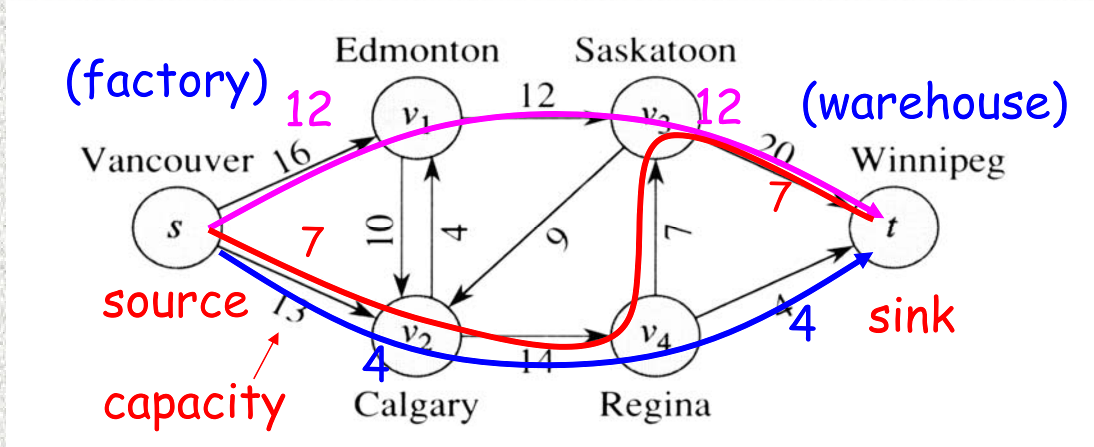
The maximum-flow problem

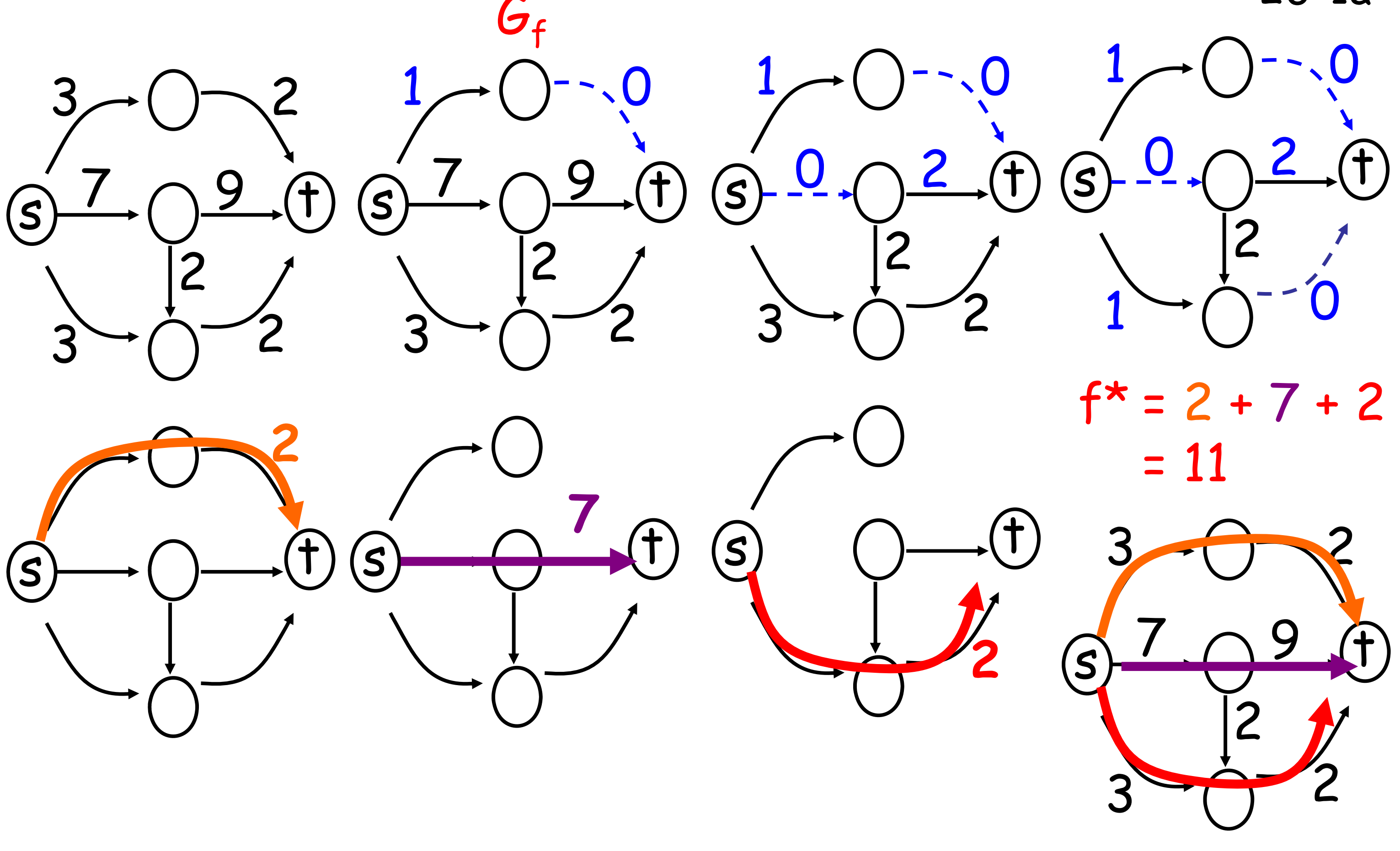
flow network (a directed graph)



Another application: file transfer
(capacity = Kbit/s)

