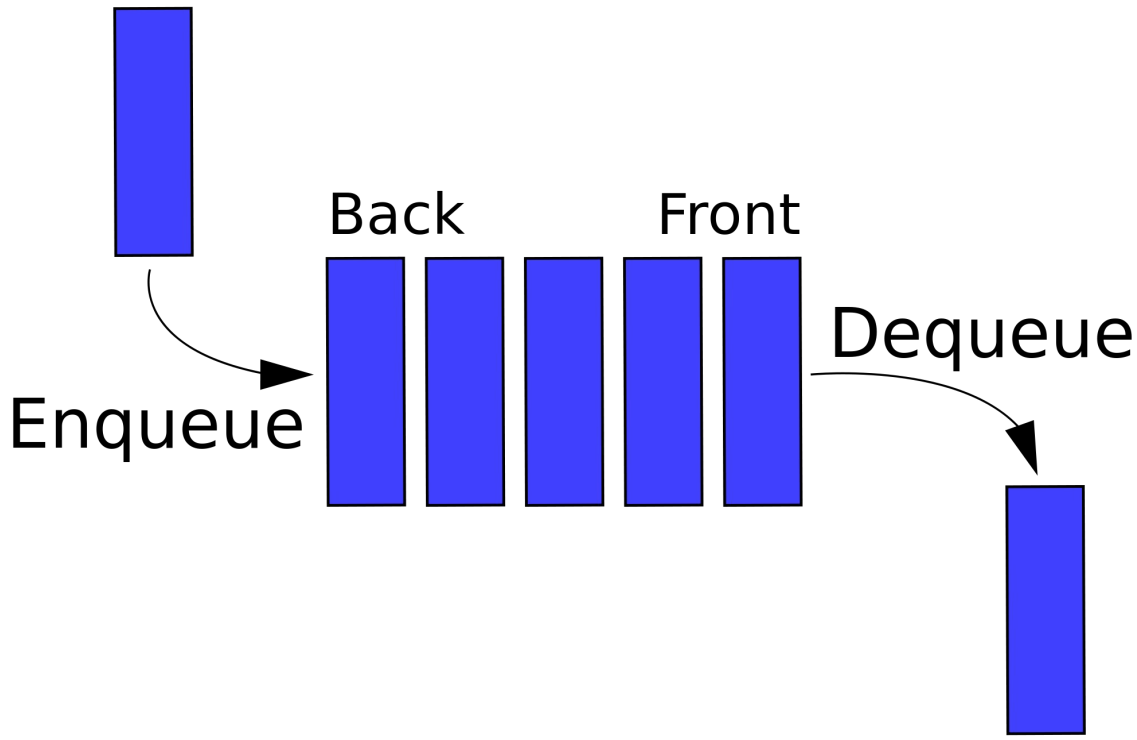
廣度優先搜尋

日月卦長

Queue

- push/ emplace(X) 加入資料
- front() 看最前面的資料
- pop() 刪掉最前面的資料
- First in first out



std::queue

front back

```cpp
#include <iostream>
#include <queue>
#include <vector>
using namespace std;
int main() {
  queue<int> Q;
  Q.emplace(1);
  Q.emplace(2);
  Q.emplace(3);
  Q.pop();
  cout << Q.front() << ' ' << Q.back();
  return 0;
}
```

```cpp
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#include <vector>
using namespace std;
int main() {
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std::queue

front back

1

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2 3

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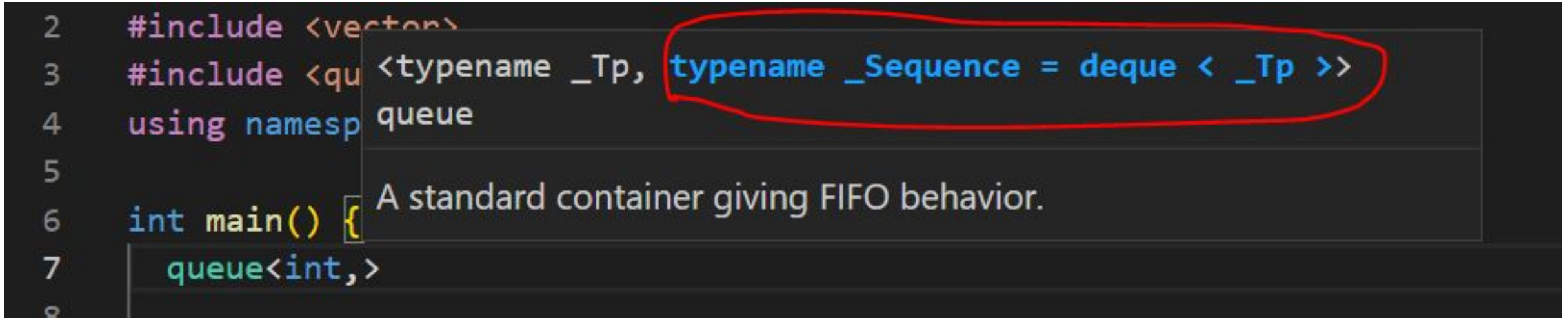
是用 deque 做出來的



