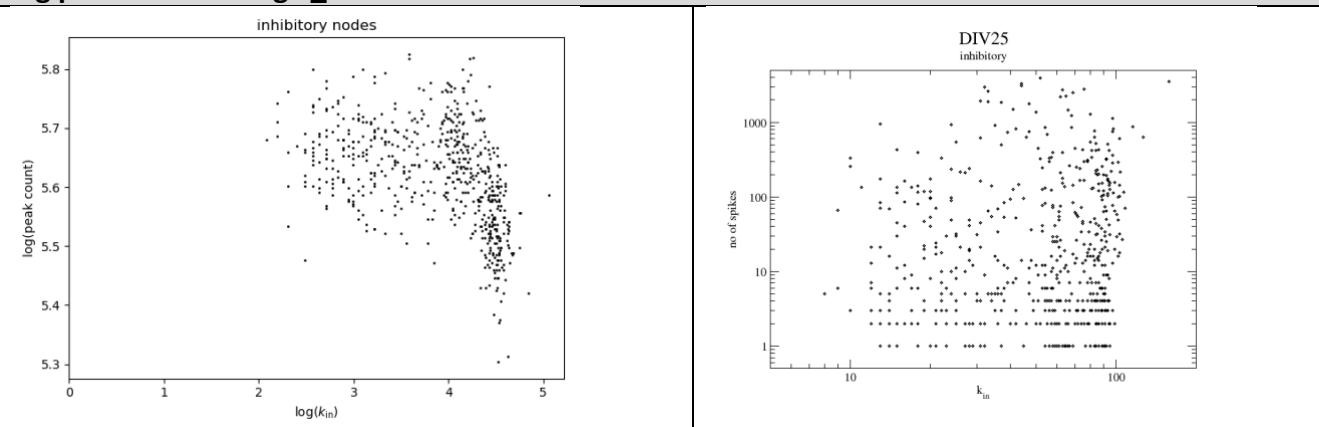


log peak count vs s^+_in

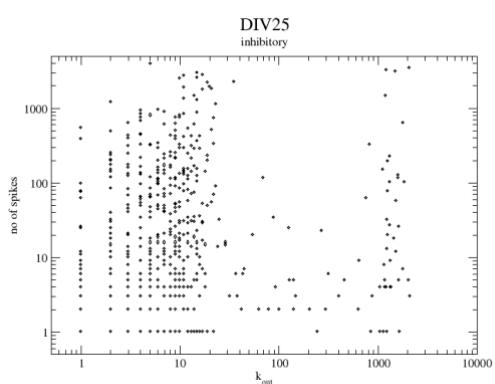
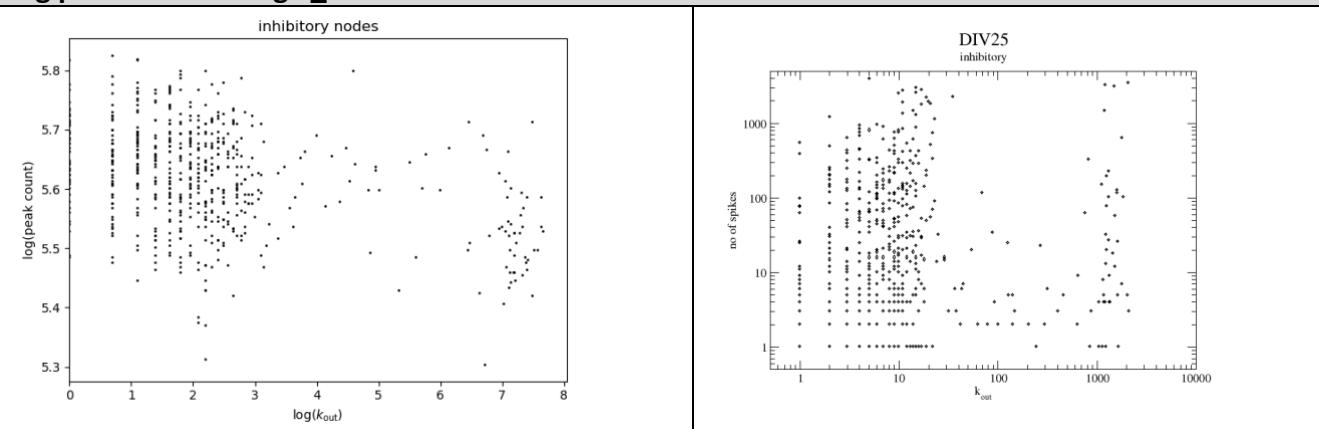


- Inhibitory nodes

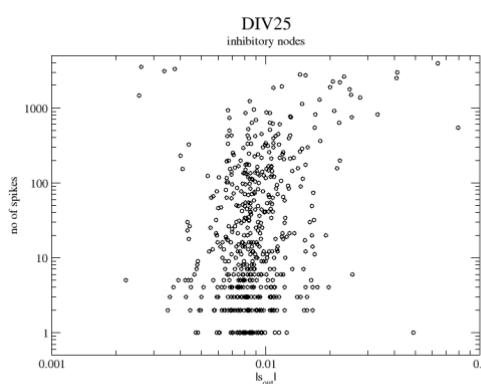
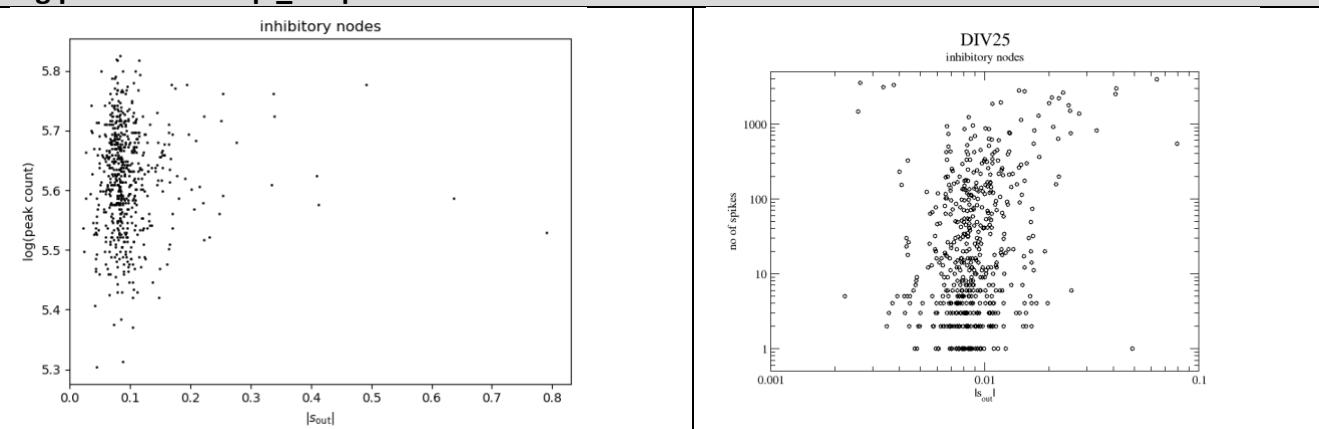
log peak count vs log k_in



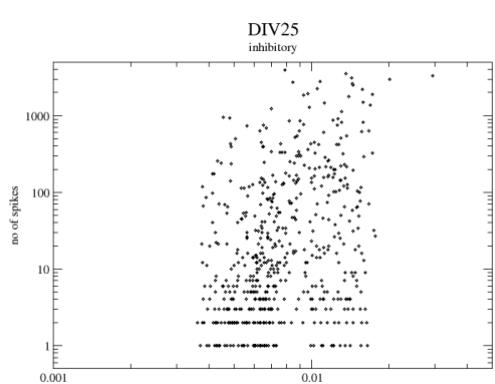
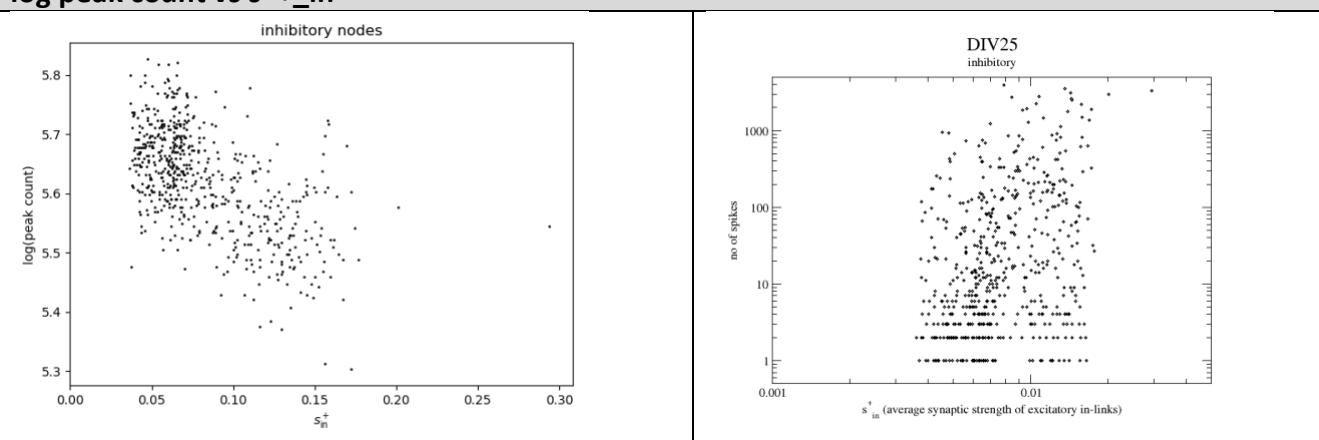
log peak count vs log k_out