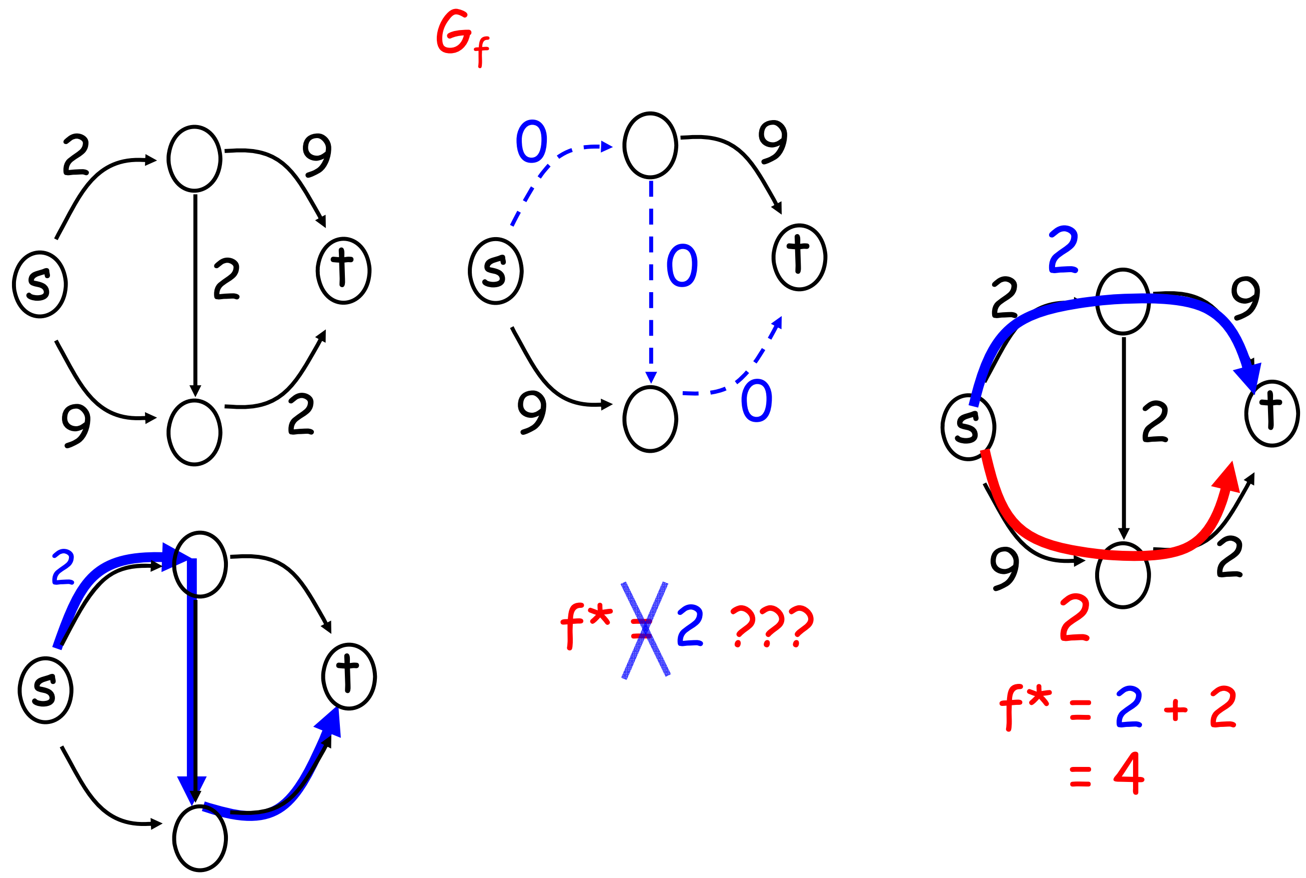
26-1x

26-1a

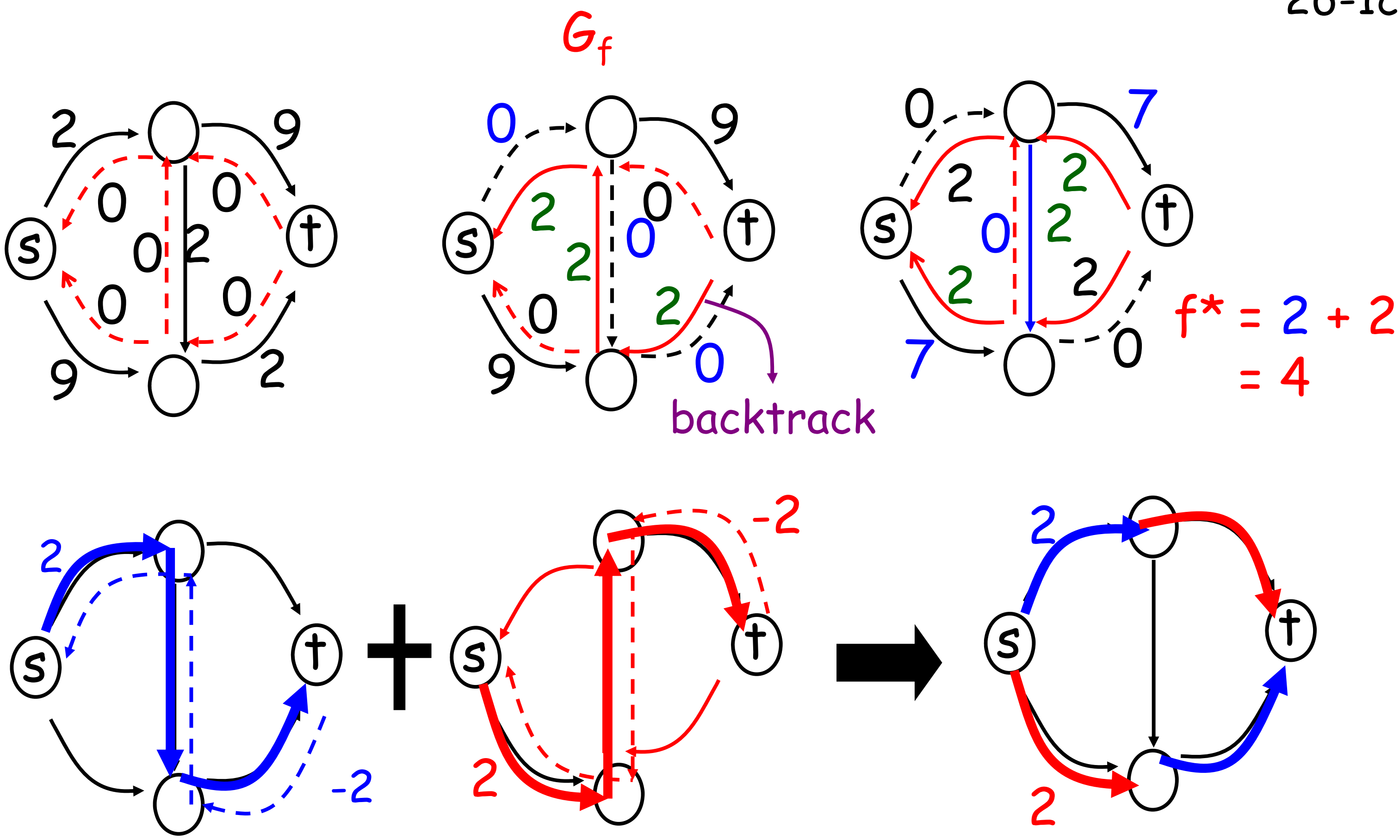


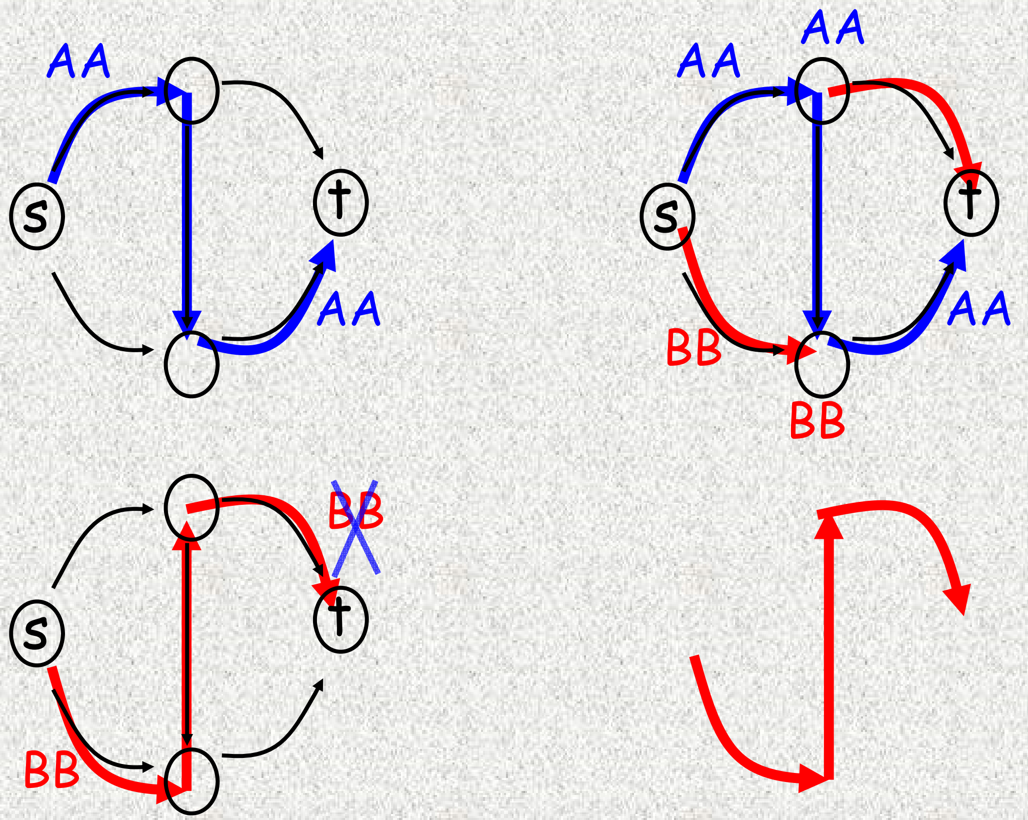
$f^* = 2 + 7 + 2$
$= 11$

26-1b



26-1c





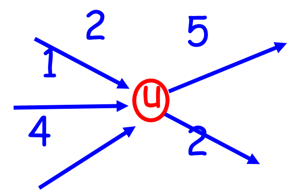
26-1y

26-2a

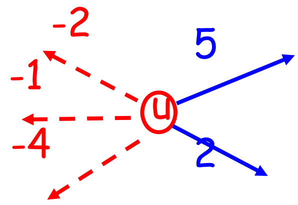
Flow Conservation: for all $u \in V - \{s, t\}$