元素存取

	queue
尾巴	back()
頭部	front()

數量資訊

	queue
裡面是不是空的	empty()
裡面有多少東西	size()

洪水算法

Flood fill algorithm

倒水問題

- 在一個 $N \times M$ 的格子圖中，選一個點倒水
- 溢出來水會往某些方向流(看題目規定水怎麼流)
- 有些時候會有障礙物無法淹沒

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假設每個格子
的水只會往上
下左右淹

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假設每個格子的水只會往上下左右淹

暴力法 $O(N^2M^2)$

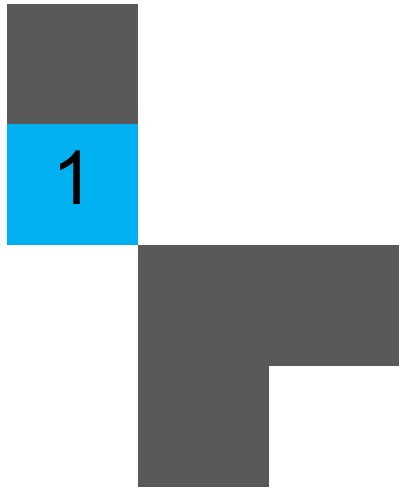
- 設起始點的level為0
- L = 1
- While True
  - 掃描所有的格子 (x,y):
    - 如果(x,y) 非障礙物 且 沒設置level 且 上下左右的格子的 level 等於 L-1
      - 設(x,y)的level為 L
  - 如果都沒有格子的level為L-1
    - Break
  - L = L + 1

能淹出水的格子

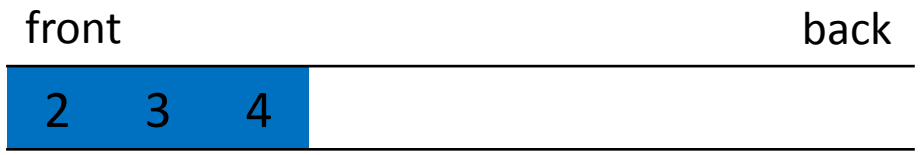
用 queue 記錄所有邊界格子

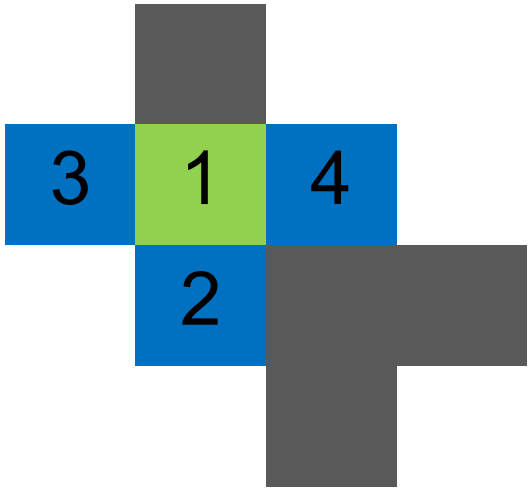
Demo

front	back
1	

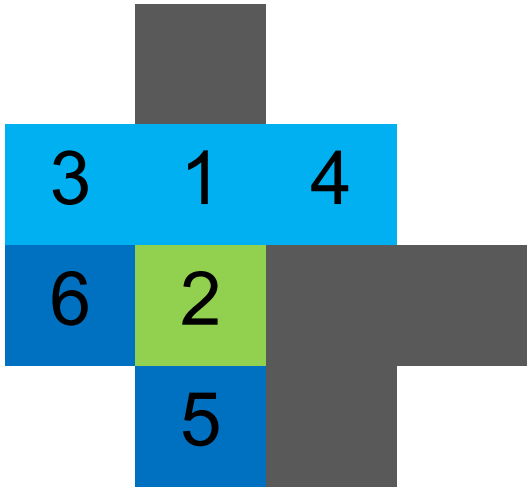


Demo