log peak count vs |s_out|

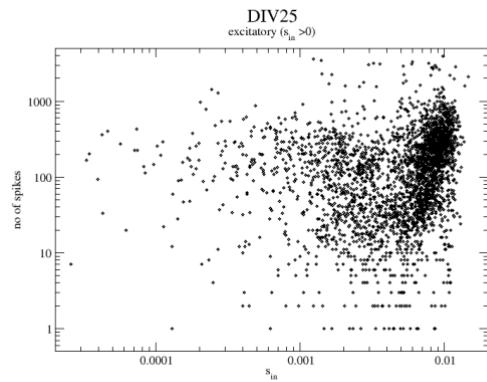
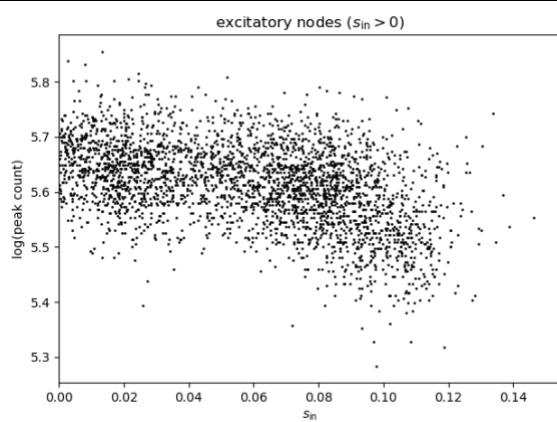


log peak count vs s^+_in

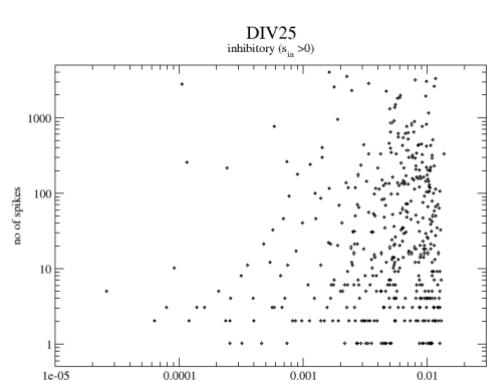
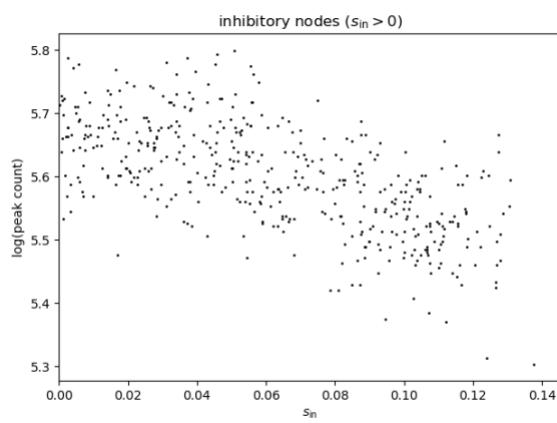


- Mixed (x-axis: s_{in} or $|s_{in}|$)**

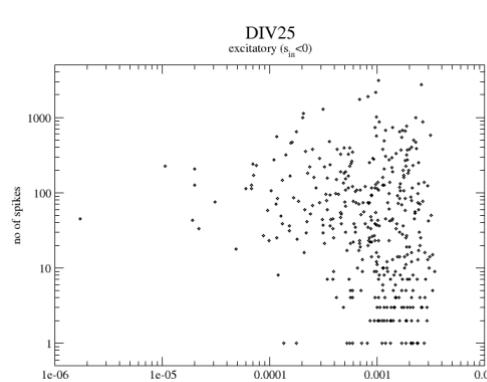
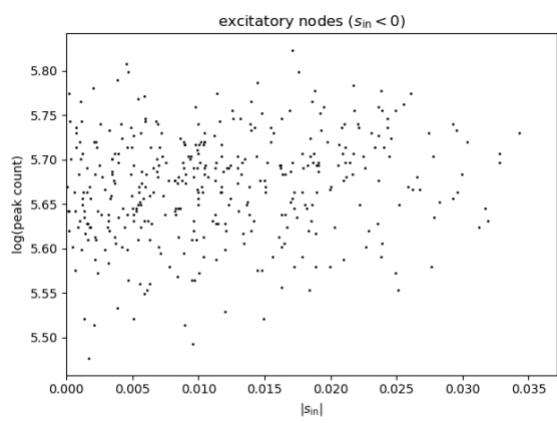
$s_{in} > 0 \& s_{out} > 0$



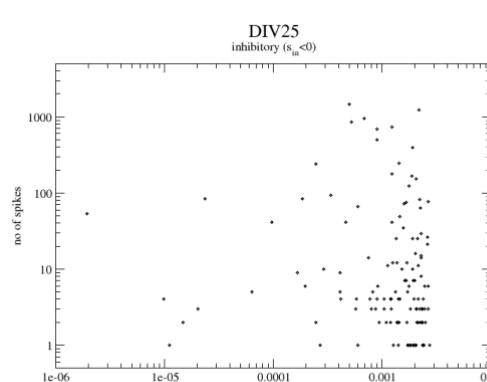
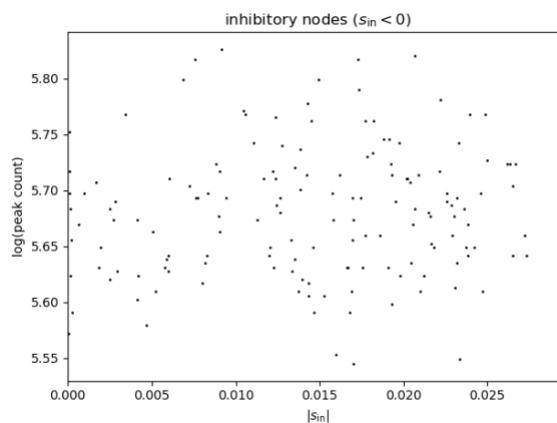
$s_{in} > 0 \& s_{out} < 0$



$s_{in} < 0 \& s_{out} > 0$

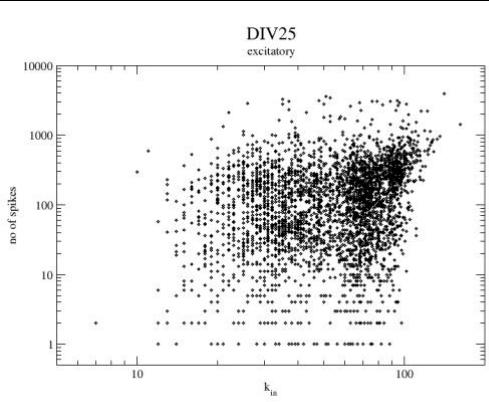
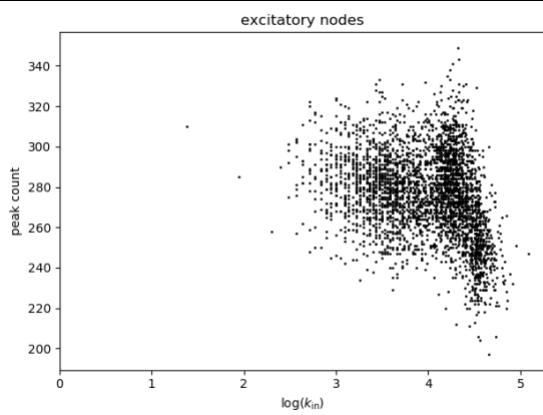


$s_{in} < 0 \& s_{out} < 0$

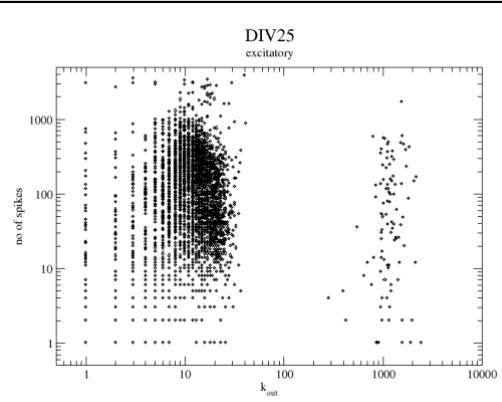
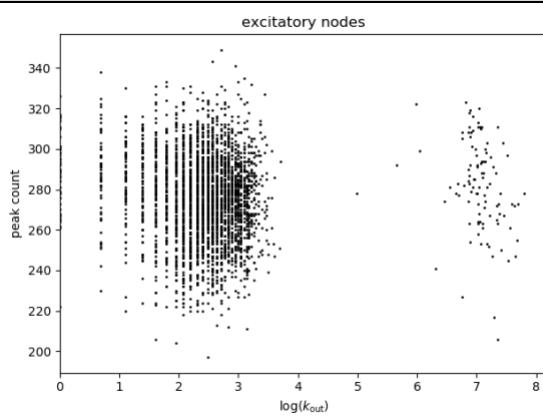


- **Excitatory nodes (log not taken for y-axis)**

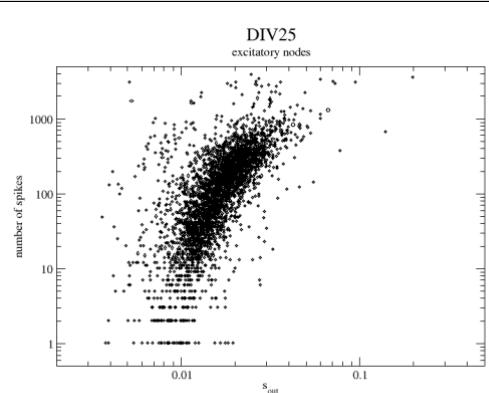
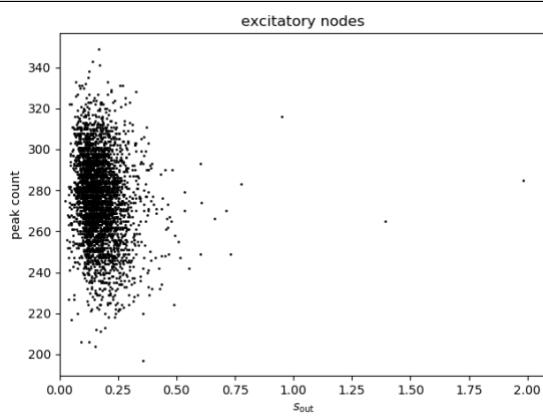
peak count vs log k_in