$$\sum_{v \in V, f(v, u) > 0} f(v, u) = \sum_{v \in V, f(u, v) > 0} f(u, v)$$

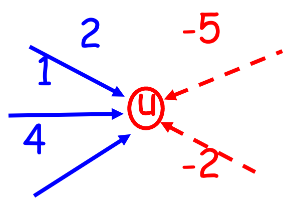
(positive in = positive out)



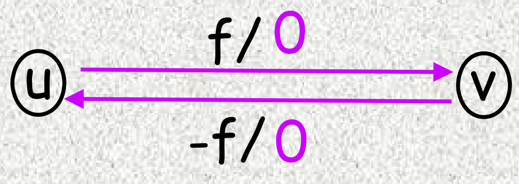
$$\sum_{v \in V} f(u, v) = 0$$ (total out = 0)



$$\sum_{v \in V} f(v, u) = 0$$ (total in = 0)

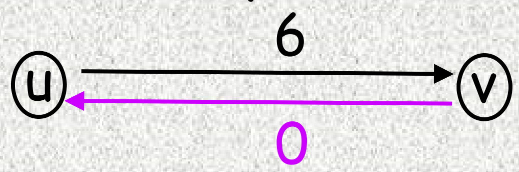


if no edge between u and v



$$\left.\begin{array}{l} f \le 0 \\ -f \le 0 \end{array}\right\} \Rightarrow f = 0$$

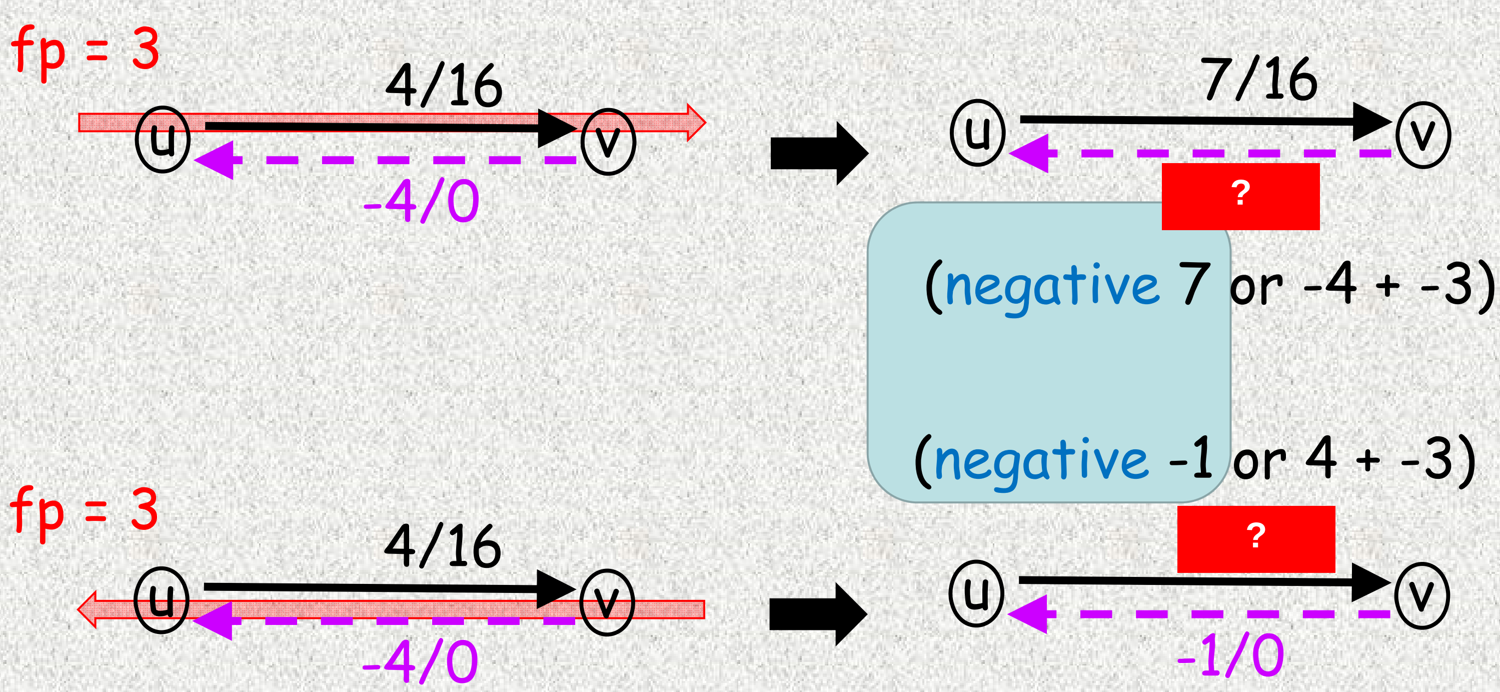
add (v, u) only if ((u, v) $\in$ E) and ((v, e) $\neq$ E)



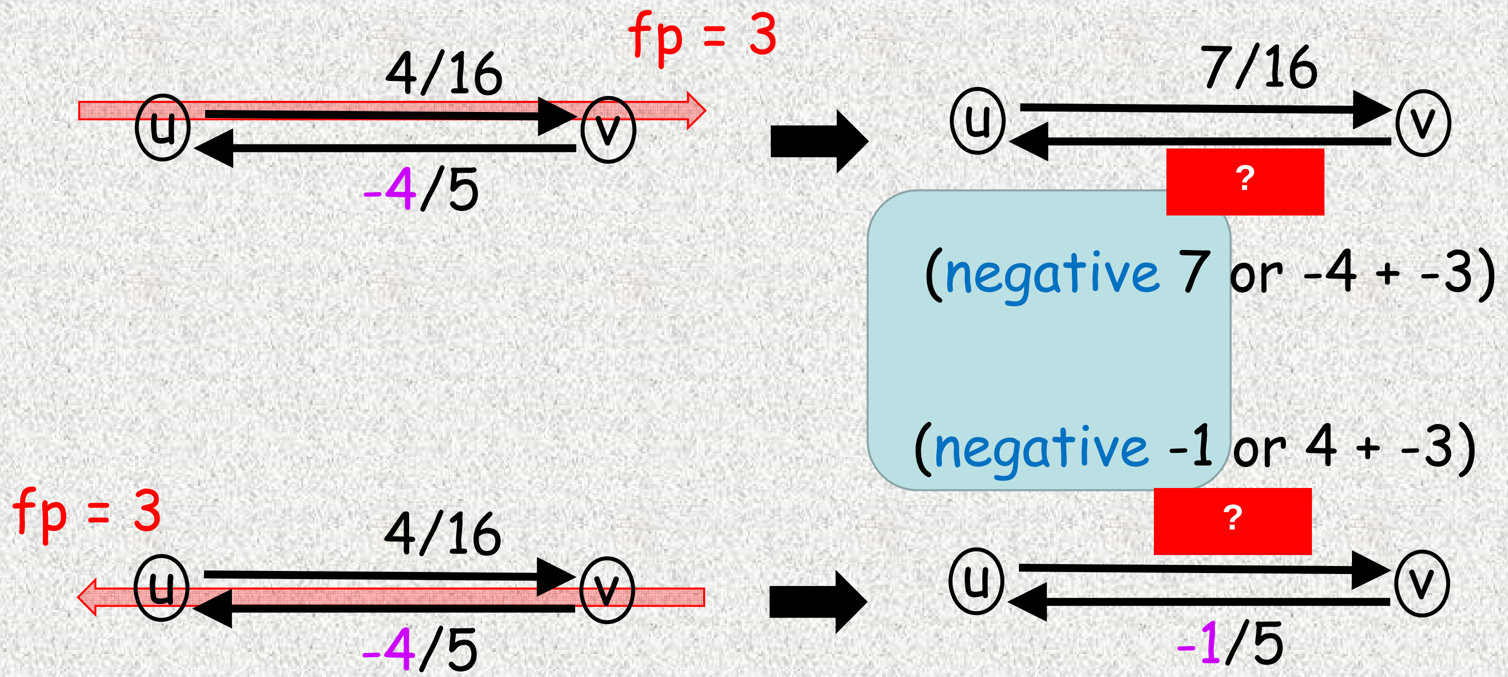
$\Rightarrow$ there are at most 2E edges