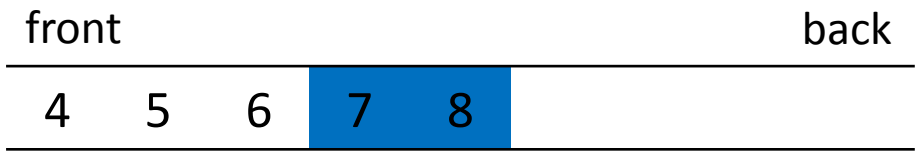


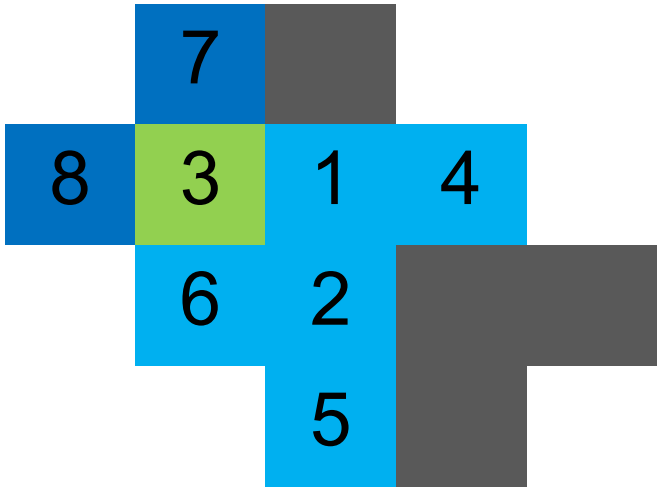


Demo

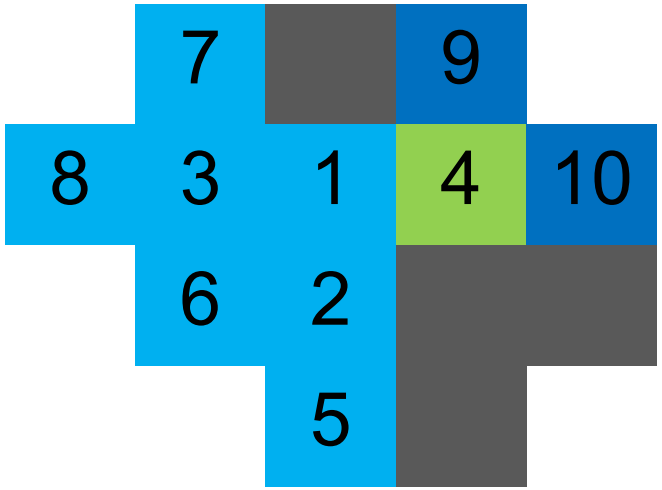


Demo





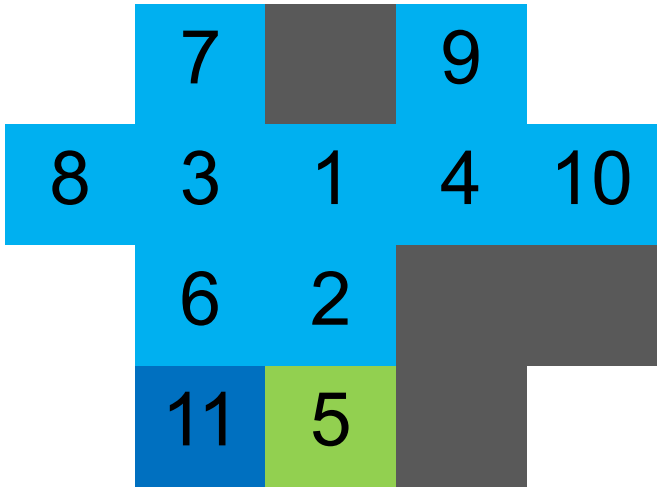
Demo



Demo

front back

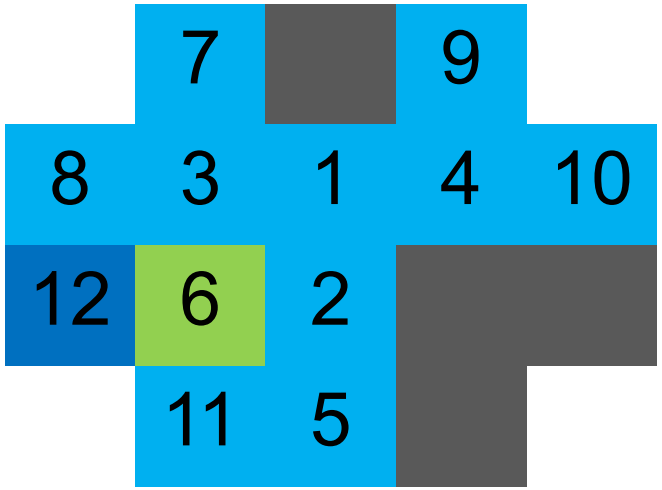
6	7	8	9	10	11	



Demo

front back

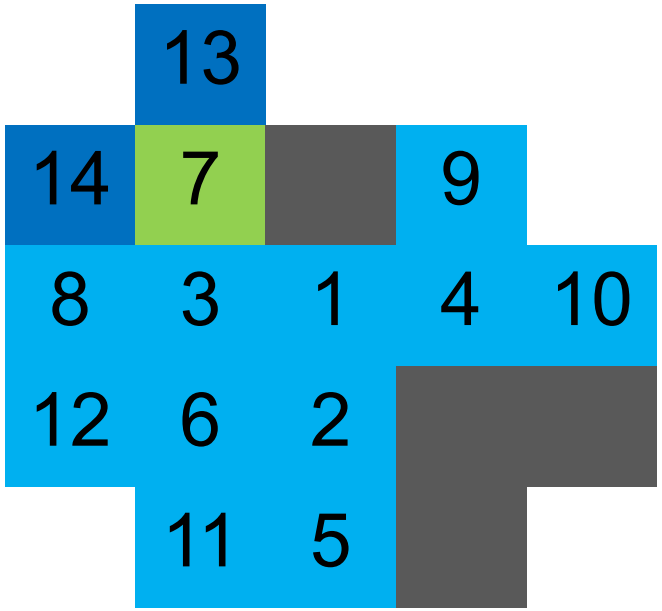
7	8	9	10	11	12	



Demo

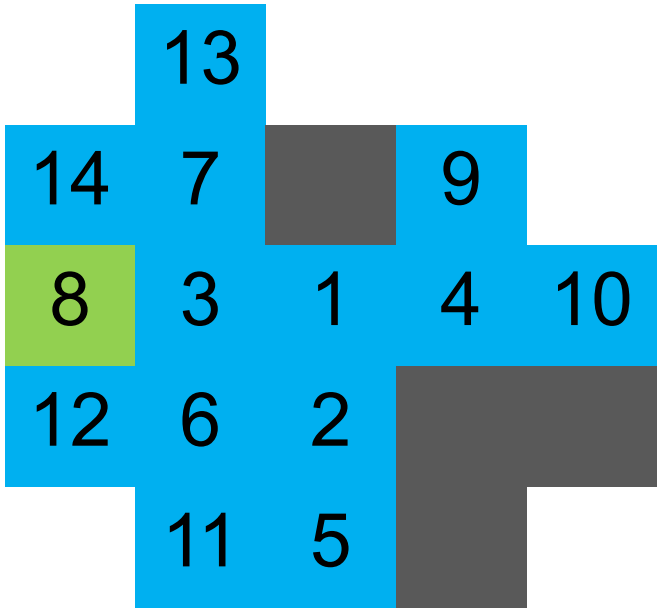
front back

8	9	10	11	12	13	14	



Demo

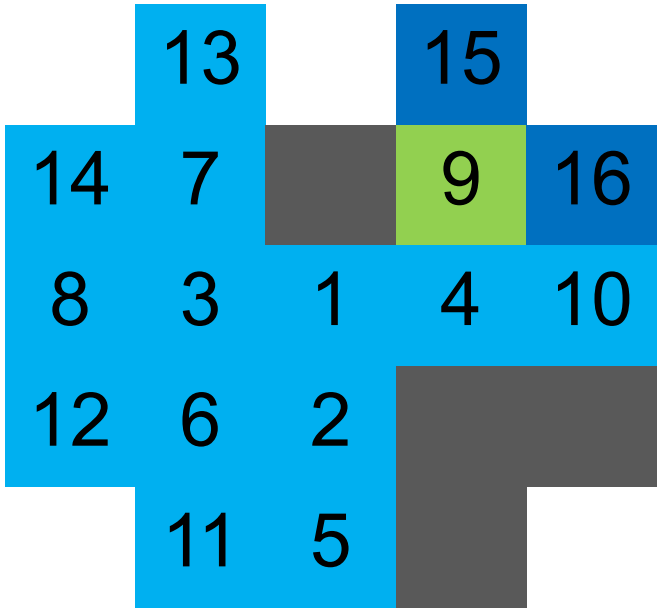
front						back
9	10	11	12	13	14	



Demo

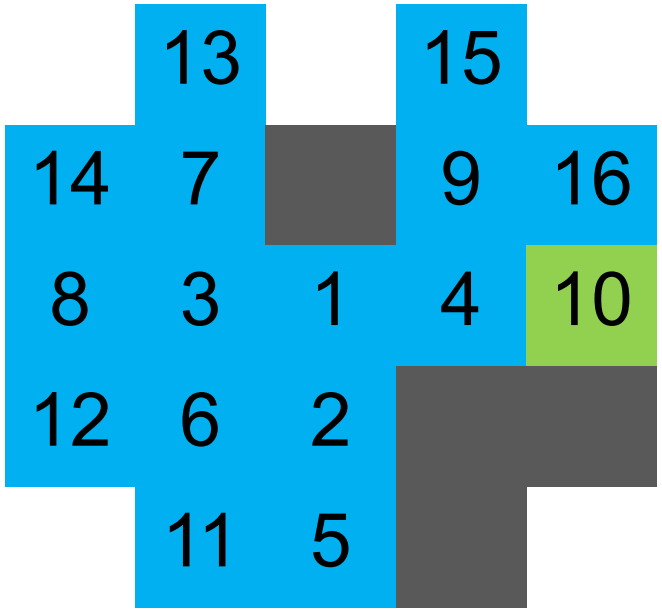
front back

10	11	12	13	14	15	16	



Demo

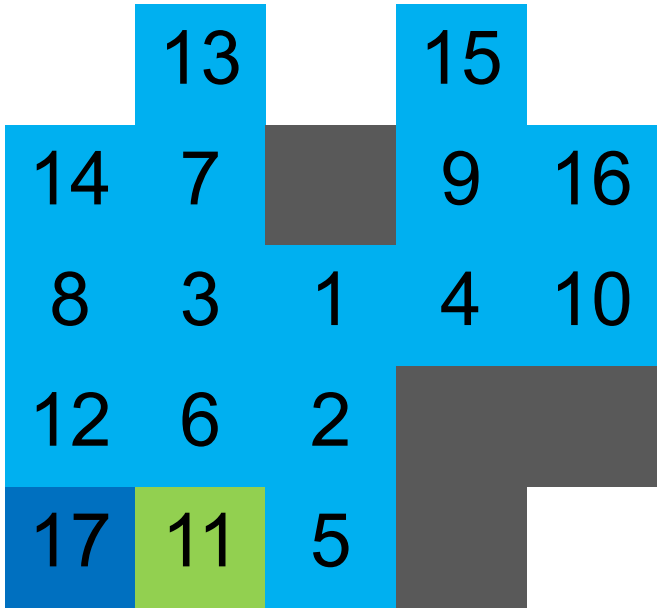
front						back
11	12	13	14	15	16	



Demo

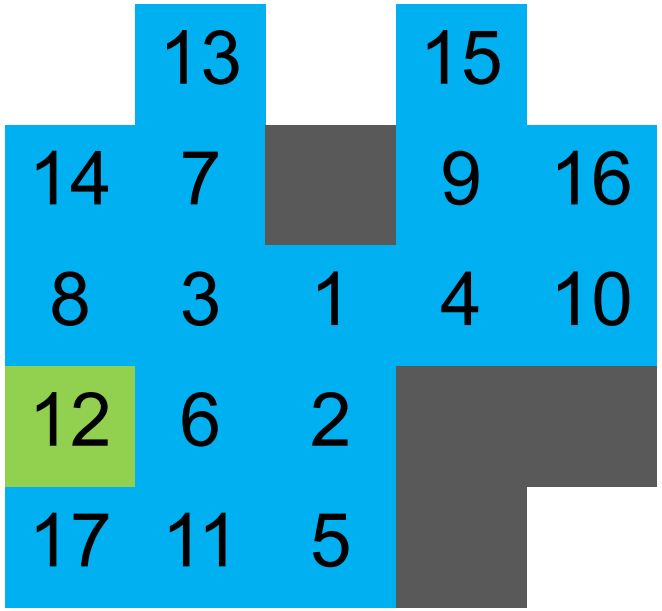
front back

12	13	14	15	16	17		