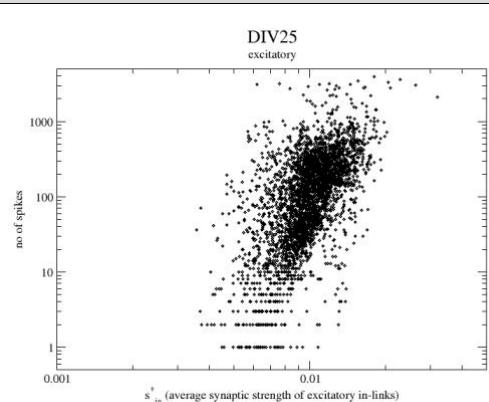
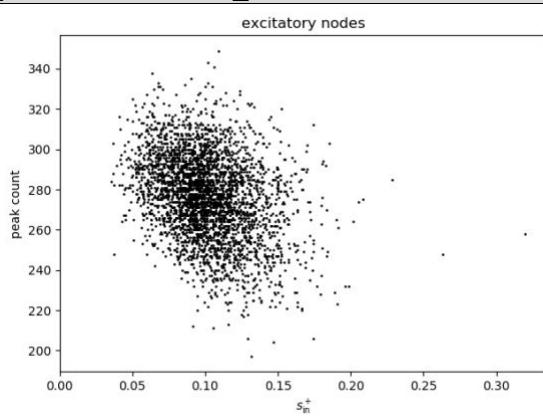
peak count vs log k_out



peak count vs s_out

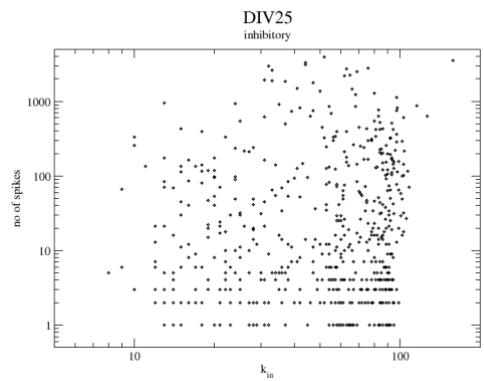
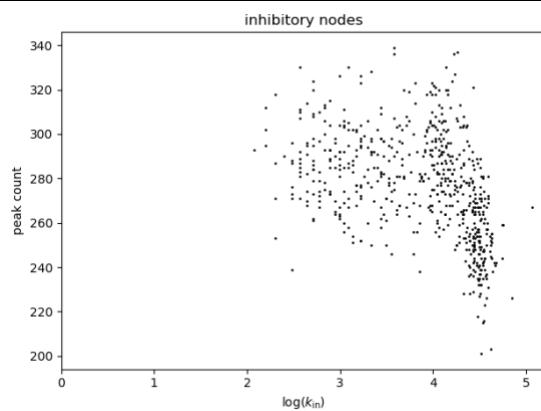


peak count vs s^+_in

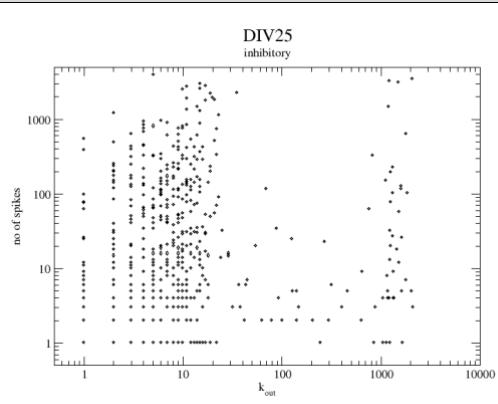
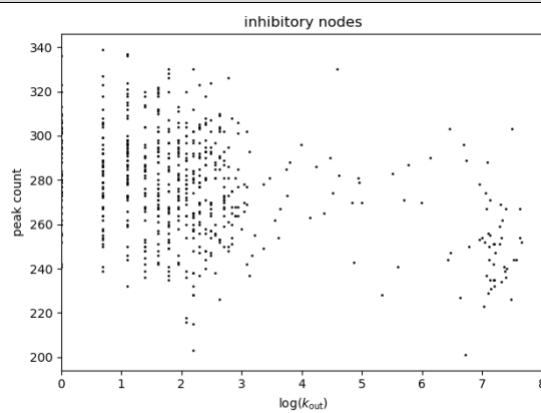


- **Inhibitory nodes (log not taken for y-axis)**

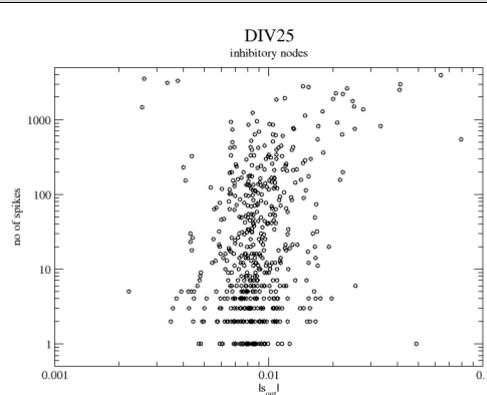
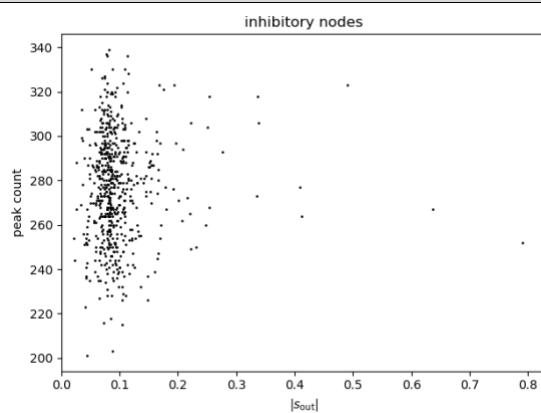
peak count vs log k_in



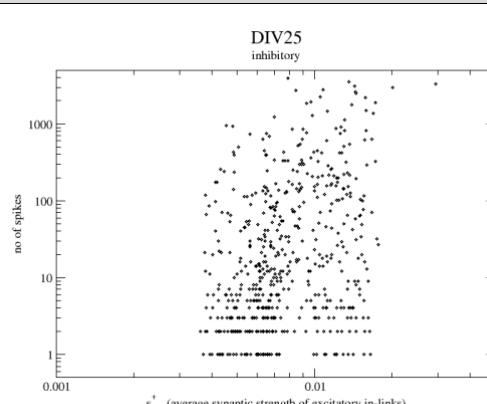
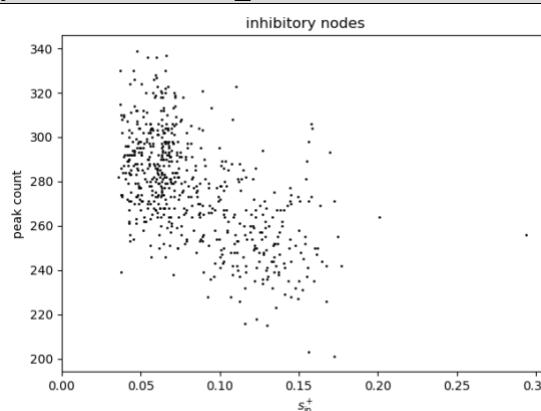
peak count vs log k_out



peak count vs |s_out|

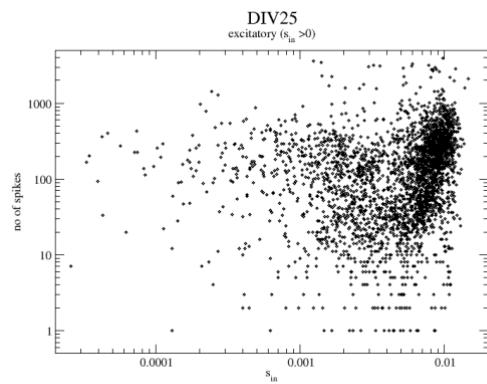
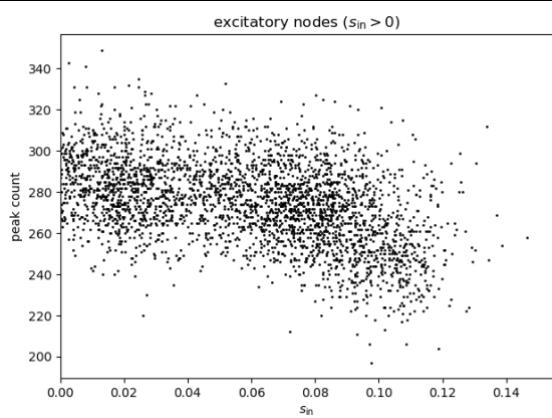


peak count vs s^+_in

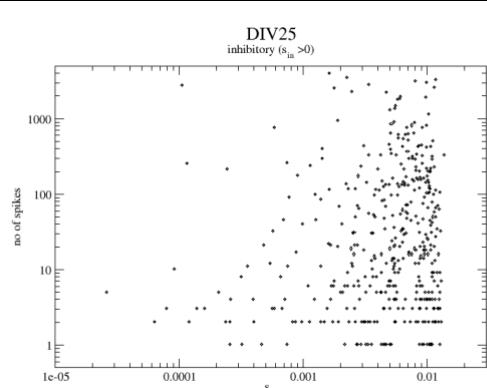
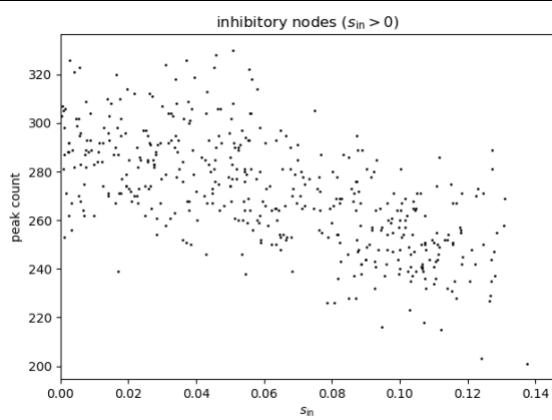