$\Rightarrow$ find a path: O(V + E) time, not O(V^2)

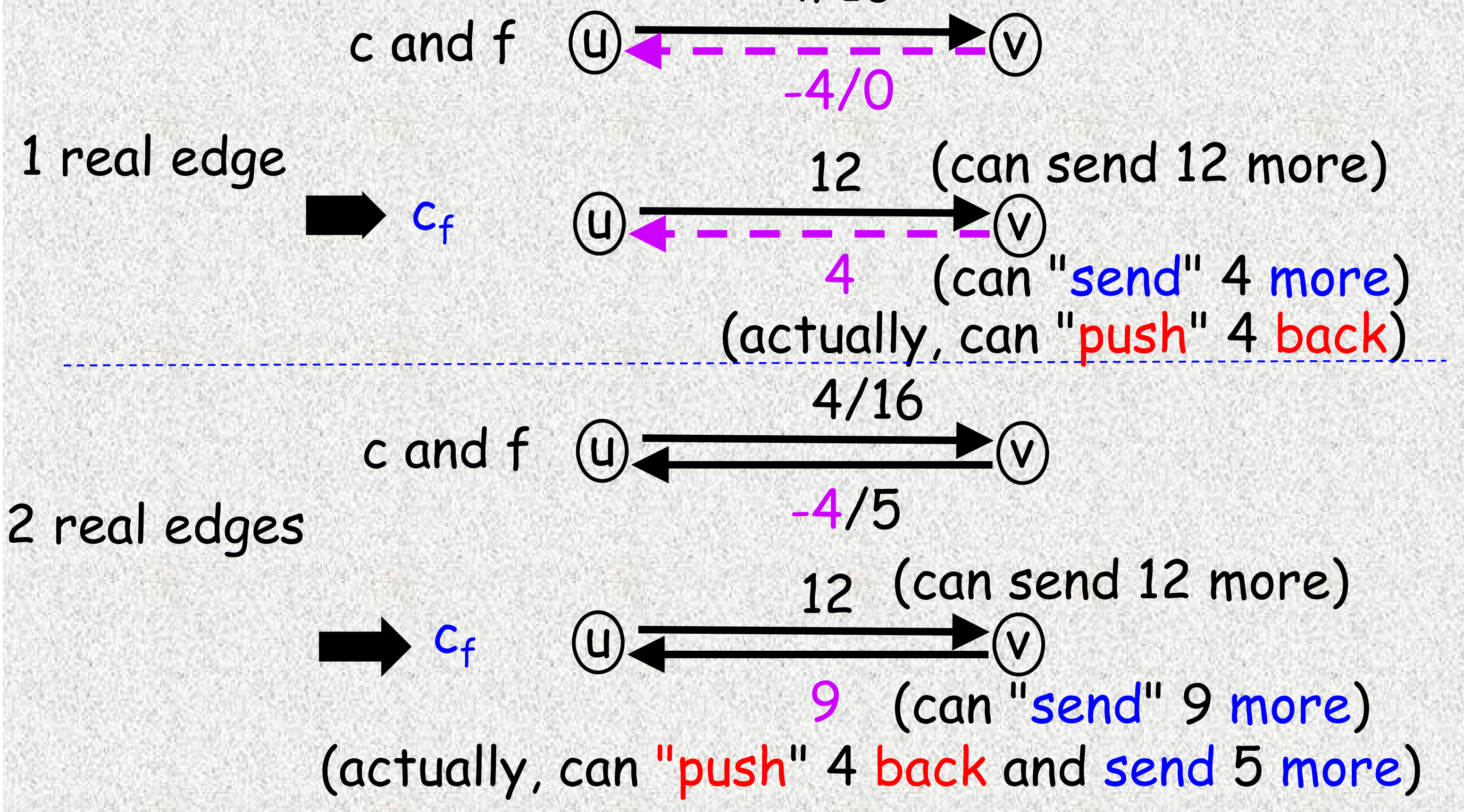
negative flow: 1 real edge



negative flow: 2 real edges



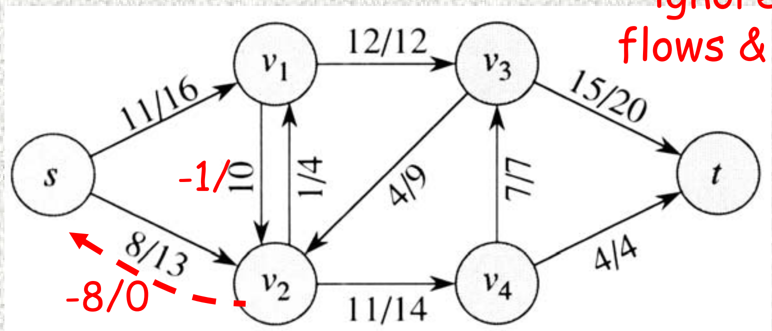
c_f of negative flow



* ignore negative flows & zero edges

Example:

G and *f*



only non-zero edges

Residual network G_f with an augmenting path p

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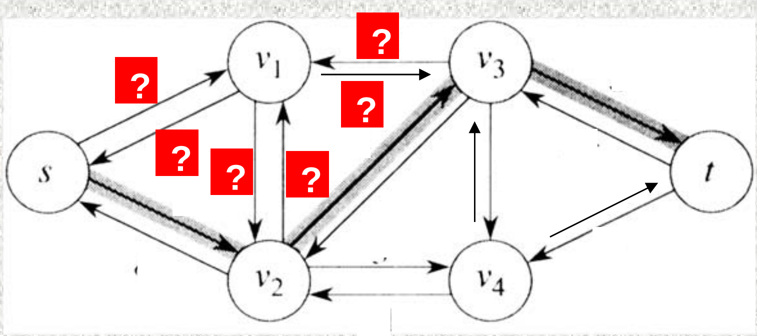
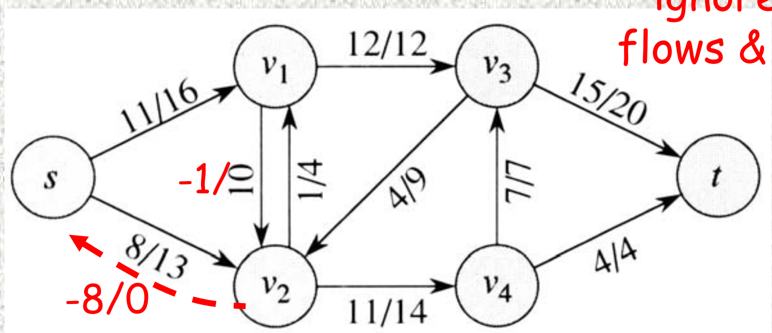