Demo

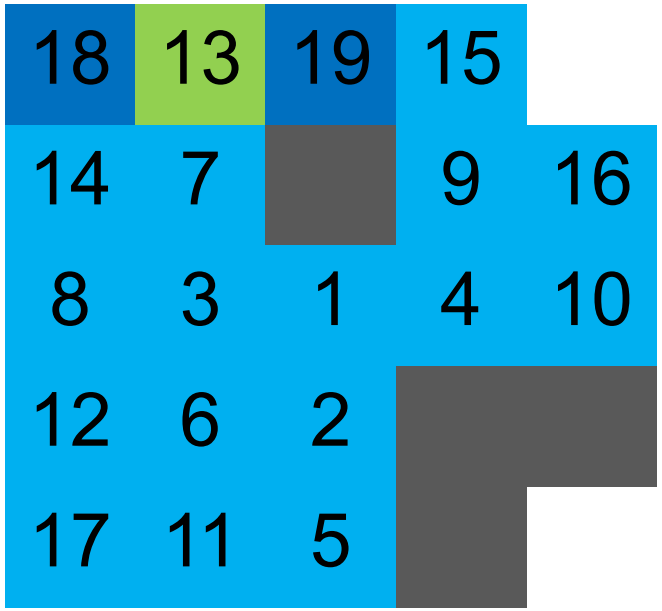
front					back
13	14	15	16	17	



Demo

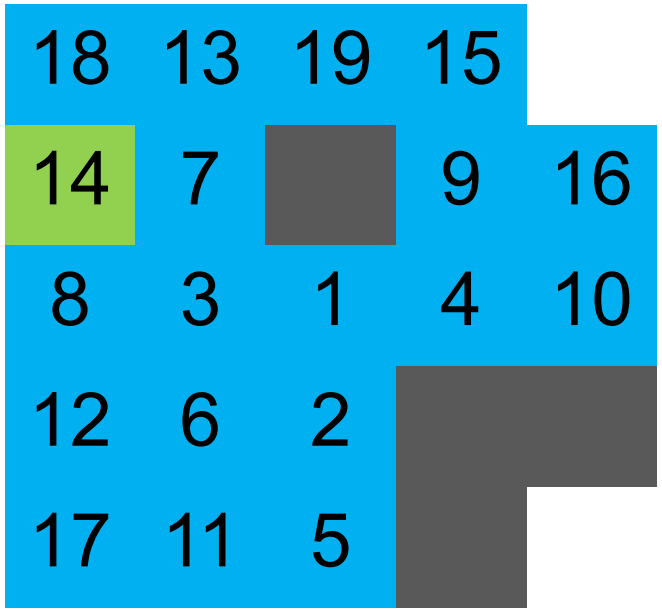
front back

14	15	16	17	18	19	



Demo

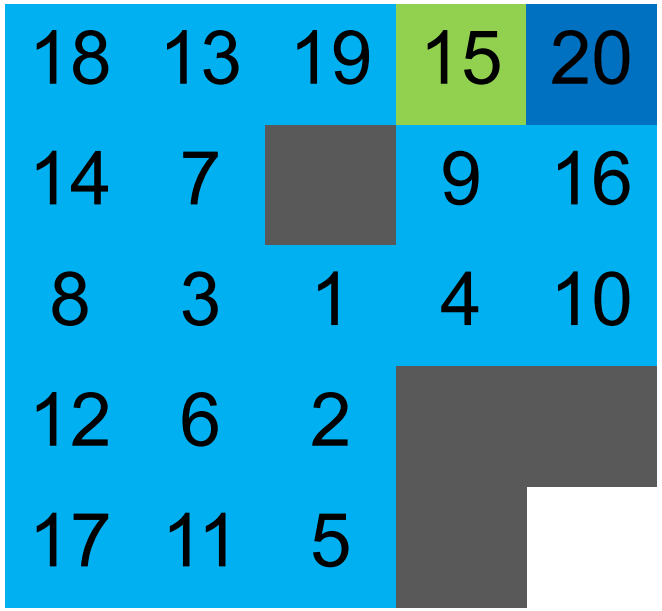
front					back
15	16	17	18	19	



Demo

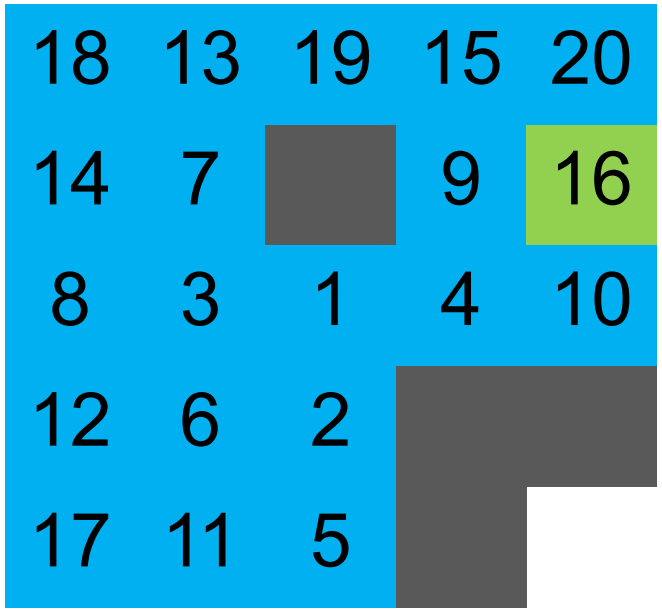
front back

16	17	18	19	20		



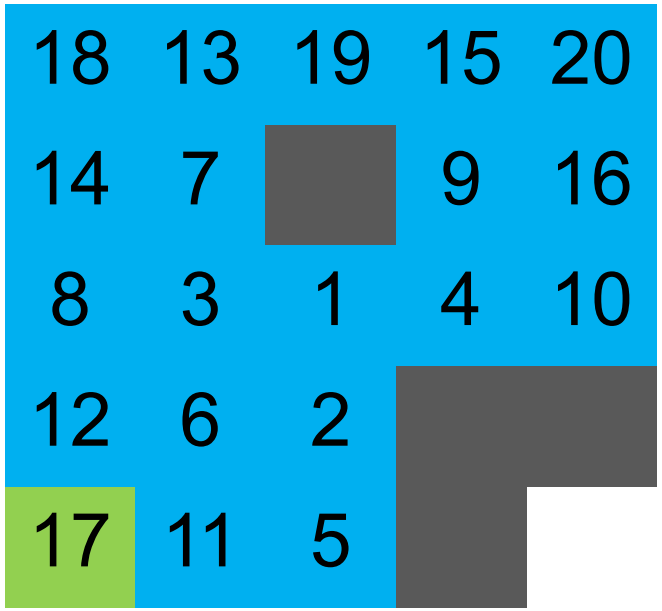
Demo

front				back
17	18	19	20	



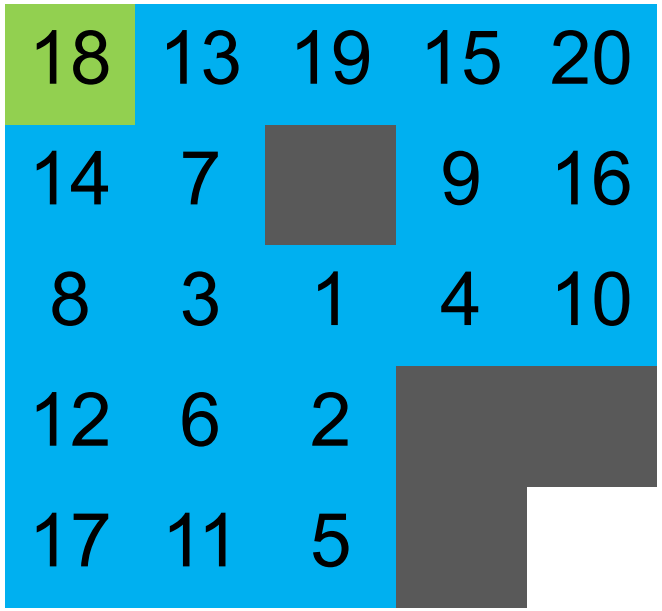
Demo

front			back
18	19	20	



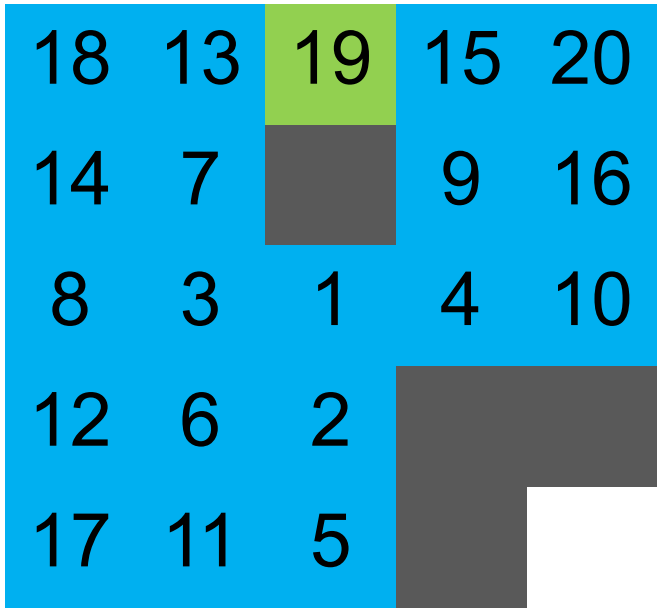
Demo

front	back
19 20	



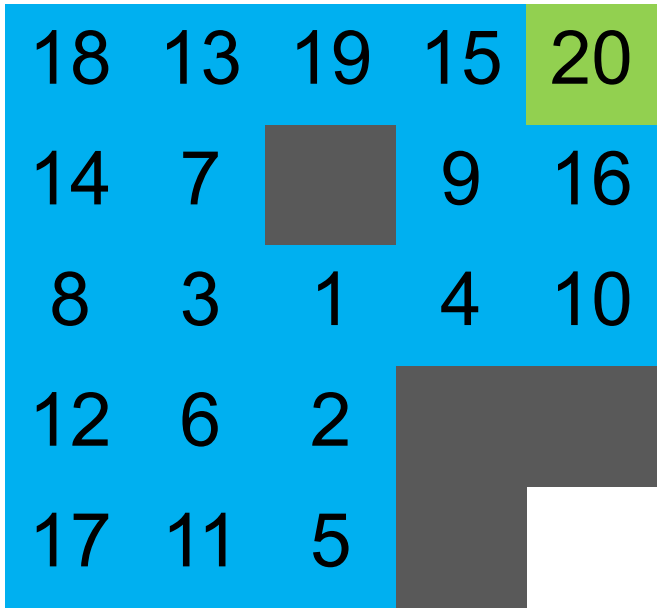
Demo

front	back
20	



Demo

front back



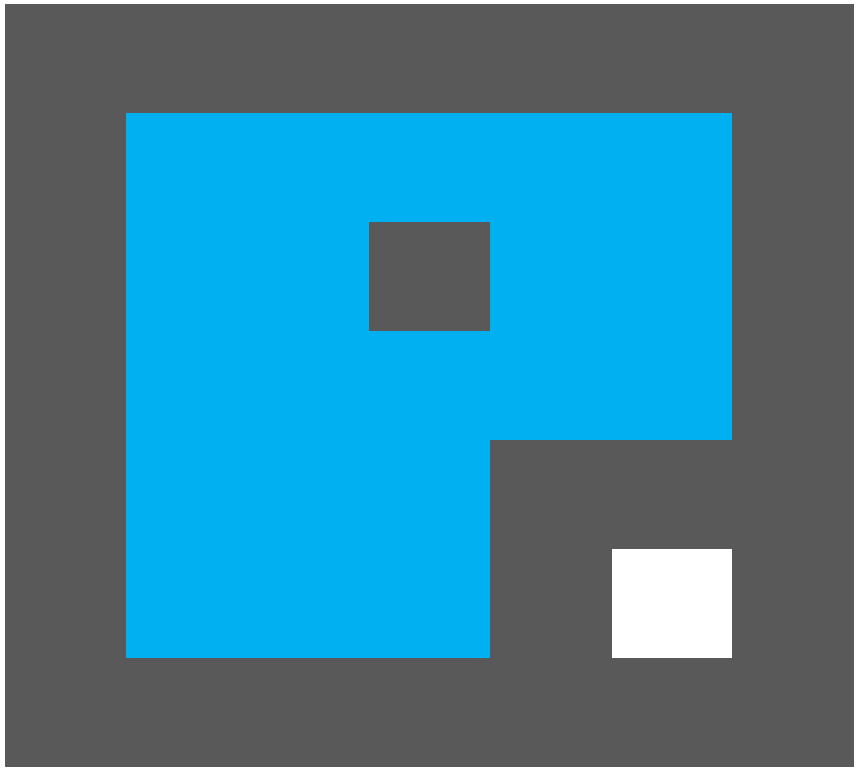
實作細節

- 有障礙物？就當作該格已經被填過了

- 同時有多個格子要倒水？
  - 一開始把所有有水的格子丟到queue中

實作細節

- 邊界好麻煩怎麼辦？