- Mixed (x-axis: s_{in}) (log not taken for y-axis)**

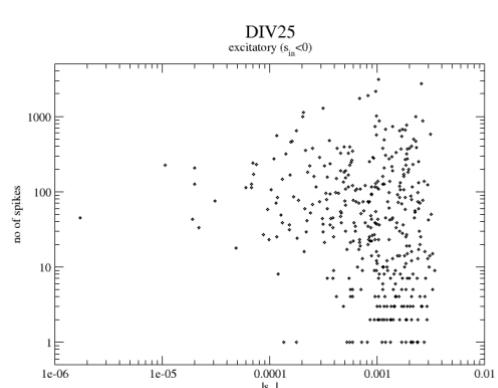
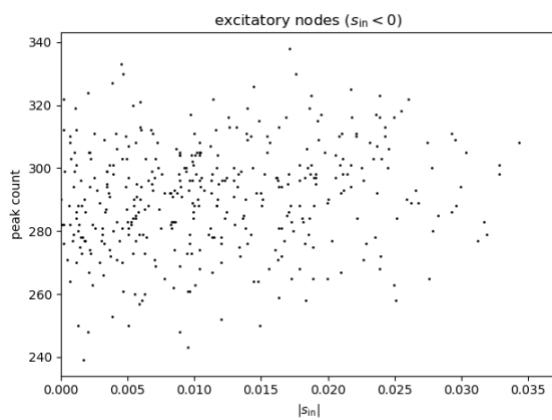
$s_{in} > 0 \& s_{out} > 0$



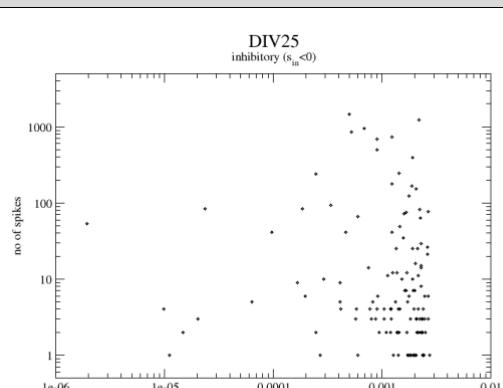
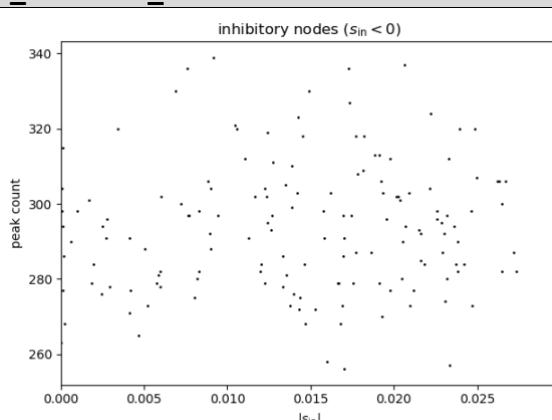
$s_{in} > 0 \& s_{out} < 0$



$s_{in} < 0 \& s_{out} > 0$



$s_{in} < 0 \& s_{out} < 0$

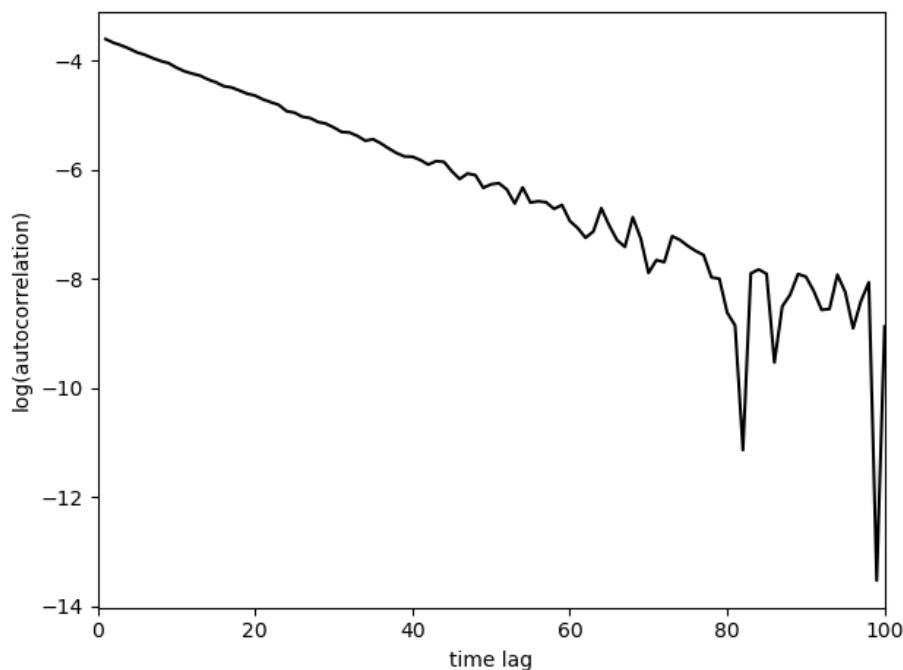
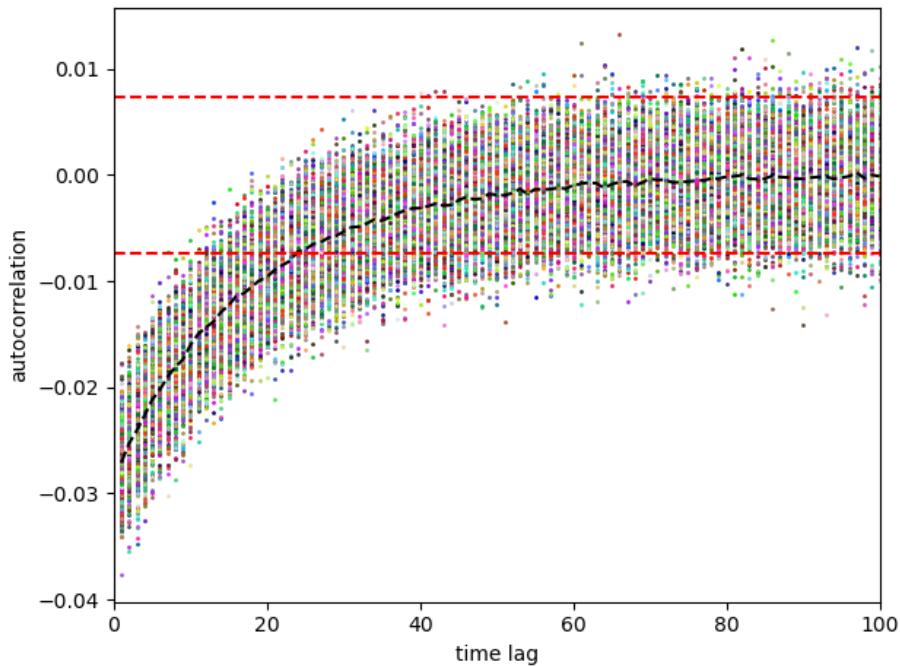


Autocorrelation analysis

- This disproves that spiking are due entirely to white noise

($r_0 = 100, g_{ij} \times 10, \sigma_i = 1.5$)