26-4x

* ignore negative flows & zero edges

Example:

G and *f*

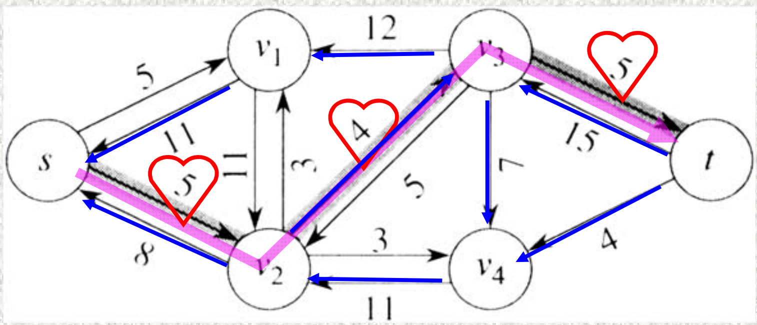


only non-zero edges

Residual network G_f with an augmenting path p

殘 餘 的

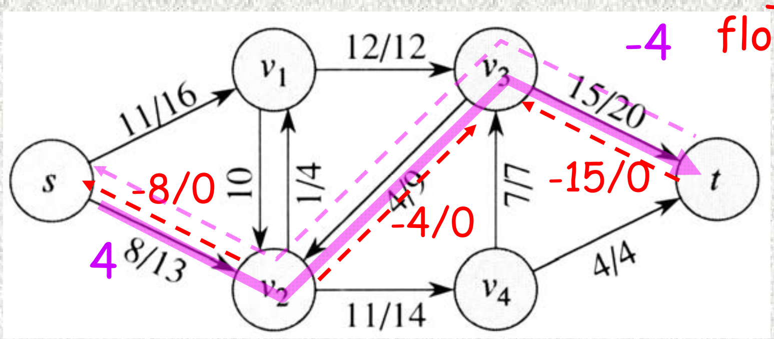
backtrack



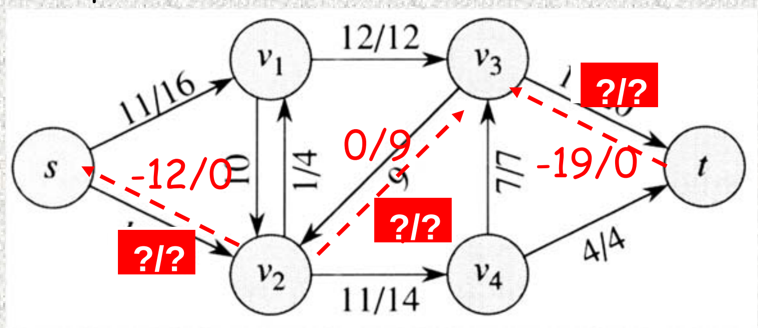
26-4x

Example:

G and *f*

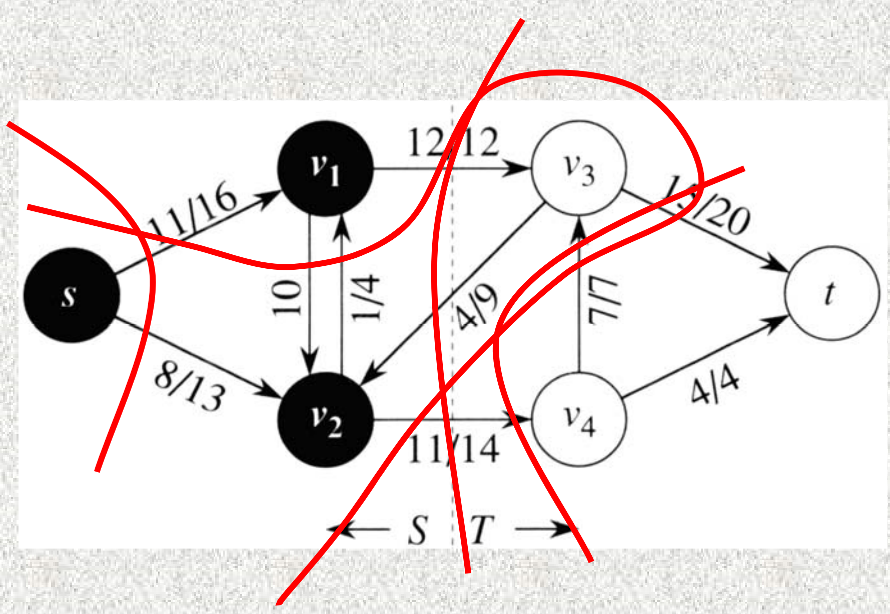


New f ← f + f_p



a cut (S, T):

a partition of V, s∈S, t∈T



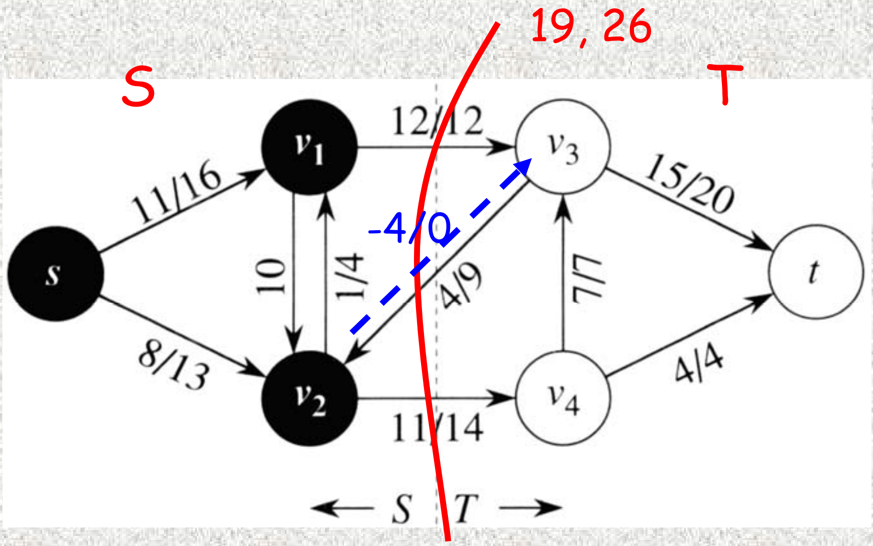
Net flow across a cut: $f(S, T)$. = 12 + (-4) + 11

19

Capacity of a cut: $c(S, T)$. = 12 + 0 + 14

26

Example: $|f|=19$, $f(S, T)=19$, and $c(S, T)=26$.

