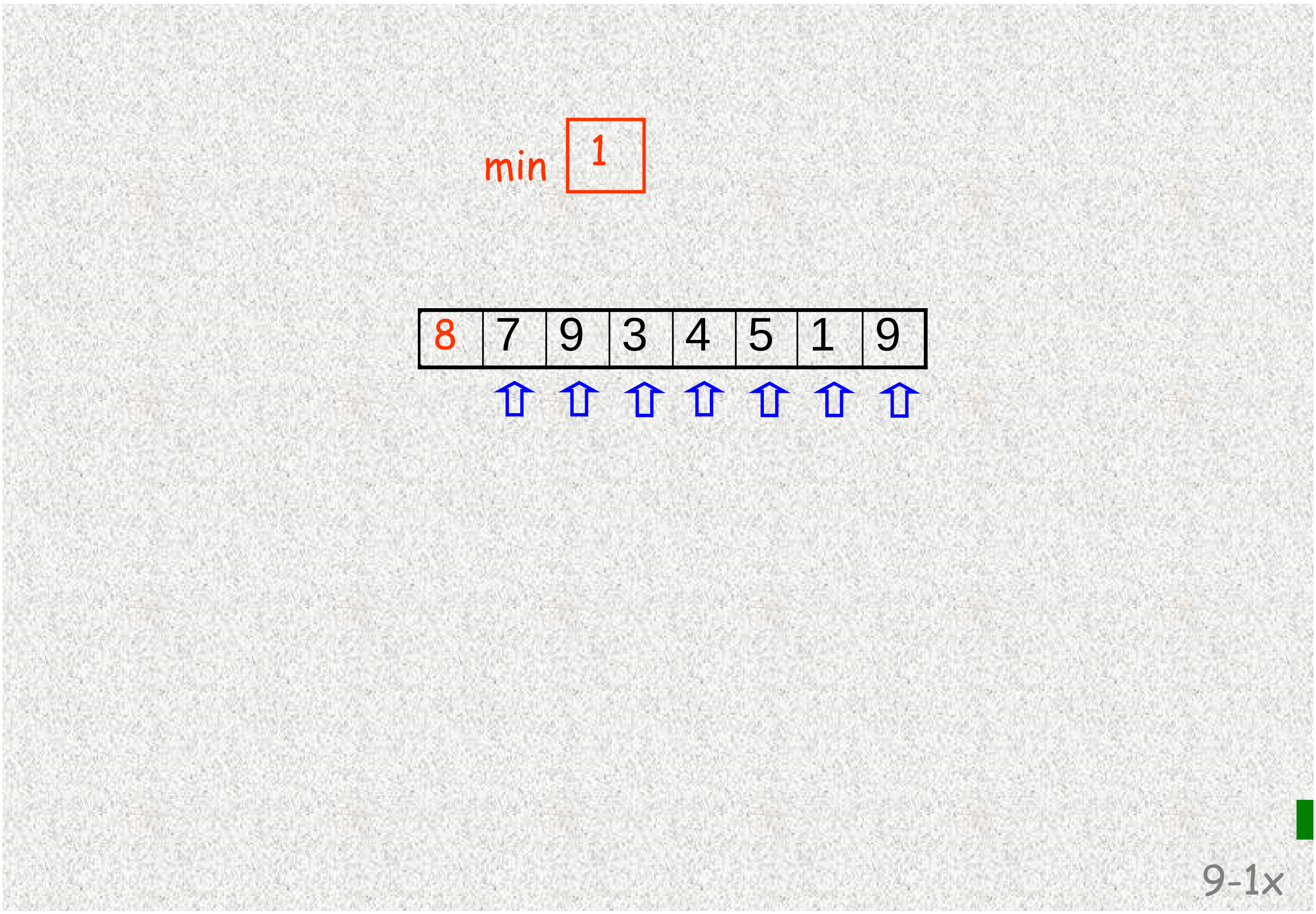
