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FHN dynamics

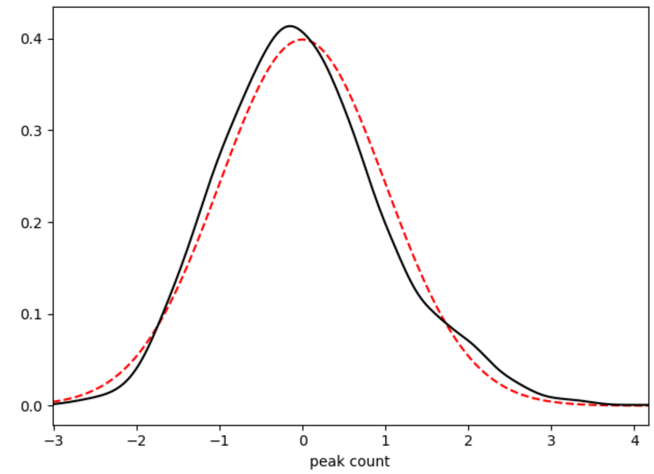
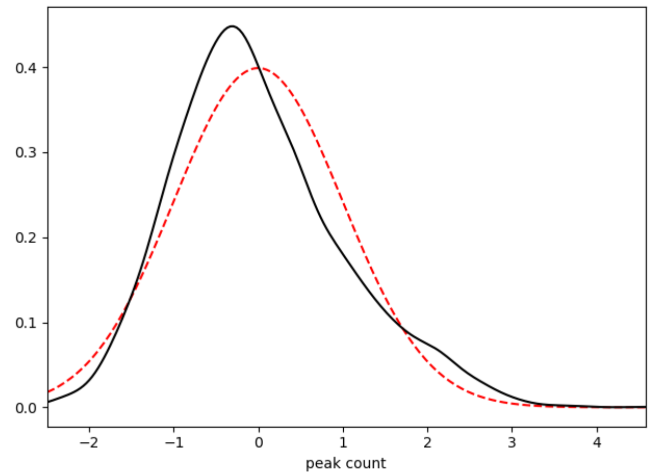
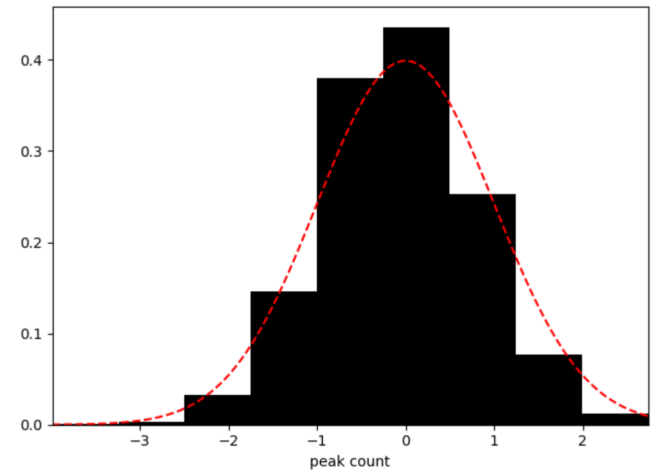
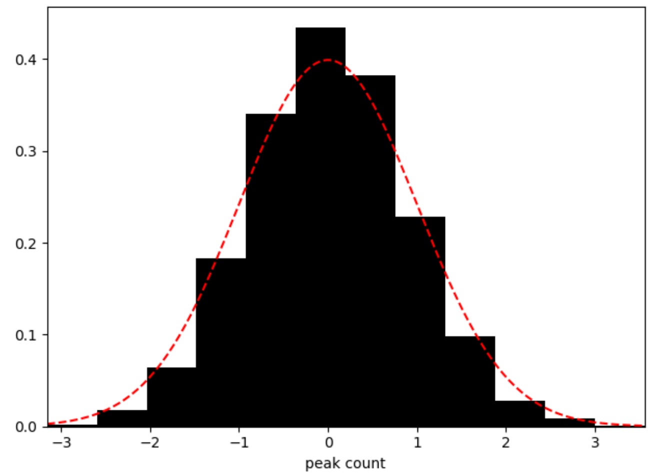
$$\begin{aligned}\dot{x}_i &= \frac{1}{\epsilon}(x_i - x_i^3/3 - y_i) + \sum_{j \neq i} g_{ij} A_{ij} h(x_i, x_j) + \eta_i \\ \dot{y}_i &= x_i + \alpha\end{aligned}$$

Function to find peaks

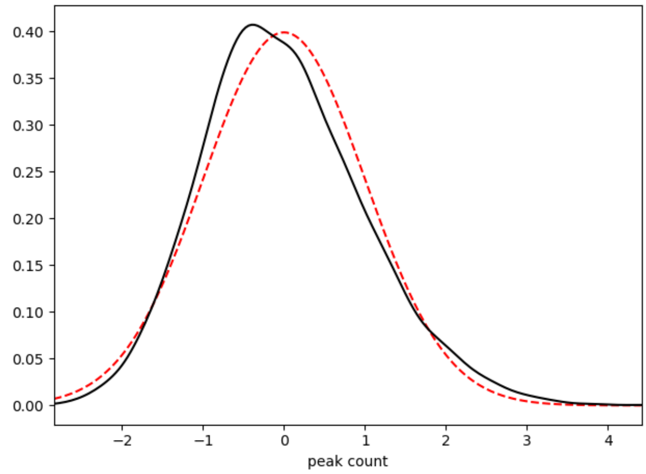
- **https://docs.scipy.org/doc/scipy/reference/generated/scipy.signal.find_peaks.html**

Standardized peak distributions

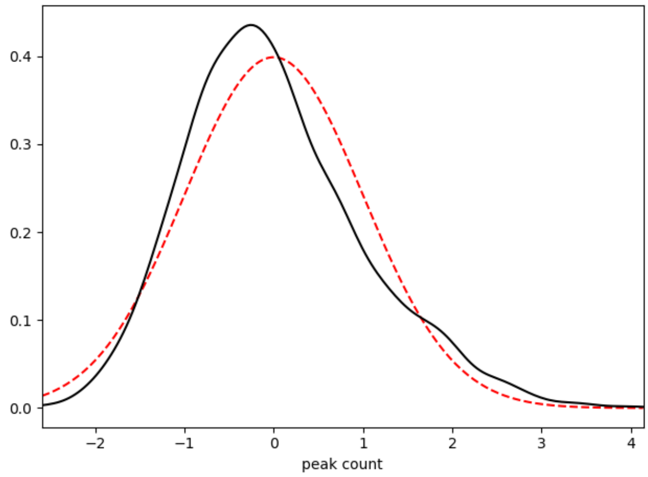
x-axis: leftmost = min, rightmost = max

1e6 steps	2e6 steps
Case 1: $\epsilon = 0.1, \alpha = 0.95, \sigma_i = 2$	
	
○ **Median = 165, Min = 149, Max = 189** ○ **Mean = 165.7985** ○ **Skewness = 0.4002 (right-skewed)** ○ **(Excess) Kurtosis = 0.2204 (heavier tails than Gaussian)**	○ **Median = 330, Min = 308, Max = 374** ○ **Mean = 331.2886** ○ **Skewness = 0.5945 (right-skewed)** ○ **(Excess) Kurtosis = 0.2684 (heavier tails than Gaussian)** ○ ***This is the most right-skewed among the other sets of params*** ○ ***Max = 4 s.d. from mean***
Case 2: $\epsilon = 0.01, \alpha = 0.95, \sigma_i = 2$	
	
○ **Median = 185, Min = 180, Max = 188** ○ **Mean = 184.7443** ○ **Skewness = -0.0100** ○ **(Excess) Kurtosis = 0.0148** ○ **Can't properly fit a curve as there are too few data points so I still use histogram**	○ **Median = 369, Min = 364, Max = 375** ○ **Mean = 369.1739** ○ **Skewness = 0.0519** ○ **(Excess) Kurtosis = 0.0033** ○ **Can't properly fit a curve as there are too few data points so I still use histogram**

Case 3: $\epsilon = 0.1, \alpha = 1, \sigma_i = 2$

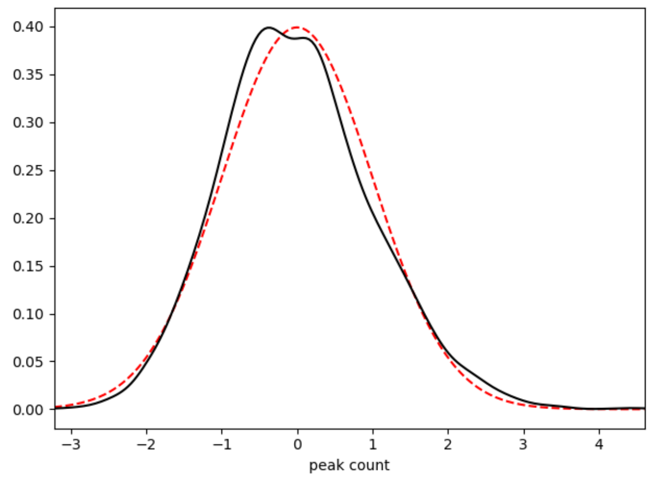


- Median = 160, Min = 145, Max = 185
- Mean = 160.6794
- Skewness = 0.4302 (right-skewed)
- (Excess) Kurtosis = 0.2224 (heavier tails than Gaussian)

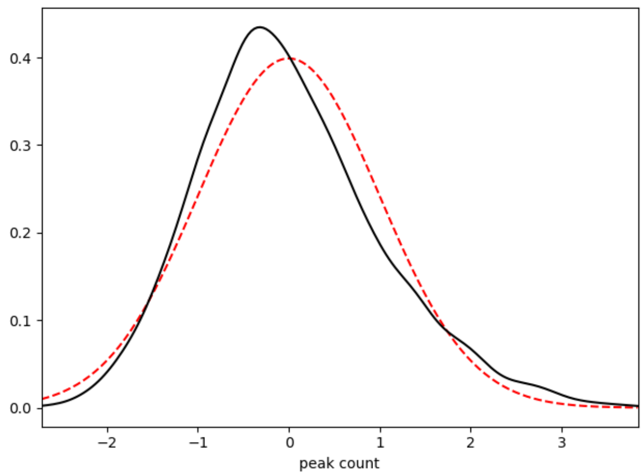


- Median = 320, Min = 297, Max = 359
- Mean = 320.9031
- Skewness = 0.5982 (right-skewed)
- (Excess) Kurtosis = 0.3511 (heavier tails than Gaussian)

Case 4: $\epsilon = 0.1, \alpha = 1.05, \sigma_i = 2$

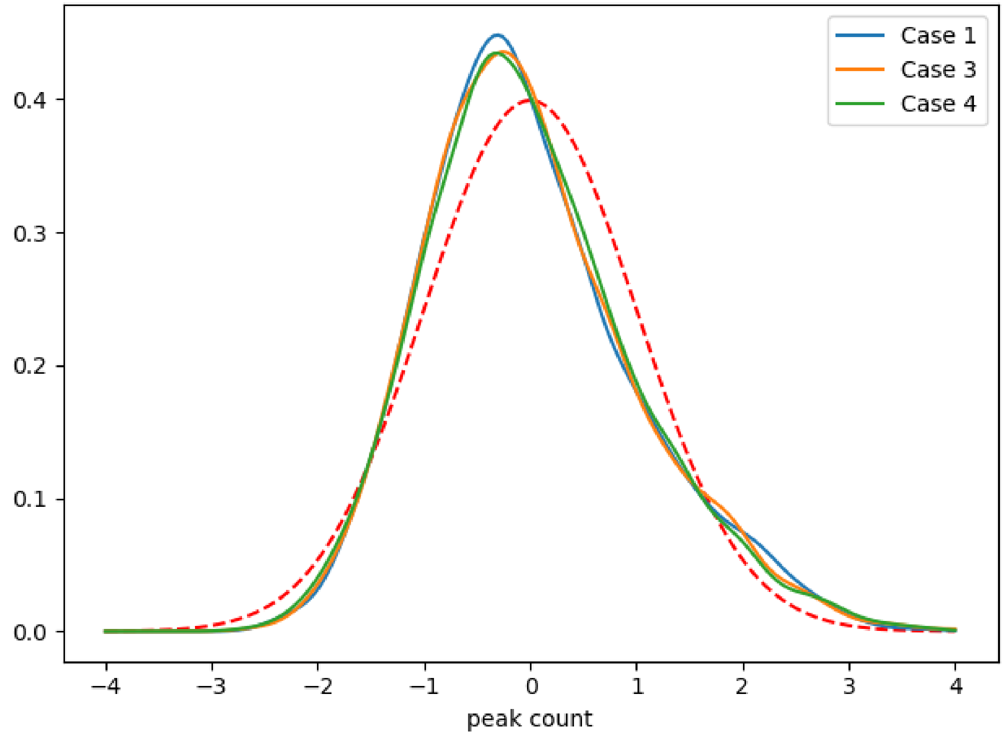


- Median = 155, Min = 137, Max = 181
- Mean = 155.1231
- Skewness = 0.3537 (right-skewed)
- (Excess) Kurtosis = 0.2562 (heavier tails than Gaussian)



- Median = 309, Min = 285, Max = 345
- Mean = 309.8799
- Skewness = 0.5384 (right-skewed)
- (Excess) Kurtosis = 0.3200 (heavier tails than Gaussian)

Case 1,3,4 combined



Analysis on reference networks

(a) reference network 1: keep Aij but replace nonzero gij by values taken from a Gaussian distribution of same mean and standard deviation (this time we do not separately consider positive and negative gij's). This network has same k_in and k_out but different s_in and s_out

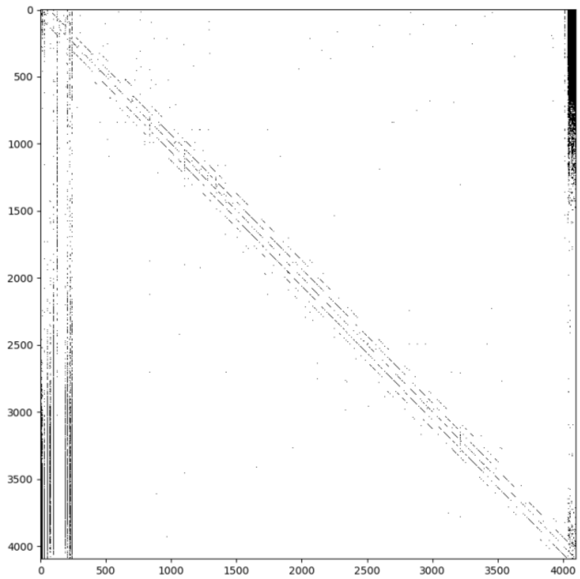
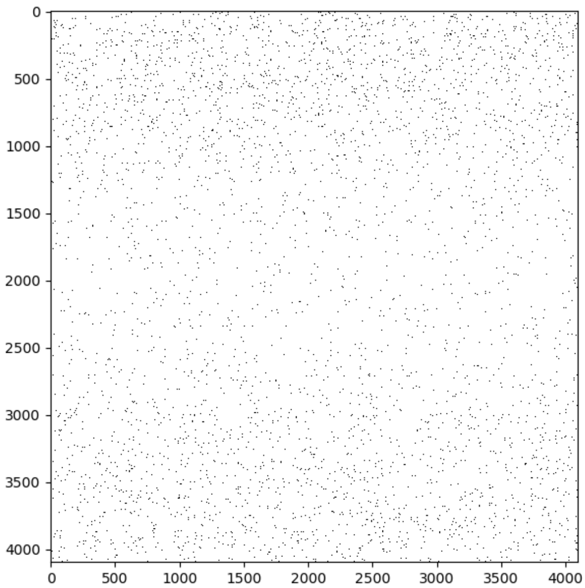
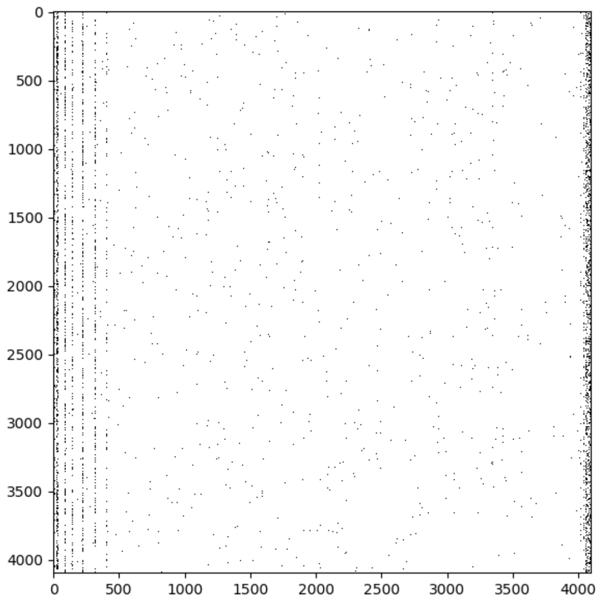
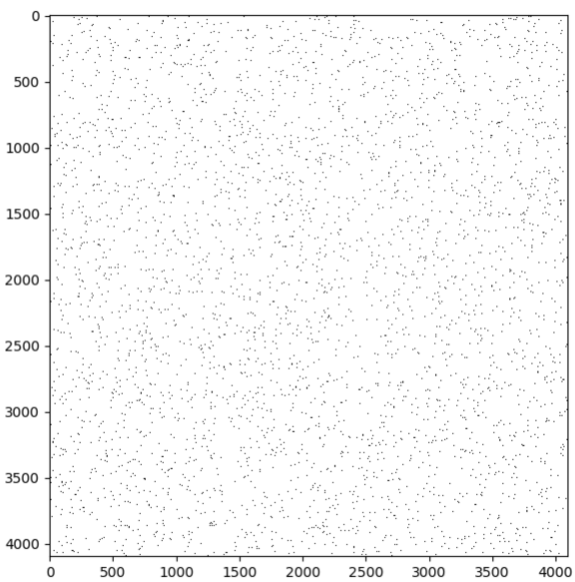
(b) reference network 2: shuffle gij for fixed i; this has same k_in and s_in but different k_out and s_out

(c) reference network 3: shuffle gij for fixed j; this has same k_out and s_out but different k_in and s_in

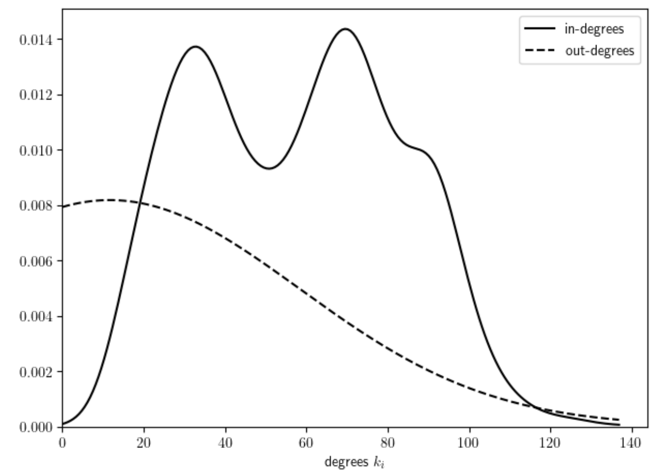
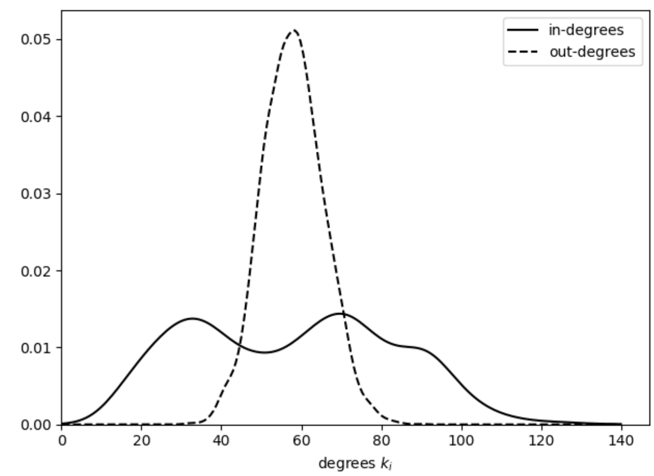
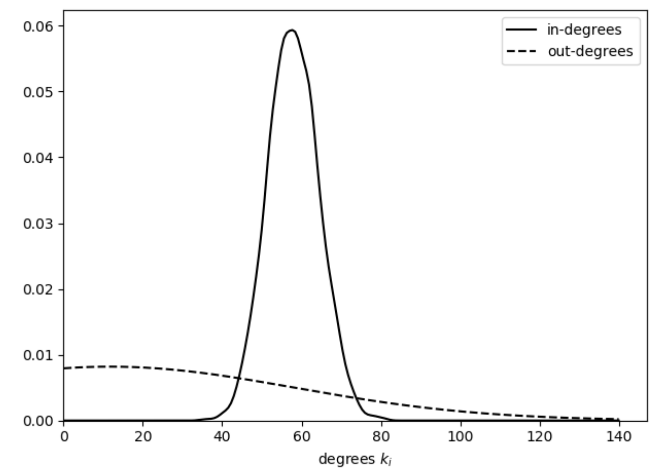
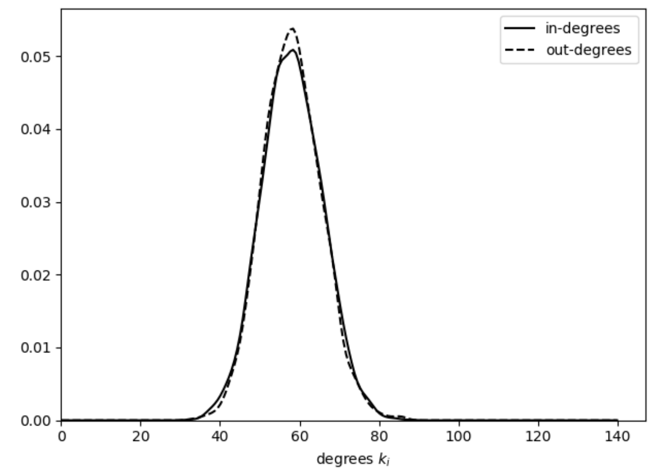
(d) reference network 4: random directed network with same connection probability p and gij from a Gaussian distribution of same mean and standard deviation

(e) reference network 5: keep Aij but shuffle non-zero gij

Adjacency matrix

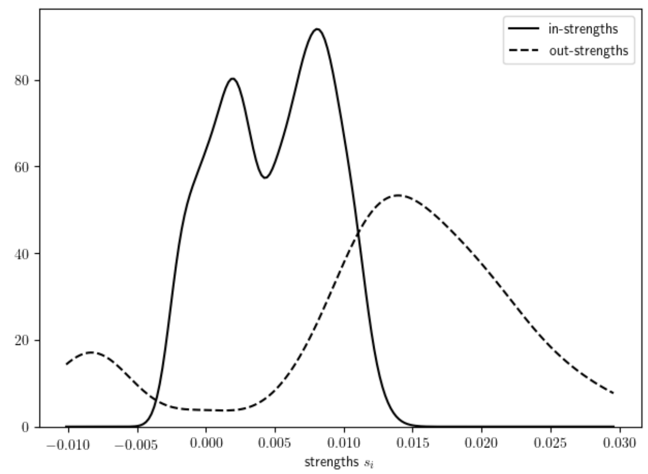
Original Network	Ref. Network 1 (non-zero gij Gaussian)
	(same as the original adjacency matrix)
Ref. Network 2 (row gij shuffled)	**Ref. Network 3 (col gij shuffled)**
	
Ref. Network 4 (random)	**Ref. Network 5 (non-zero gij shuffled)**
	(same as the original adjacency matrix)

Degree distribution

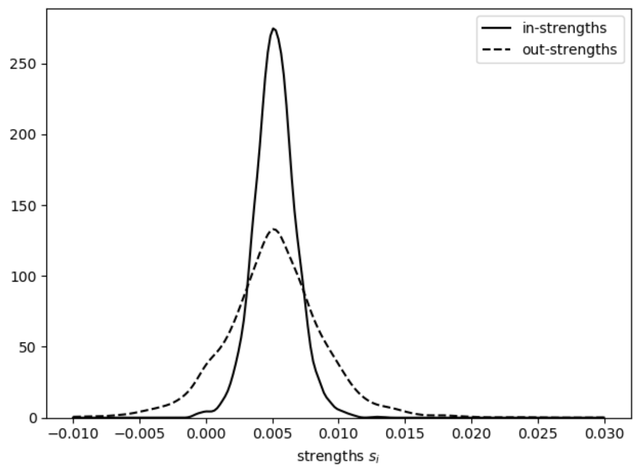
Original Network	Ref. Network 1 (non-zero gij Gaussian)
	(same as the original degree distribution)
Ref. Network 2 (row gij shuffled)	**Ref. Network 3 (col gij shuffled)**
 • In-degrees identical to original	 • Out-degrees identical to original
Ref. Network 4 (random)	**Ref. Network 5 (non-zero gij shuffled)**
 • In-degrees & out-degrees have same distribution	(same as the original degree distribution)