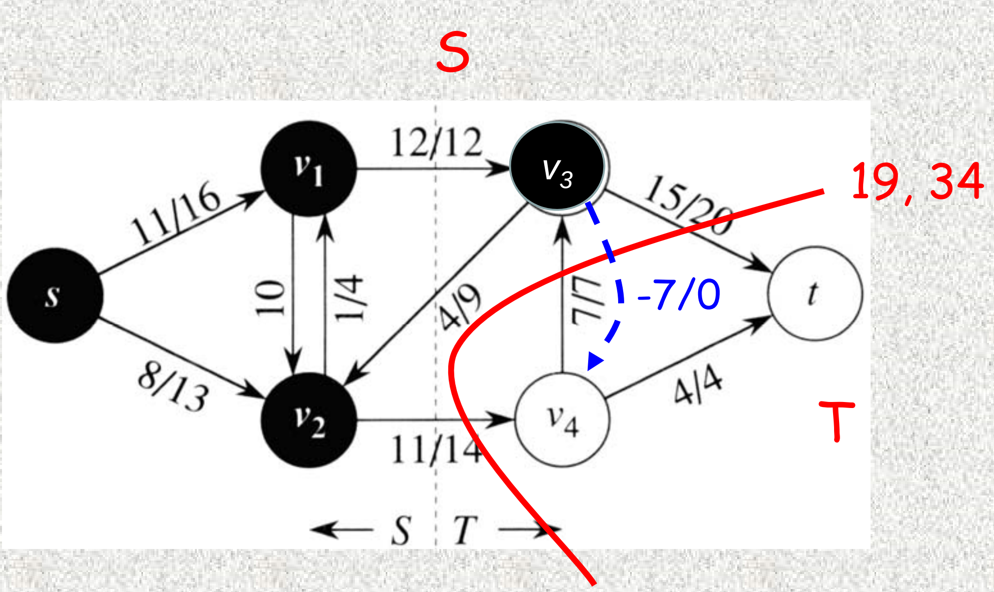


Net flow across a cut: $f(S, T)$. = 15 + (-7) + 11

19

Capacity of a cut: $c(S, T)$. = 20 + 0 + 14

34

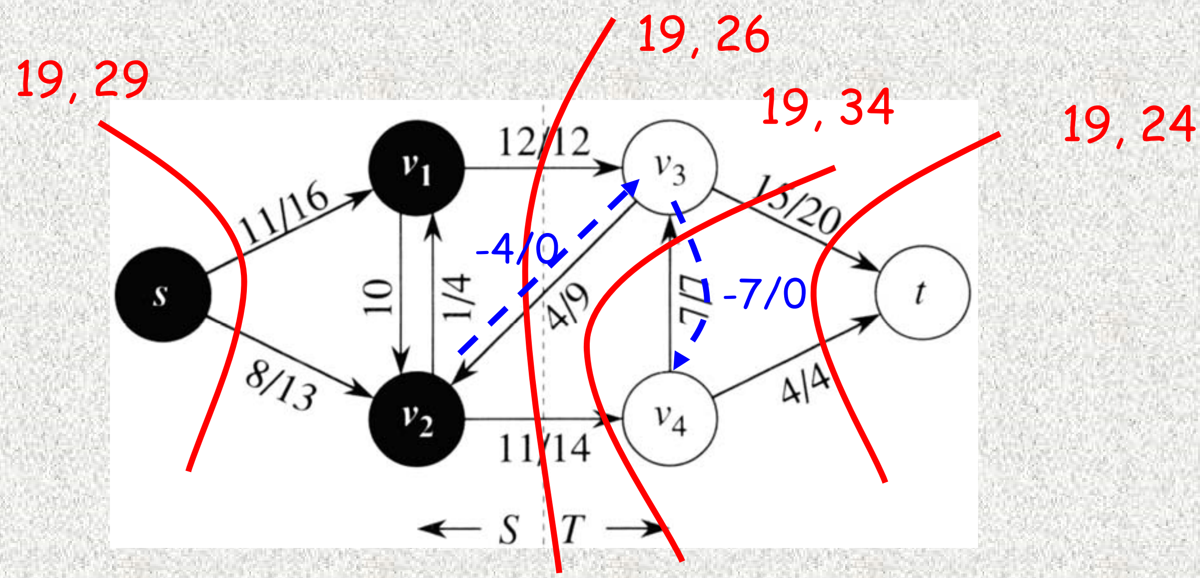


Lemma 26.5: f(S, T) = |*f*|

(flow conservation)

Corollary 26.6: |*f*| = f(S, T) ≤ *c*(*S*, *T*)

(capacity constraint)



Lemma 26.5: f(S, T) = |*f*|

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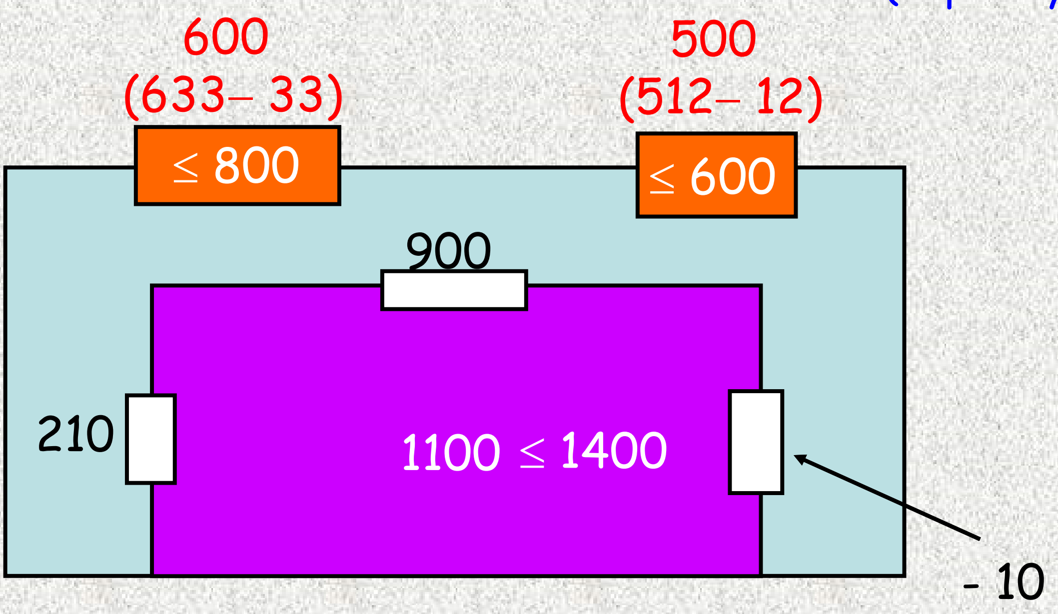
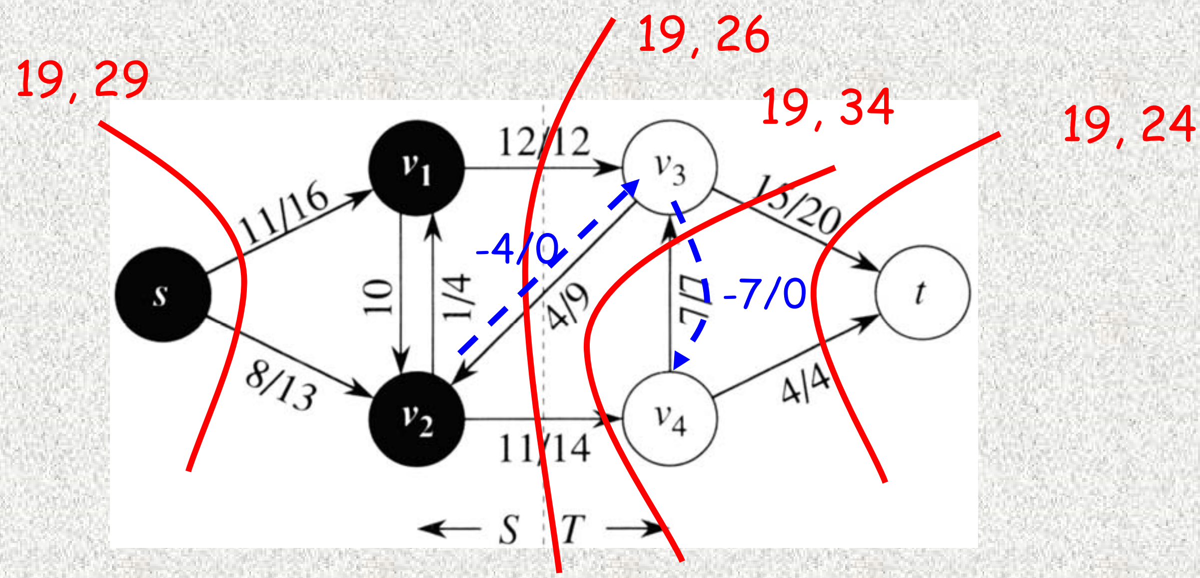


Lemma 26.5: f(S, T) = |f|

(flow conservation)