  - 弄一個外框，然後把外框都標記成障礙物



實作細節

```
void bfs(int x, int y) {
  std::queue<std::pair<int,int>> Q;
  Q.emplace(x, y);
  while(Q.size()) {
    std::tie(x, y) = Q.front();
    Q.pop();
    if(grid[x][y]) continue;
    grid[x][y] = true;
    Q.emplace(x+1, y);
    Q.emplace(x, y+1);
    Q.emplace(x-1, y);
    Q.emplace(x, y-1);
  }
}
```

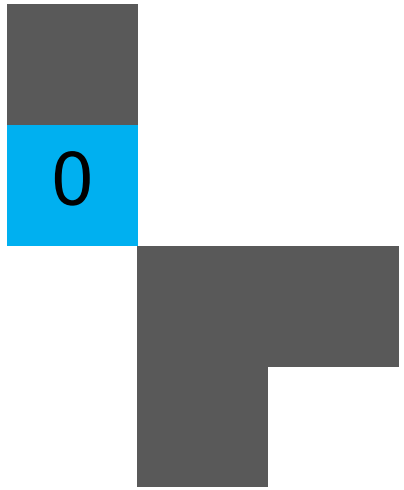
嶄新的複雜度

- 每一格最多只會被丟進queue一次
  - 進入queue內元素不超過格子個數=$O(NM)$
- 每個格子向鄰近的格子溢出只需要常數的操作$O(1)$
- 整體複雜度:$O(NM)$

同時擴散

- 觀察我們queue的執行結果，可以發現queue裡面「梯數」一定是非嚴格遞增的
- 只要在queue裡面多紀錄每個格子的「梯數」，然後每回合一口氣把同一個梯數的一起做完

同時擴散



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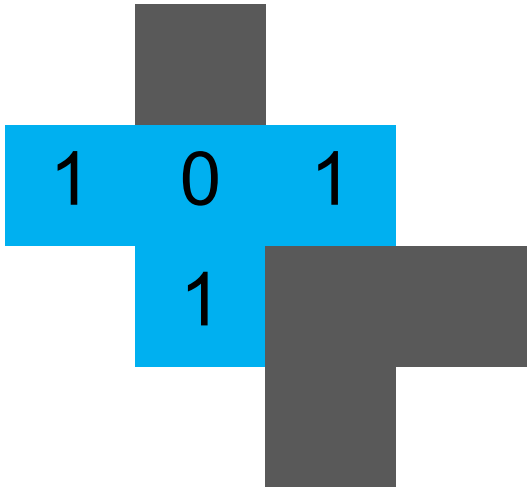

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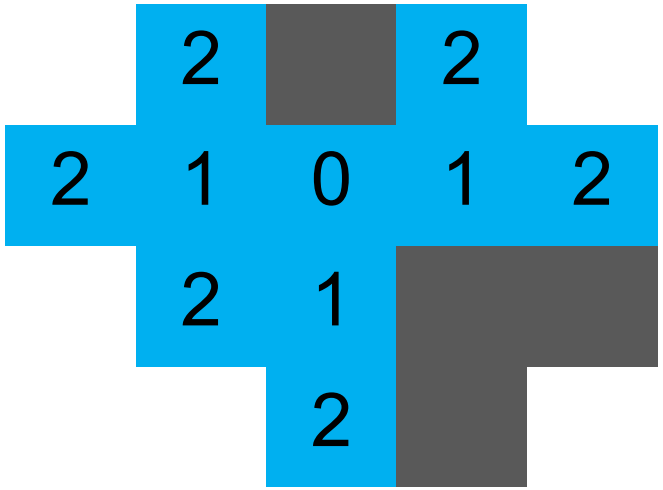

