1.2 Let $X_1, ...X_n$ be uncorrelated random variables with common expectation $\theta$ and variance $\sigma^2$. Then, among all linear estimators $\sum \alpha_i X_i$ of $\theta$ satisfying $\sum \alpha_i = 1$, the mean $\overline{X}$ has the smallest variance.

1.8 If $\phi(a) = E|X - a| < \infty$ for some $a$, show that $\phi(a)$ is minimized by any median of X.

[Hint: If $m_0 \leq m \leq m_1$ and $m_1 < c$, then

$$E|X-c|-E|X-m| = (c-m)[P(X \leq m)-P(X > m)]+2\int_{m<x<c}(c-x)dP(x)]. \tag{1}$$

4.1 If the distributions of a positive random variable X form a scale family, show that the distributions of $logX$ form a location family.

4.13 Let $U$ be a positive random variable, and let

$$X = bU^{1/c}, b > 0, c > 0. \tag{2}$$

(a) Show that this defines a group family.

(b) If U is distributed as exponential distribution, then X is distributed according to the Weibull distribution with density

$$\frac{c}{b}(\frac{x}{b})^{c-1}e^{-(x/b)^c}, x > 0. \tag{3}$$

5.1 Determine the natural parameter space of a family $P_\theta$ with densities of the form

$$p(x|\eta) = exp[\sum_{i=1}^{s} \eta_i T_i(x) - A(\eta)]h(x). \tag{4}$$

when $s = 1$,$T_1(x) = x$, $\mu$ is Lebesgue measure, and $h(x)$ is $(1)e^{-|x|}$ and $(2)e^{-|x|}/(1+x^2)$.