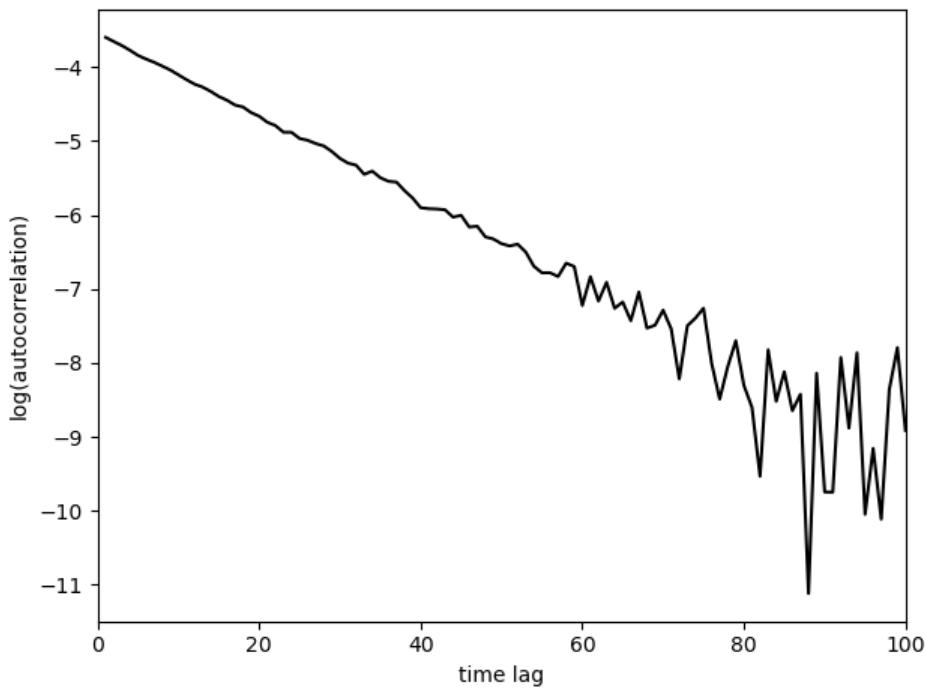
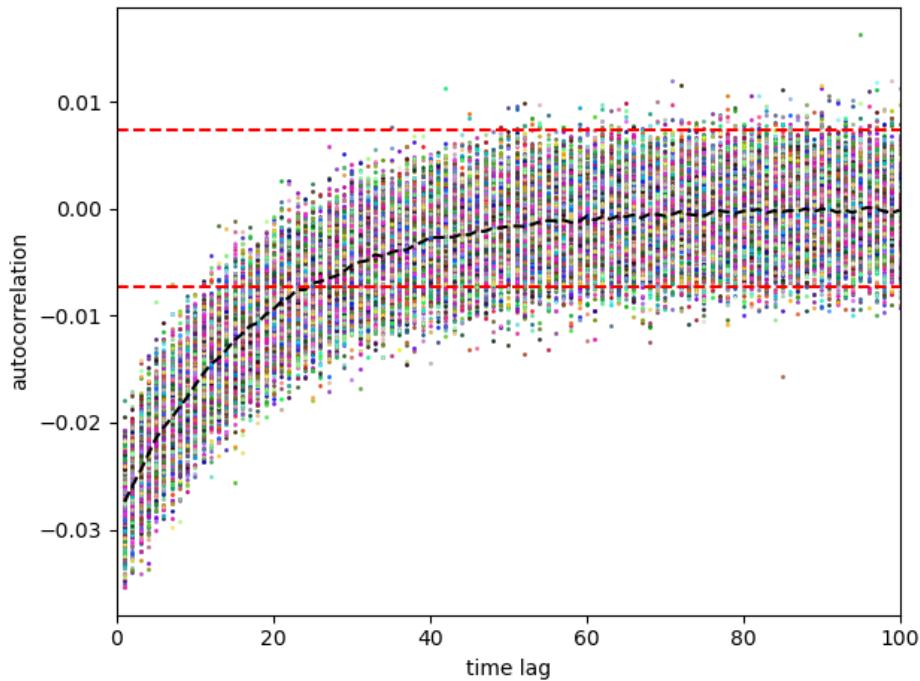
- Autocorrelation of dx_i (node 0 to 500)
 - Statistically significant but **not strong**
 - **Red dotted lines** ($\pm 2.33/\sqrt{T}$): 98% likelihood bound (for a pure white noise process)
 - **Black dotted line**: mean of autocorrelations over all nodes
 - Autocorrelation $< 0 \because x(1 - x)$ term makes time series mean-reverting
 - Autocorrelation $\sim 10^{-2}$, only a small multiple larger than the likelihood bound \rightarrow autocorrelation not strong
 - Autocorrelation obeys an exponential decay \rightarrow ARMA process, makes sense $\because dx_i$ is a linear combination of node values



$$(r_0 = 100, g_{ij} \times 10, \sigma_i = 0.5)$$

- Autocorrelation of dx_i (node 0 to 500)

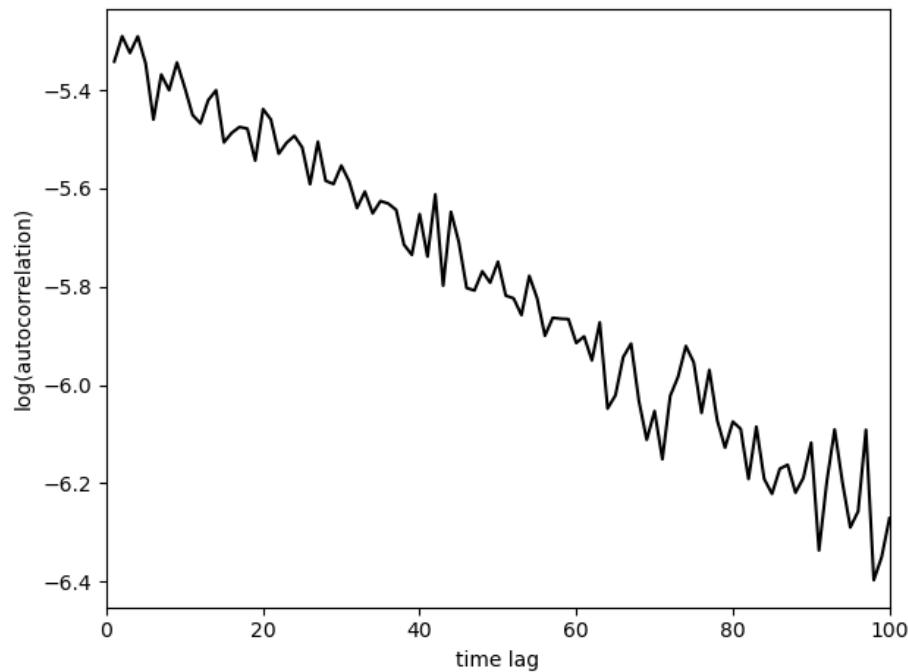
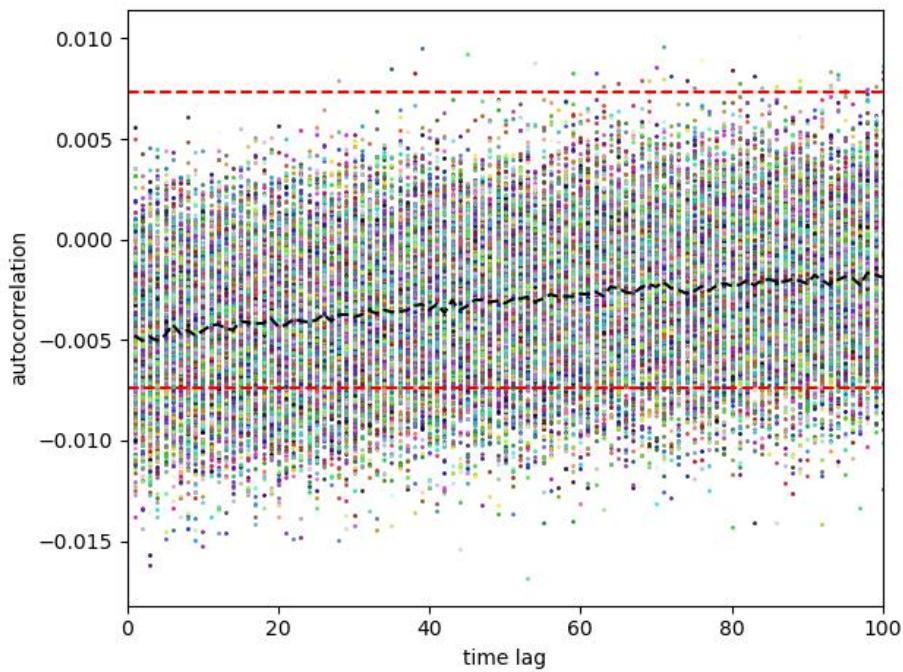
- Here smaller $\sigma_i = 0.5$ is used
- Autocorrelation plot has similar shape & values compared to $\sigma_i = 1.5$ case
- **Original guess:** large noise “fuzzes out” dependence of time series (and peaks) on g_{ij} hence making autocorrelation small; but it turns out noise doesn’t affect autocorrelation at all (maybe because of the averaging when computing autocorrelation)
- **Guess:** autocorrelation is small because g_{ij} is small



$$(r_0 = 10, g_{ij} \times 10, \sigma_i = 0.25)$$

- Autocorrelation of dx_i (node 0 to 500)

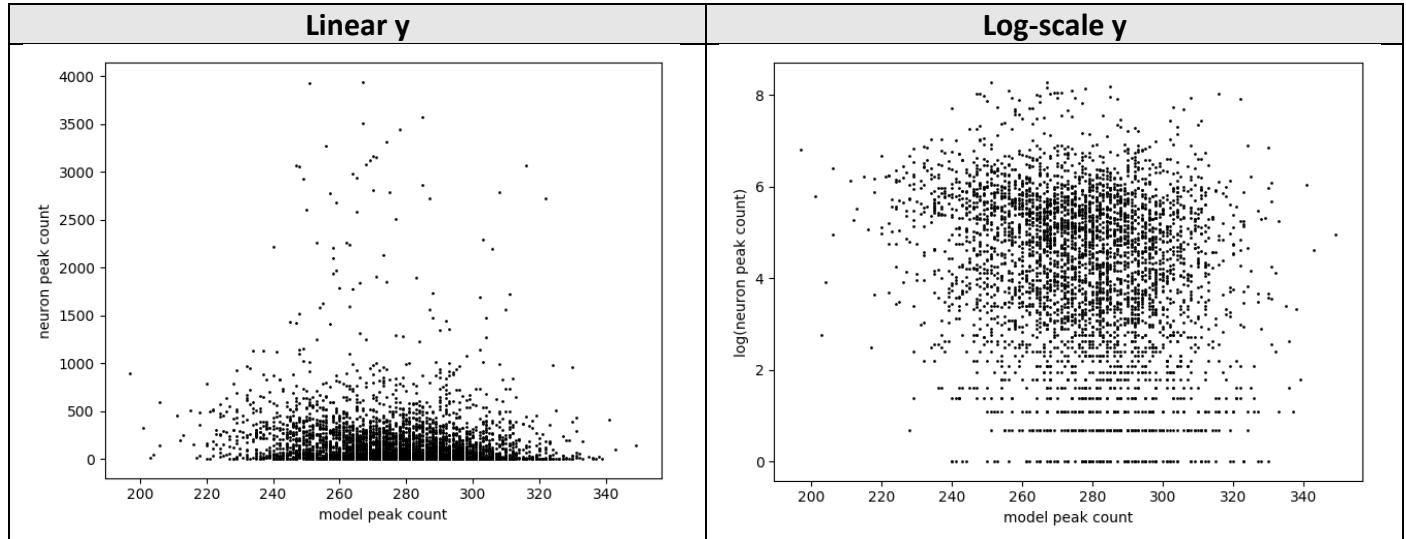
- Here smaller $r_0 = 10$ is used
- Autocorrelation becomes an order smaller (e.g. $-0.03 \rightarrow -0.005$)