Strength distribution

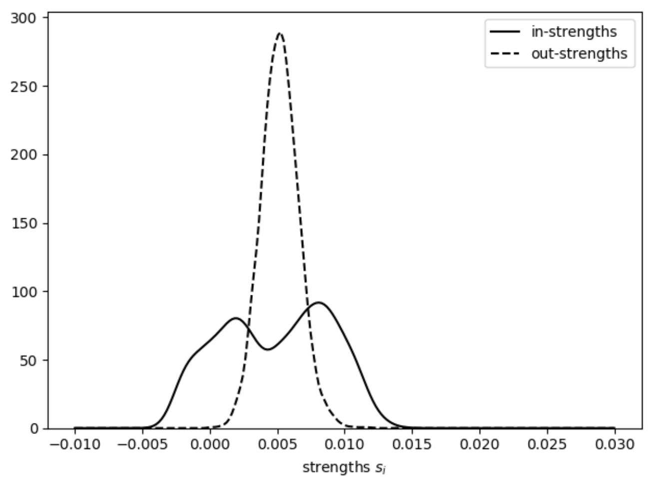
Original Network



Ref. Network 1 (non-zero gij Gaussian)

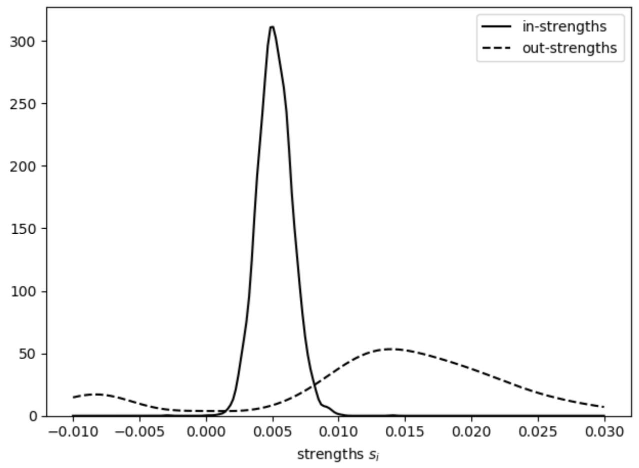


Ref. Network 2 (row gij shuffled)



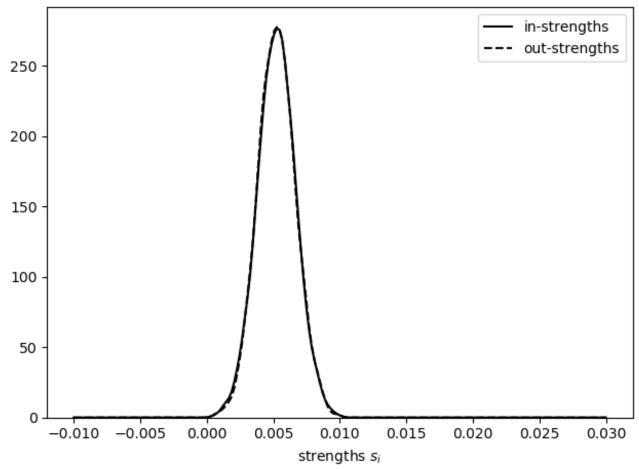
- **In-strengths identical to original**

Ref. Network 3 (col gij shuffled)



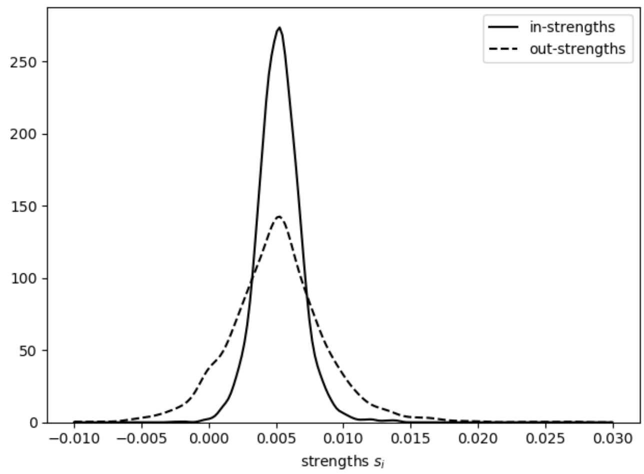
- **Out-strengths identical to original**

Ref. Network 4 (random)

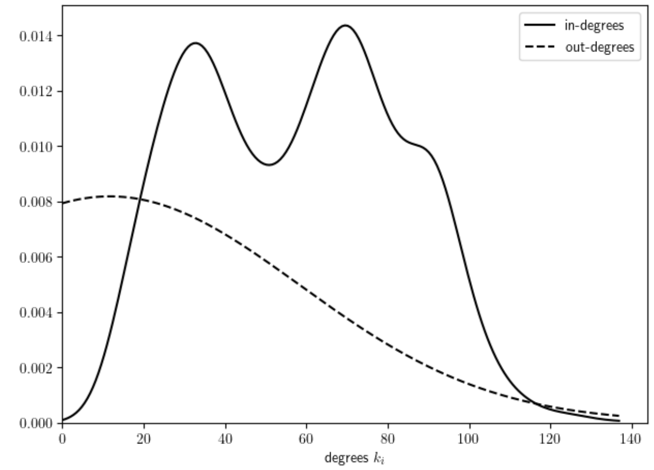
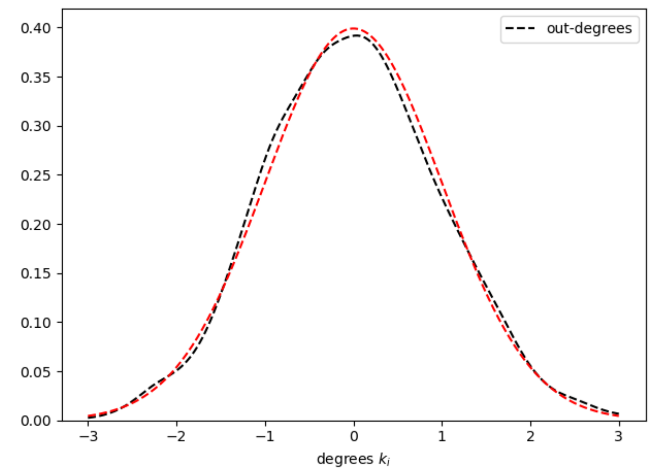
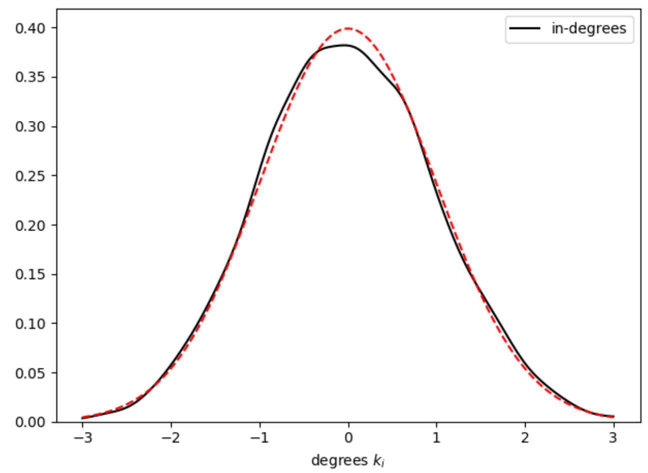
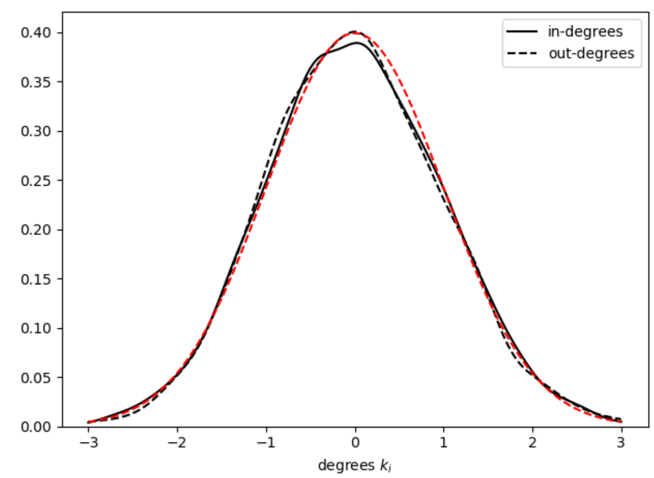


- **In-strengths & out-strengths have same distribution**

Ref. Network 5 (non-zero gij shuffled)

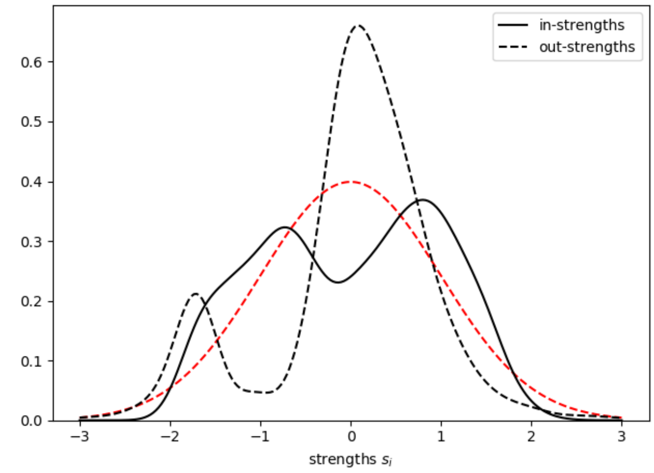


Degree distribution (standardized)
Red dotted = standard Gaussian

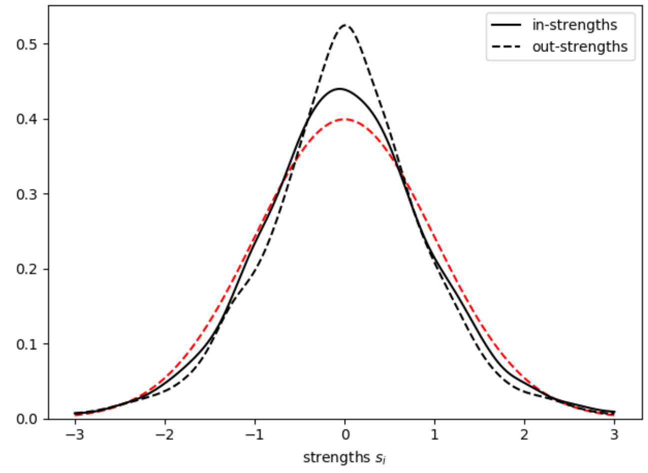
Original Network	Ref. Network 1 (non-zero gij Gaussian)
	**(same as the original degree distribution)**
Ref. Network 2 (row gij shuffled)	**Ref. Network 3 (col gij shuffled)**
	
Ref. Network 4 (random)	**Ref. Network 5 (non-zero gij shuffled)**
	**(same as the original degree distribution)**

Strength distribution (standardized)
Red dotted = standard Gaussian

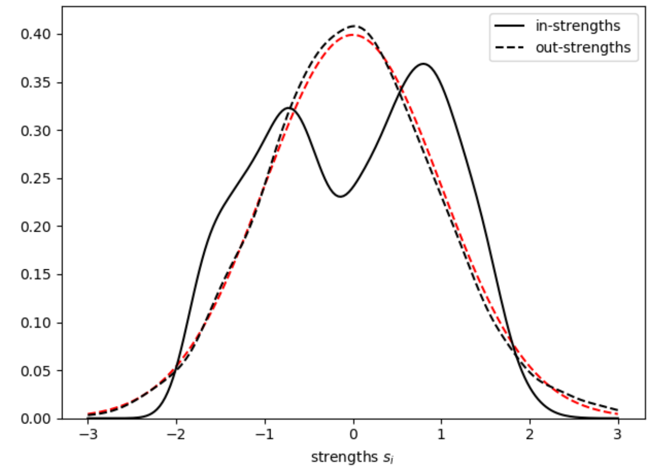
Original Network



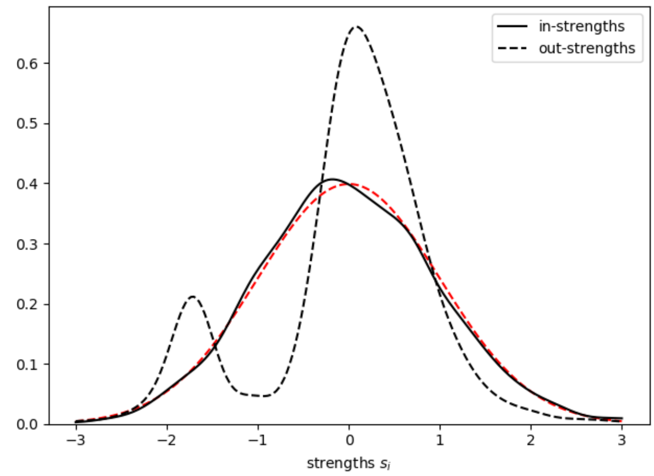
Ref. Network 1 (non-zero gij Gaussian)



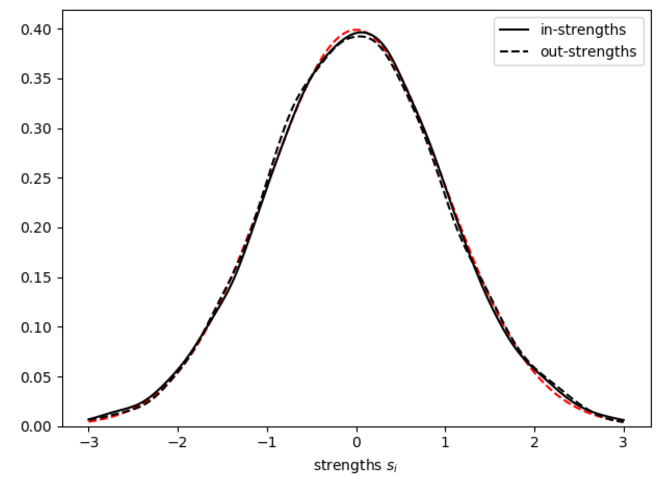
Ref. Network 2 (row gij shuffled)



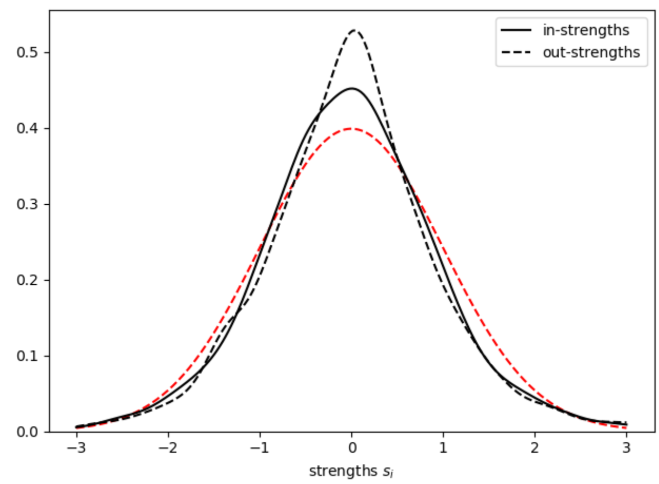
Ref. Network 3 (col gij shuffled)



Ref. Network 4 (random)

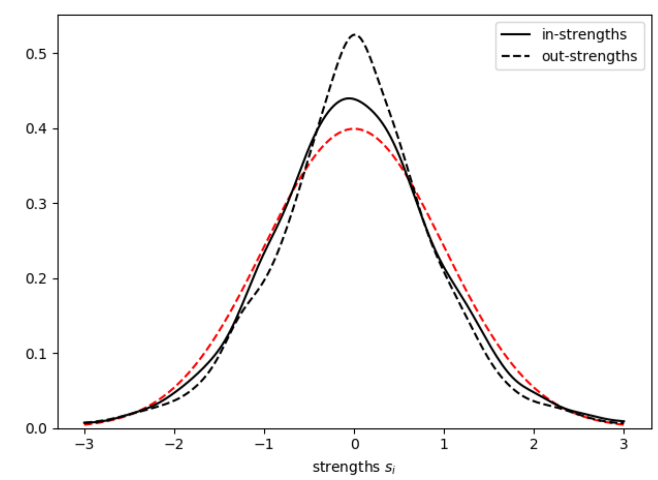


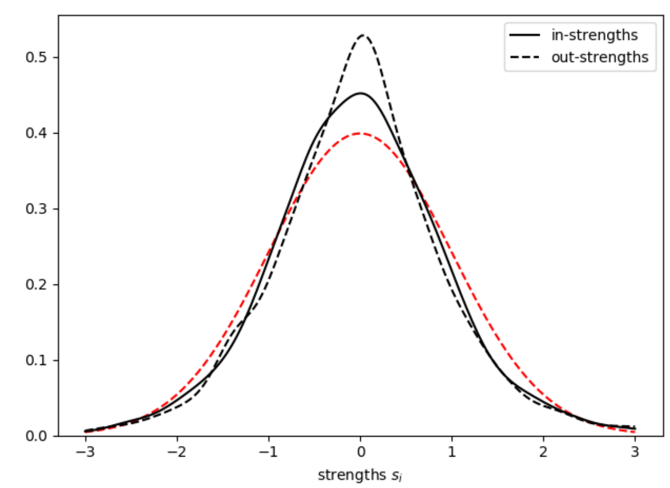
Ref. Network 5 (non-zero gij shuffled)



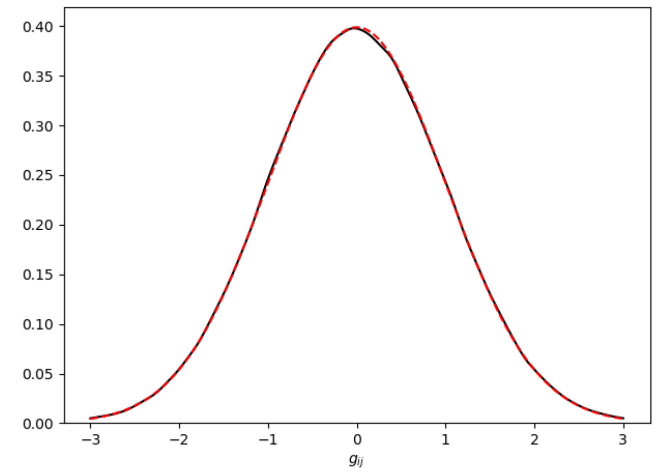
Ref. Network 1 (non-zero gij Gaussian)	Ref. Network 5 (non-zero gij shuffled)

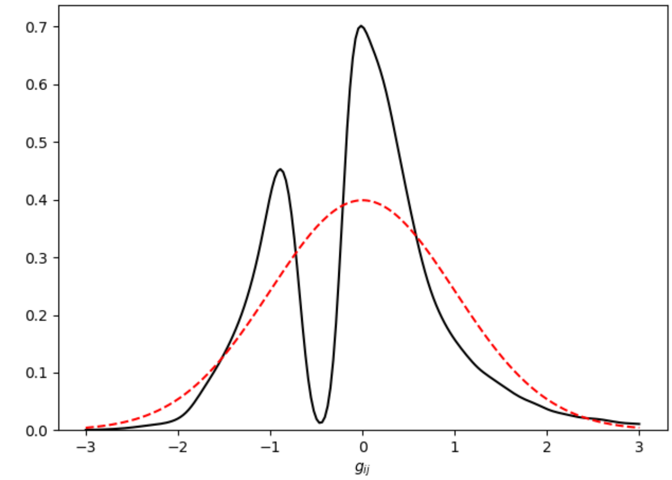
Strength distribution (standardized)





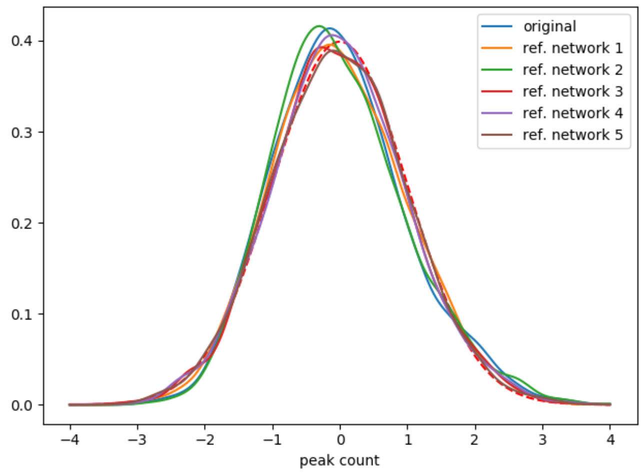
gij distribution (standardized)

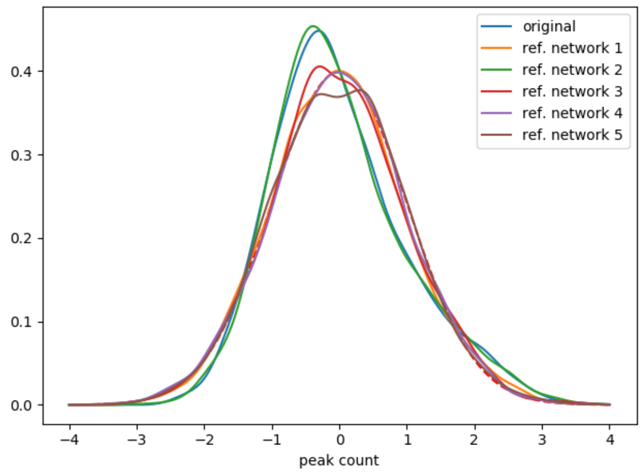




(same as original network's)

Peak count distribution comparison (2e6 steps)

<table>
<tr><th colspan="2">Distributions of original & 5 ref. networks combined</th></tr>
<tr><th>1e6 steps</th><th>2e6 steps</th></tr>
<tr><td>