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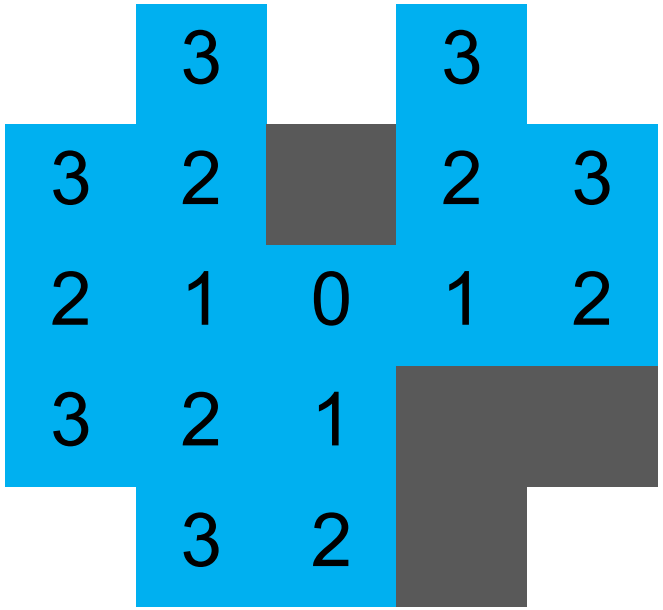


同時擴散

```
void bfs(int x, int y) {
  std::queue<std::pair<int, int>> Q;
  Q.emplace(x, y);
  int L = 0;
  while (Q.size()) {
    for (int Num = Q.size(); Num--;) {
      std::tie(x, y) = Q.front();
      Q.pop();
      if (grid[x][y])
        continue;
      grid[x][y] = true;
      Level[x][y] = L;
      Q.emplace(x + 1, y);
      Q.emplace(x, y + 1);
      Q.emplace(x - 1, y);
      Q.emplace(x, y - 1);
    }
    L += 1;
  }
}
```

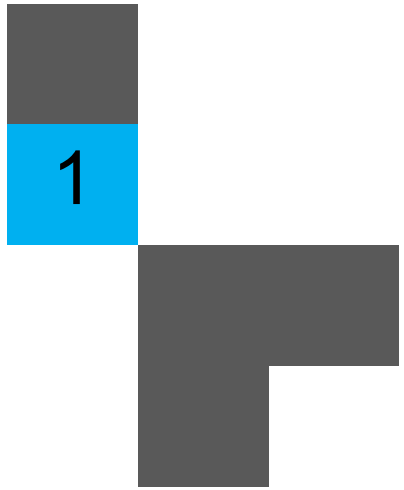
想一下

- 剛剛的淹水演算法中，用的資料結構是queue
- 如果用stack代替queue，會發生什麼事呢？

Demo

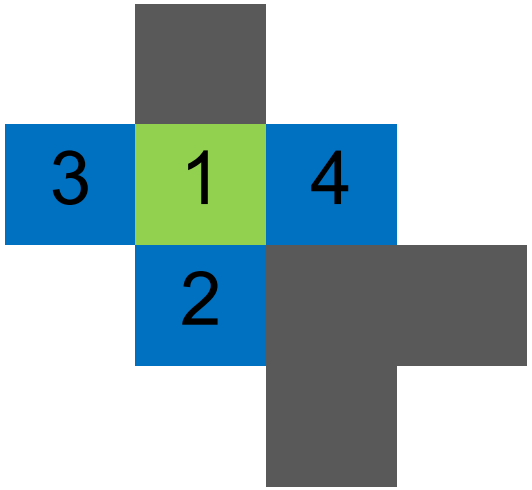
stack : 1

STK | 1



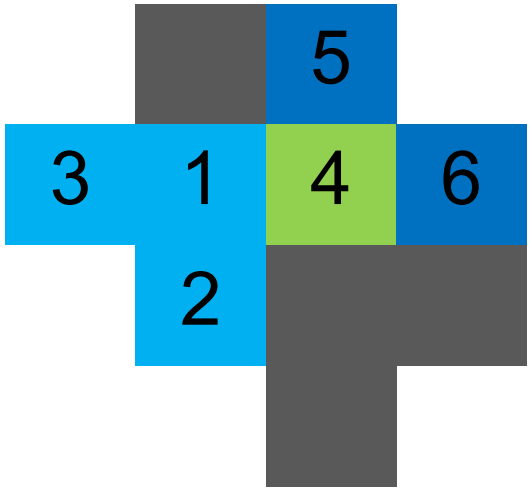
Demo

stack : 2,3,4