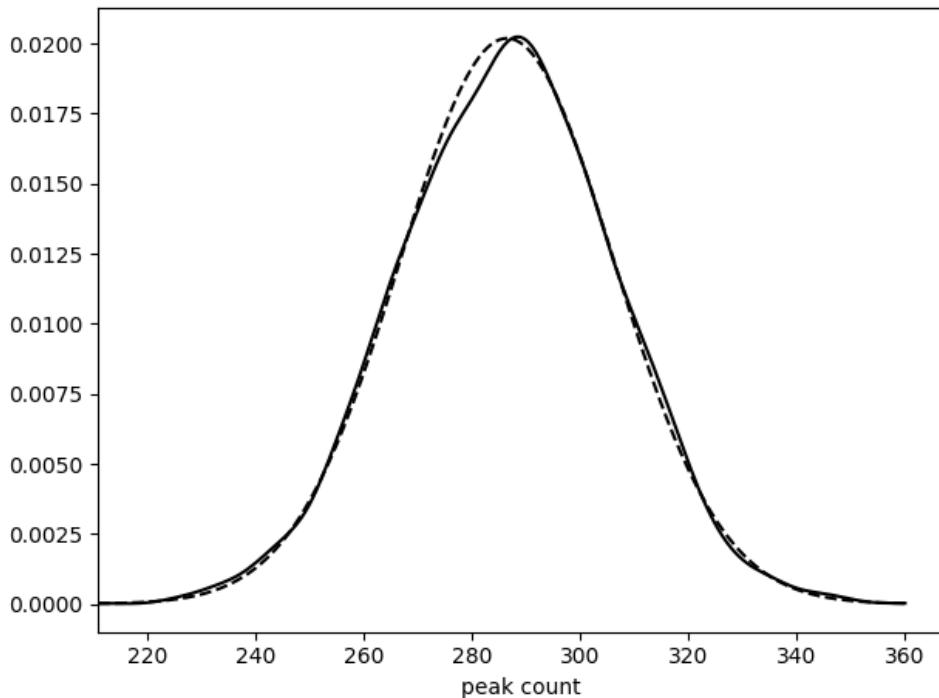
Spike analysis (Gaussian reference network)

- Model params: diffusive, ($r_0 = 100$, $g_{ij} \times 10$, $\sigma_i = 1.5$)
- Draw comparison btw. simulation data & reference data (reference Gaussian network)

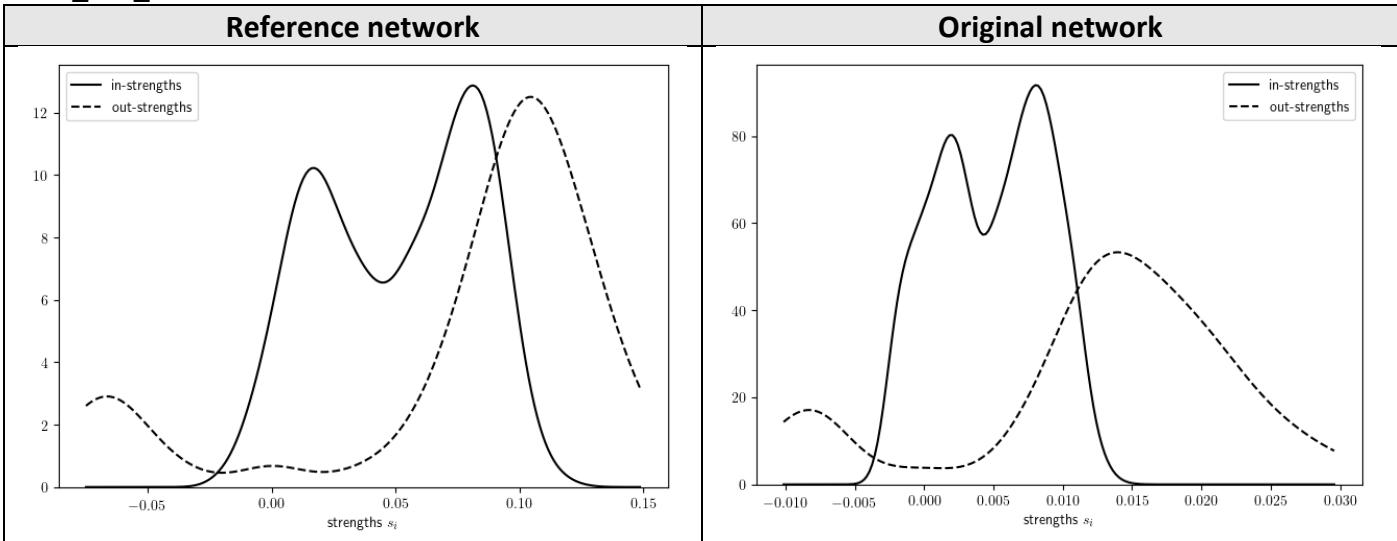
- Model peaks vs experimentally measured peaks



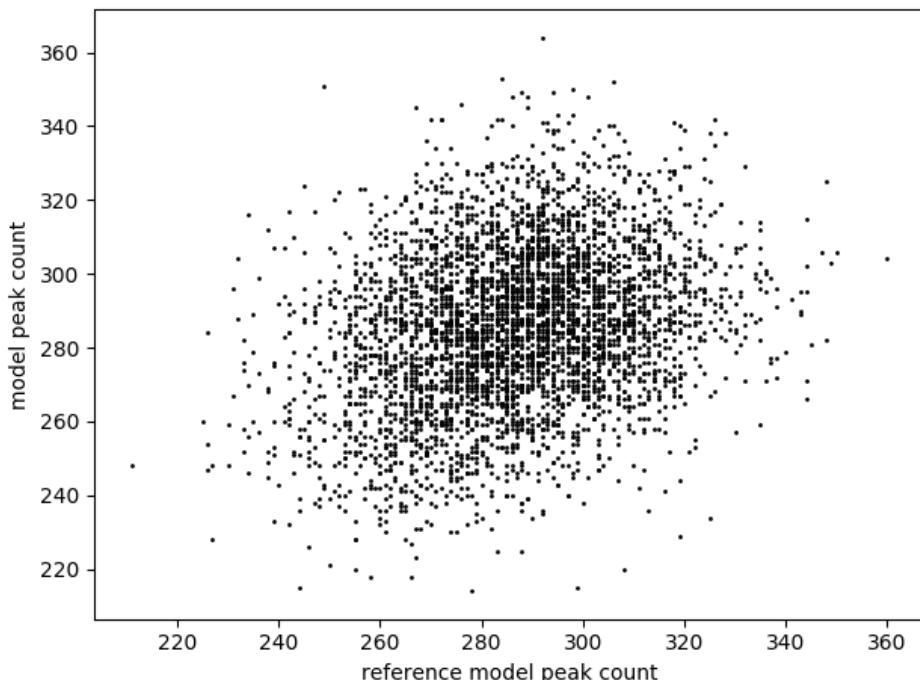
- **With-noise case: params**
 $(r_0, g_{ij} \times \text{multiplier}, \sigma_i) = (r_0 = 100, g_{ij} \times 10, \sigma_i = 1.5)$
- **Question to address:** do heavy-tailed strengths explain any behaviour of peaks?
- **Reference network with Gaussian couplings**
 - (1) Calculate the mean and standard deviation of all positive g_{ij} (μ_1 , σ_1) and all negative g_{ij} (μ_2 , σ_2).
 - (2) For every link with a positive g_{ij} , replace it from a value randomly picked from a Gaussian distribution of mean μ_1 and standard deviation σ_1 but with the additional requirement that it has to be positive.
 - (3) Similarly, for every link with a negative g_{ij} , replace it from a value randomly picked from a Gaussian distribution of mean μ_2 and standard deviation σ_2 but with the additional requirement that it has to be negative.
- **(Continuing from last time) (Numerical) spiking detection**
Criteria:
 - (a) Peak value > steady value + $(\alpha = 2 \text{ or } 2.5) * \text{s.d. of fluctuations}$ (of all nodes) over $[0, T]$,
s.d. of fluctuations ~ 0.1
 - (b) Time steps btw. peak values $> d = 20$ (avoid multiple-counting for very close peaks)
- **Distribution of number of peaks (near normal)**