▪ Original & ref. network 2 are the most skewed
▪ Their peak count vs index plots are also similar</td><td>
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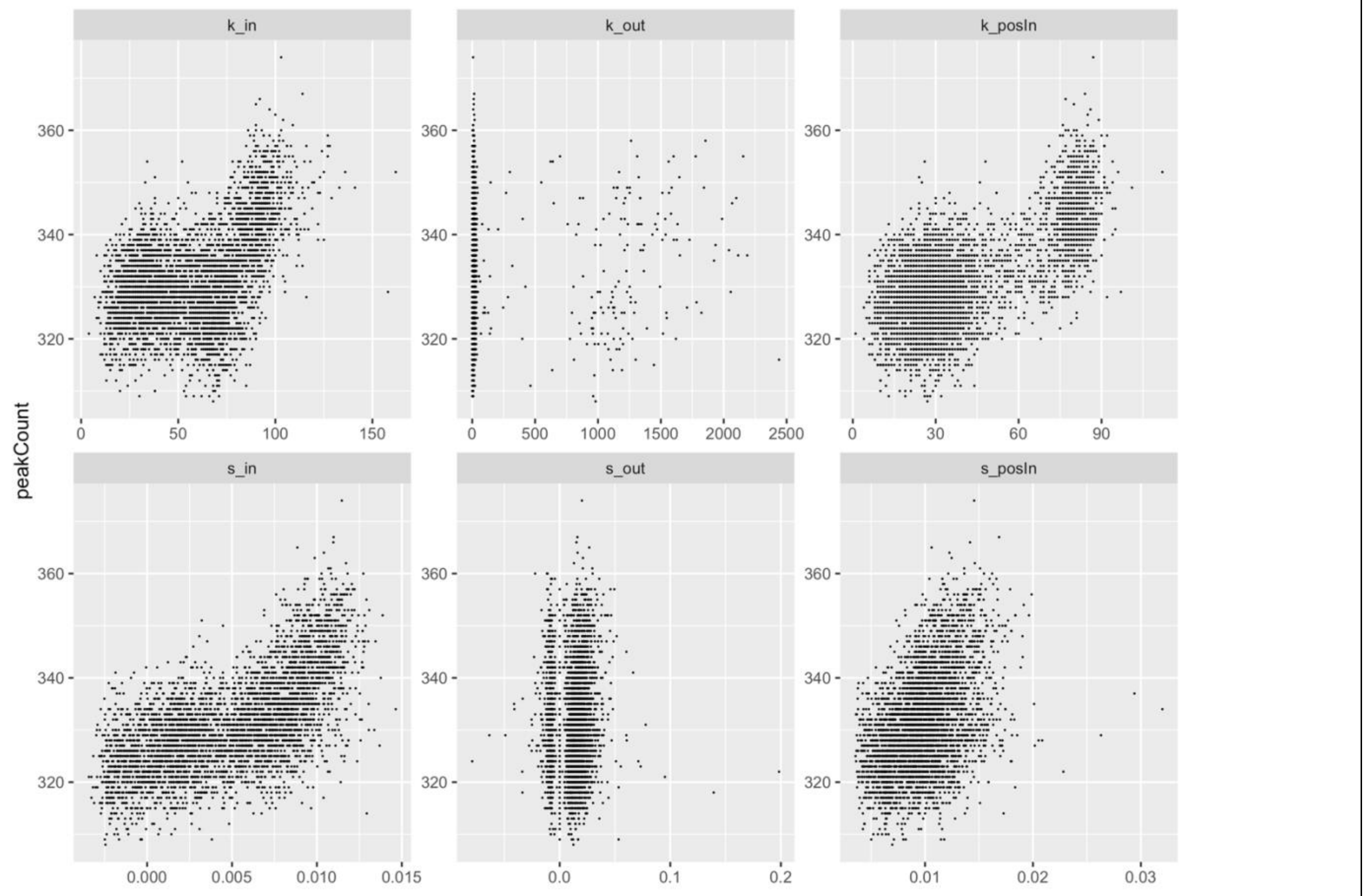
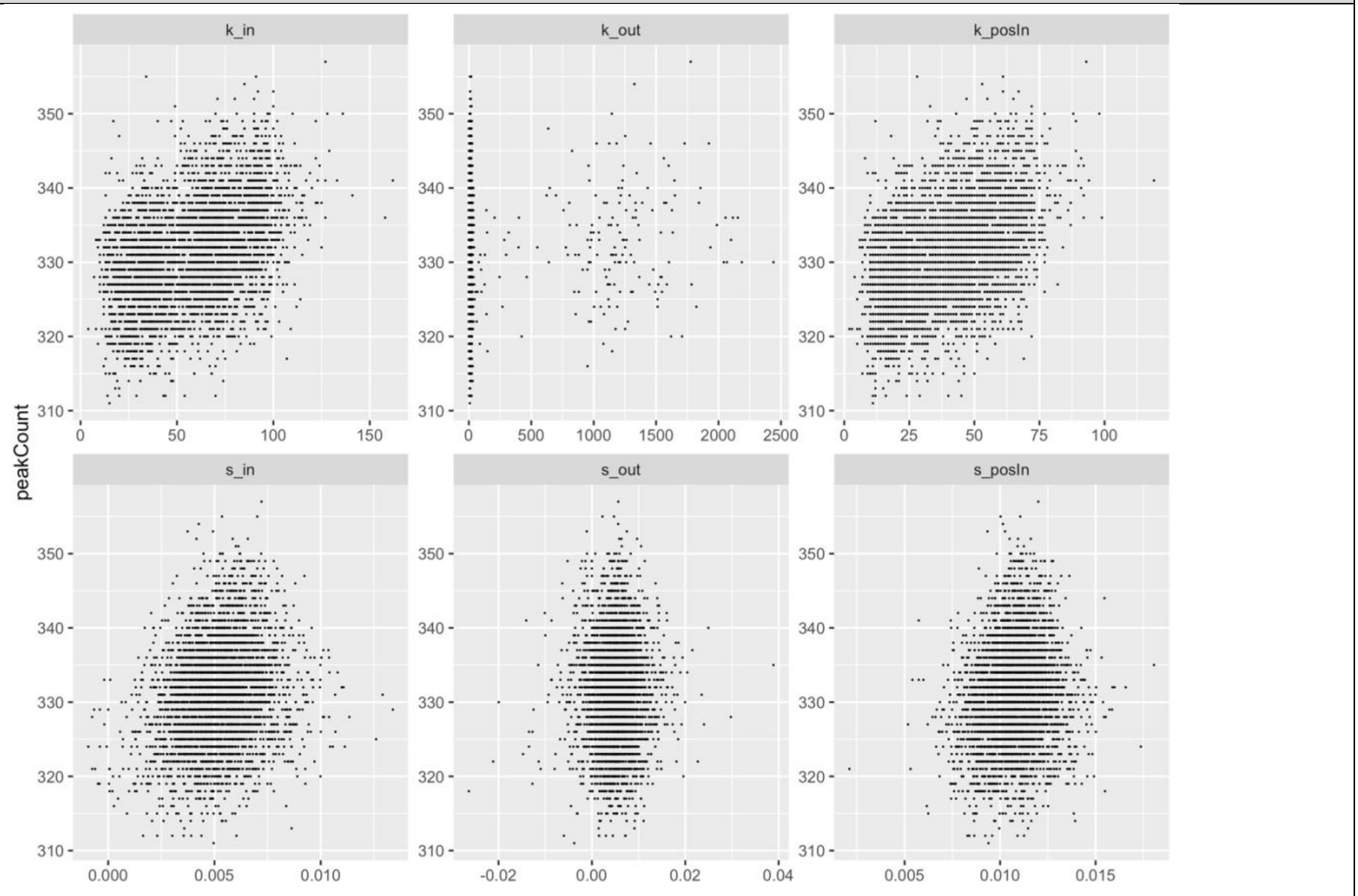
Exploratory analysis (2e6 steps)

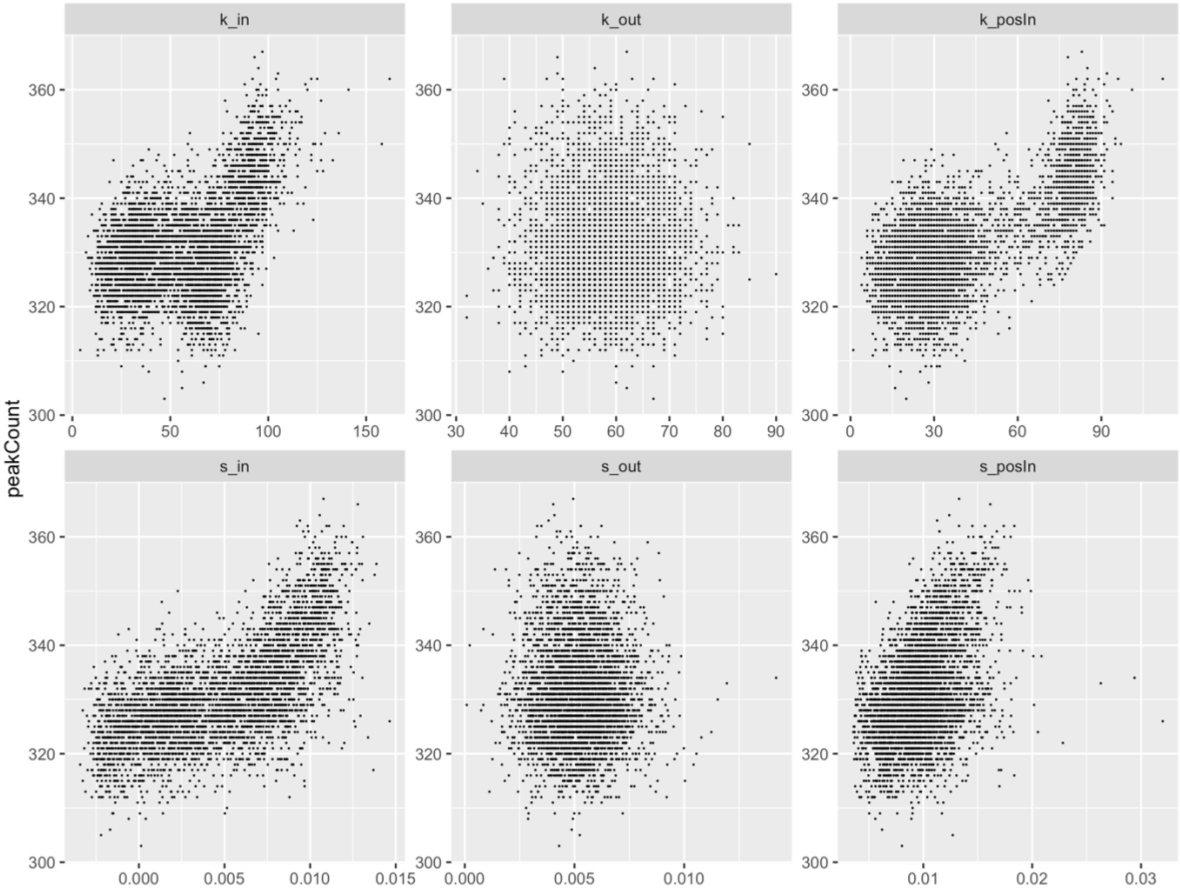
Original Network



Ref. Network 1 (non-zero gij Gaussian)

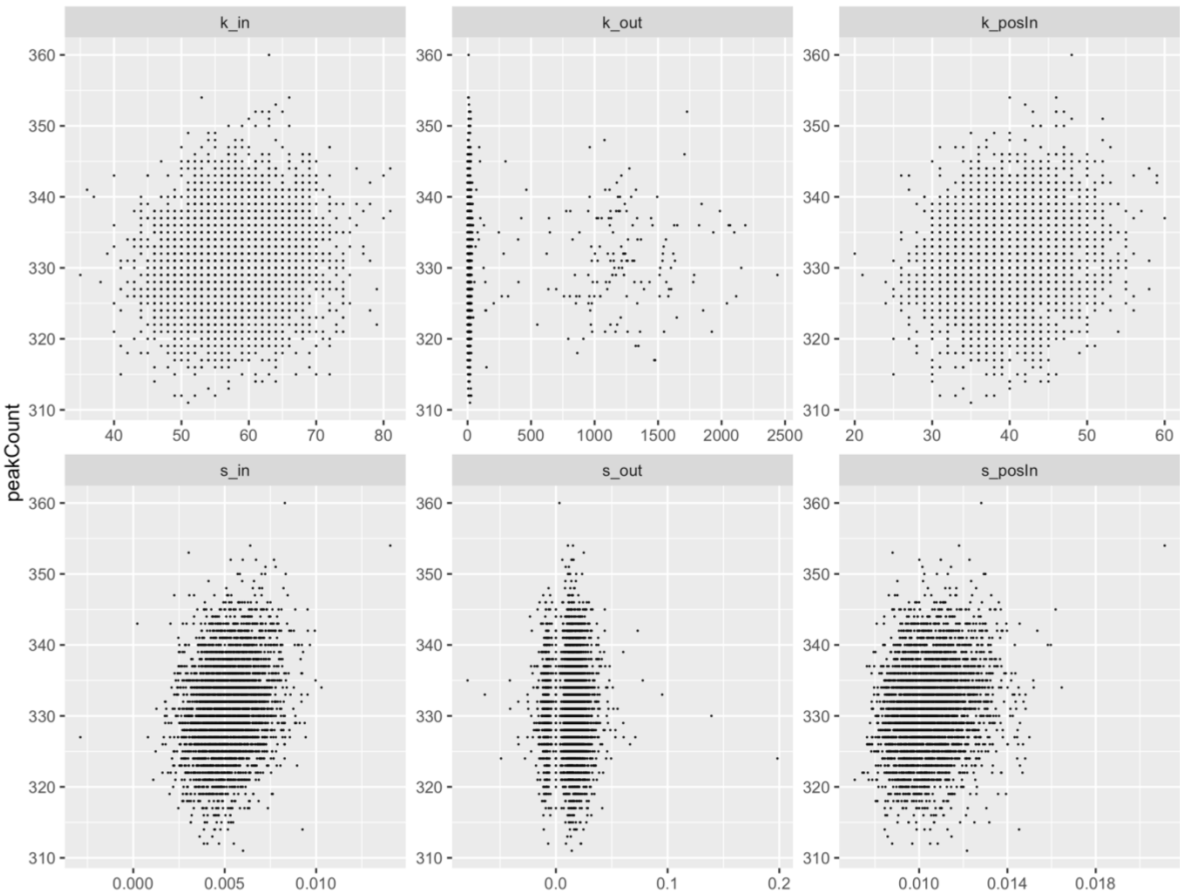


Ref. Network 2 (row gij shuffled)

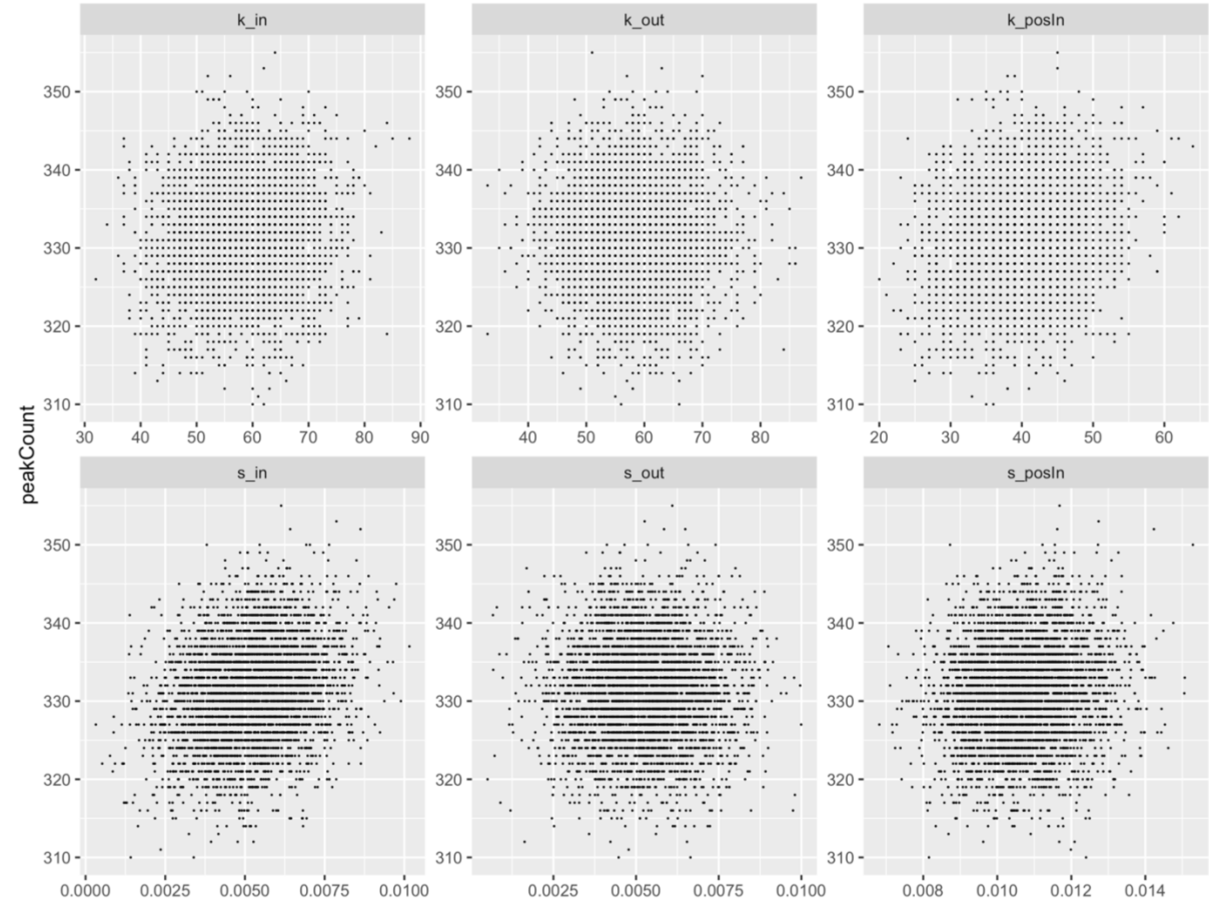


- **Very similar to original**

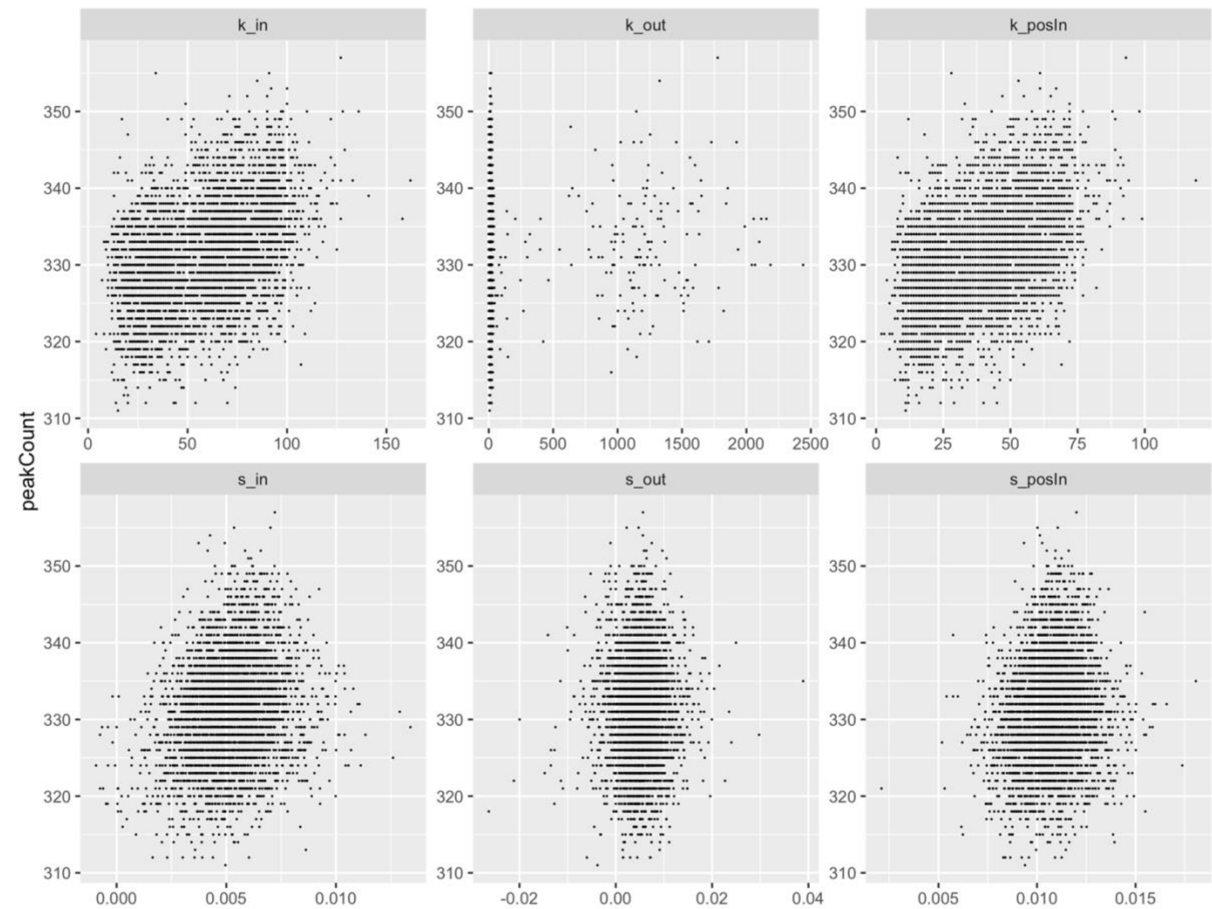
Ref. Network 3 (col gij shuffled)



Ref. Network 4 (random)



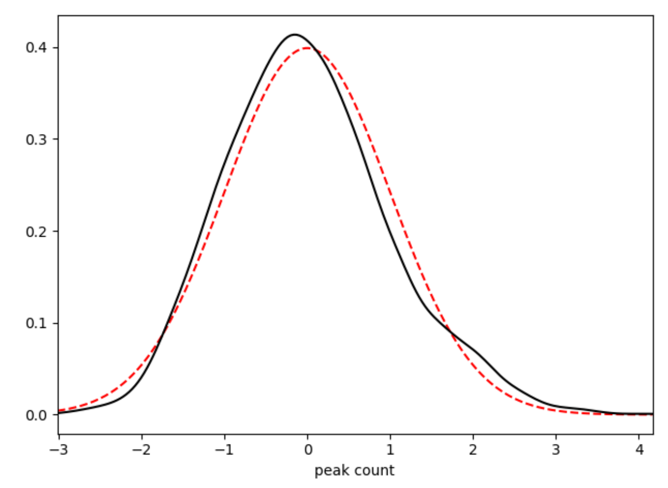
Ref. Network 5 (non-zero gij shuffled)



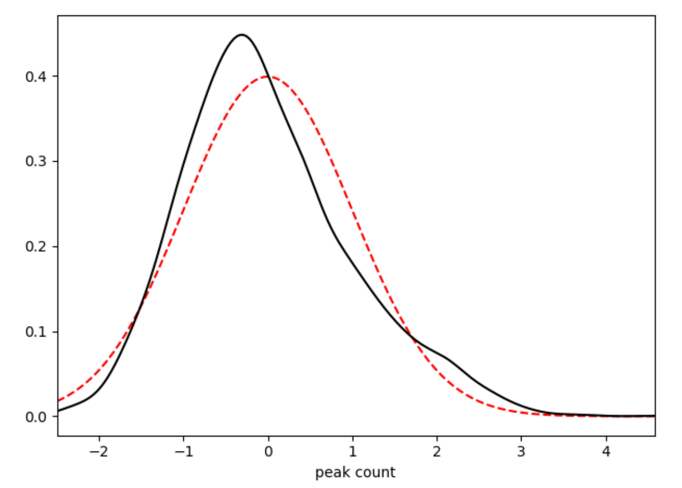
Peak count distribution comparison of Original Network (3e6 steps)