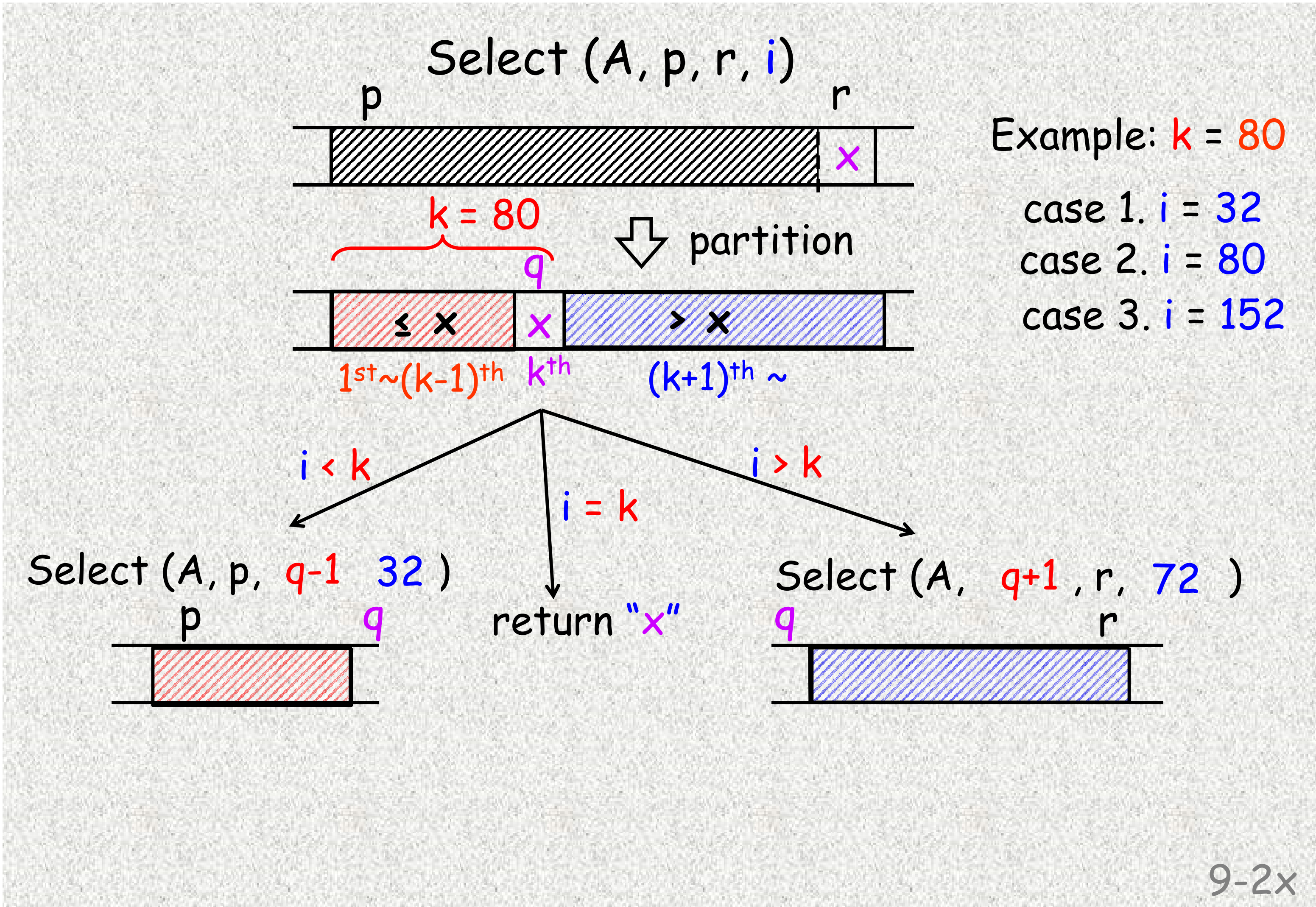
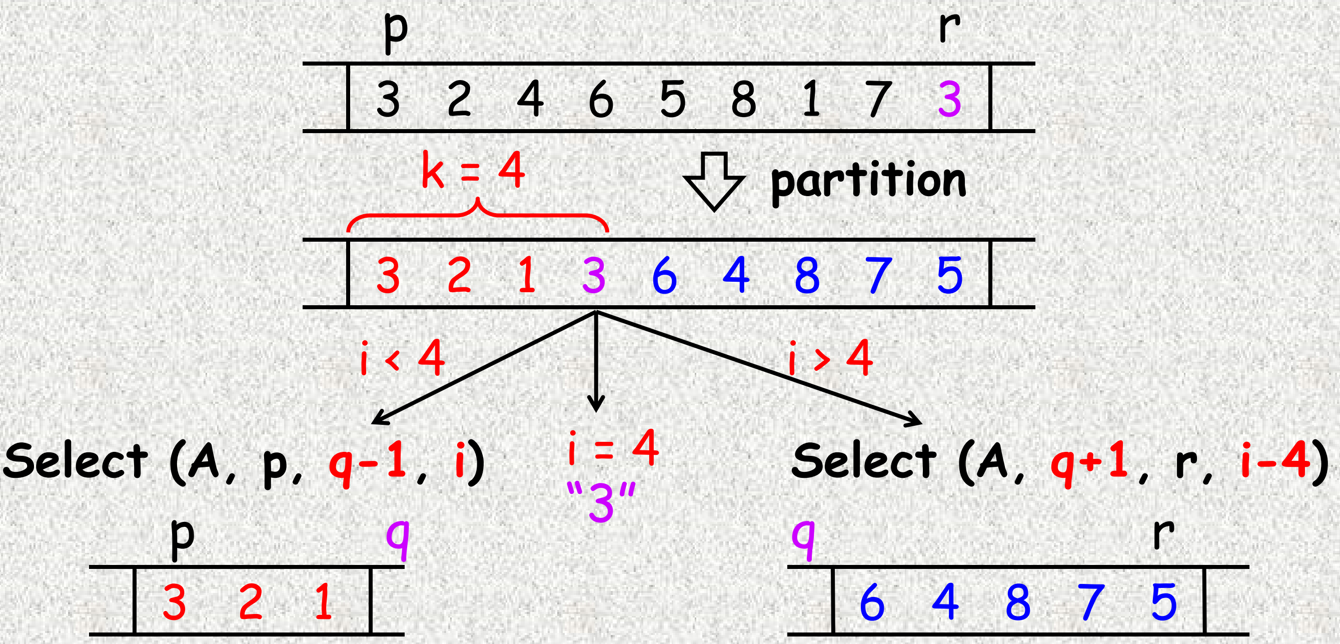