Corollary 26.6: |f| = f(S, T) ≤ c(S, T)

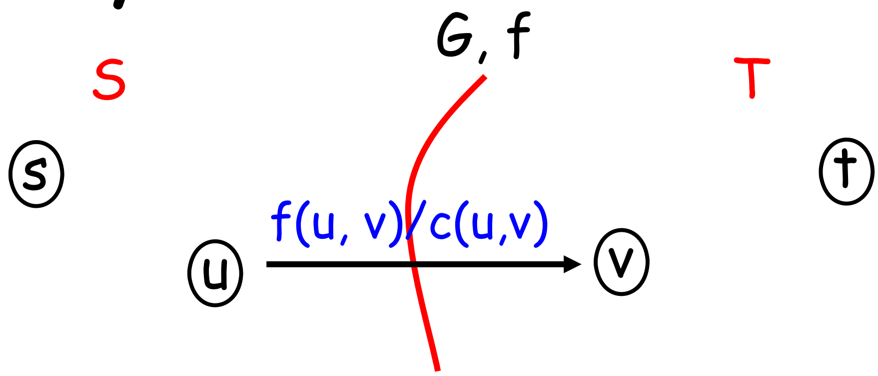
(capacity constraint)



26-6z

Corollary 26.6

26-6a



$$f(u, v) \leq c(u, v)$$

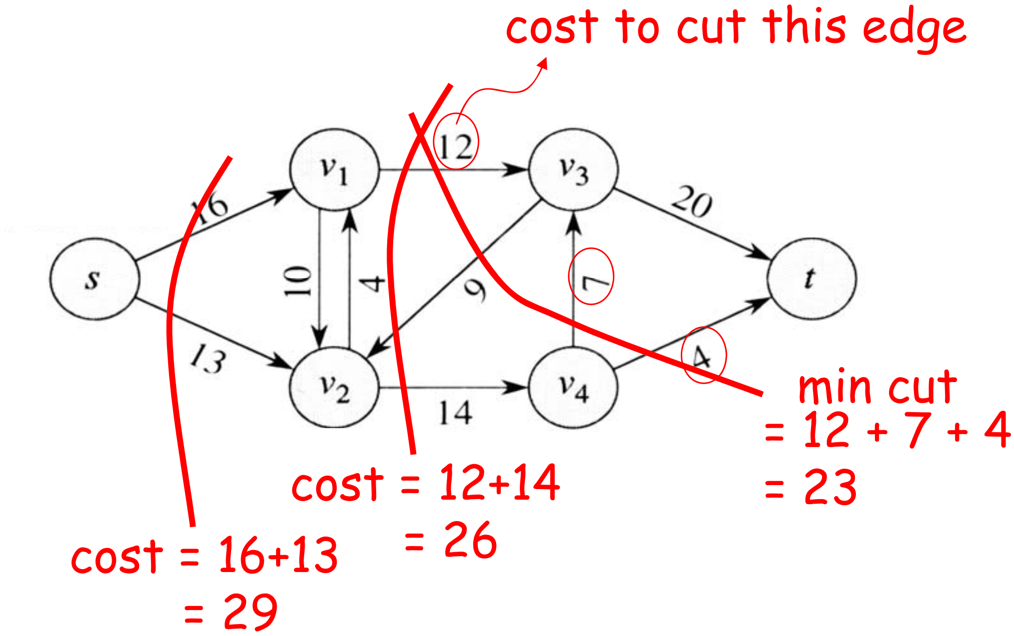
$$\Rightarrow \sum_{u \in S, v \in T} f(u, v) \leq \sum_{u \in S, v \in T} c(u, v)$$

$$\Rightarrow f(S, T) \leq c(S, T)$$

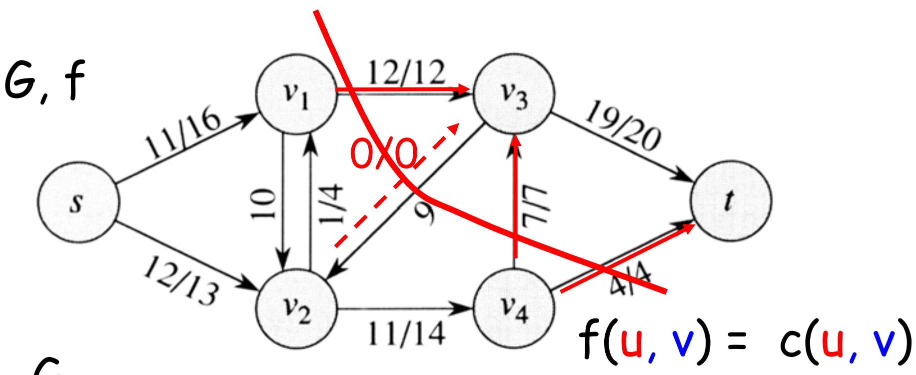
$$\Rightarrow |f| \leq c(S, T)$$

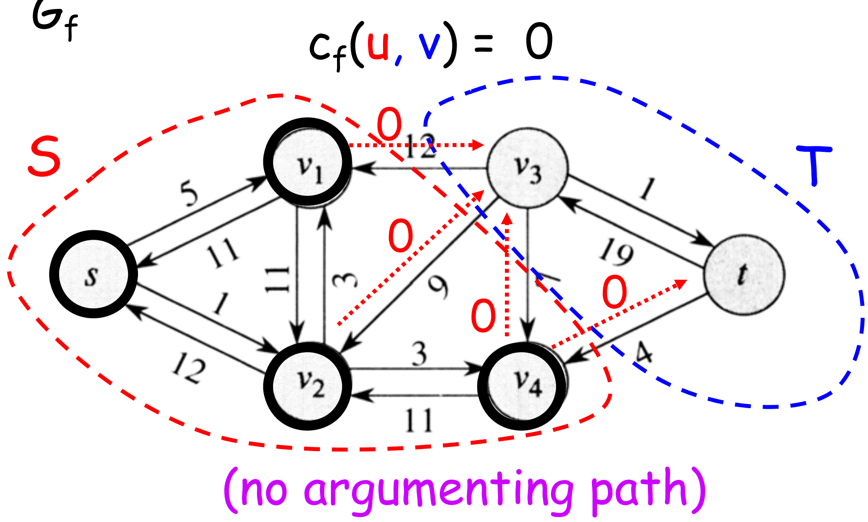
(**by Lemma 26.5:** f(S, T) = |f|)

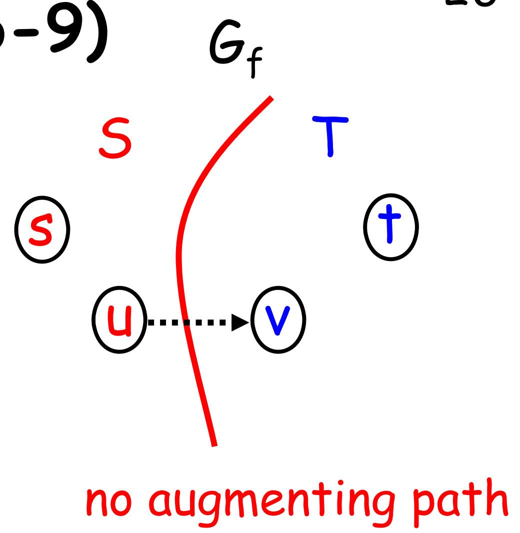
The min cut problem



Thm. 26.7 (2)-->(3) (See 26-9)