- **To check if peak count distribution is stationary**

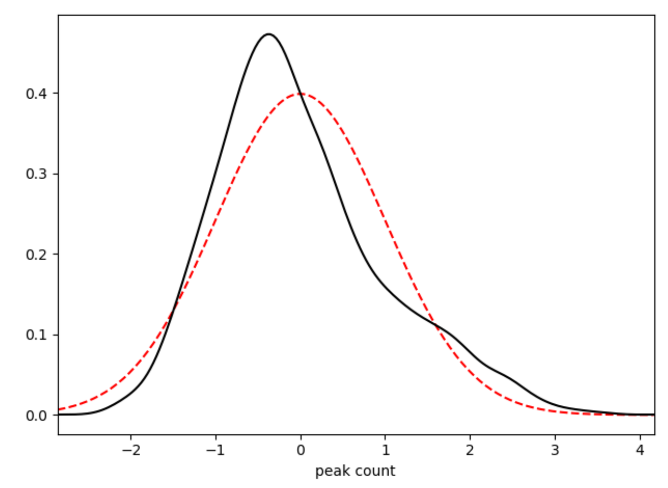
1e6 steps



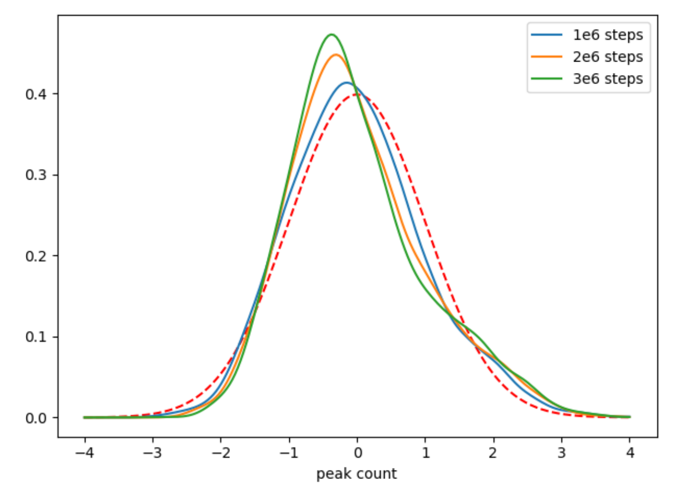
2e6 steps



3e6 steps

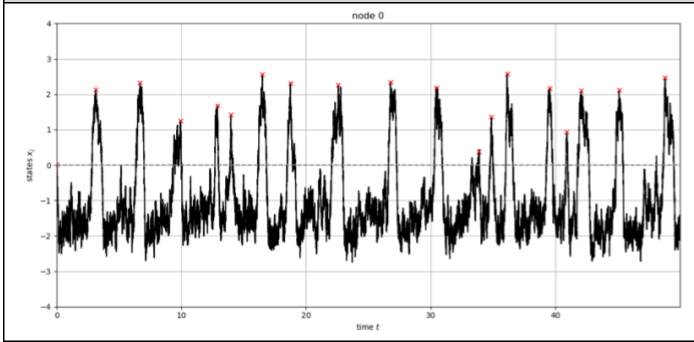
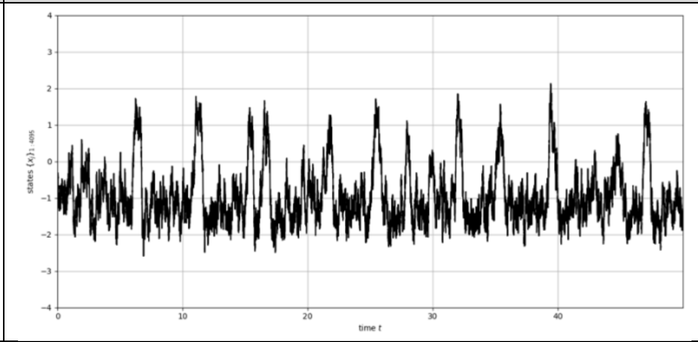
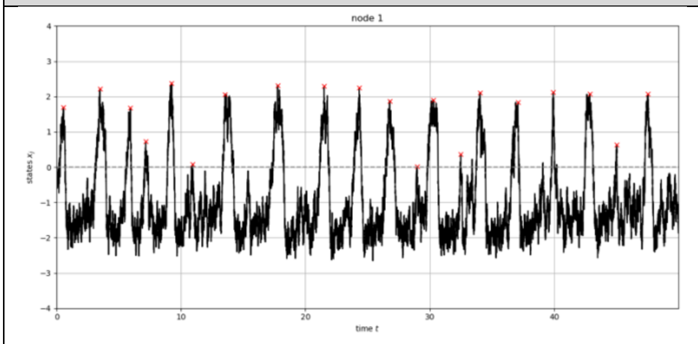
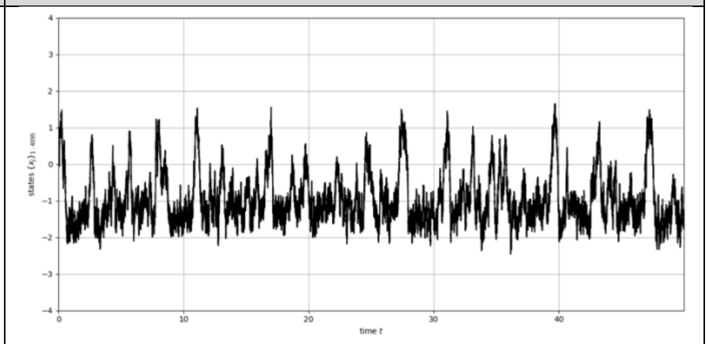
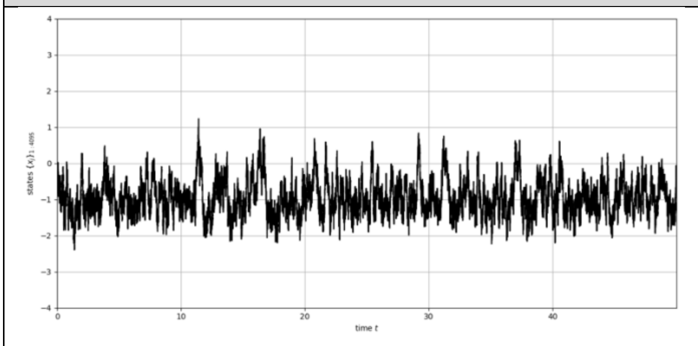
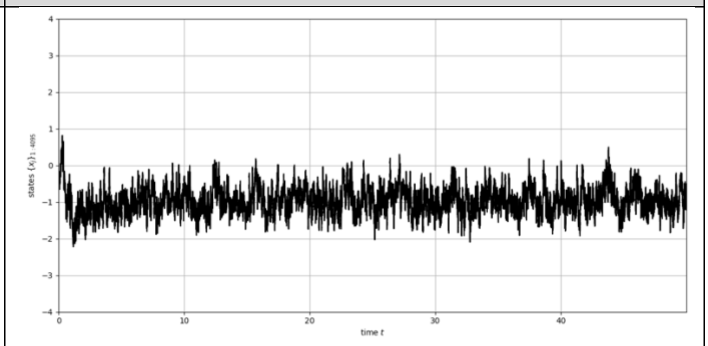
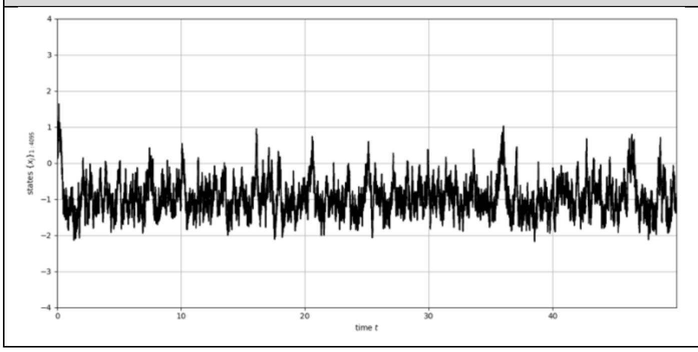
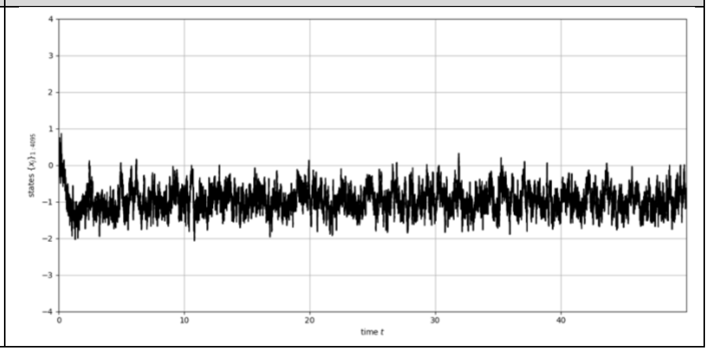


All three

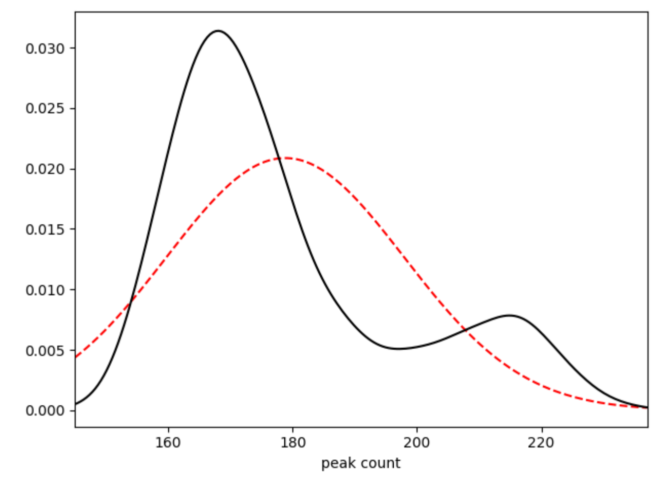
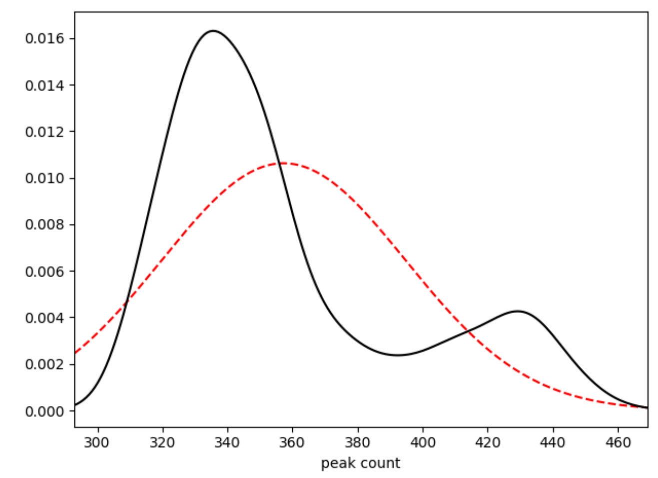
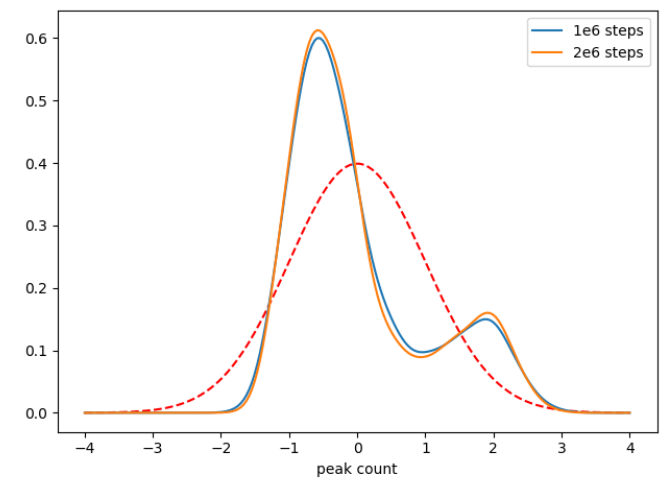
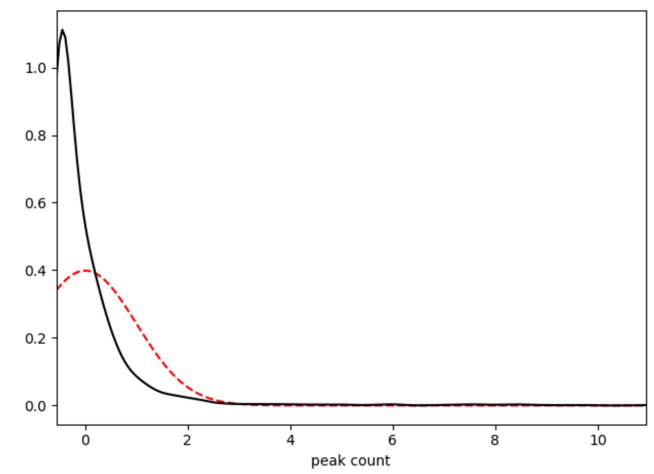
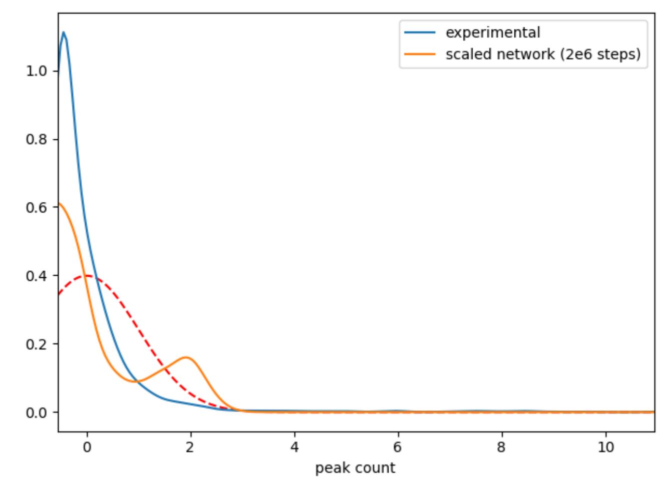


Model with scaled gij

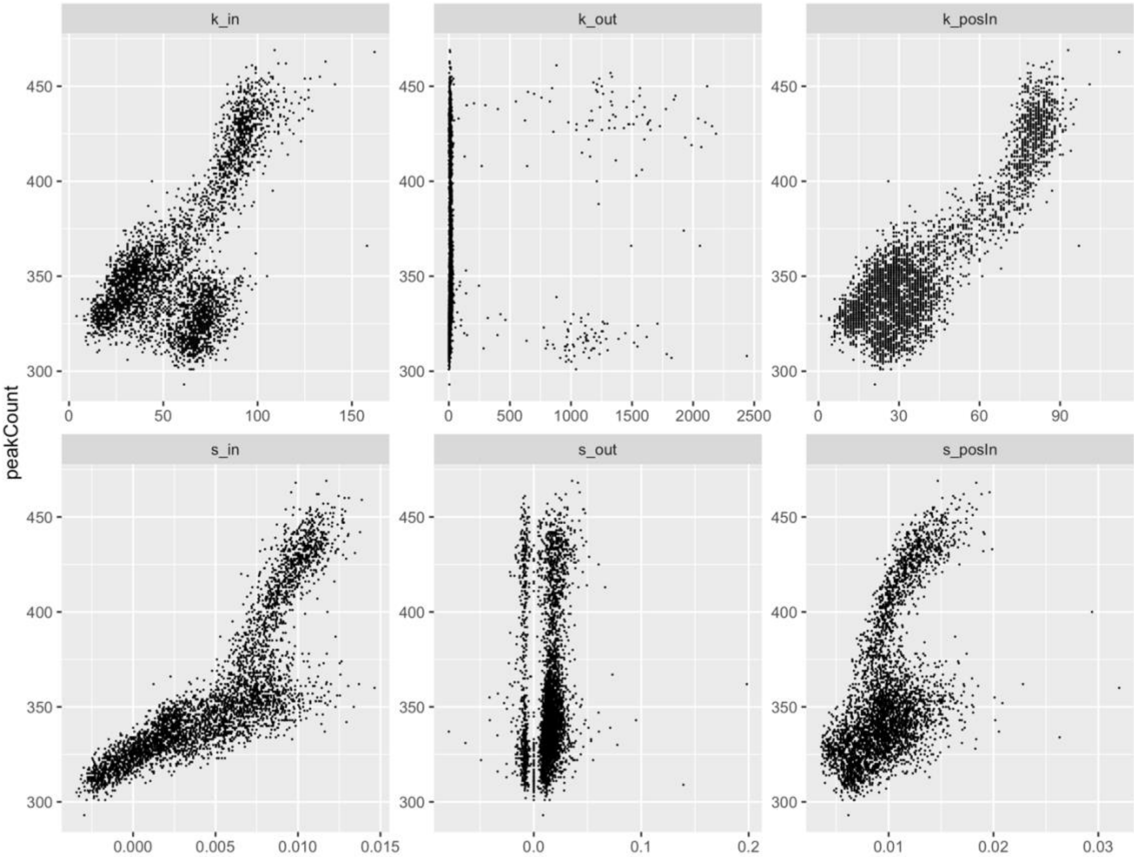
- Parameters $\epsilon = 0.1, \alpha = 0.95, \sigma_i = 2$

gij not scaled	gij * 5
Node 0	Node 0
	
Node 1	Node 1
	
gij * 10	gij *20
Node 0	Node 0
	
Node 1	Node 1
	

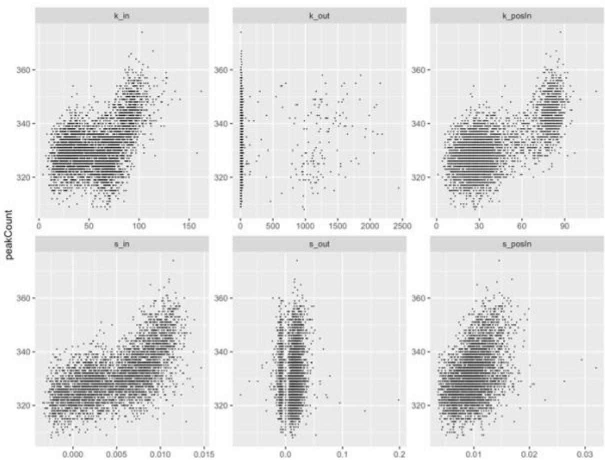
Peak count distribution of gij * 5 network

1e6 steps	2e6 steps
	
1e6 & 2e6	**Experimental**
	
Comparison	
	

Exploratory analysis (2e6 steps)



Original network without scaling gij (P59)



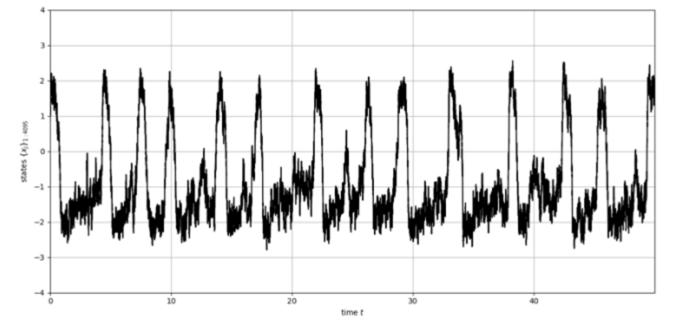
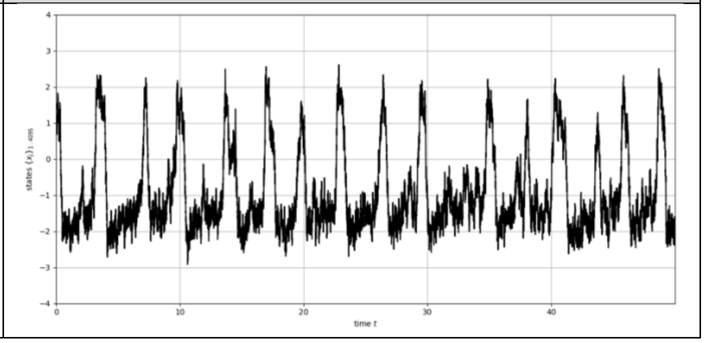
Scaled gij makes patterns more obvious and more "exaggerated"

Synaptic FHN model

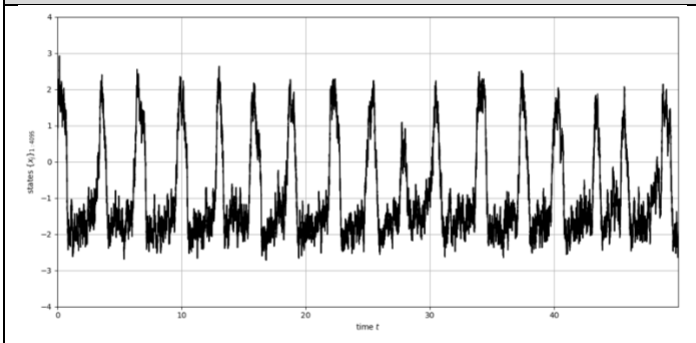
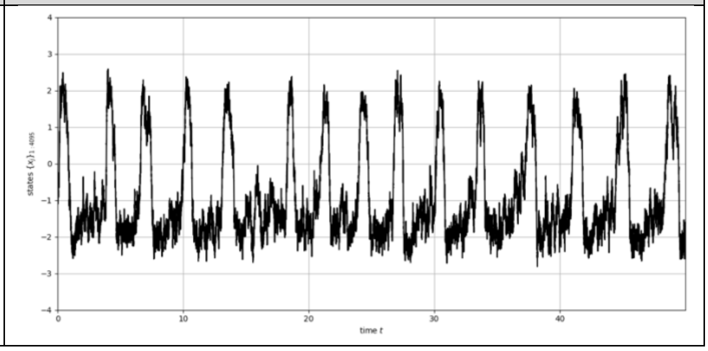
- **Play around with different parameters**

$$h^{\mathrm{syn}}(x,y) = (1/\beta_1)\{1+\tanh[\beta_2(y-y_0)]\}.$$

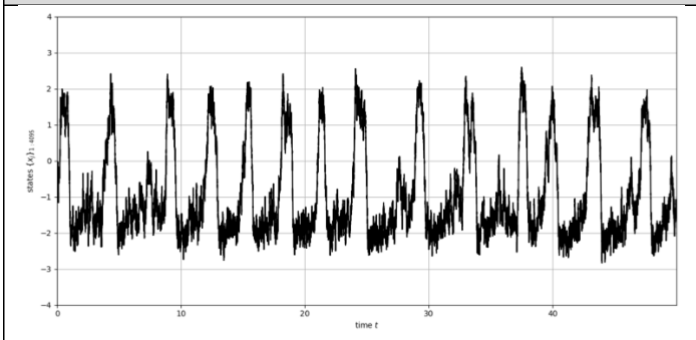
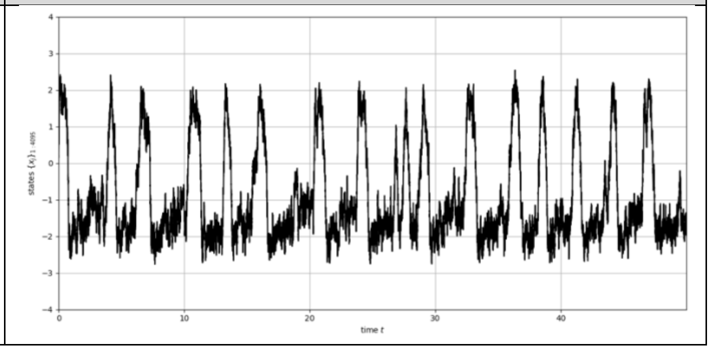
- $(\boldsymbol{\beta_1}, \boldsymbol{\beta_2}, \boldsymbol{y_0}) = (\mathbf{0.5}, \mathbf{1}, \mathbf{1})$

Node 0	Node 1
	

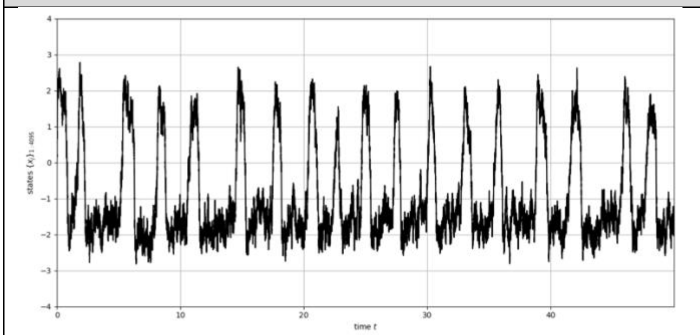
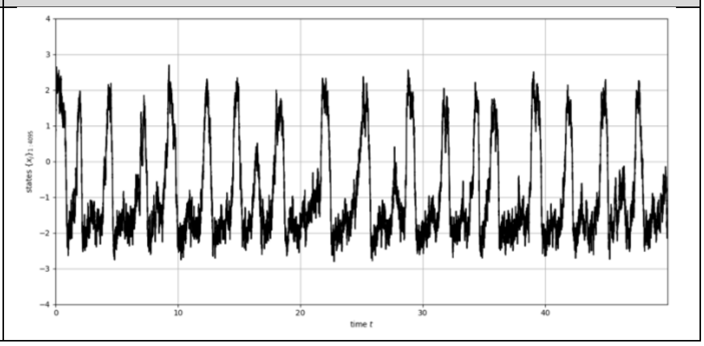
- $(\boldsymbol{\beta_1}, \boldsymbol{\beta_2}, \boldsymbol{y_0}) = (\mathbf{0.1}, \mathbf{1}, \mathbf{1})$