- s_{in} s_{out} distribution



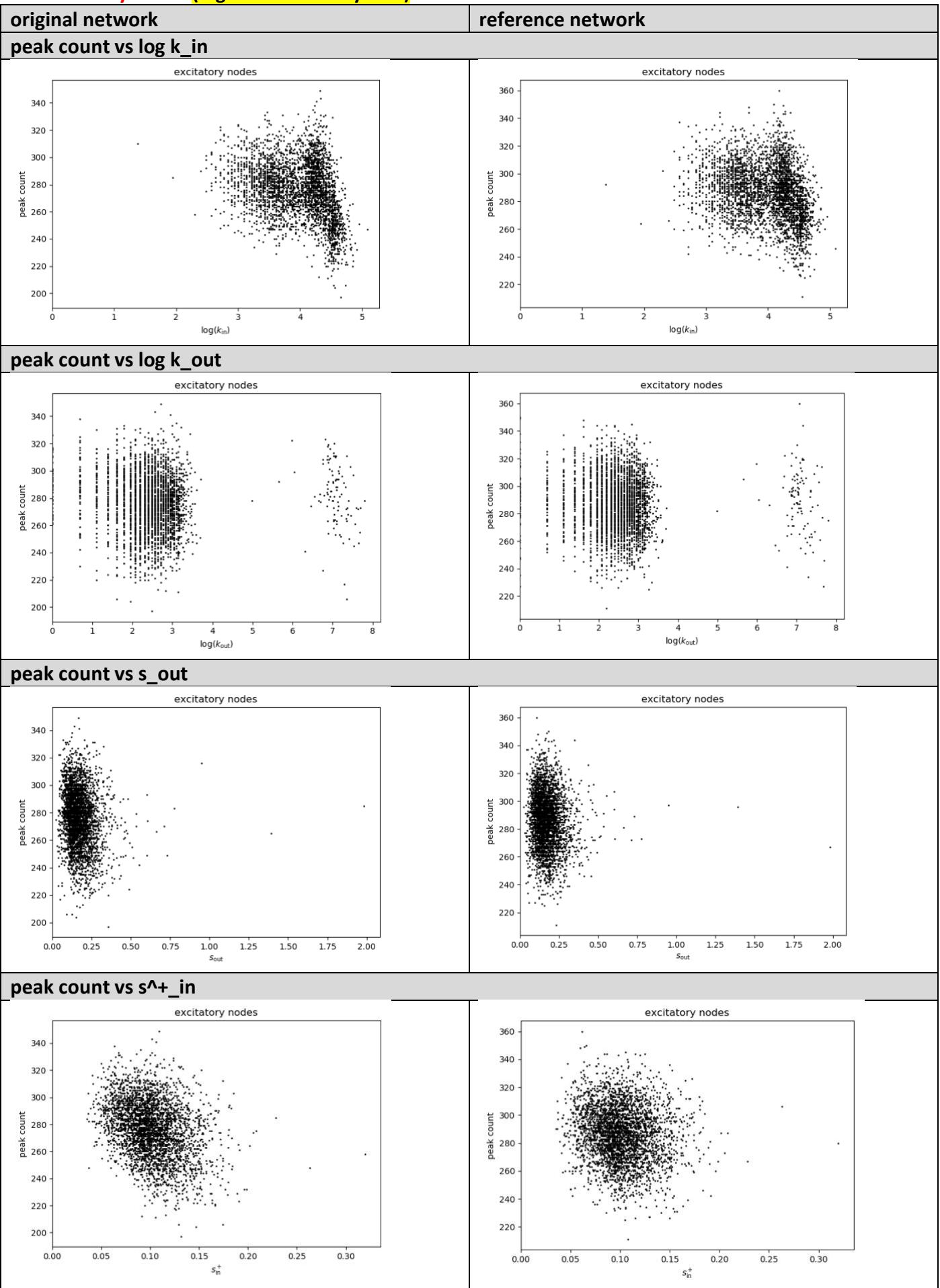
- How peaks in reference network correlate with original network (near linear, similar range)



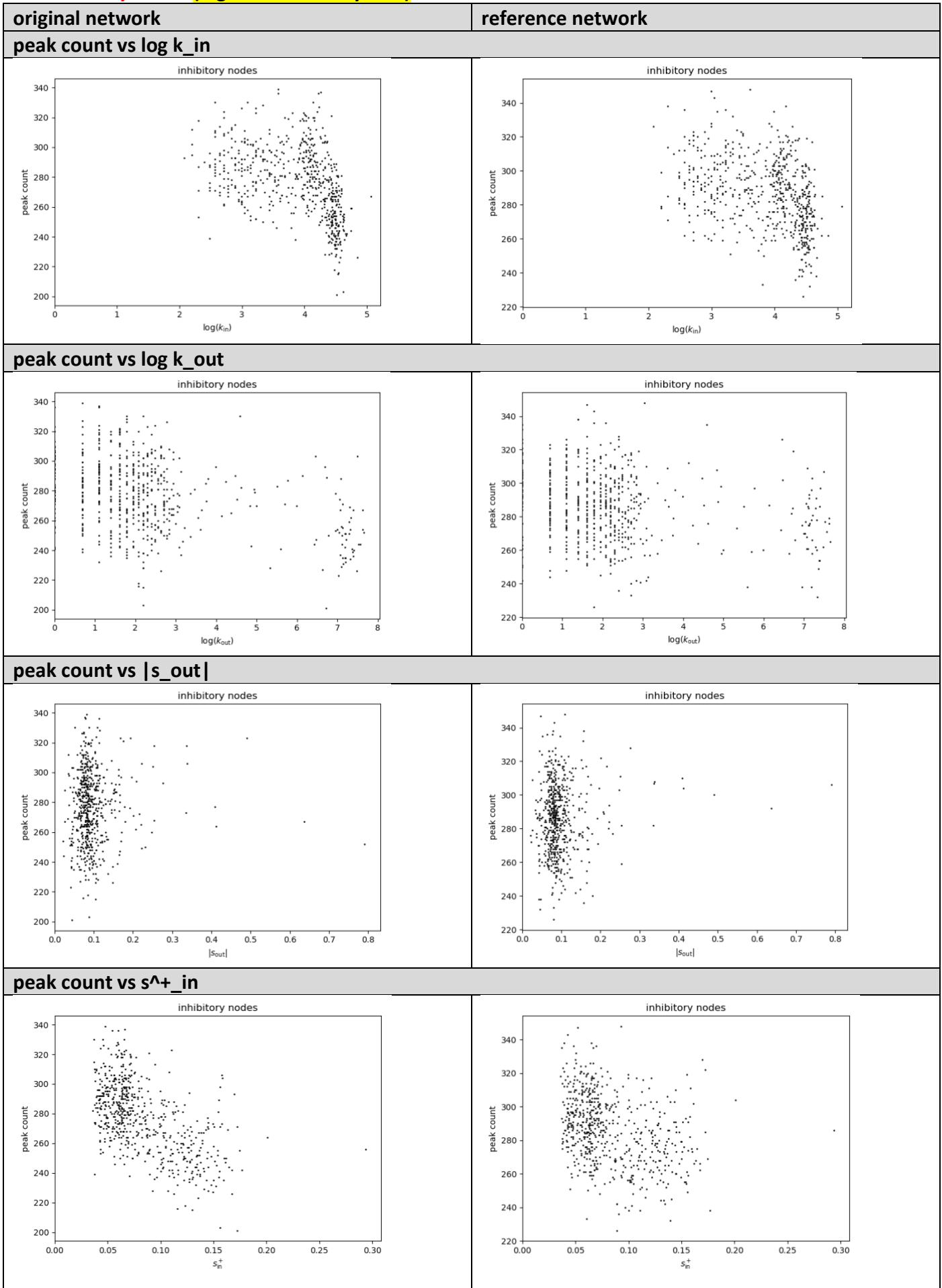
- Note for the following plots:

- Peak criteria: $\alpha = 2, d = 20$
- Number of peaks range from 200 to 360 (**1e5 steps**)
- Left column: original network
Right column: reference network

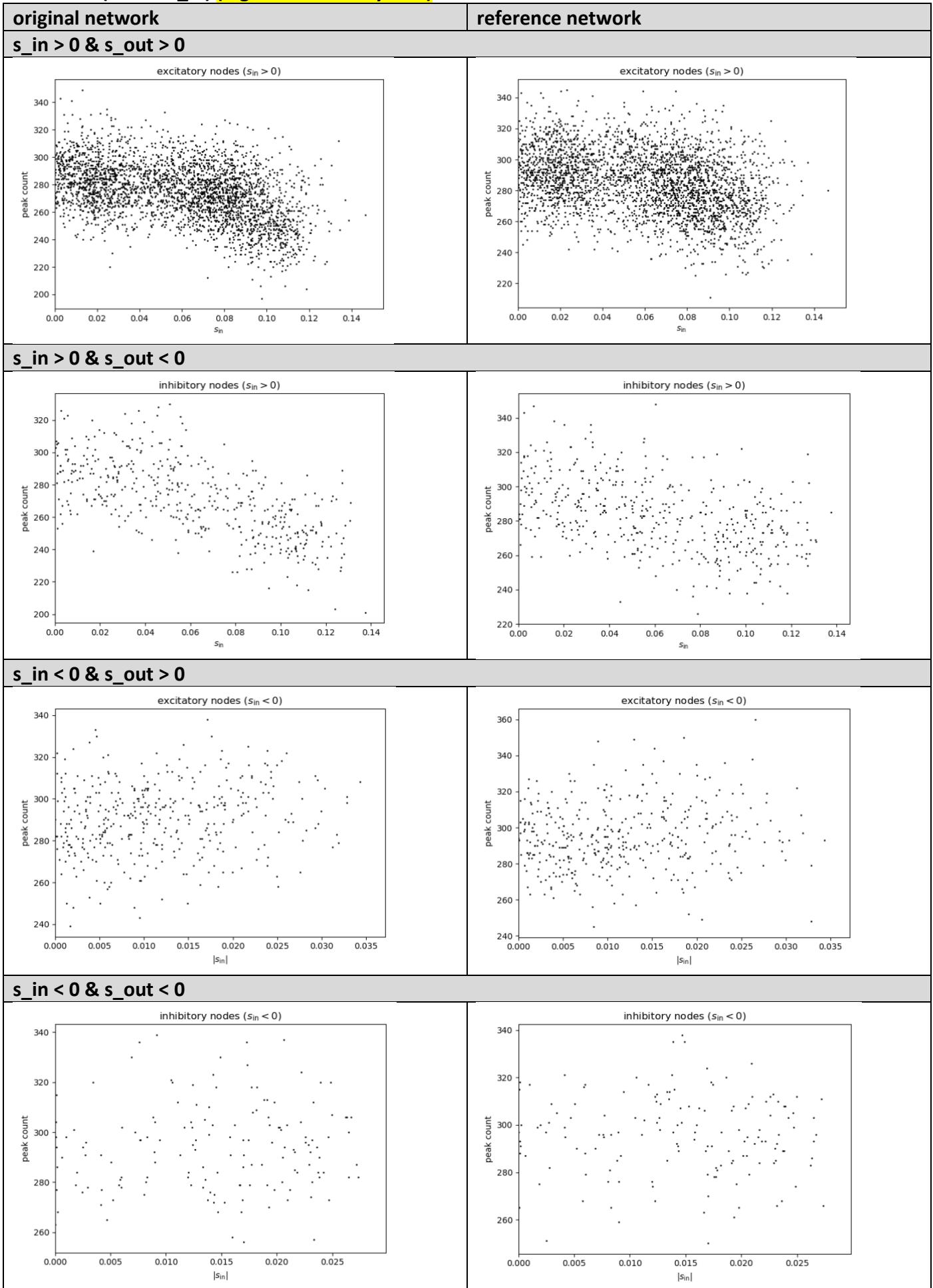
- **Excitatory nodes (log not taken for y-axis)**