⇨ $c_f(u, v) = 0$

⇨ $f(u, v) = c(u, v)$

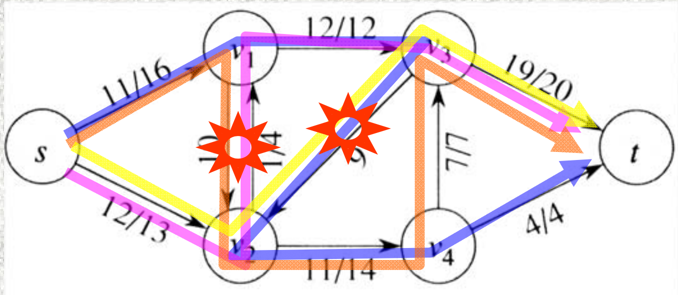
⇨ $\sum_{u \in S, v \in T} f(u, v) = \sum_{u \in S, v \in T} c(u, v)$

⇨ $f(S, T) = c(S, T)$

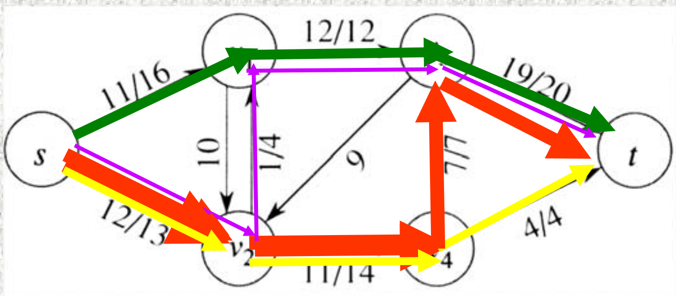
Constructing flow paths

4
7
8
4
$f^* = 23$

G & f

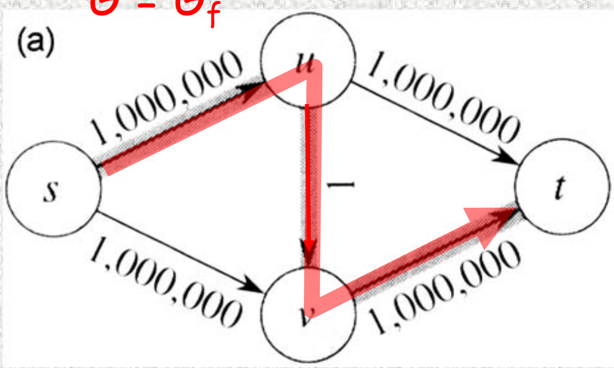


11
1
4
7



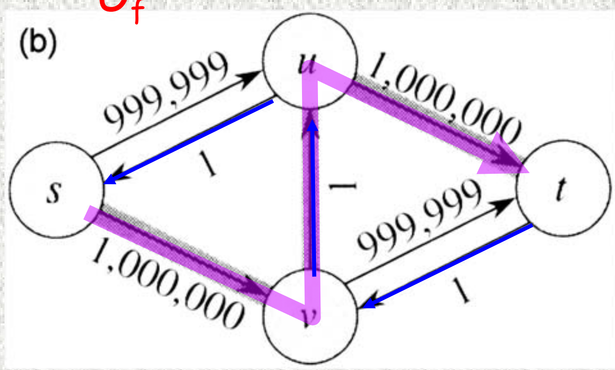
P_1

$G = G_f$

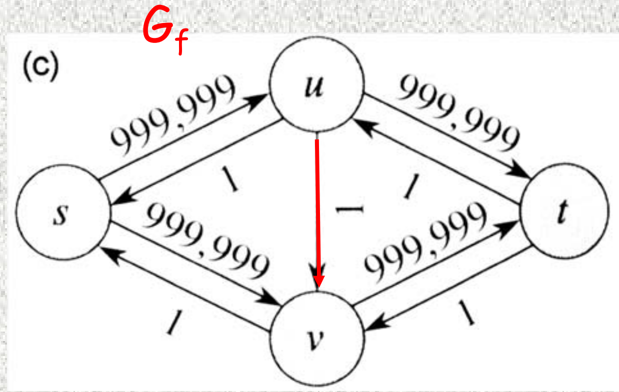


P_2

G_f



G_f



2×10^6 iterations

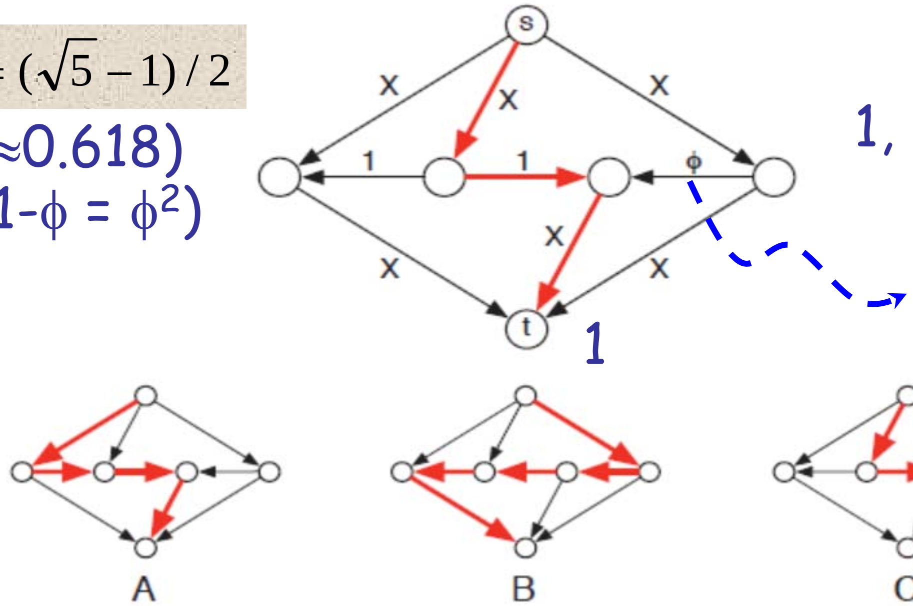
Appendix (See course-page, x is a larger integer)

about 0.372 left each time

$\phi = (\sqrt{5} - 1)/2$

($\approx$0.618)

($1-\phi = \phi^2$)

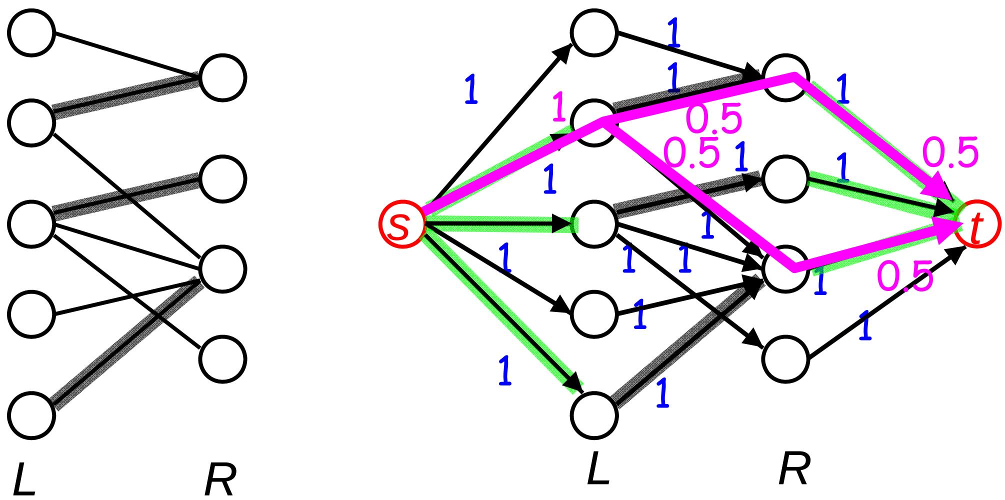


1, (B,C,B,A)*

$\phi, \phi^3, \phi^5, \phi^7, \ldots$

1

Uri Zwick's non-terminating flow example, and three augmenting paths.



* matching ---> flow

* flow ~~--~~-> matching ?

* integer flow ----> matching

* integer flow <----> matching

* max integer flow <----> max matching

Max flow on undirected G