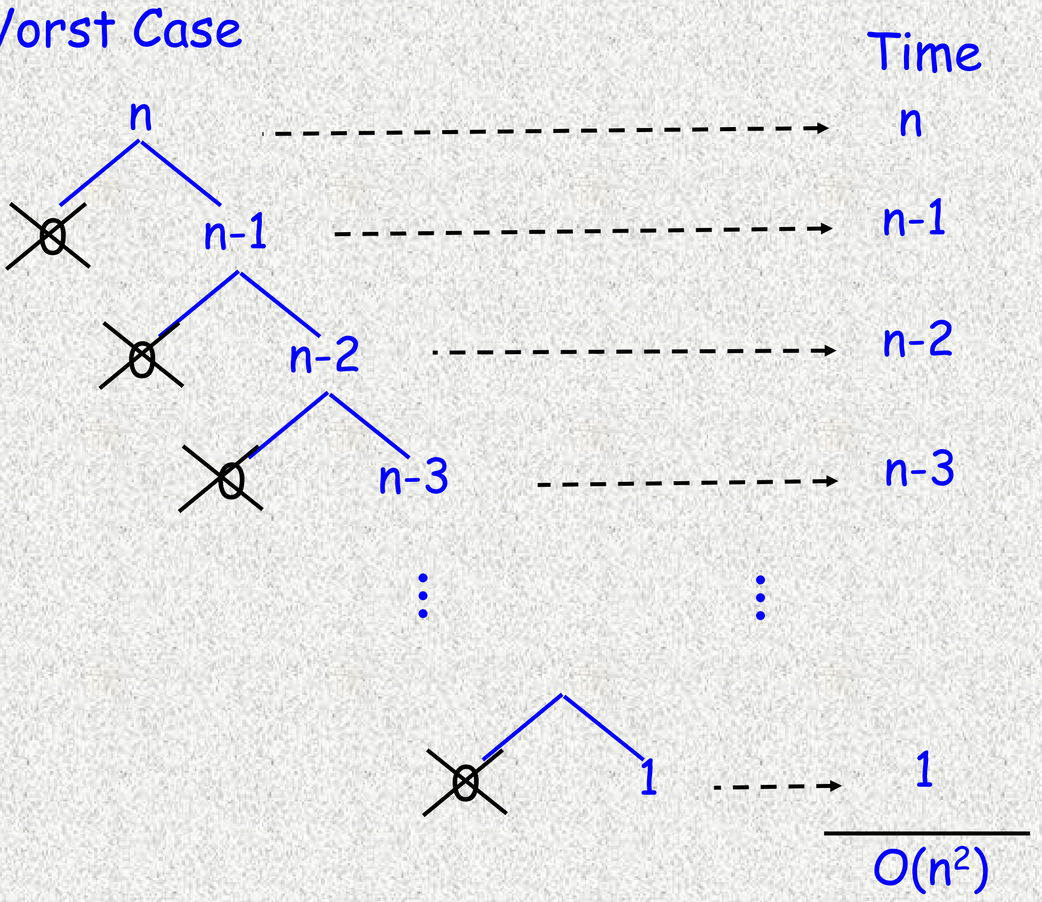