Node 0	Node 1
	

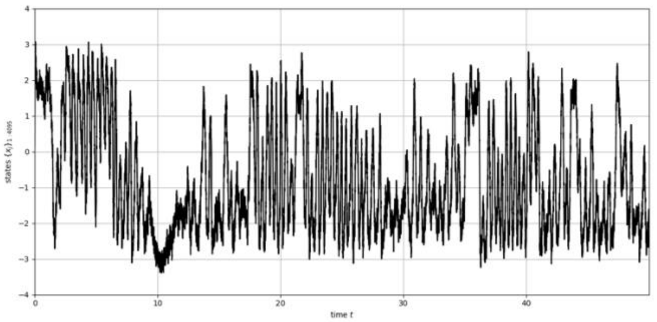
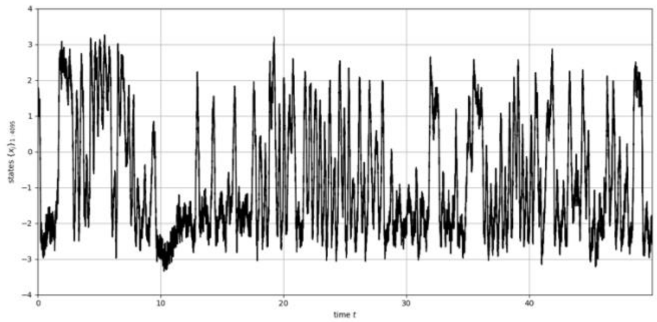
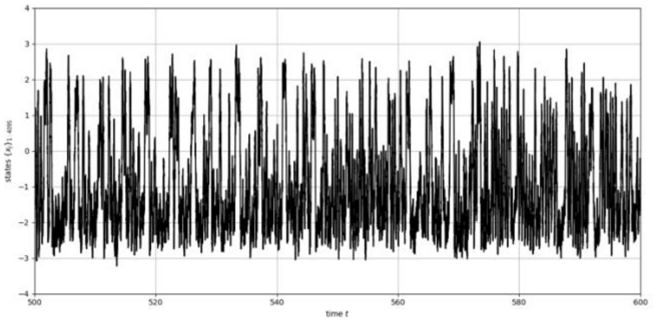
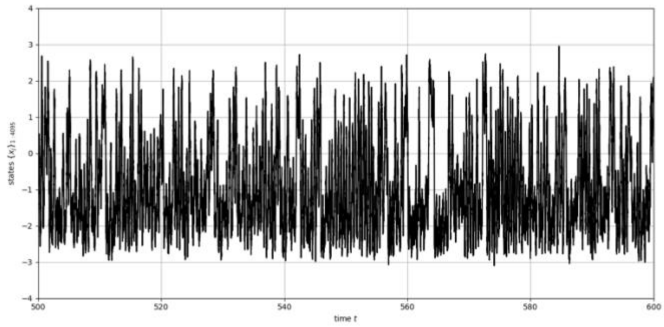
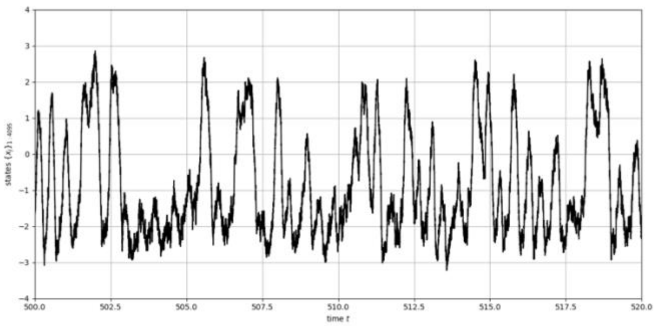
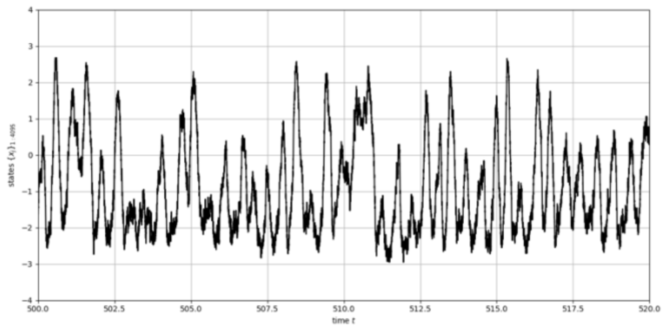
- $(\boldsymbol{\beta_1}, \boldsymbol{\beta_2}, \boldsymbol{y_0}) = (\mathbf{0.1}, \mathbf{1}, \mathbf{0})$

Node 0	Node 1
	

- $(\boldsymbol{\beta_1}, \boldsymbol{\beta_2}, \boldsymbol{y_0}) = (\mathbf{0.05}, \mathbf{1}, \mathbf{0})$

Node 0	Node 1
	

- $(\boldsymbol{\beta_1}, \boldsymbol{\beta_2}, \boldsymbol{y_0}) = (\mathbf{0.01}, \mathbf{1}, \mathbf{0})$

<table>
<tr><th colspan="2">Starting from t=0 to 50</th></tr>
<tr><th>Node 0</th><th>Node 1</th></tr>
<tr><td></td><td></td></tr>
<tr><th colspan="2">Starting from t=500 to 600</th></tr>
<tr><th>Node 0</th><th>Node 1</th></tr>
<tr><td></td><td></td></tr>
<tr><th colspan="2">Starting from t=500 to 520</th></tr>
<tr><th>Node 0</th><th>Node 1</th></tr>
<tr><td></td><td></td></tr>
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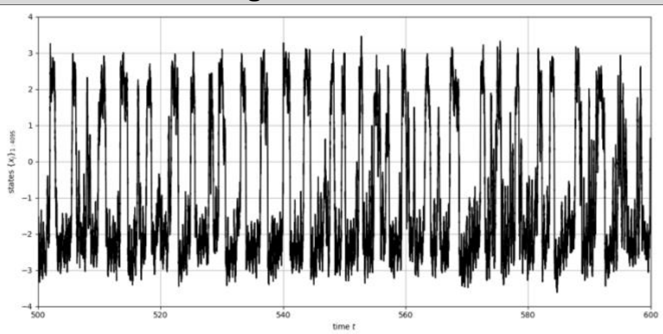
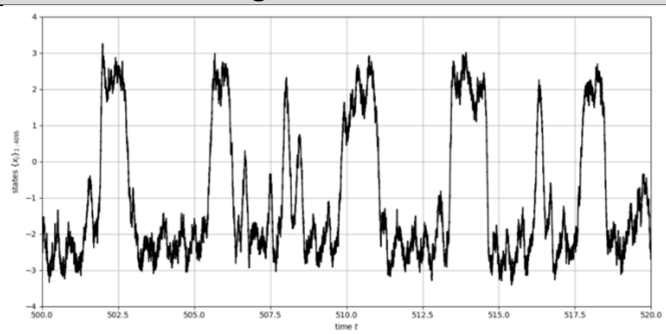
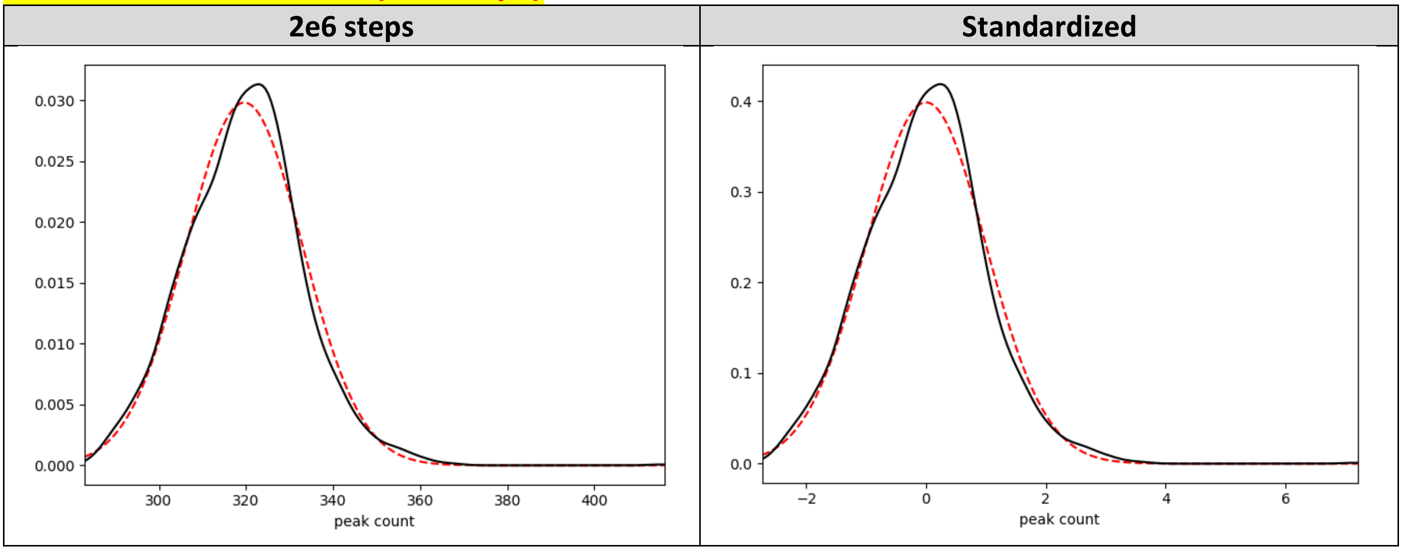
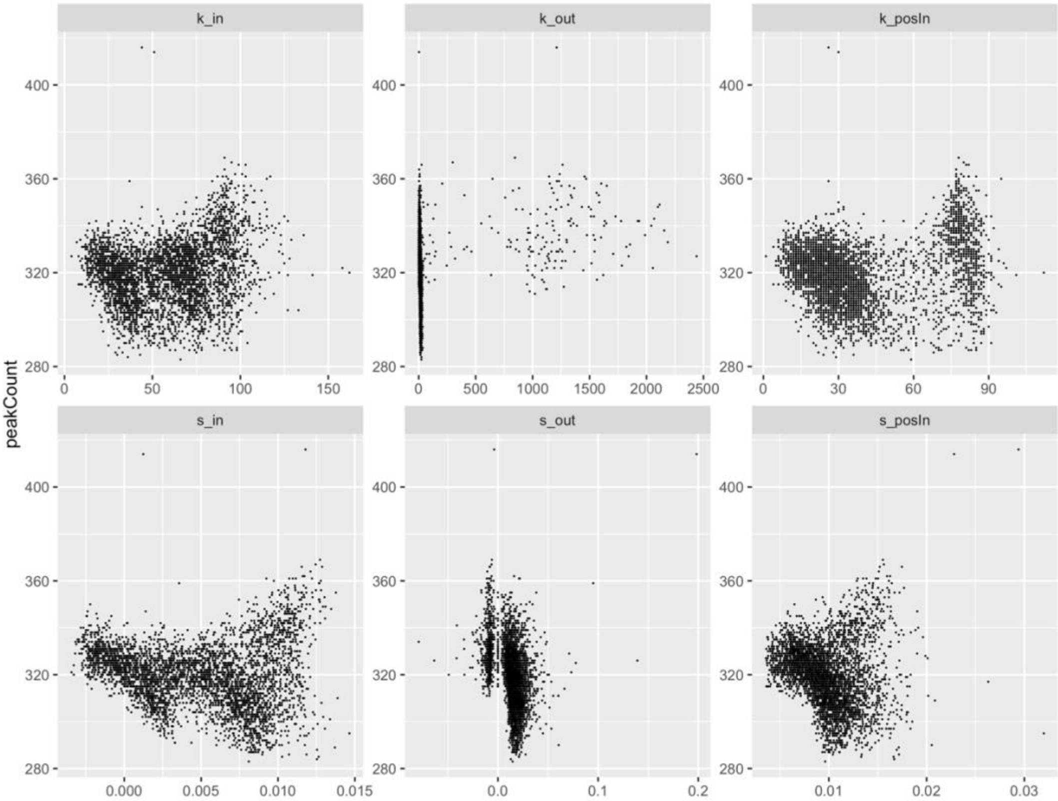
<table>
<tr><th colspan="2">Node 40</th></tr>
<tr><th>Starting from t=500 to 600</th><th>Starting from t=500 to 520</th></tr>
<tr><td></td><td></td></tr>
</table>

$(\beta_1, \beta_2, y_0) = (0.05, 1, 0); \epsilon = 0.1, \alpha = 0.95$

Peak count distribution (2e6 steps)



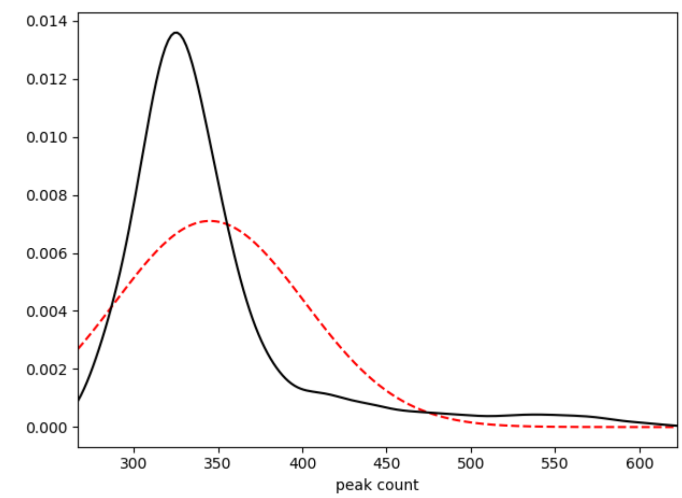
Exploratory analysis (2e6 steps)



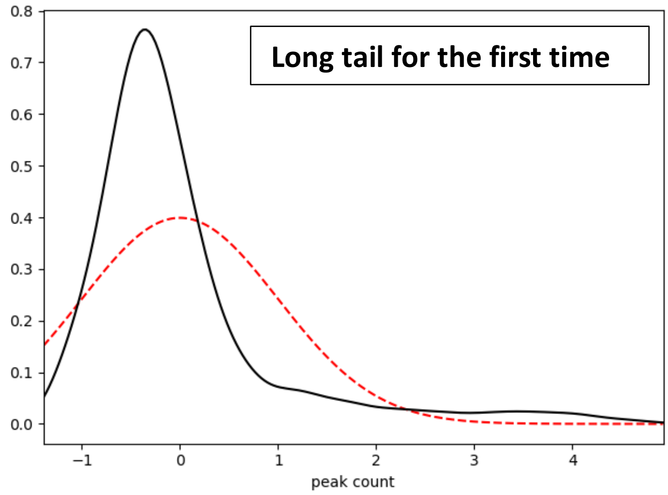
$(\beta_1, \beta_2, y_0) = (0.01, 1, 0); \epsilon = 0.1, \alpha = 0.95$

Peak count distribution (2e6 steps)

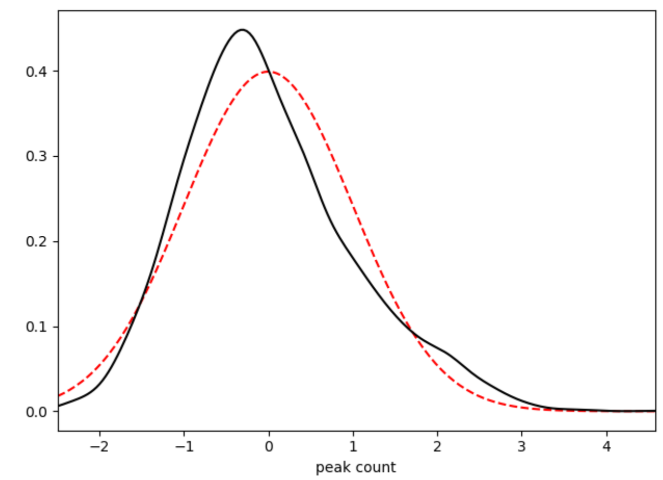
2e6 steps



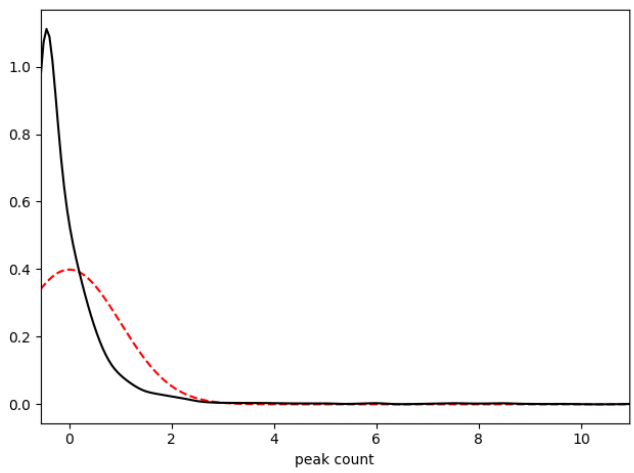
Standardized



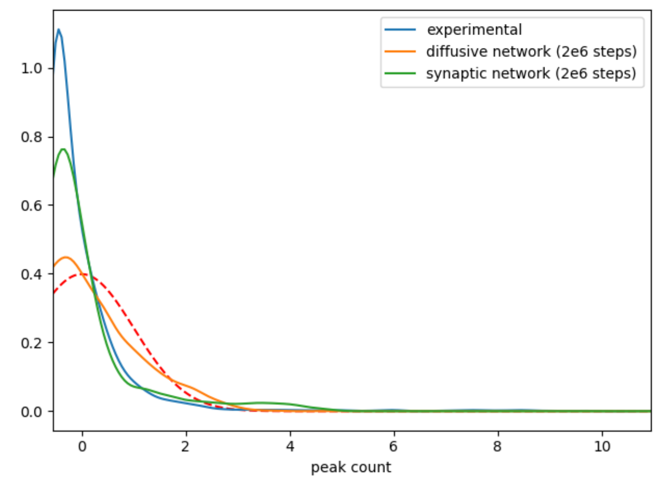
Diffusive coupling ("Original network")



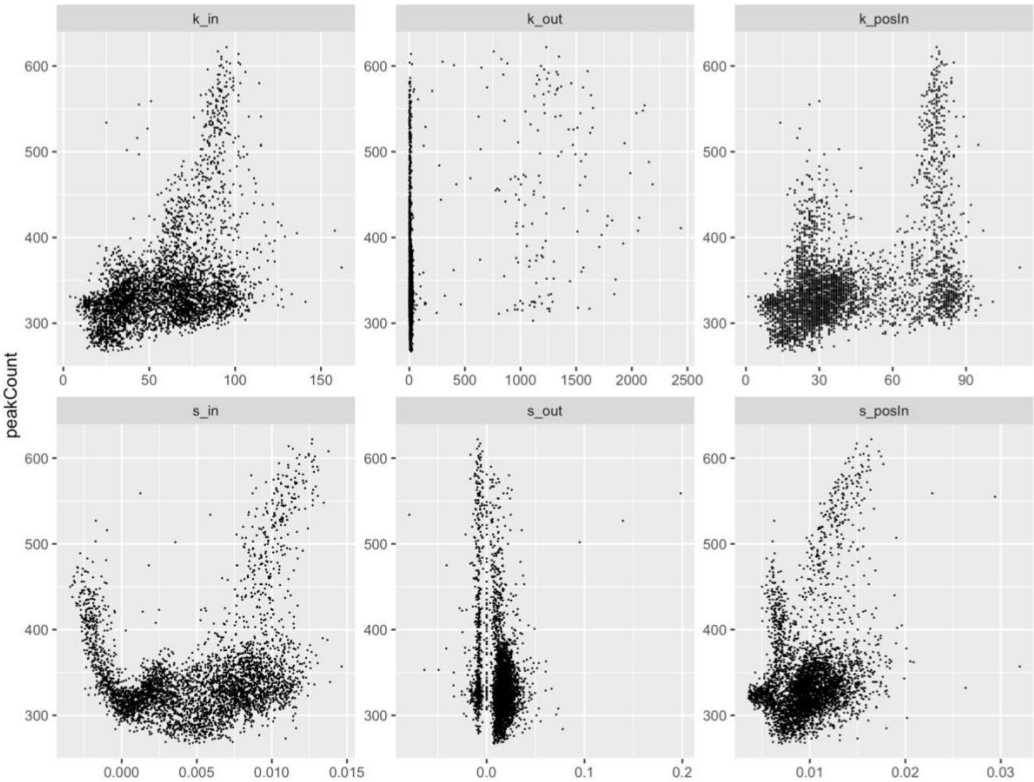
Experimental

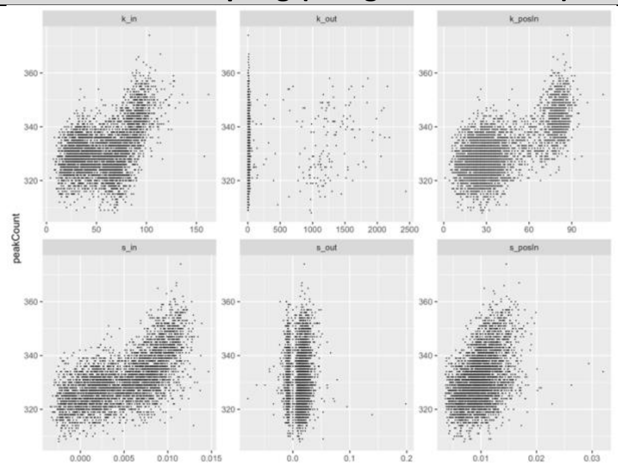
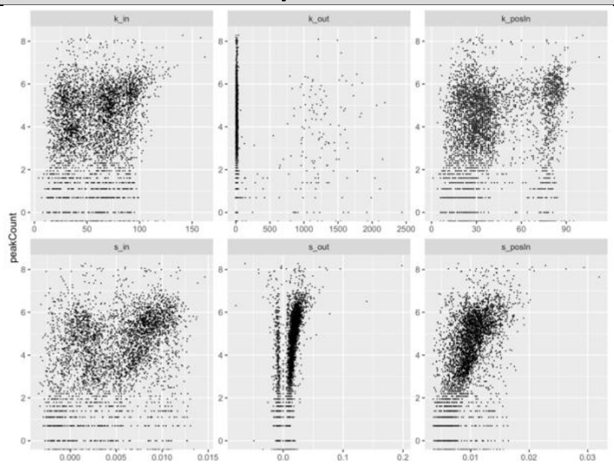


Comparison

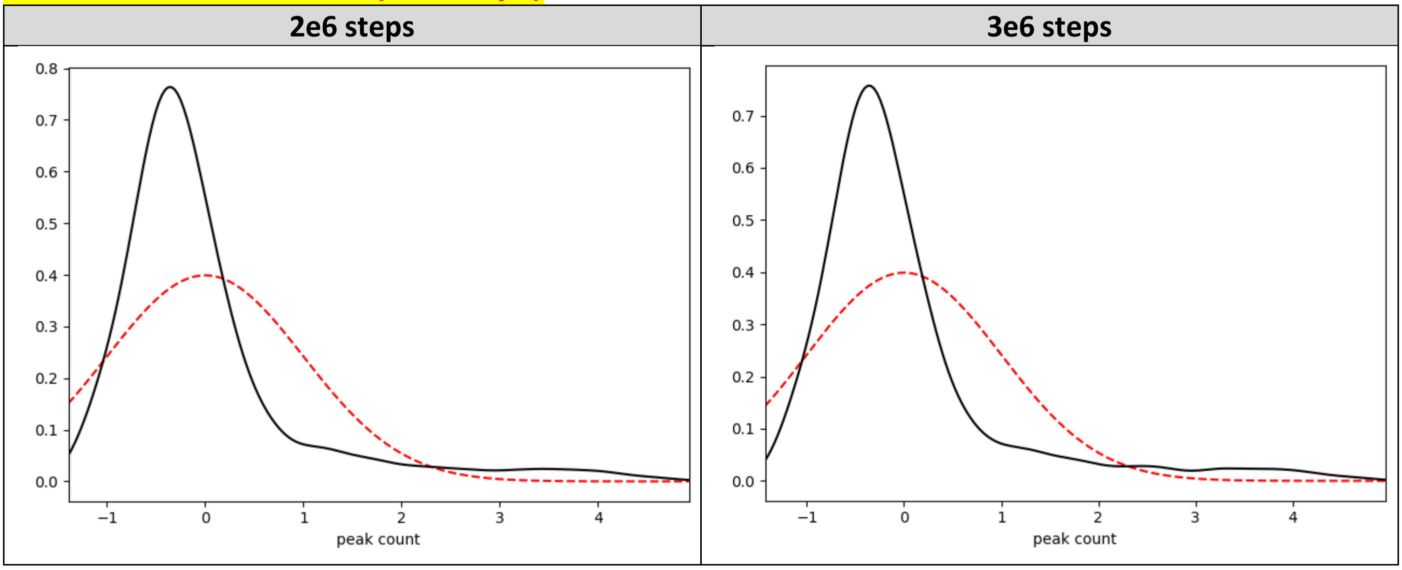


Exploratory analysis (2e6 steps)

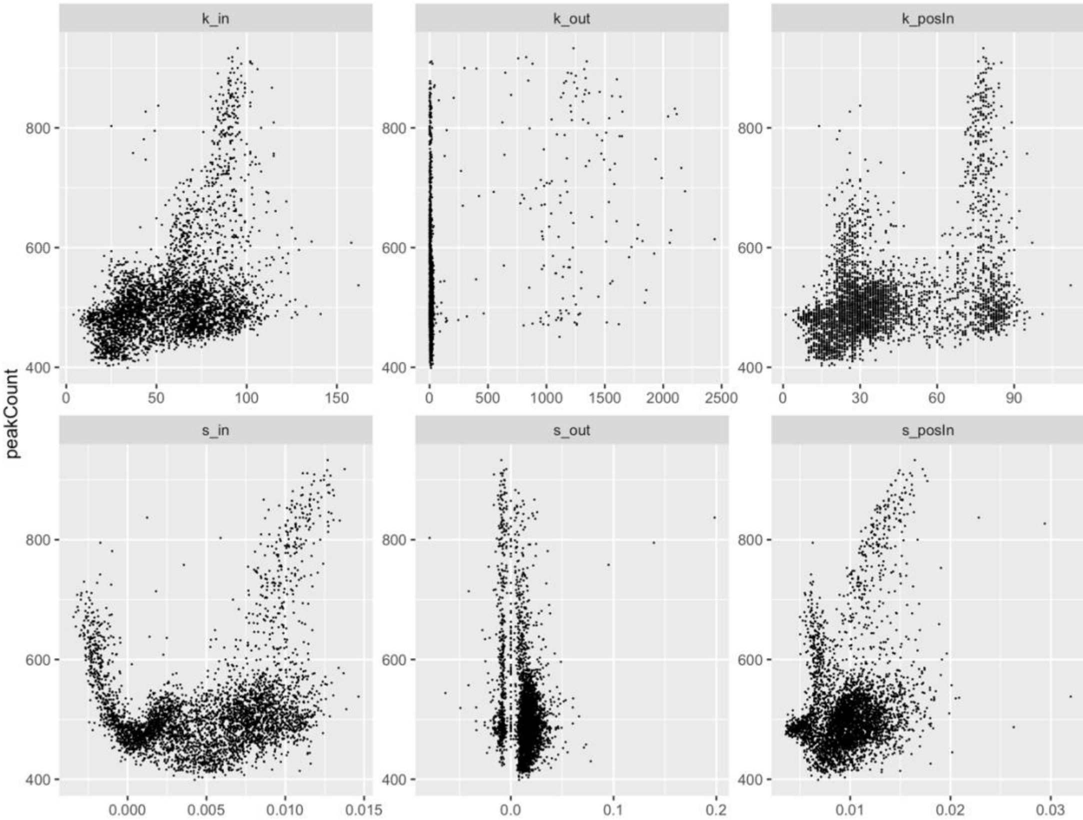


Diffusive coupling ("Original network")	Experimental
	

Peak count distribution (3e6 steps)