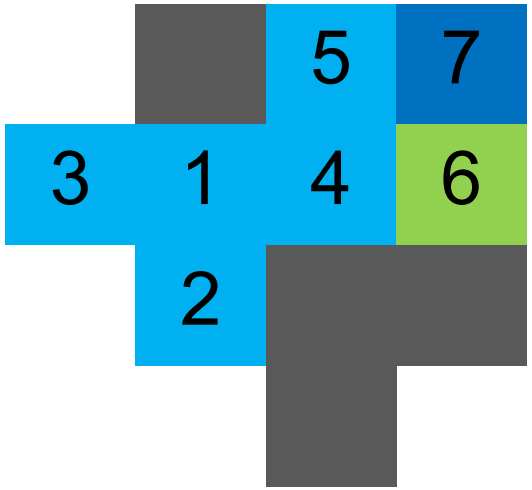
Demo

stack : 2,3,5,6



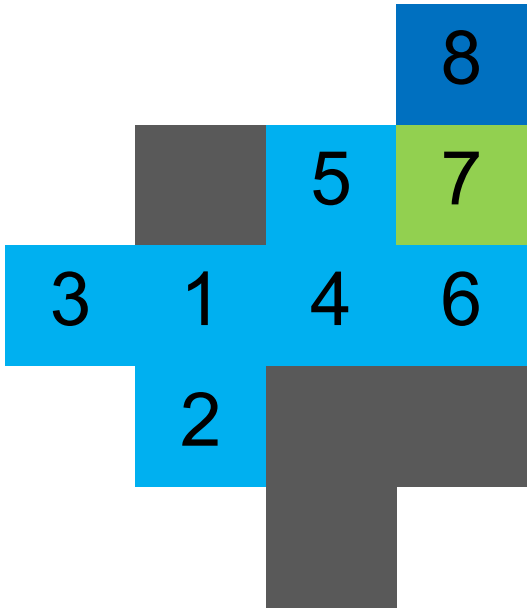
Demo

stack : 2,3,5,7



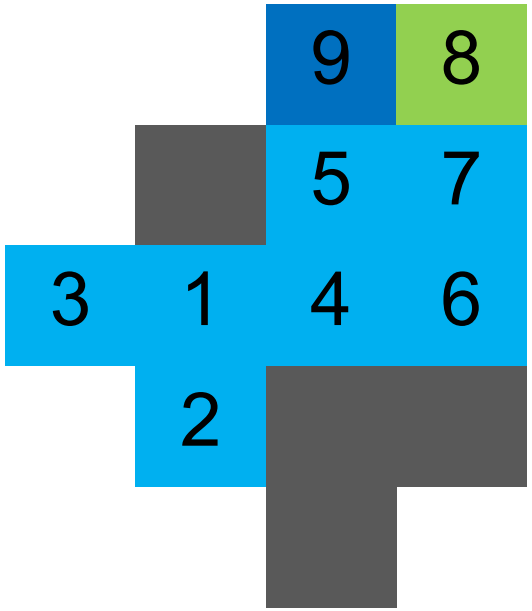
Demo

stack : 2,3,5,8



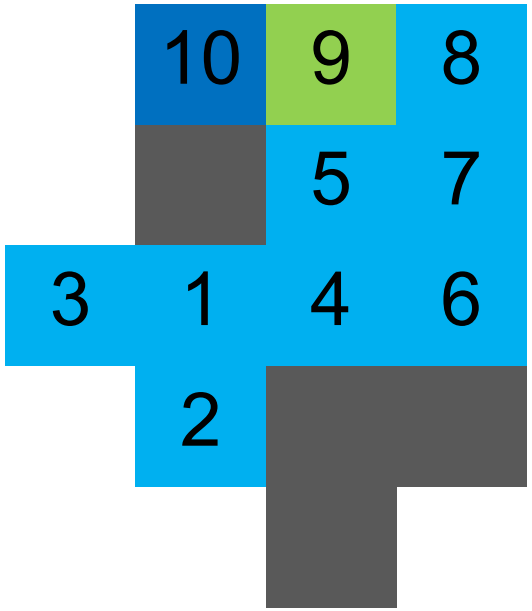
Demo

stack : 2,3,5,9



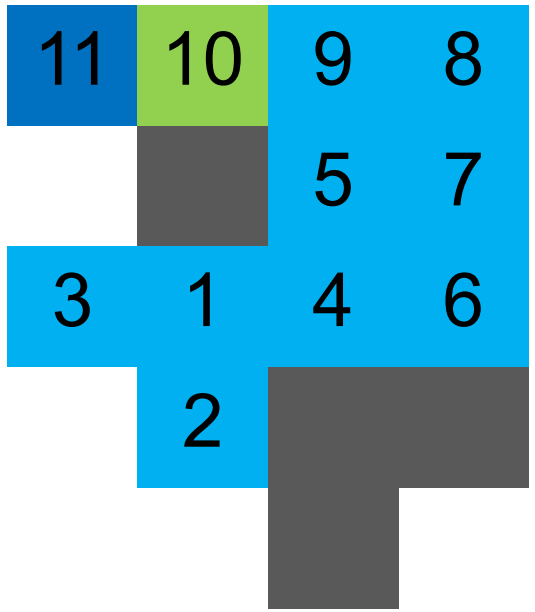
Demo

stack : 2,3,5,10



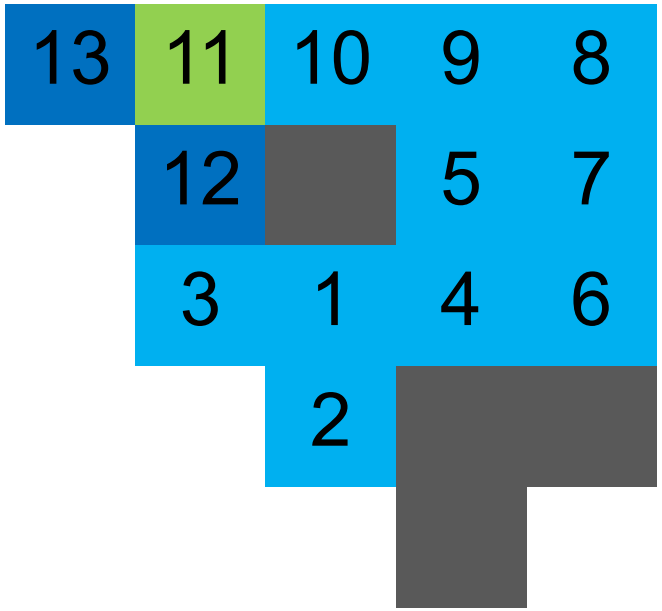
Demo

stack : 2,3,5,11



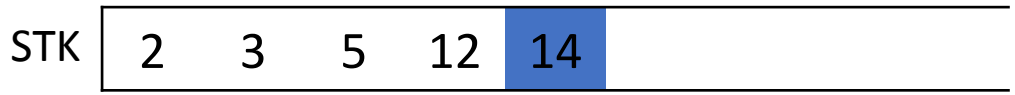
Demo

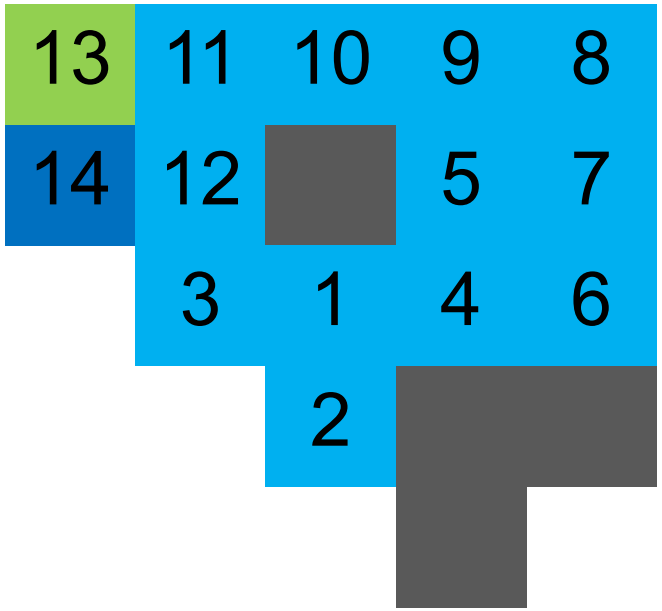
stack : 2,3,5,12,13



Demo

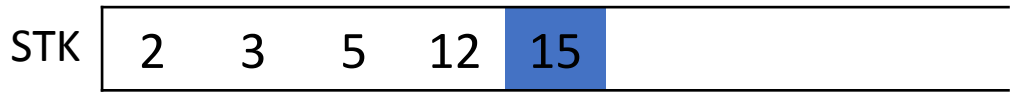
stack : 2,3,5,12,14

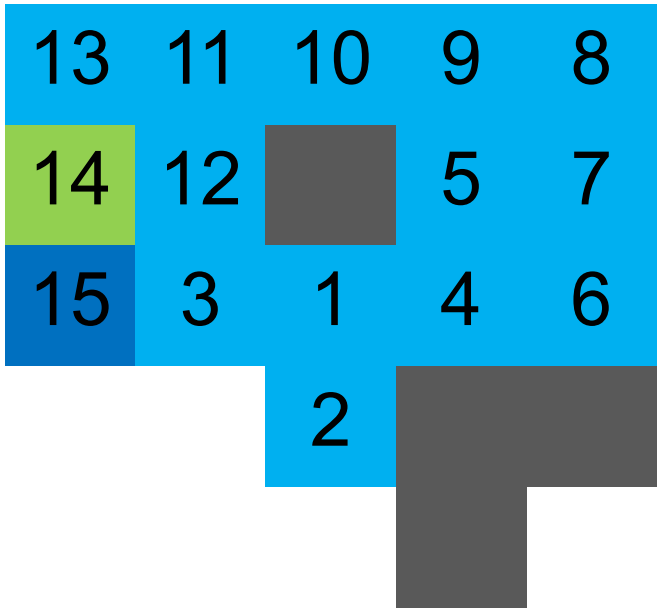




Demo

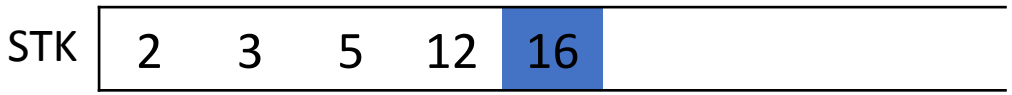
stack : 2,3,5,12,15

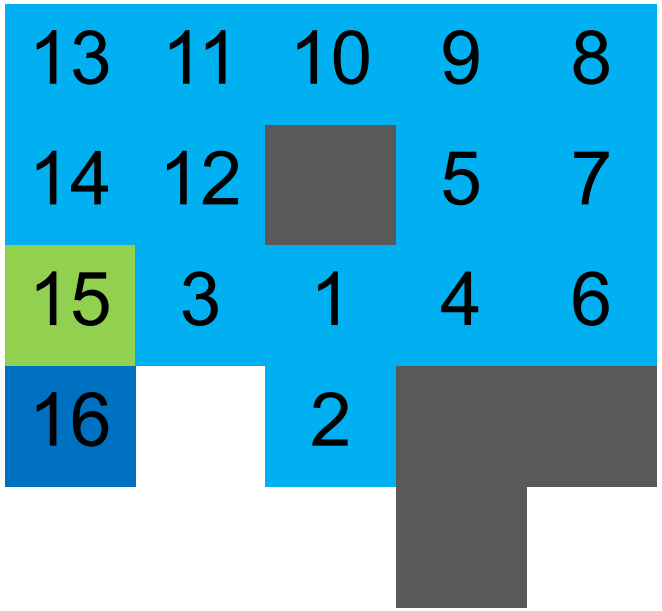




Demo