min
1
8 7 9 3 4 5 1 9
9-1x


Select (A, p, r, i)
p
r
x
k = 80
q
partition
≤ x
x
> x
1st~(k-1)th
kth
(k+1)th ~
i < k
i = k
i > k
Select (A, p, q-1 32)
p
q
return "x"
Select (A, q+1 , r, 72)
q
r
Example: k = 80
case 1. i = 32
case 2. i = 80
case 3. i = 152
9-2x

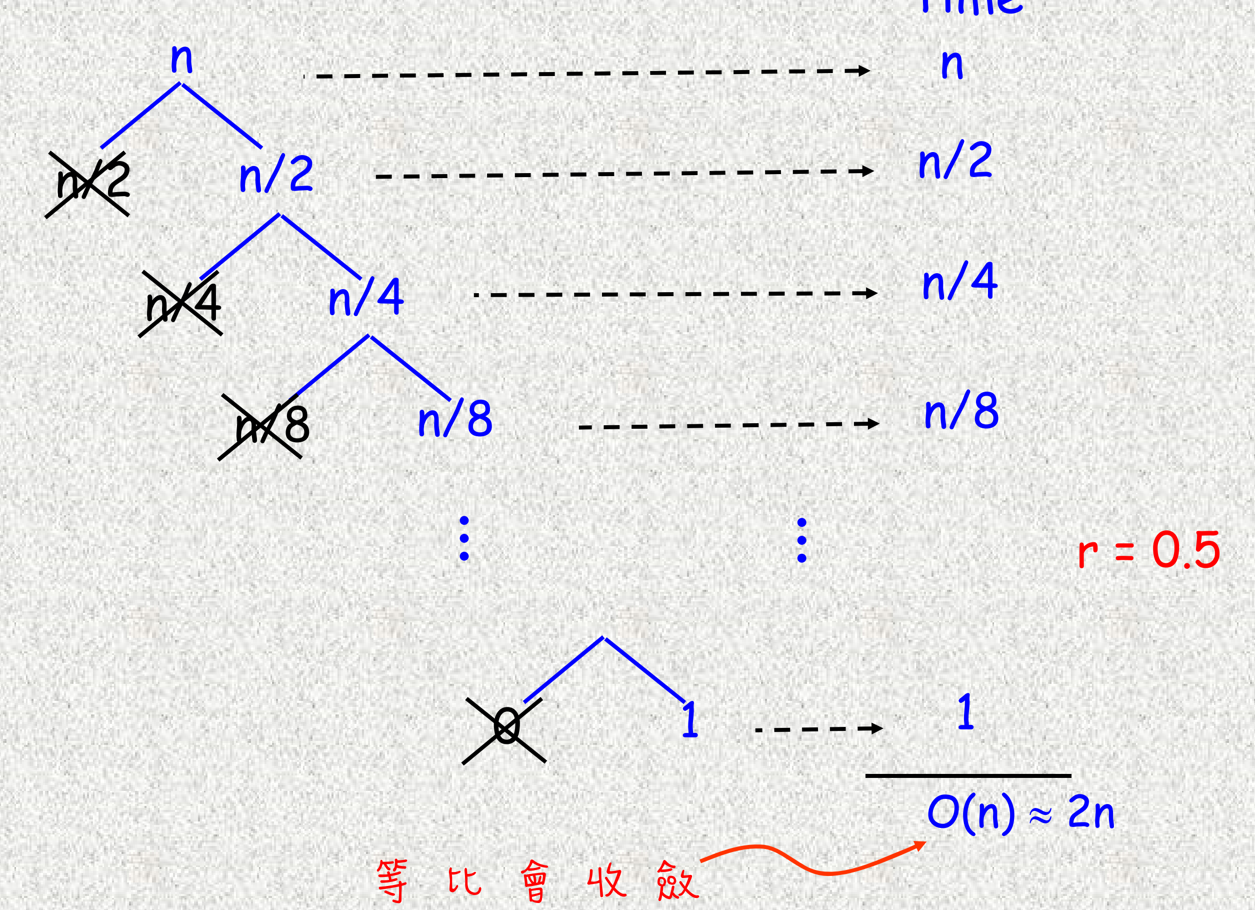
Select (A, p, r, i)