Exploratory analysis (3e6 steps)



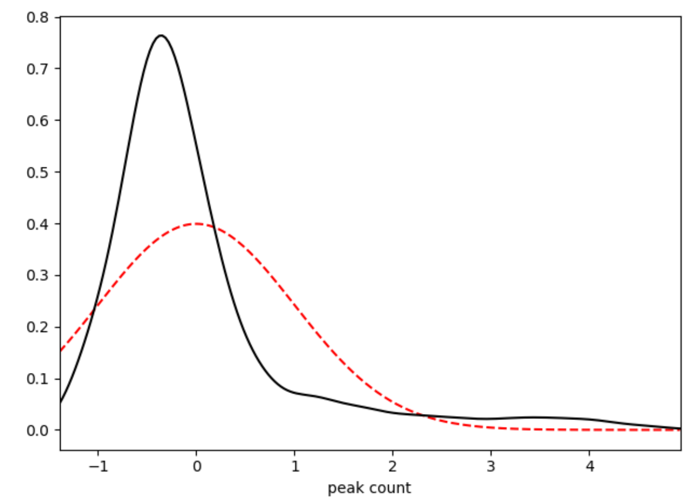
Analysis on reference networks

- **$(\beta_1, \beta_2, y_0) = (0.01, 1, 0); \epsilon = 0.1, \alpha = 0.95$**

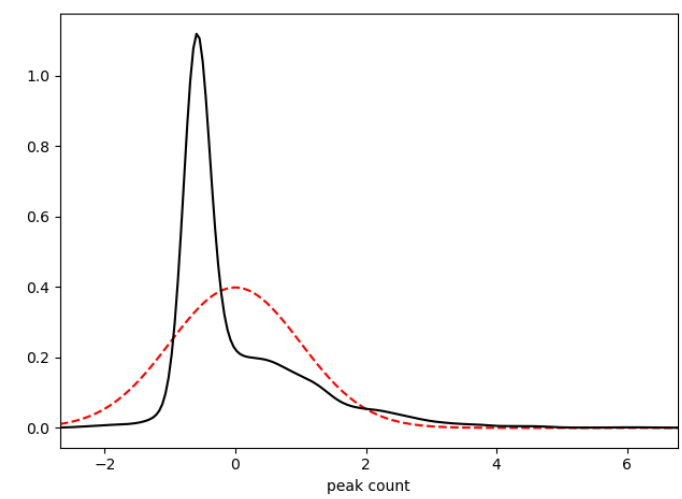
(a) reference network 1: keep Aij but replace nonzero gij by values taken from a Gaussian distribution of same mean and standard deviation (this time we do not separately consider positive and negative gij's). This network has same k_in and k_out but different s_in and s_out
(b) reference network 2: shuffle gij for fixed i; this has same k_in and s_in but different k_out and s_out
(c) reference network 3: shuffle gij for fixed j; this has same k_out and s_out but different k_in and s_in
(d) reference network 4: random directed network with same connection probability p and gij from a Gaussian distribution of same mean and standard deviation
(e) reference network 5: keep Aij but shuffle non-zero gij

Peak count distribution comparison (2e6 steps)

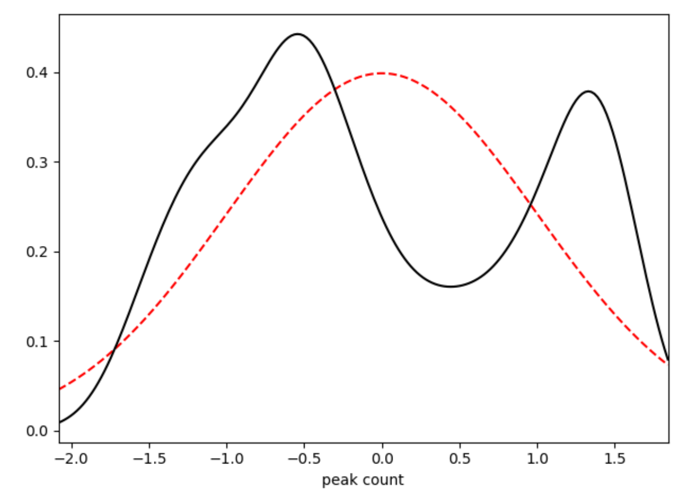
Original Network



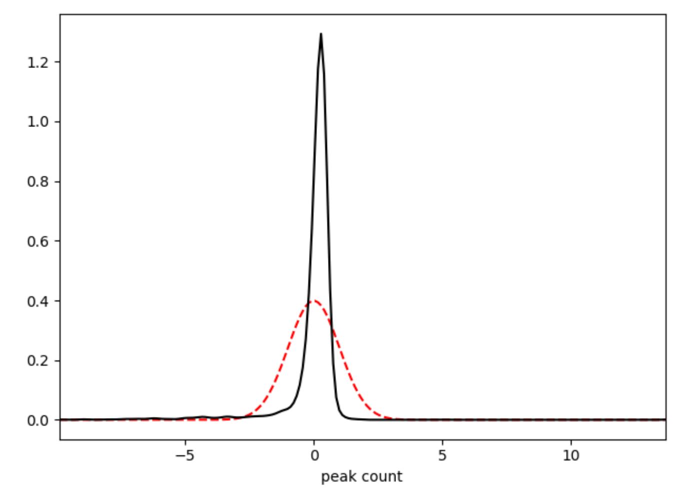
Ref. Network 1 (non-zero gij Gaussian)



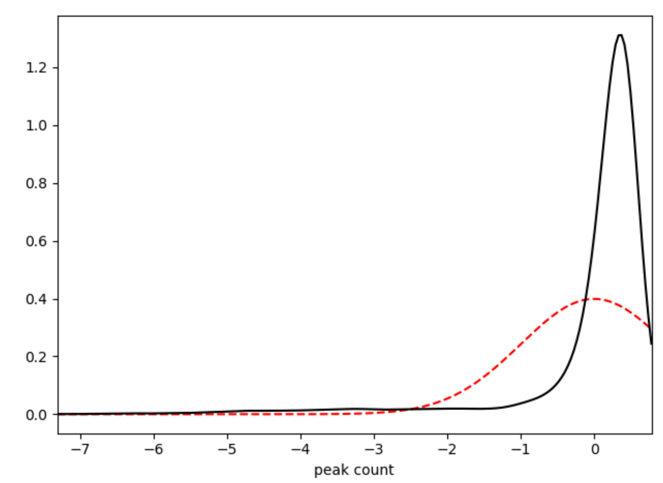
Ref. Network 2 (row gij shuffled)



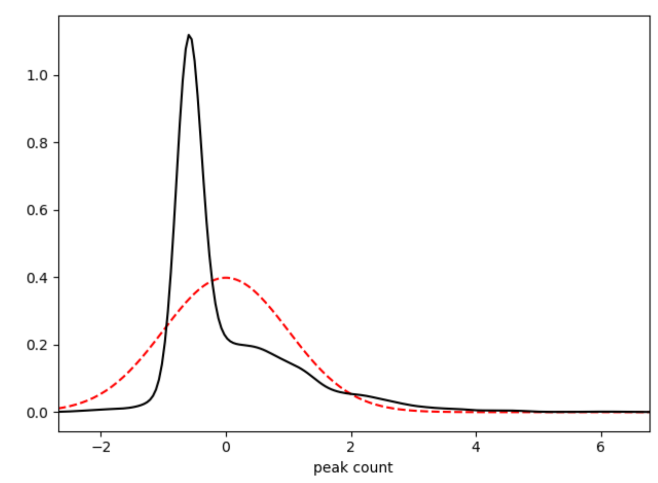
Ref. Network 3 (col gij shuffled)



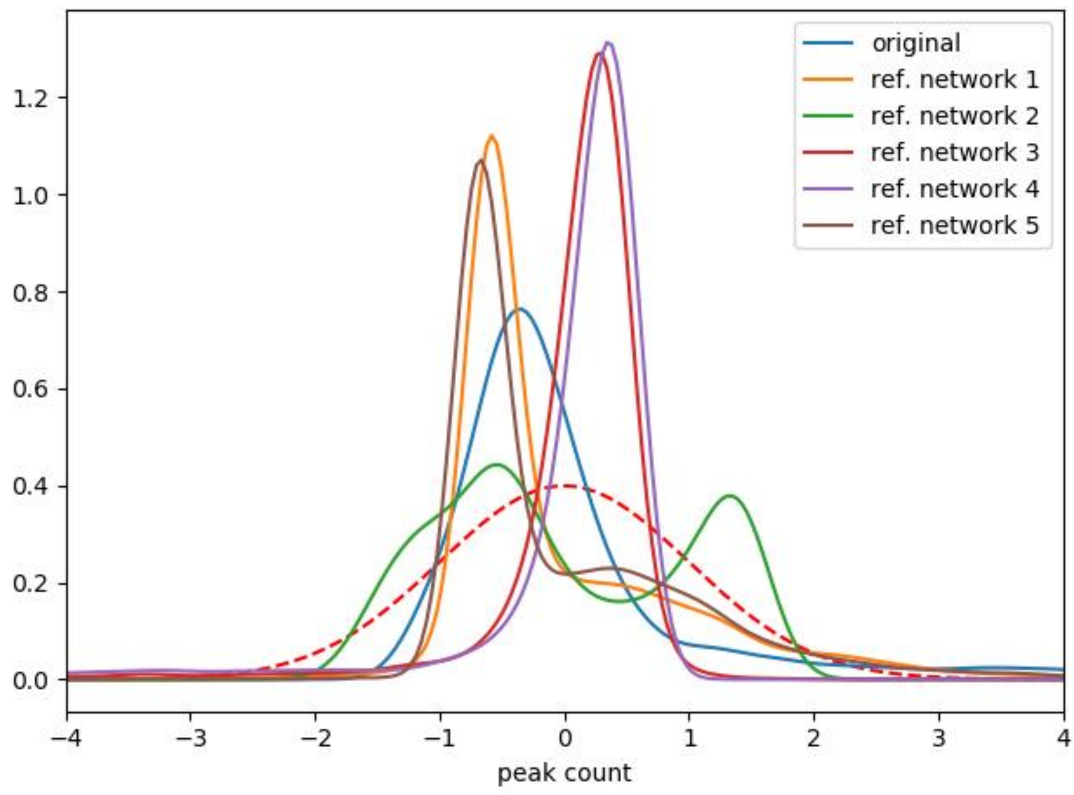
Ref. Network 4 (random)



Ref. Network 5 (non-zero gij shuffled)



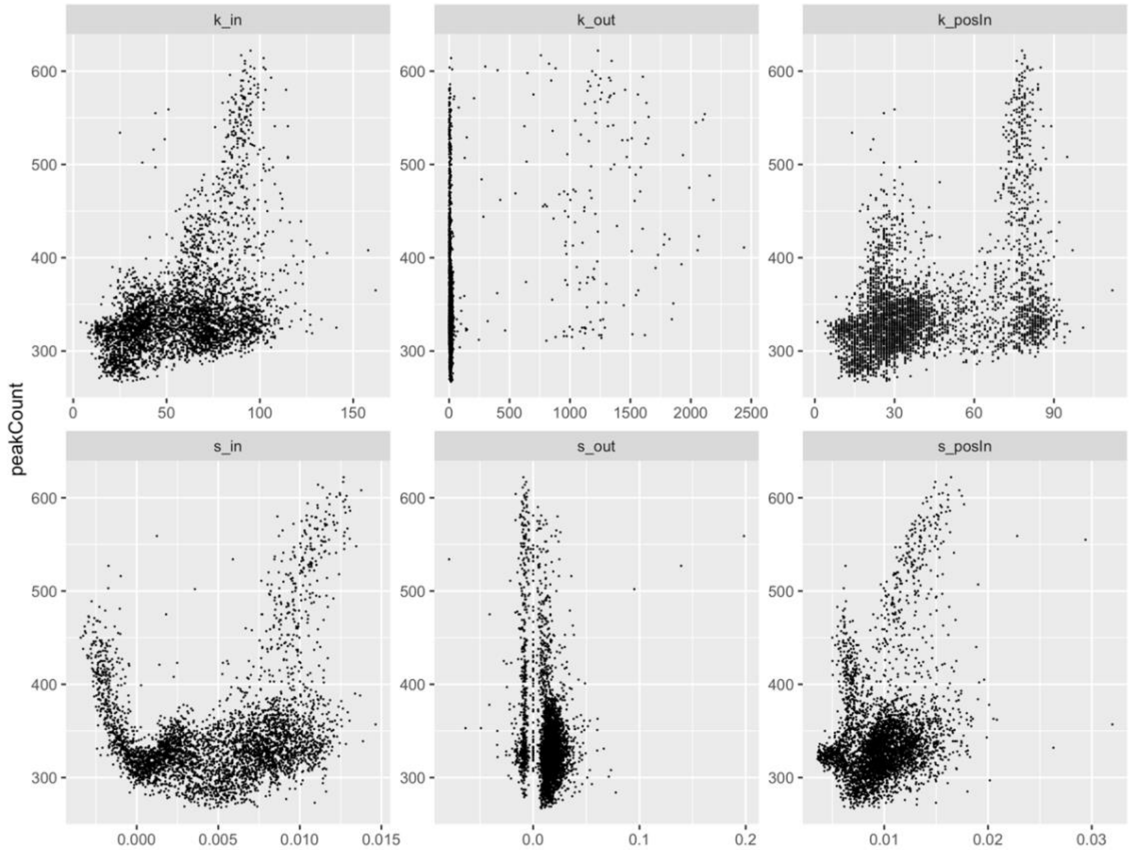
Combined



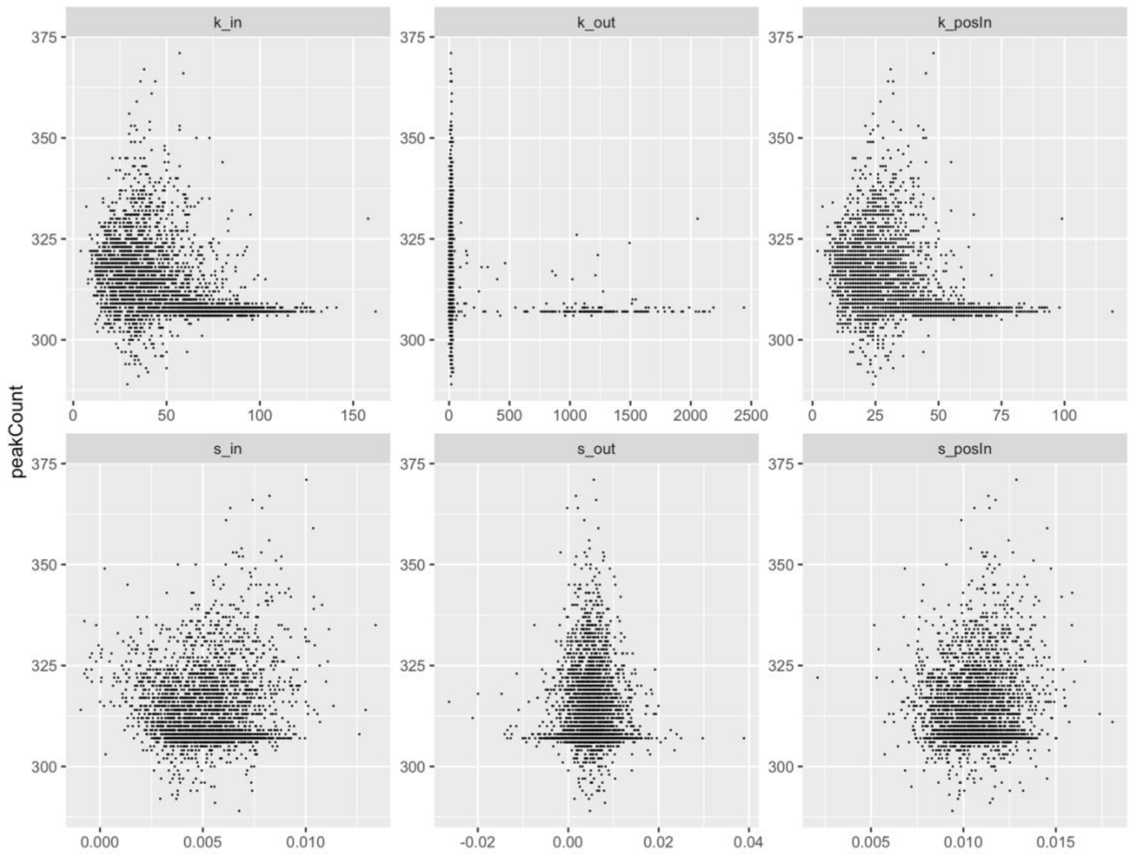
original
ref. network 1
ref. network 2
ref. network 3
ref. network 4
ref. network 5
1.2
1.0
0.8
0.6
0.4
0.2
0.0
−4
−3
−2
−1
0
1
2
3
4
peak count

Exploratory analysis (2e6 steps)

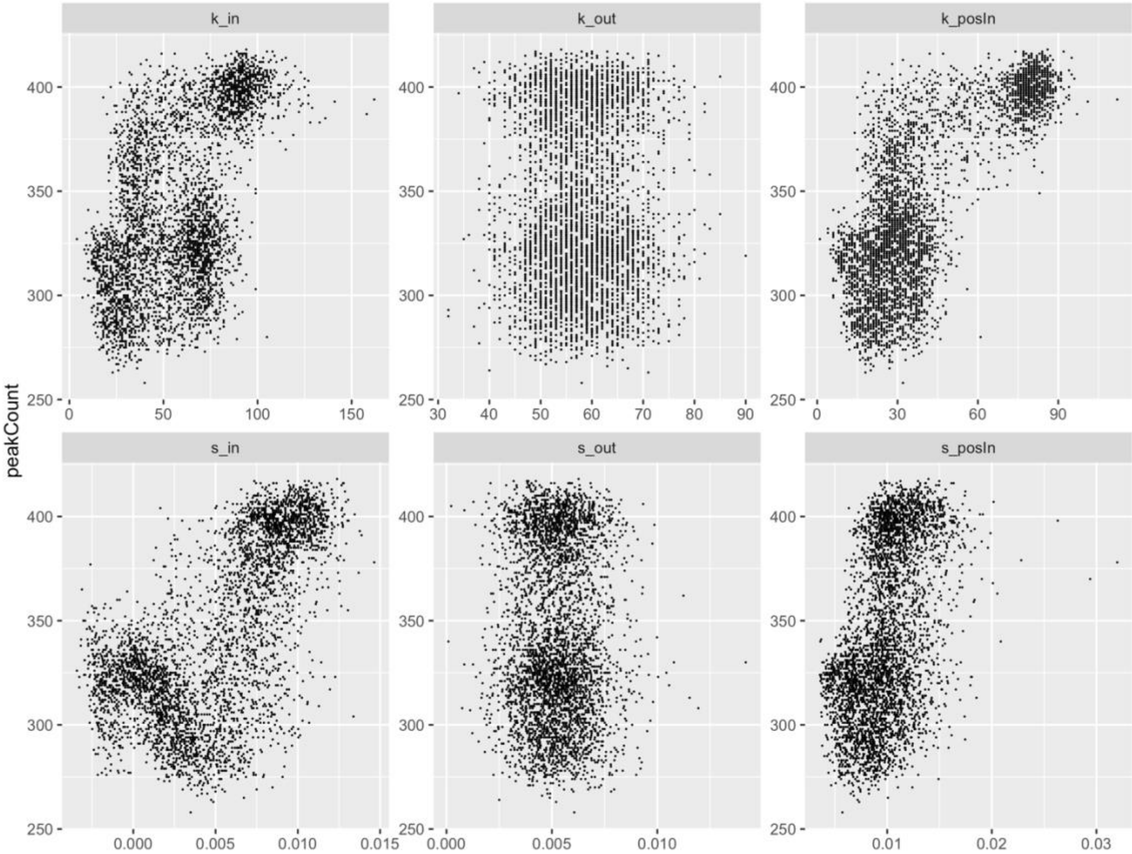
Original Network



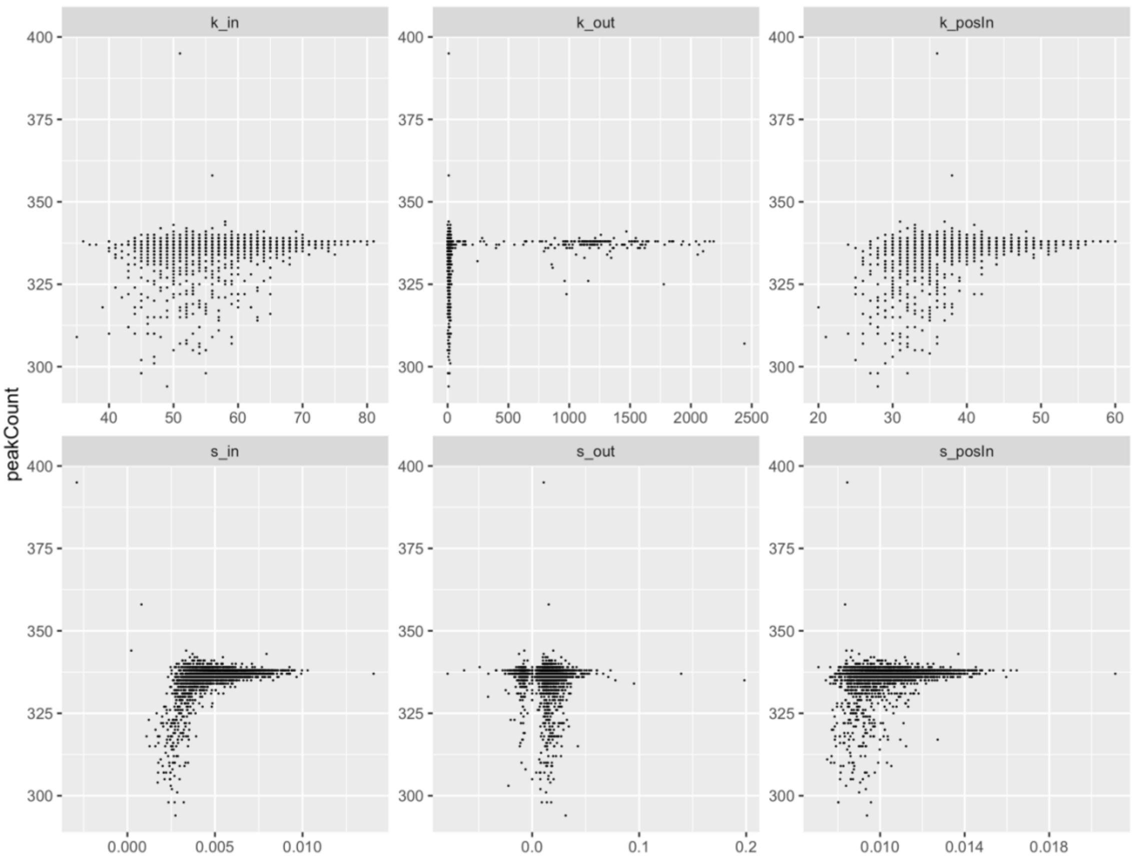
Ref. Network 1 (non-zero gij Gaussian)