- **Inhibitory nodes (log not taken for y-axis)**



- Mixed (x-axis: s_{in}) (log not taken for y-axis)

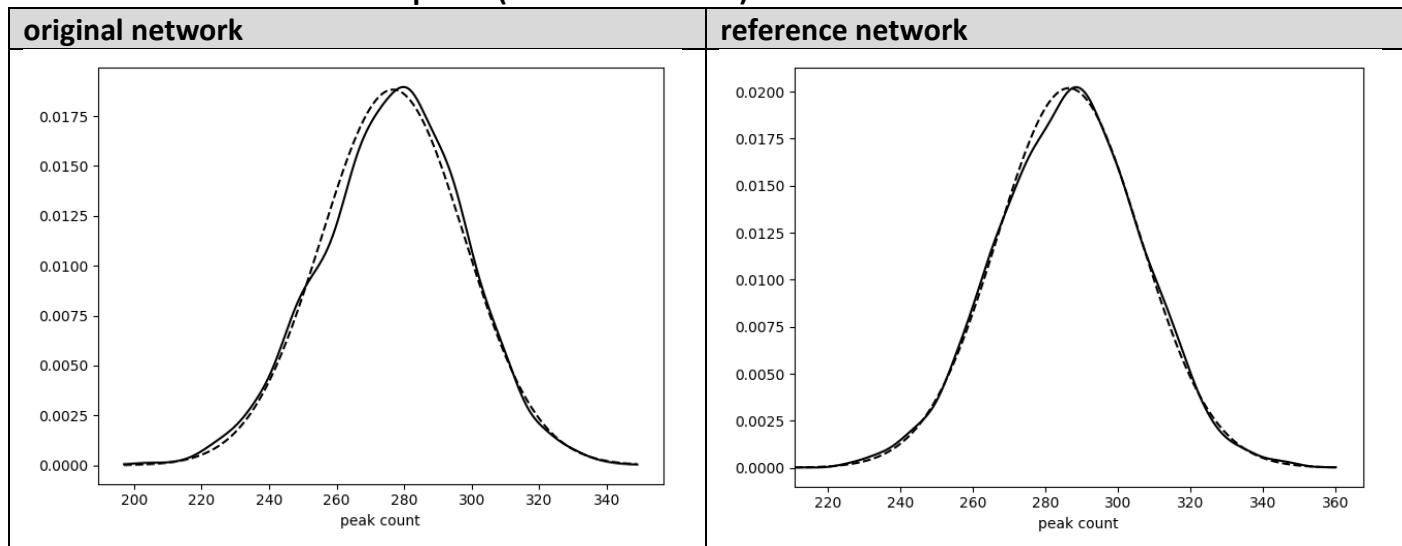


- **Guess**

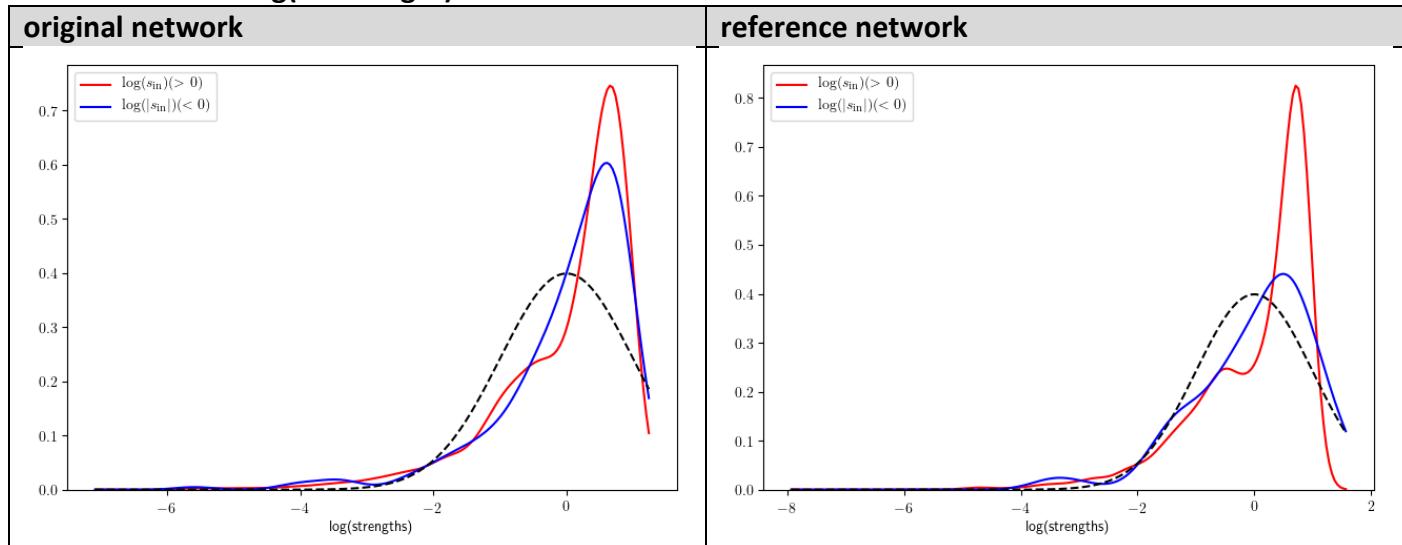
- (1) Noise is too large \Rightarrow network structure (g_{ij}) has little to do with the noise & peaks are completely due to random Gaussian white noise, which explains why peak counts are Gaussian
- (2) But choosing smaller noise makes peaks less well-defined

Reference network analysis

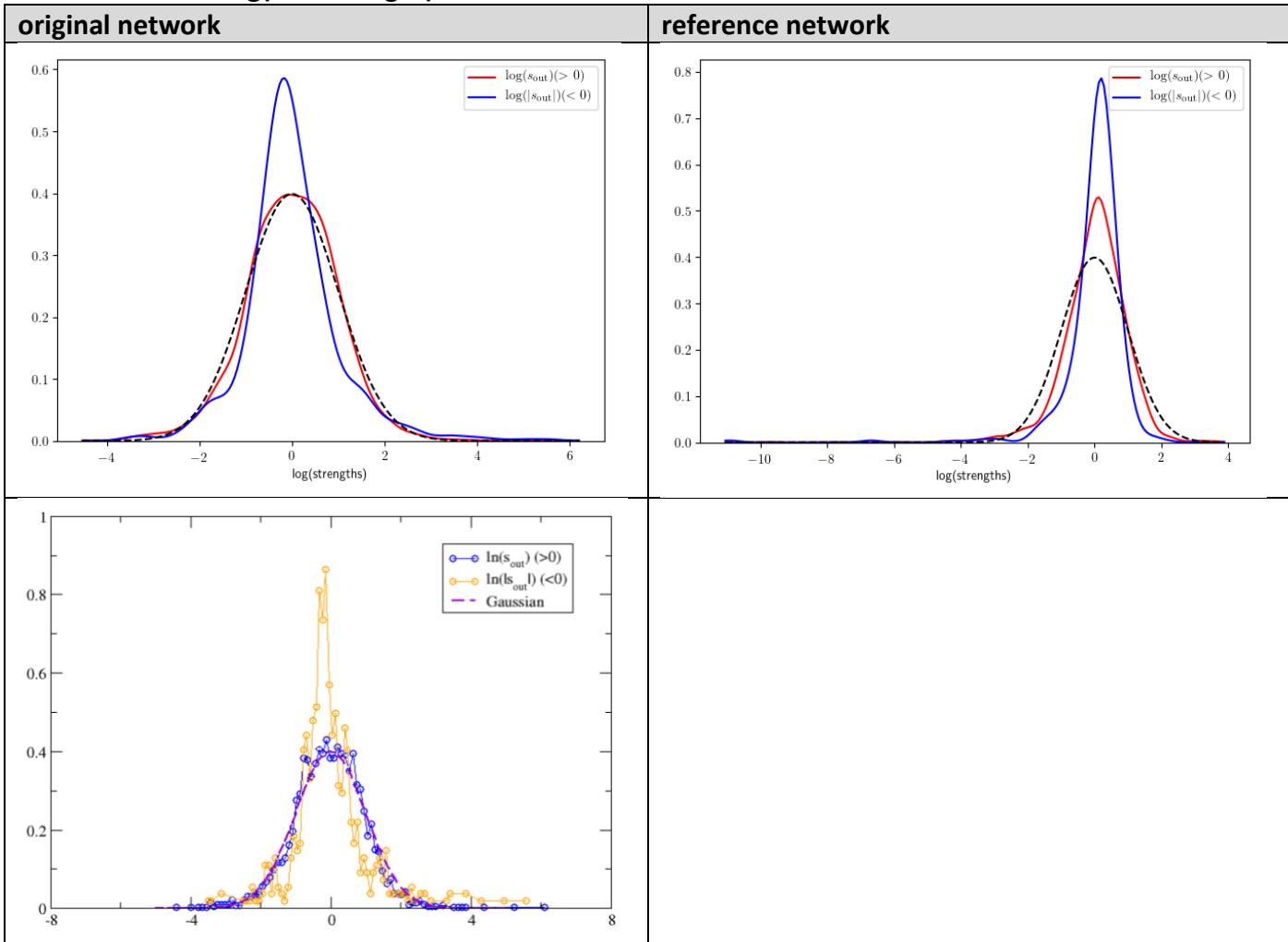
- Distribution of number of peaks (both near normal)



- Standardized log(in-strength) distribution



- Standardized log(out-strength) distribution



- $|g_{ij}|$ distribution (2 cases: $g_{ij} > 0$ and $g_{ij} < 0$)
Classified as “heavy-tailed” if mean \gg median