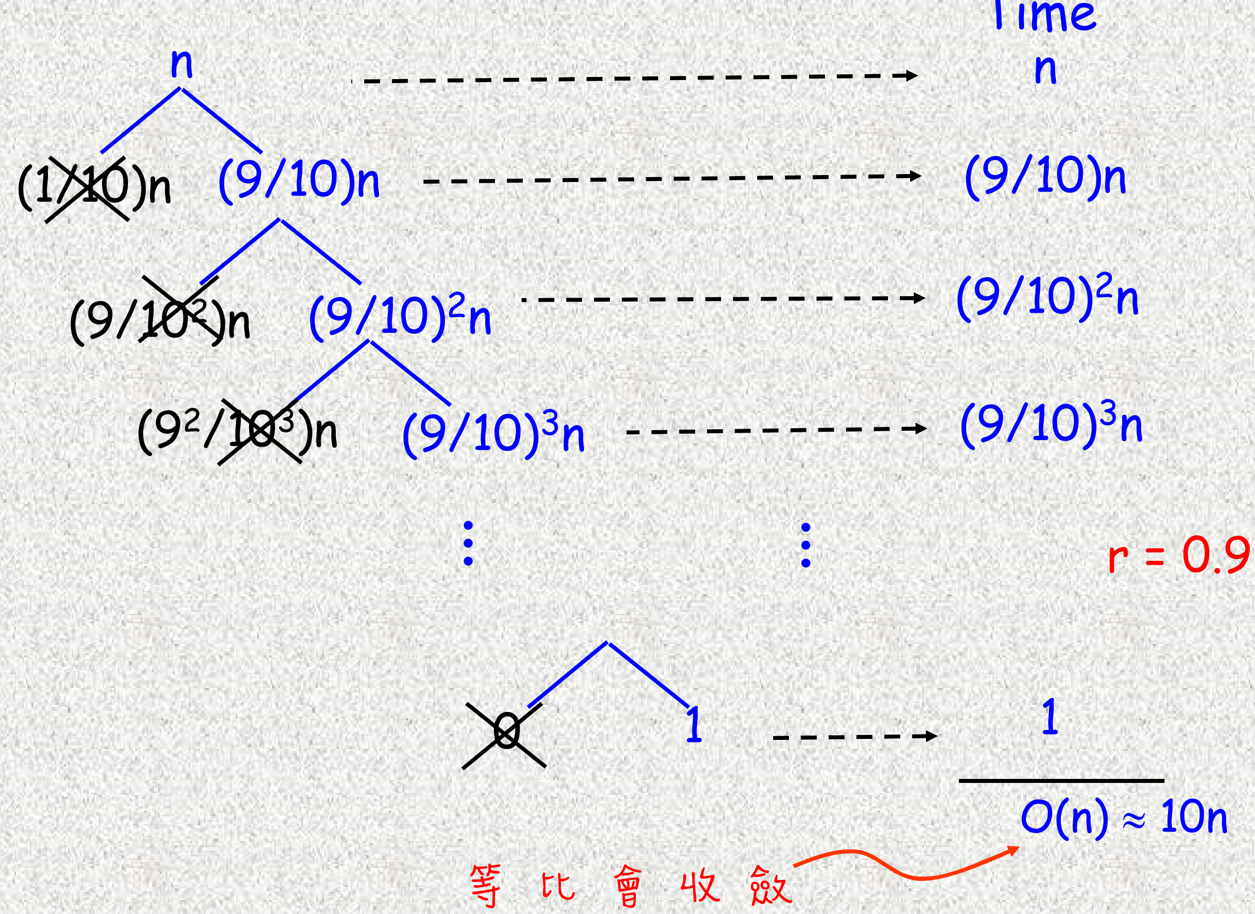
Selection: Worst Case



Selection: Balanced Case



Selection: Assume 1/10 candidates are removed

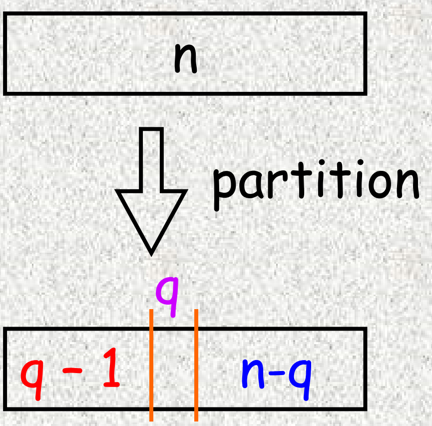


Quick Sort: Average Case

$\cancel{0}$,1,2...,n-1 　　 n-1,...,2,1,$\cancel{0}$

$$E(n) = (n-1) + \underset{\text{機率}}{\frac{1}{n}} \sum_{q=1}^{n} \big(E(q-1) + E(n-q) \big)$$

兩邊都要做



0	1	n-1
1	2	n-2
2	3	n-3
⋮	⋮	⋮
n-1	n	0

$$= (n-1) + \frac{2}{n} (E(1)+E(2)+\ldots+E(n-1))$$

$$= (n-1) + \frac{2}{n} \sum_{k=1}^{n-1} E(k)$$

$$= O(n \lg n)$$ (substitution method or Knuth's approach)

9-3z

Quicksort: Average Case

9-3a

$$E(n) = n\text{-}1 + \frac{1}{n} \sum_{q=1}^{n} \{ E(q-1) + E(n-q) \} \qquad = n\text{-}1 + \frac{2}{n} \sum_{k=1}^{n-1} E(k)$$

兩邊都要做

substitution method or Knuth's approach ⇨ $O(n \lg n)$

Selection: Average Case

永遠都做大的那一邊

(多算沒關係)

$$E(n) = n\text{-}1 + \frac{1}{n} \sum_{q=1}^{n} \{ E(\max\{q-1, n-q\}) \}$$

substitution method ⇨ $O(n)$

只做一邊，機率按比例

$$E(n) = n\text{-}1 + \frac{1}{n} \sum_{q=1}^{n} \left\{ \frac{q-1}{n-1} E(q-1) + \frac{n-q}{n-1} E(n-q) \right\}$$