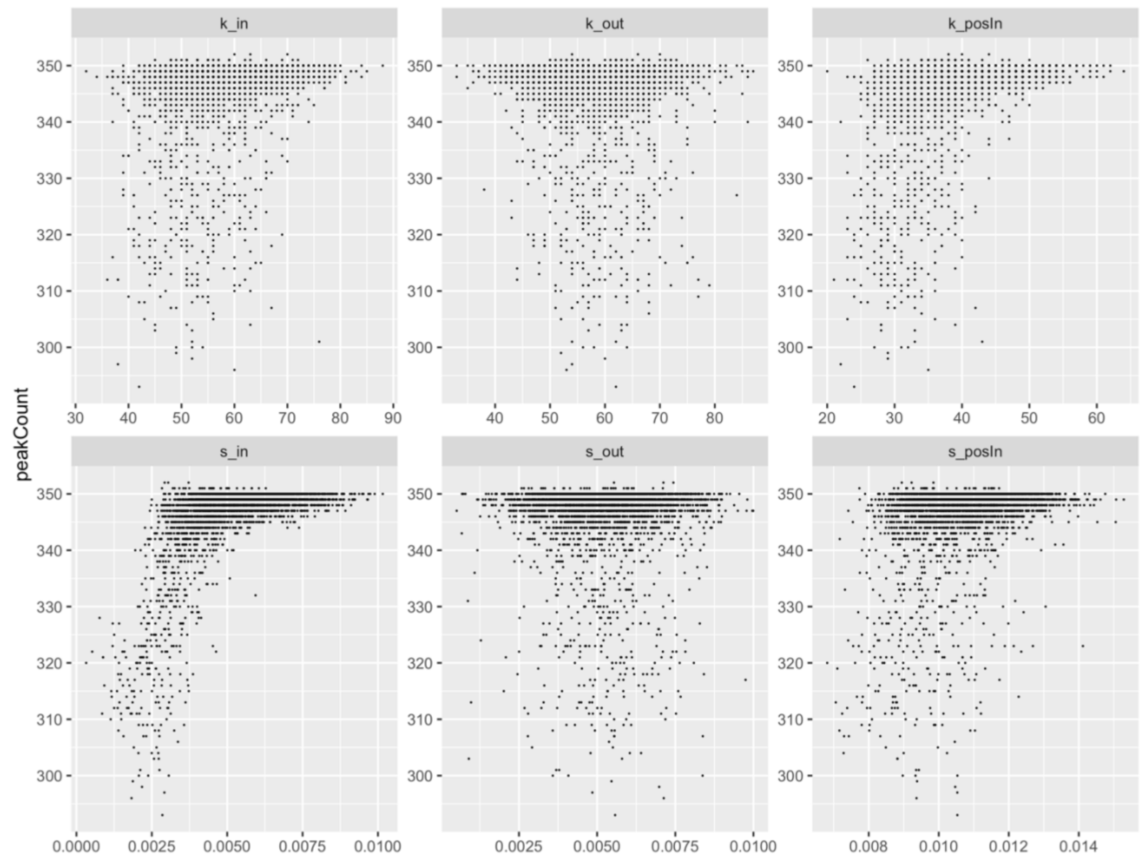
Ref. Network 2 (row gij shuffled)



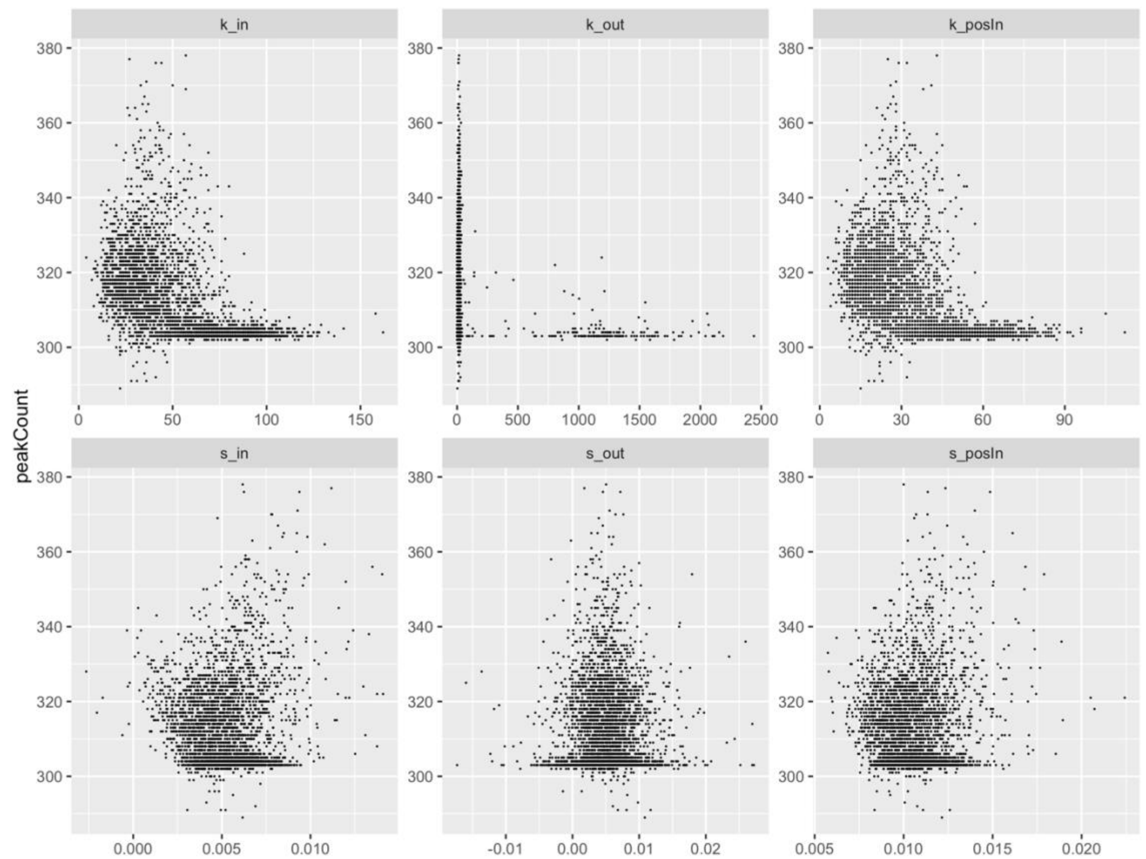
Ref. Network 3 (col gij shuffled)



Ref. Network 4 (random)

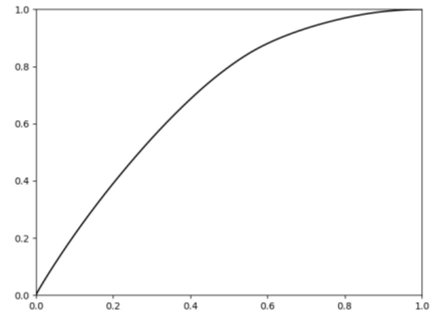
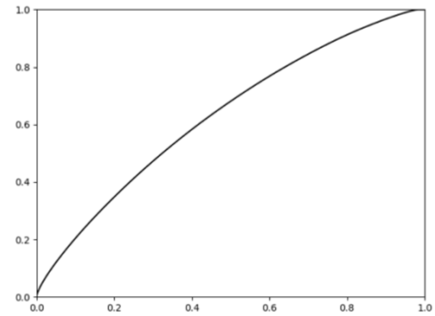
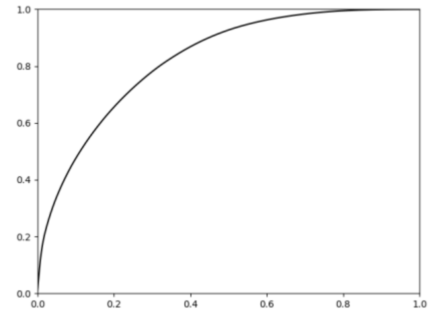
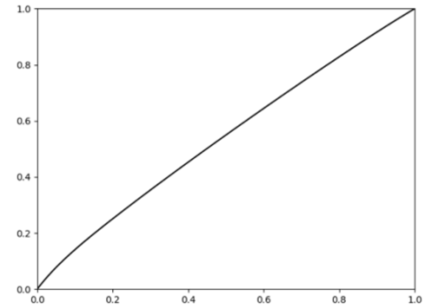
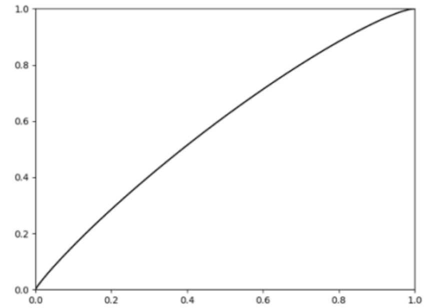
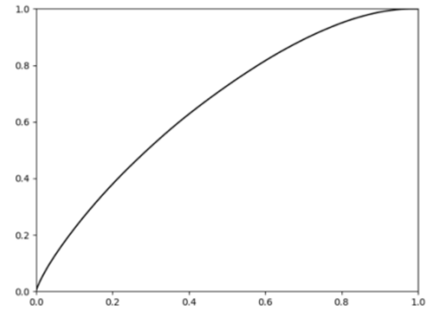
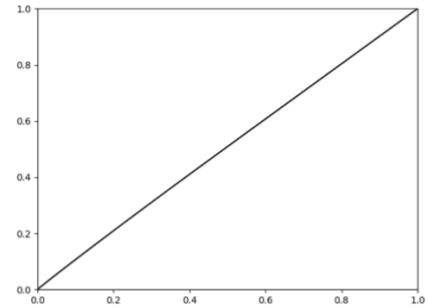
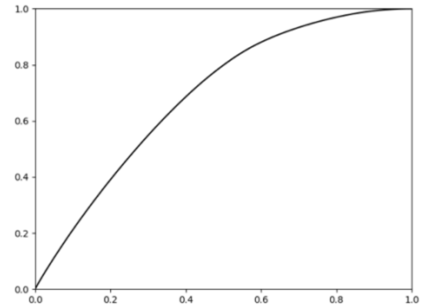
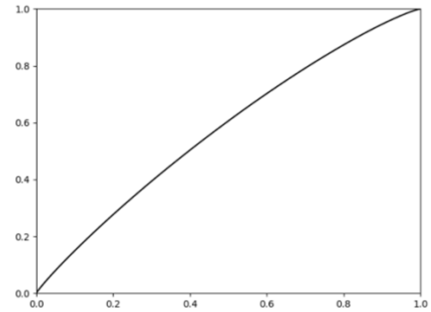
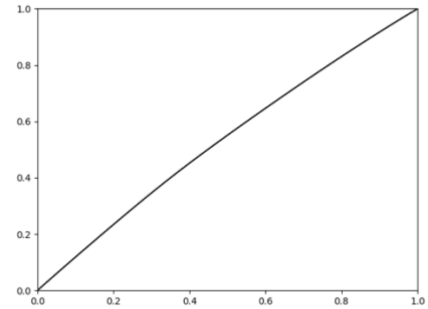


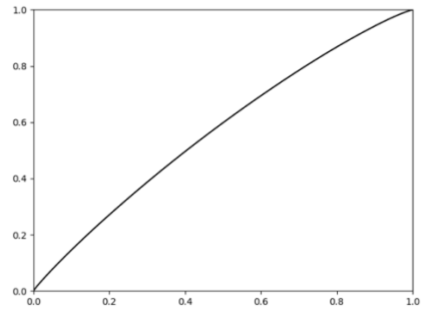
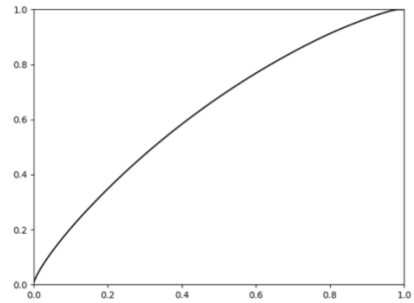
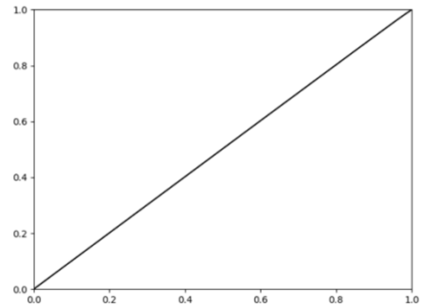
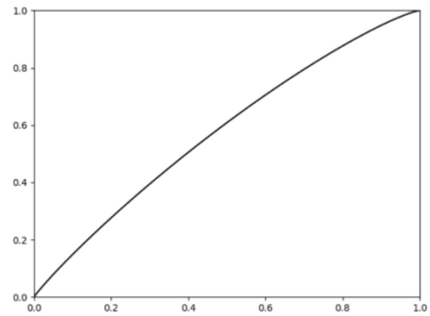
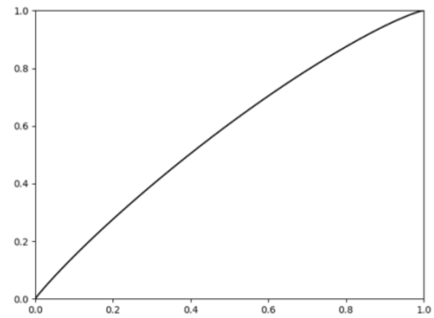
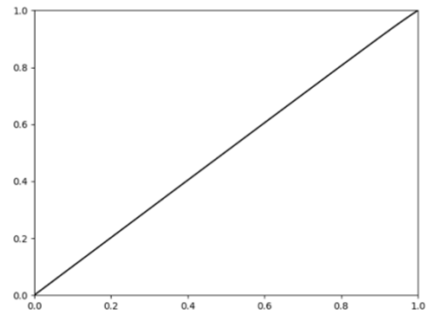
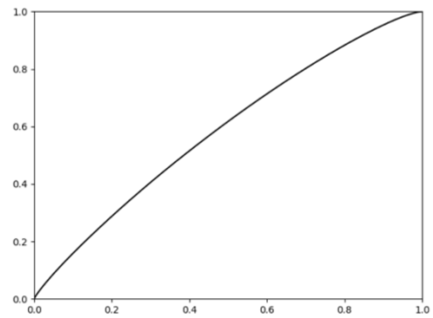
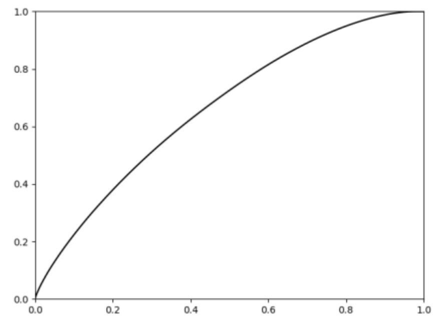
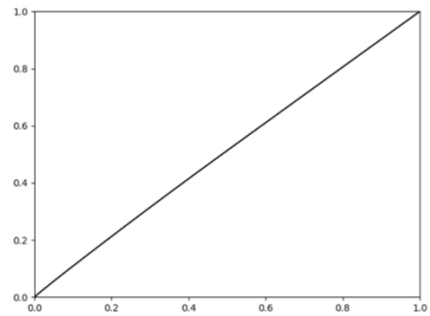
Ref. Network 5 (non-zero gij shuffled)

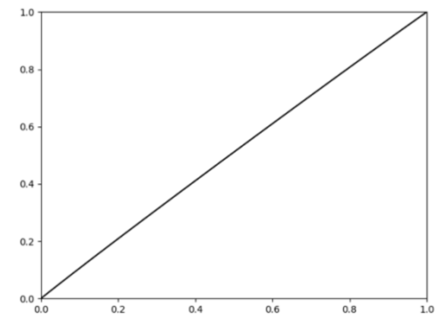
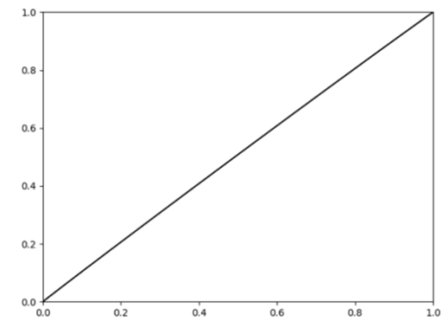
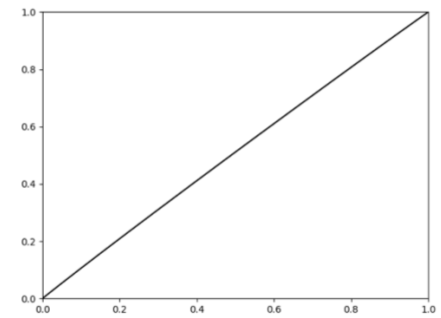
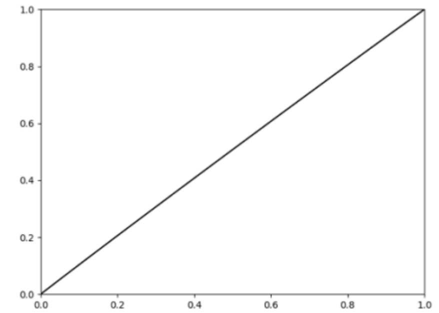
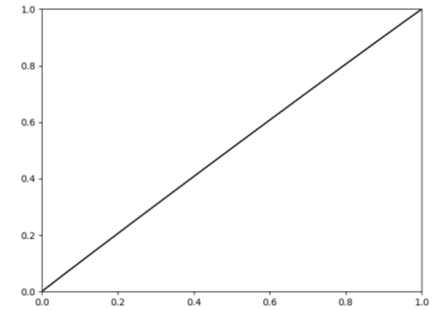
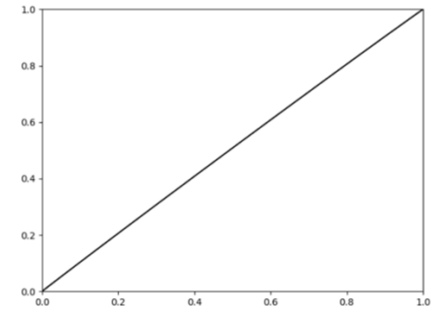


Analysis on peak counts

- **Fraction of total accounted for by the top quantiles**

In-strengths (Magnitude)	Out-strengths (Magnitude)	Peak counts
Experimental		
		 **(Very large curvature)**
Original Network (Synaptic)		
Same as above	**Same as above**	 **(Little curvature)**
Ref. Network 1 (non-zero gij Gaussian)		
		 **(Basically straight)**
Ref. Network 2 (row gij shuffled)		
		 **(Little curvature but most curved among all ref networks)**

Ref. Network 3 (col gij shuffled)		
		 **(Basically straight)**
Ref. Network 4 (random)		
		 **(Basically straight)**
Ref. Network 5 (non-zero gij shuffled)		
		 **(Basically straight)**

<table>
<tr><th colspan="3">Diffusive (All basically straight)</th></tr>
<tr><th>Original Network (Diffusive)</th><th>Ref. Network 1 (non-zero gij Gaussian)</th><th>Ref. Network 2 (row gij shuffled)</th></tr>
<tr><td></td><td></td><td></td></tr>
<tr><th>Ref. Network 3 (col gij shuffled)</th><th>Ref. Network 4 (random)</th><th>Ref. Network 5 (non-zero gij shuffled)</th></tr>
<tr><td></td><td></td><td></td></tr>
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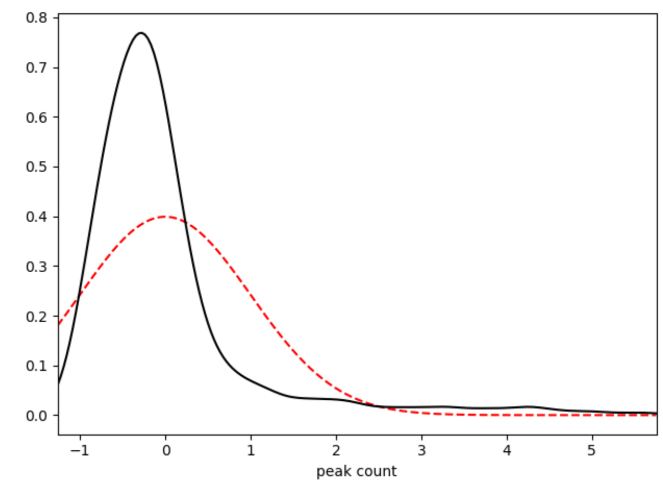
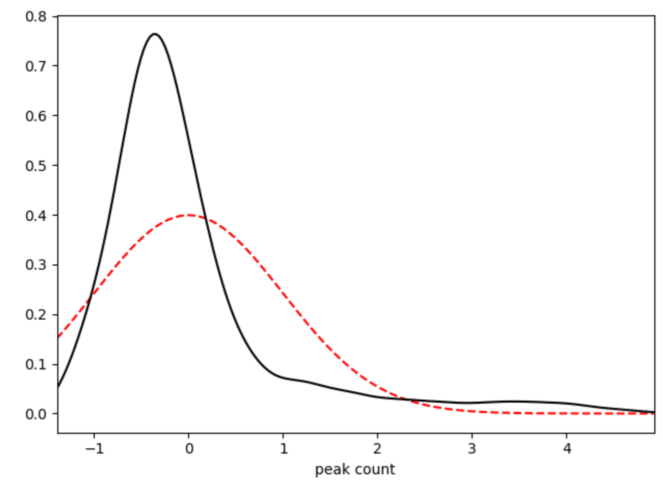
Peak detection method

<table>
<tr><th colspan="2">Original Network (Synaptic)</th></tr>
<tr><th>d=1000 time steps btw. peaks</th><th>d=2000 time steps btw. peaks (previously used)</th></tr>
<tr><td>
</td><td>
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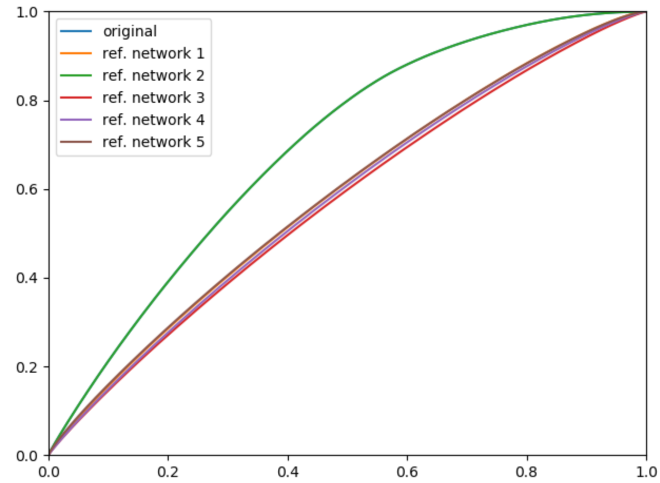
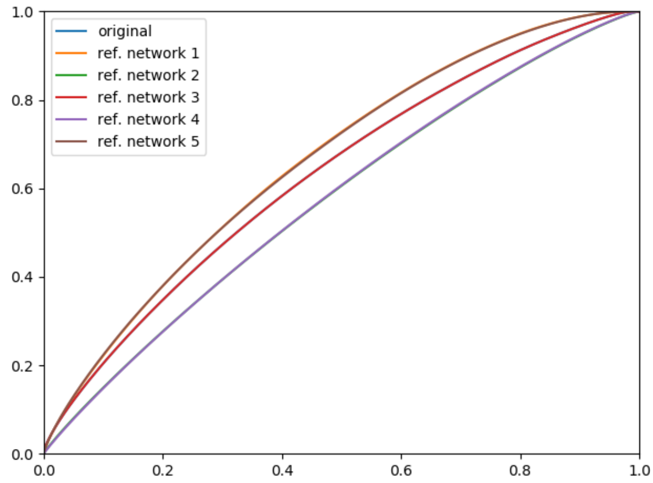
Dominance plot comparison

In-strengths (Magnitude)

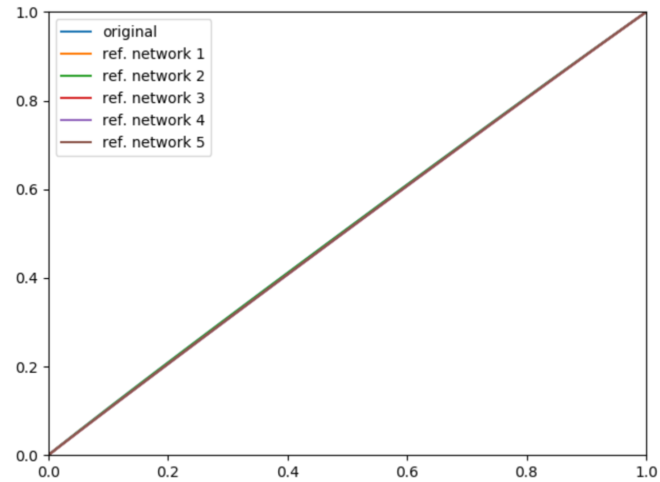


Out-strengths (Magnitude)