Knuth's approach ⇨ $O(n)$

Selection: Average Case

只做一邊，機率按比例

$$E(n) = n\text{-}1 + \frac{1}{n}\sum_{q=1}^{n}\left\{\frac{q-1}{n-1}E(q-1) + \frac{n-q}{n-1}E(n-q)\right\}$$

$$E(n) = n\text{-}1 + \frac{1}{n(n-1)}\{0E(0) + 1E(1) + 2E(2) + \dots + (n\text{-}1)E(n\text{-}1) + (n\text{-}1)E(n\text{-}1) + (n\text{-}2)E(n\text{-}2) + \dots + 0E(0)\}$$

$$E(n) = n\text{-}1 + \frac{2}{n(n-1)}\sum_{k=1}^{n-1} kE(k)$$

Knuth's approach

$$E(n) = n+1 + \frac{2}{n(n-1)}\sum_{k=1}^{n-1} kE(k) \qquad (1)$$

不換也可以，但推導比較比較不簡潔漂亮

$$E(n) = n+1 + \frac{2}{n(n-1)}\sum_{k=1}^{n-1} kE(k) \qquad (1)$$

$$n(n\text{-}1)E(n) = (n+1)n(n\text{-}1) + 2\sum_{k=1}^{n-1} kE(k) \qquad (2): (1) \times n(n\text{-}1)$$

$$(n\text{-}1)(n\text{-}2)E(n\text{-}1) = n(n\text{-}1)(n\text{-}2) + 2\sum_{k=1}^{n-2} kE(k) \qquad (3): (2) \text{ with } n = n\text{-}1$$

$$n(n\text{-}1)E(n) = n(n\text{-}1)(3) + 2(n\text{-}1)E(n\text{-}1) + (n\text{-}1)(n\text{-}2)E(n\text{-}1)$$

$$n(n\text{-}1)E(n) = 3n(n\text{-}1) + n(n\text{-}1)E(n\text{-}1)$$

$$E(n) = 3 + E(n\text{-}1) = 3n = O(n) \quad \text{(by iteration method)}$$

1. 4 1 6 3 1 2 5 7 8 7 6 5 9 1 7 1 3 5 7 6 9 2 4 3 5 $(r = 5)$

2. $m_1 \begin{array}{|c}1\\1\\3\\4\\6\end{array} \quad m_2 \begin{array}{|c}2\\5\\7\\7\\8\end{array} \quad m_3 \begin{array}{|c}1\\5\\6\\7\\9\end{array} \quad m_4 \begin{array}{|c}1\\3\\5\\6\\7\end{array} \quad m_5 \begin{array}{|c}2\\3\\4\\5\\9\end{array}$ $\qquad M = \{3, 7, 6, 5, 4\}$

3. $m = \text{Select}(M, \lceil |M|/2 \rceil) = 5$ (median of medians)

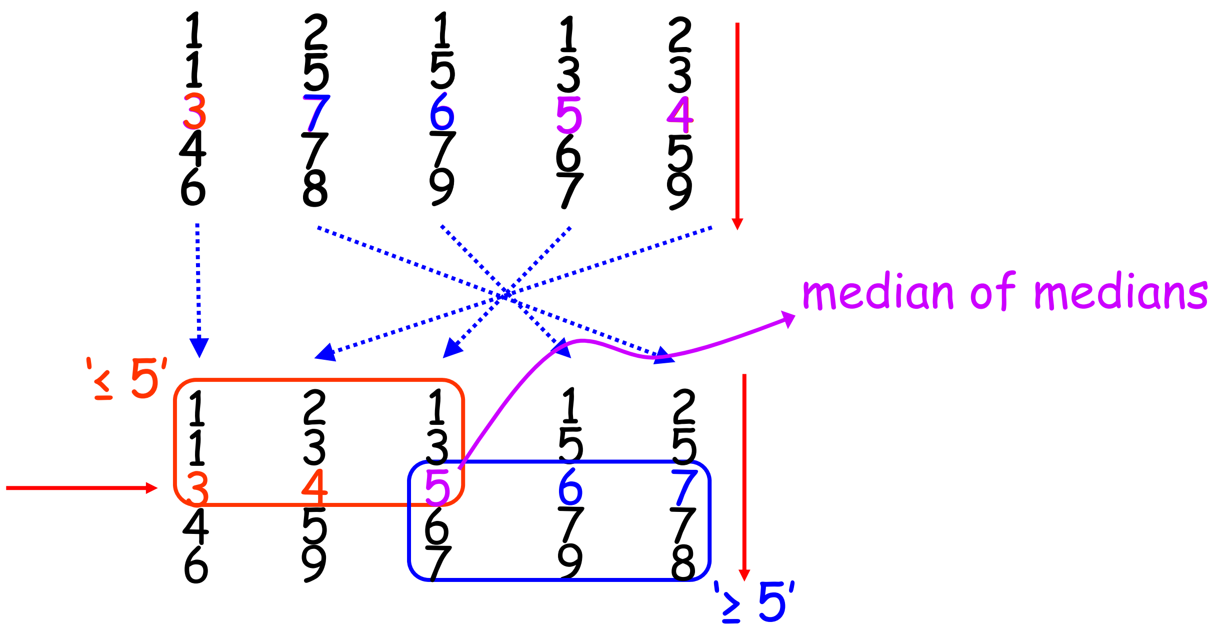
4. $S_1 = \{4\ 1\ 3\ 1\ 2\ 1\ 1\ 3\ 2\ 4\ 3\}$
$S_2 = \{5\ 5\ 5\ 5\}$
$S_3 = \{6\ 7\ 8\ 7\ 6\ 9\ 7\ 7\ 6\ 9\}$

$\boxed{|S_1| = 11,\ |S_2| = 4,\ |S_3| = 10}$

5. $\boxed{\text{case 1. } i = 7}$ Select(S_1, 7)

$\boxed{\text{case 2. } i = 13}$ return $m = 5$

$\boxed{\text{case 3. } i = 22}$ Select(S_3, 22 – 15)



⇨ '≤ 5' 超過 $\frac{1}{4}$ ⇨ '> 5' 最多 $\frac{3}{4}$ ⇨ $|S_3| \le \frac{3}{4}n$

⇨ '≥ 5' 超過 $\frac{1}{4}$ ⇨ '< 5' 最多 $\frac{3}{4}$ ⇨ $|S_1| \le \frac{3}{4}n$

--

1. 4 1 6 3 1 2 5 7 8 7 6 5 9 1 7 1 3 5 7 6 9 2 4 3 5 (r = 5)

$O(r^2 x(n/r)) = O(nr) = O(n)$

2.

	m_1	m_2	m_3	m_4	m_5
	1	2	1	1	2
	1	5	5	3	3
median	3	7	6	5	4
	4	7	7	6	5
	6	8	9	7	9

M = {3, 7, 6, 5, 4}

T(n/5)

3. m = Select(M, |M|/2) = 5 (median of medians)

O(n)

4. S_1 = {4 1 3 1 2 1 1 3 2 4 3}
S_2 = {5 5 5 5}
S_3 = {6 7 8 7 6 9 7 7 6 9}

$|S_1| = 11$, $|S_2| = 4$, $|S_3| = 10$

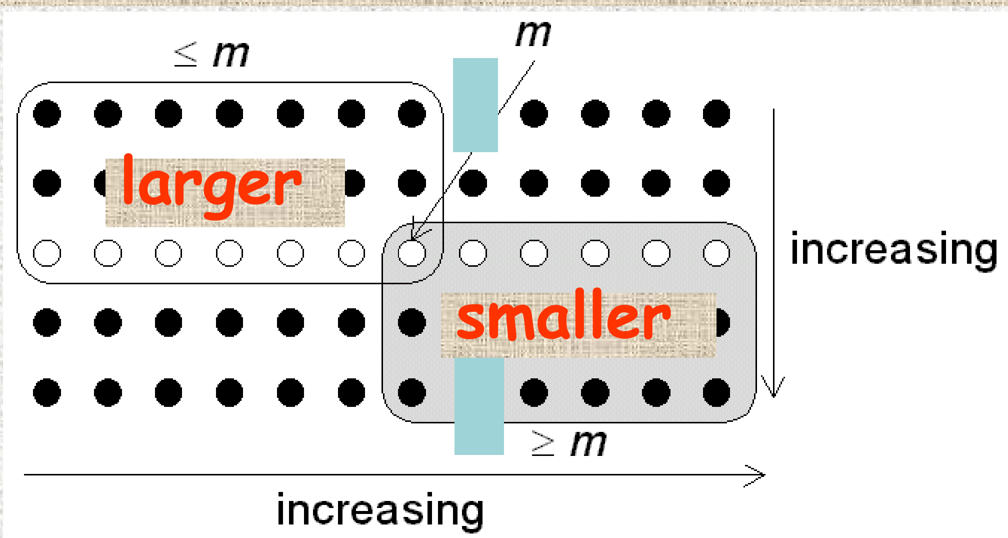
T(3n/4)