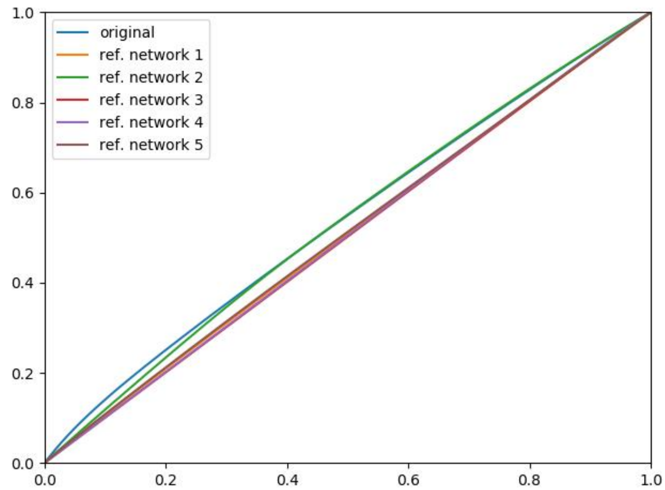


FHN

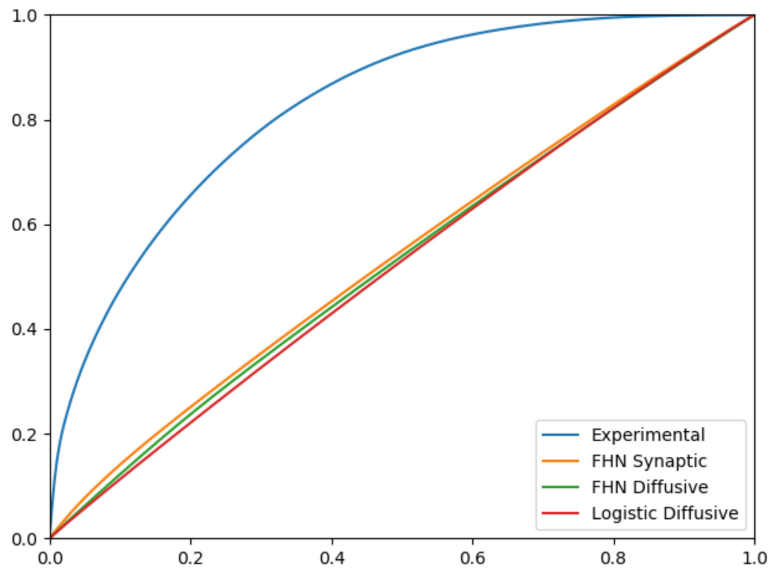
Diffusive



Synaptic

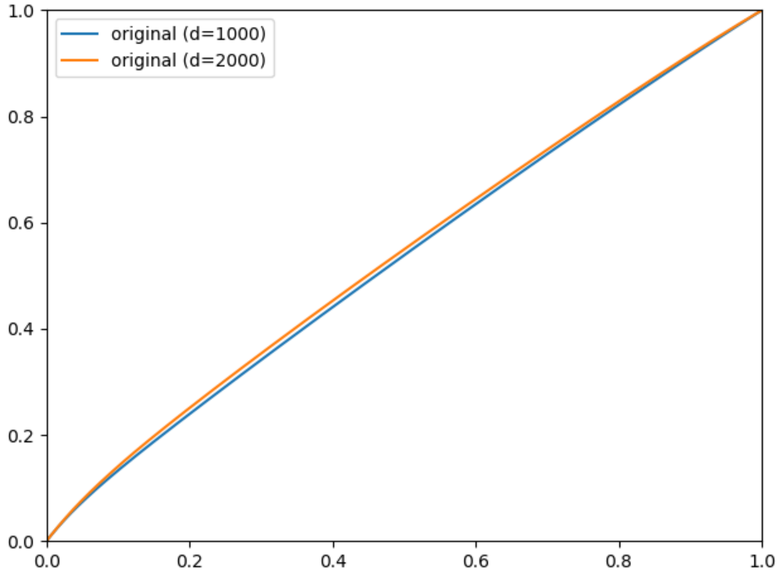


(Model comparison) Experimental, Logistic, FHN Diffusive (gij multiplied), FHN Synaptic

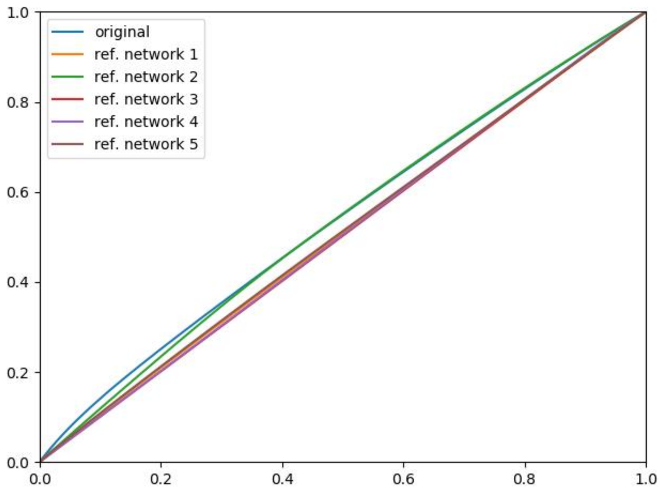


Effect of min sep d on peak counts

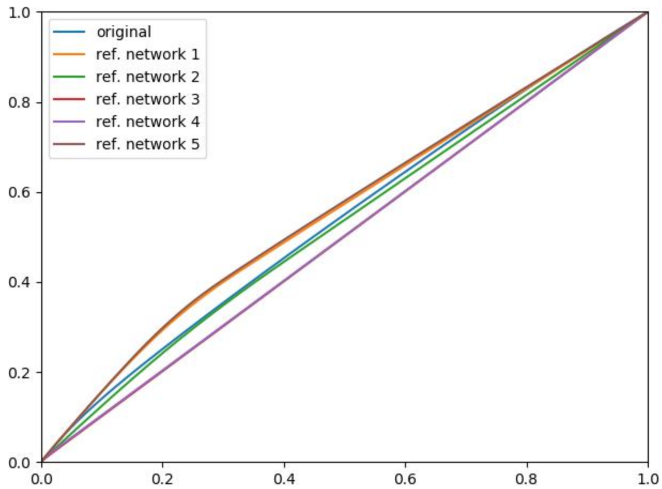
Original Network (d=1000 vs d=2000)



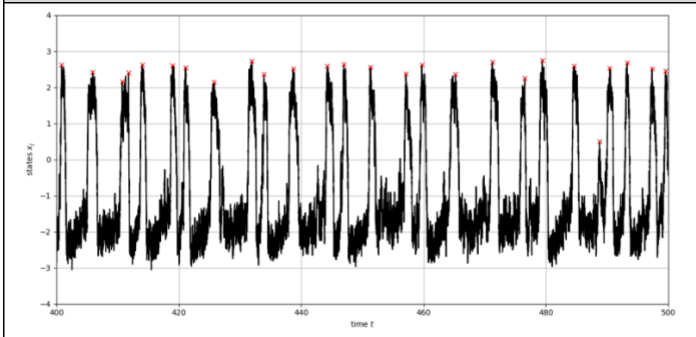
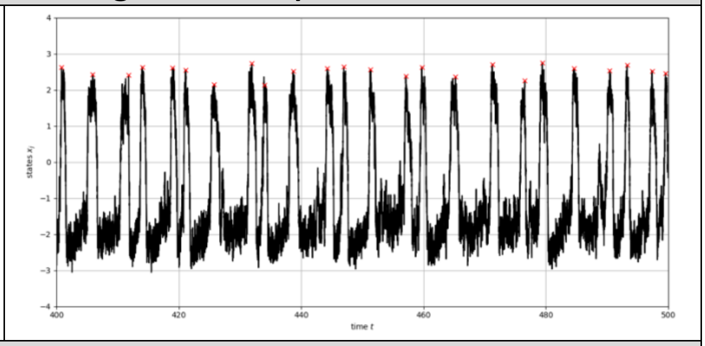
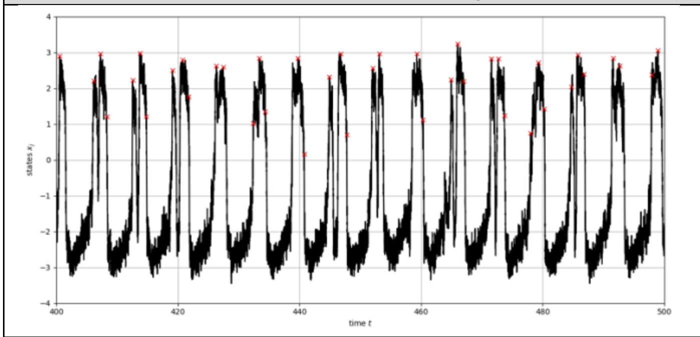
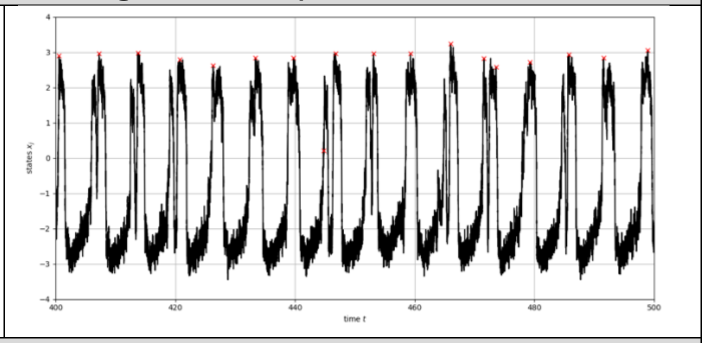
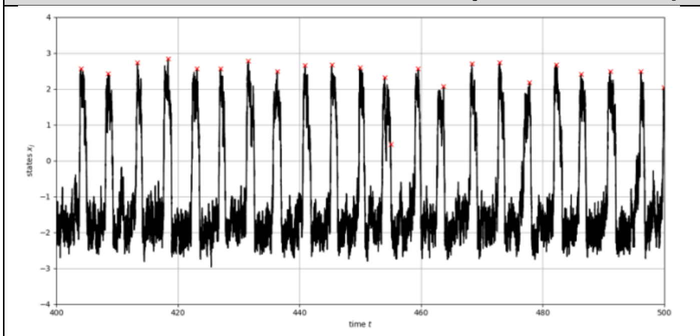
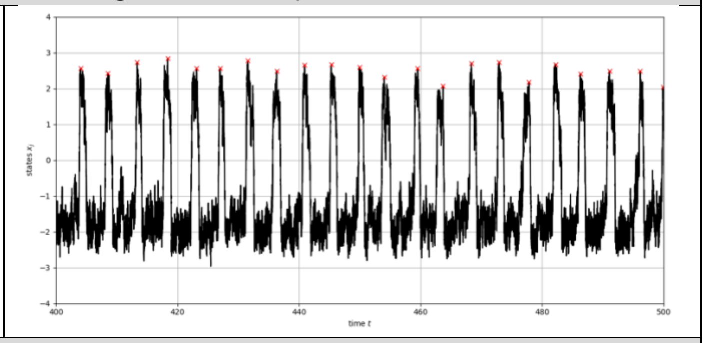
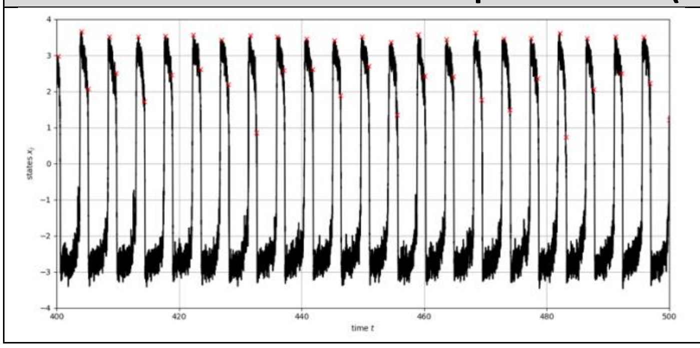
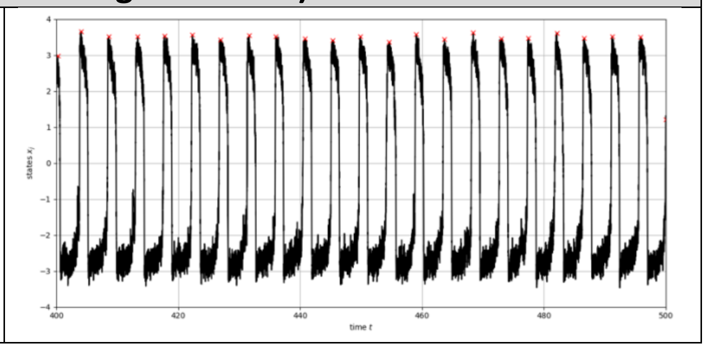
Ref. Network (d=2000)



Ref. Network (d=4000)



Refence network detected peaks

<table>
<tr><th>d=2000</th><th>d=4000</th></tr>
<tr><td colspan="2">Ref. Network 1 (non-zero gij Gaussian)</td></tr>
<tr><td colspan="2">Min peak counts (according to d=2000)</td></tr>
<tr><td></td><td></td></tr>
<tr><td colspan="2">Max peak counts (according to d=2000)</td></tr>
<tr><td></td><td></td></tr>
<tr><td colspan="2">Ref. Network 2 (row gij shuffled)</td></tr>
<tr><td colspan="2">Min peak counts (according to d=2000)</td></tr>
<tr><td></td><td></td></tr>
<tr><td colspan="2">Max peak counts (according to d=2000)</td></tr>
<tr><td></td><td></td></tr>
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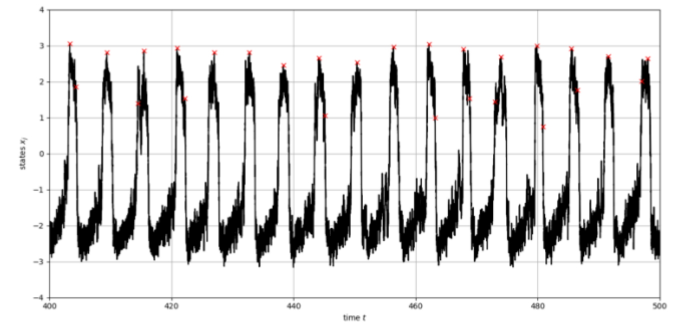
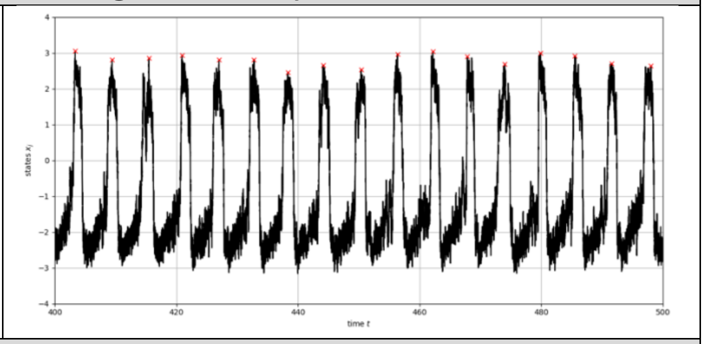
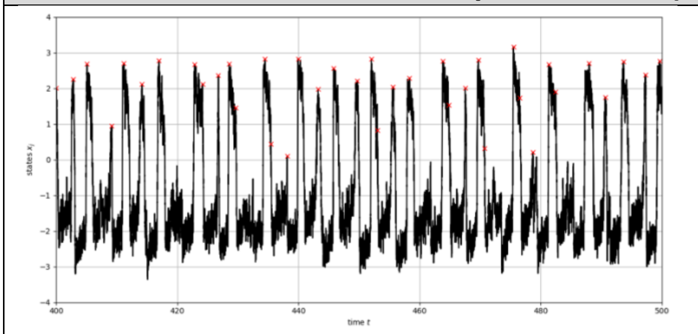
Ref. Network 3 (col gij shuffled)

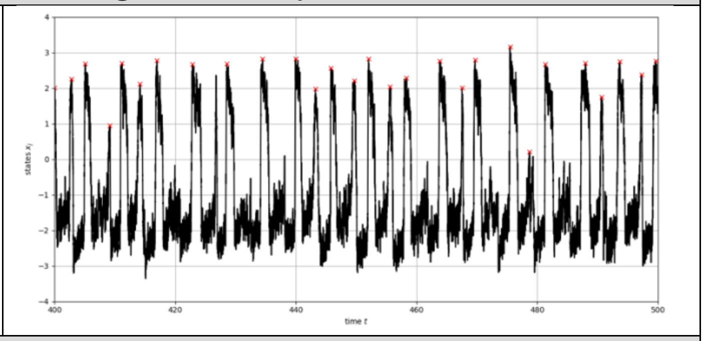
Min peak counts (according to d=2000)





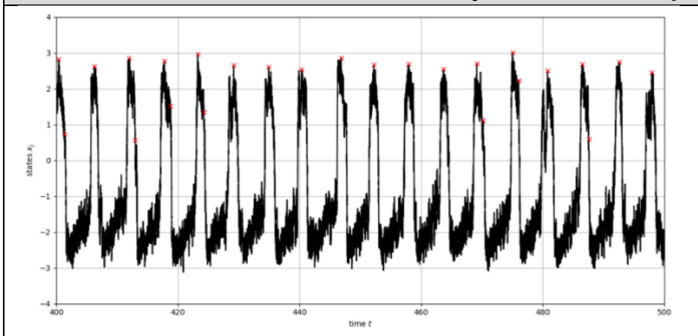
Max peak counts (according to d=2000)

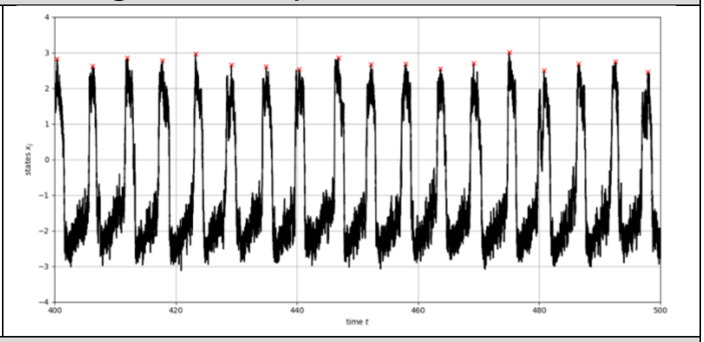




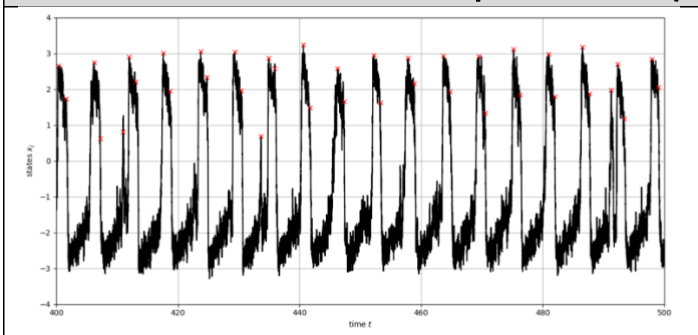
Ref. Network 4 (random)

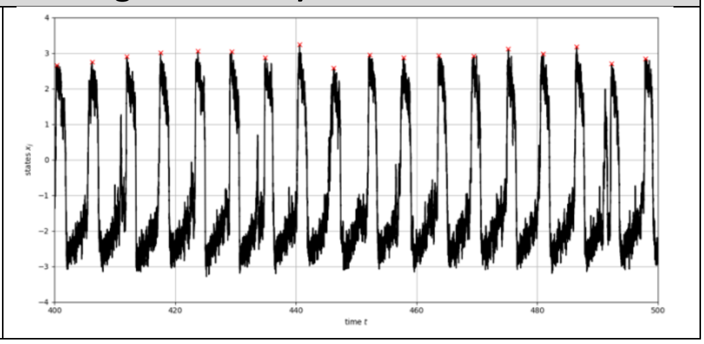
Min peak counts (according to d=2000)

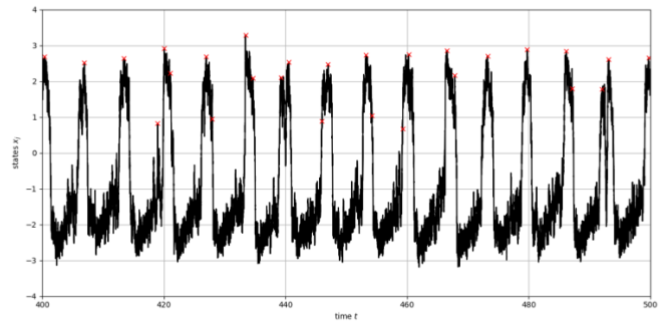
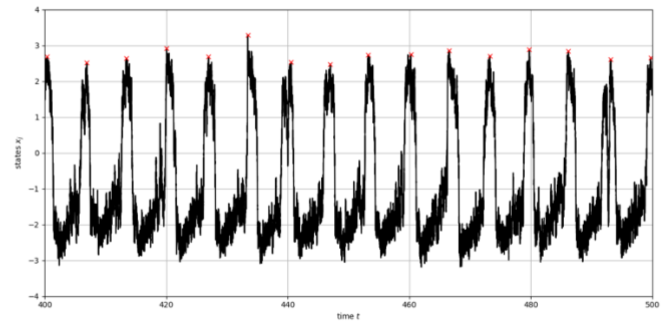
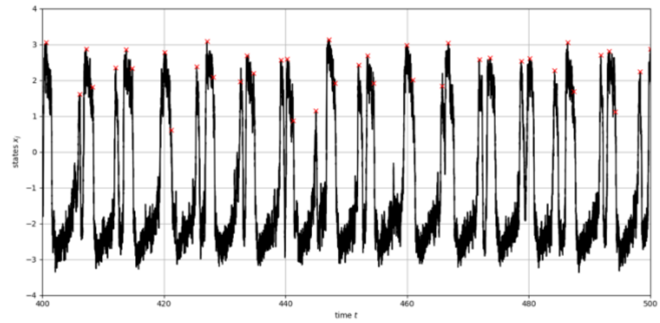
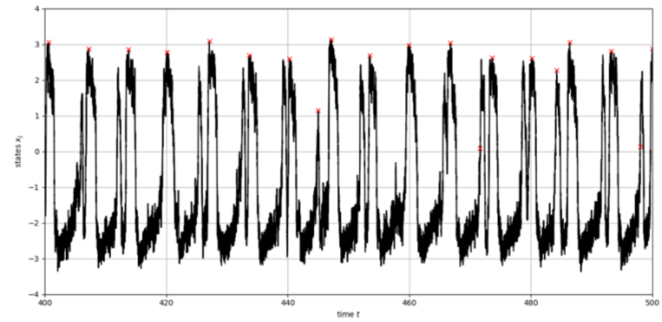




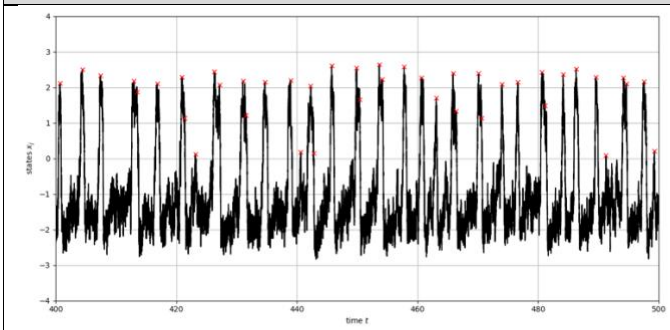
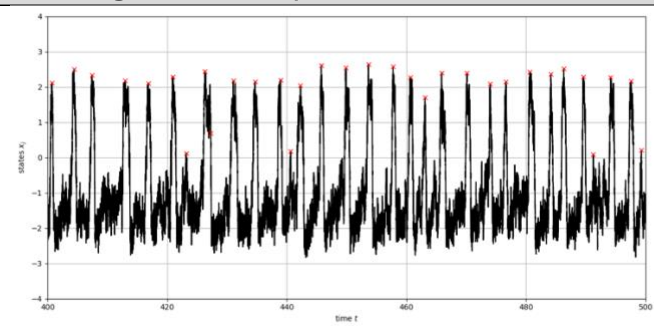
Max peak counts (according to d=2000)





Ref. Network 5 (non-zero gij shuffled)	
Min peak counts (according to d=2000)	
	
Max peak counts (according to d=2000)	
	

Original network detected peaks

d=1000	d=2000
Min peak counts (according to d=1000)	
	
Max peak counts (according to d=1000)