5. case 1. i = 7 — Select(S_1, 7)

case 2. i = 13 — return m = 5

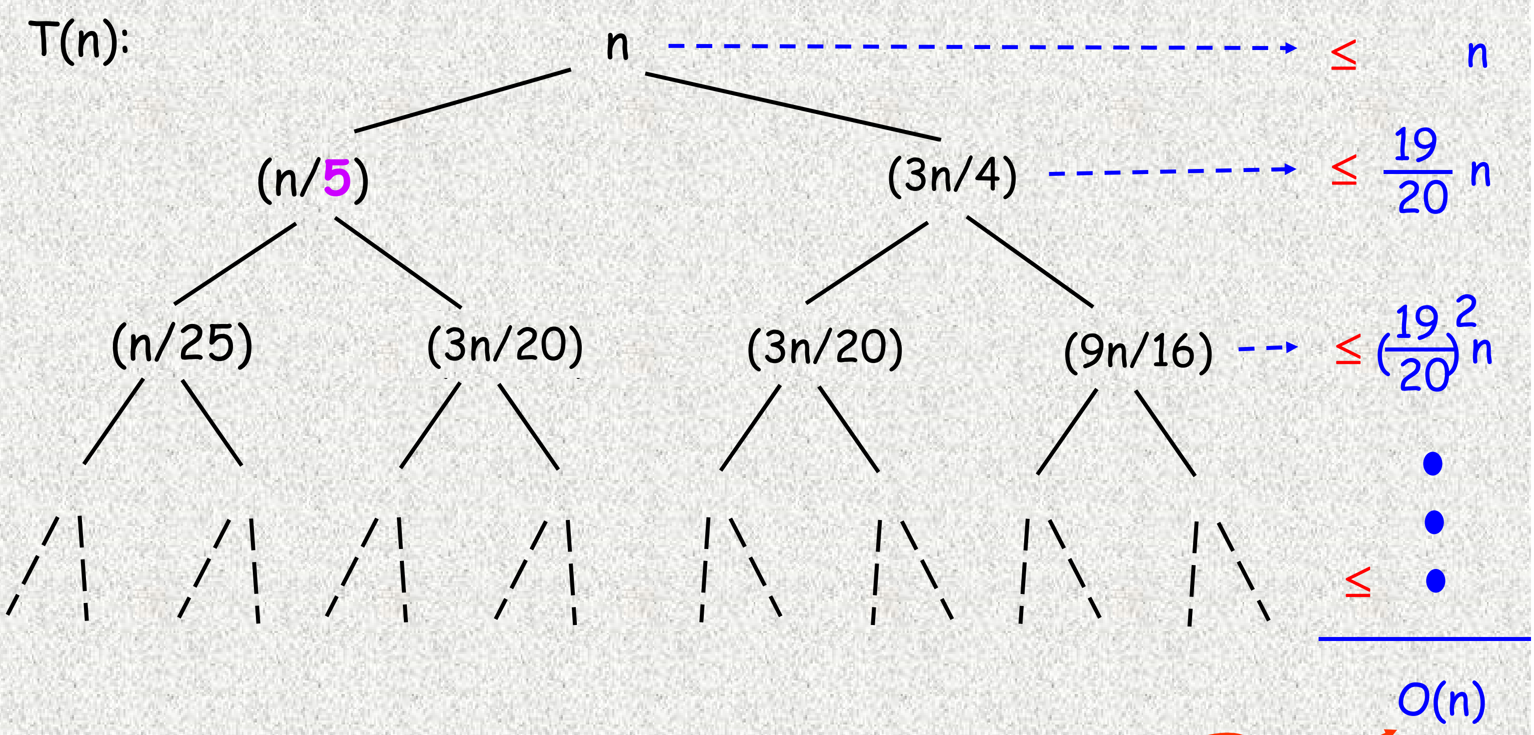
case 3. i = 22 — Select(S_3, 22 – 15)

Remark:
1. n/5 may not be odd
2. n may not be a multiple of 5



* textbook shows: $|S_1|, |S_2| \le (7/10)n + 6$
(refer to it for the details)

$T(n) = T(n/5) + T(3n/4) + n$ **Why $T(n) = O(n)$?**

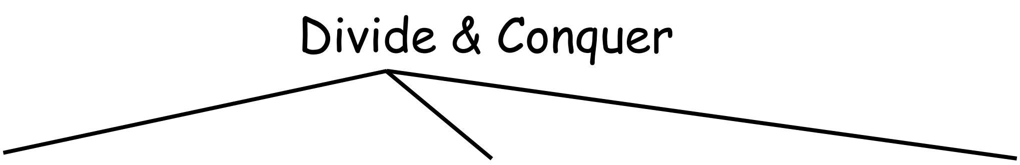


等 比 會 收 斂

Remark:
$r \geq 5$ makes $r' \leq 19/20 < 1$

9-4z

9-4b



D & C	**Partition**	**Prune & Search**
1. partition the input into same subproblems	1. break the problem into independent subproblems	repeatedly remove invalid candidates
2. recursively solve the subproblems	2. solve the subproblems	
3. combine subsolutions		
(Ex. merge-Sort)	(Ex. quick-sort)	(Ex. selection) (Ex. binary search)
$T(n) = \sum T(n_i) + C(n)$	$T(n) = P(n) + \sum T_i(n_i)$	$T(n) = r(n) + T(n')$