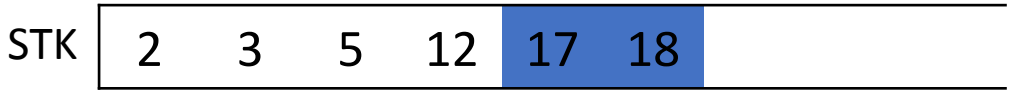
stack : 2,3,5,12,16

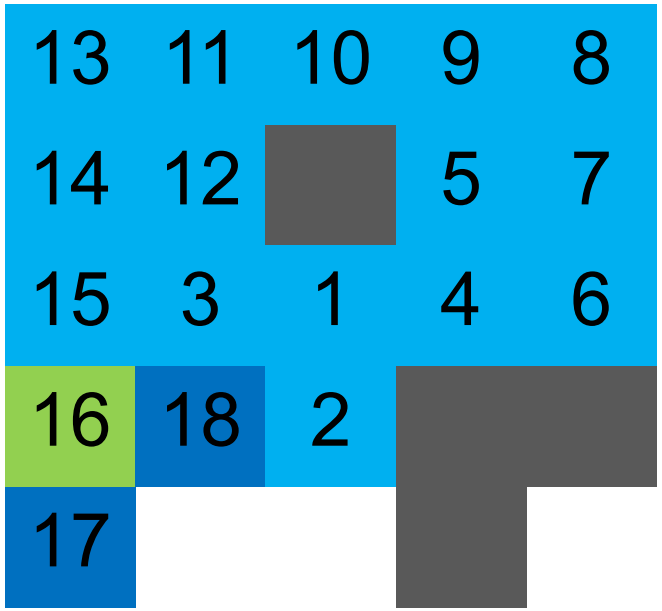




Demo

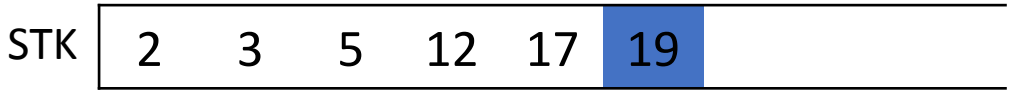
stack : 2,3,5,12,17,18

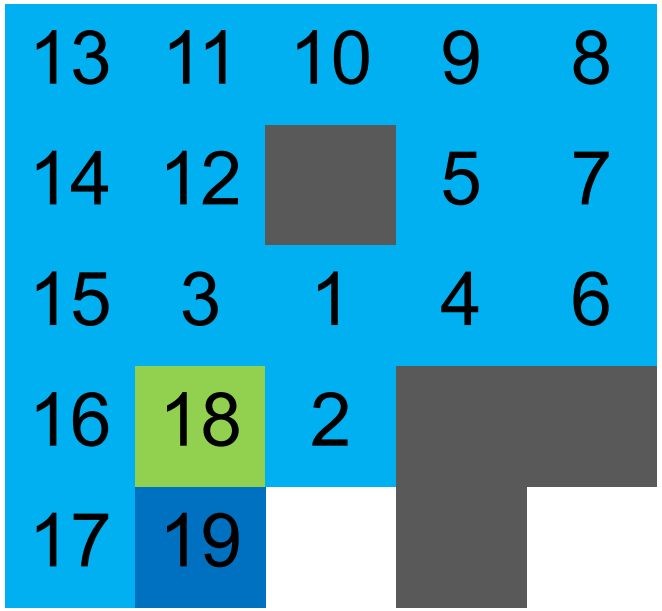




Demo

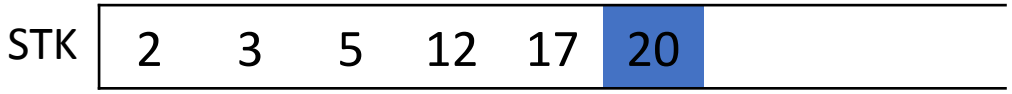
stack : 2,3,5,12,17,19

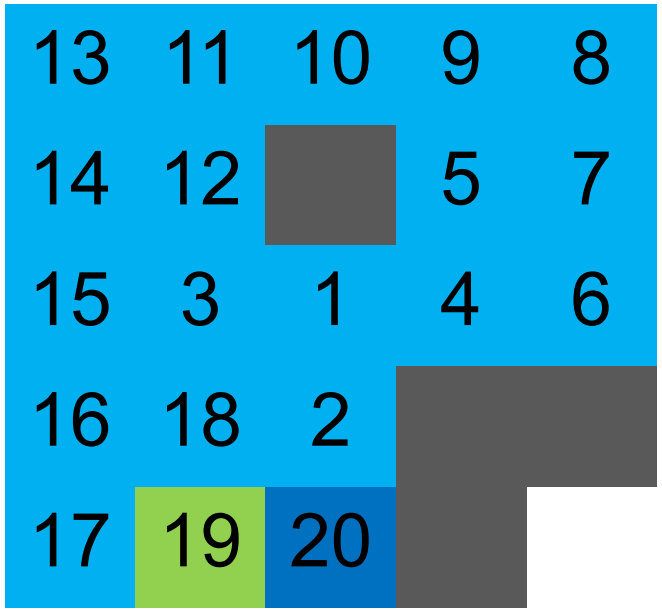




Demo

stack : 2,3,5,12,17,20

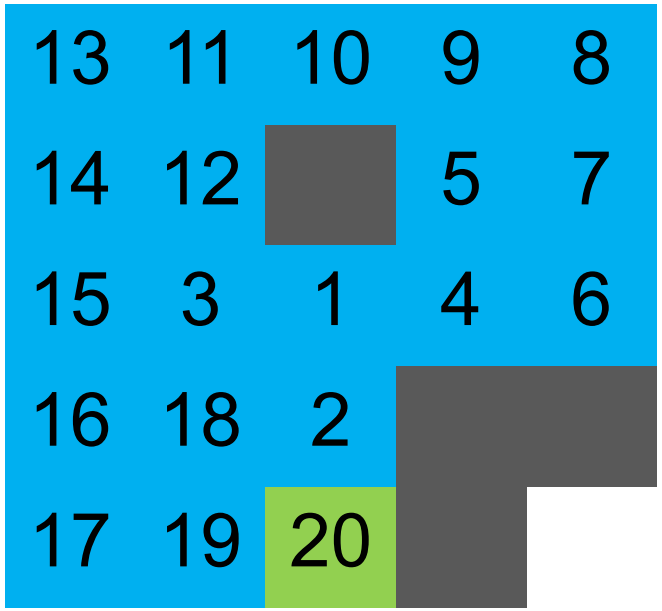




Demo

STK | 2 | 3 | 5 | 12 | 17 |

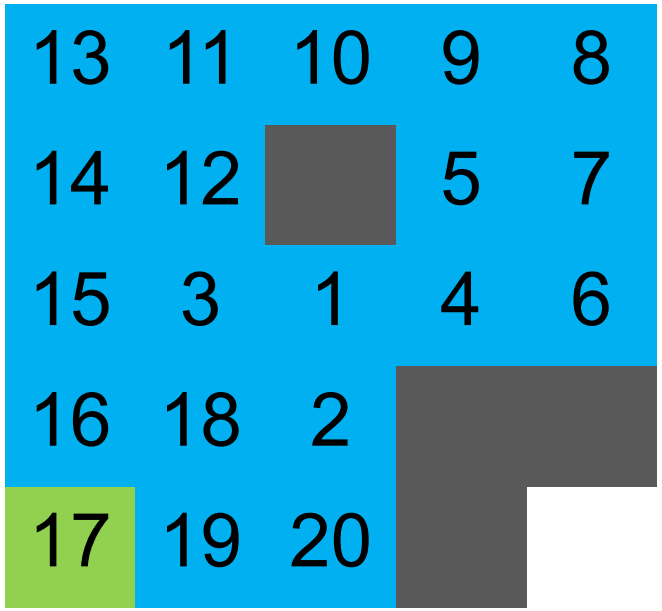
stack : 2,3,5,12,17



Demo

stack : 2,3,5,12

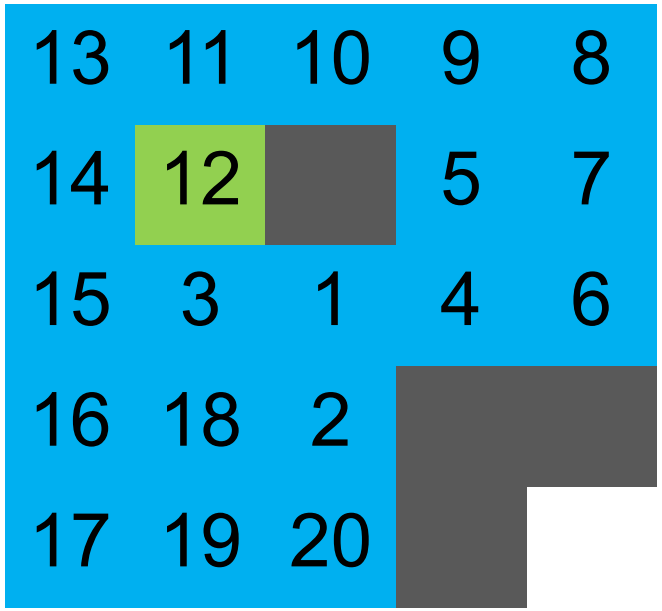
STK | 2 | 3 | 5 | 12 |



Demo

stack : 2,3,5

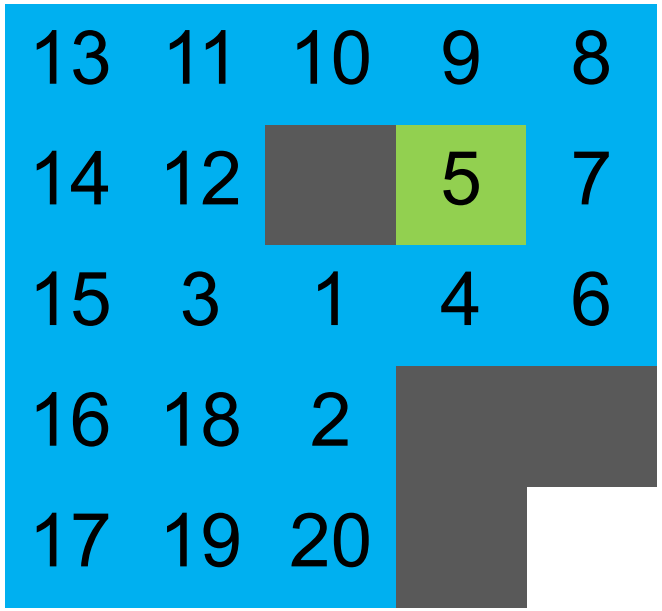
STK | 2 | 3 | 5 |



Demo

stack : 2,3

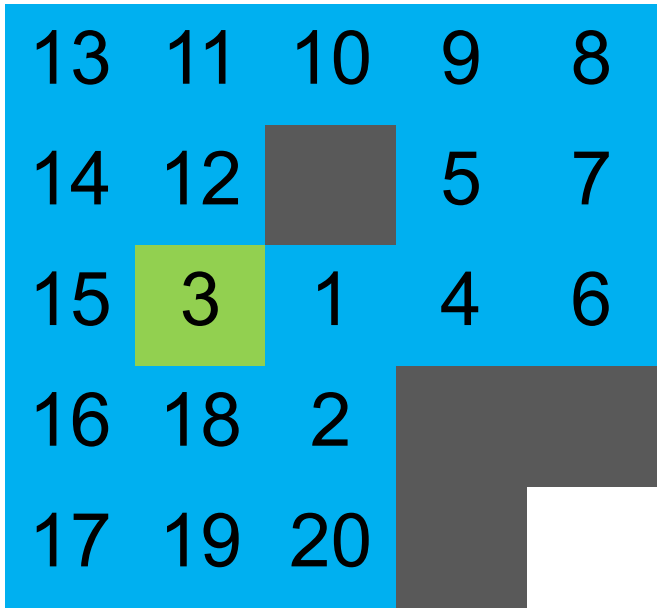
STK | 2 | 3 |



Demo

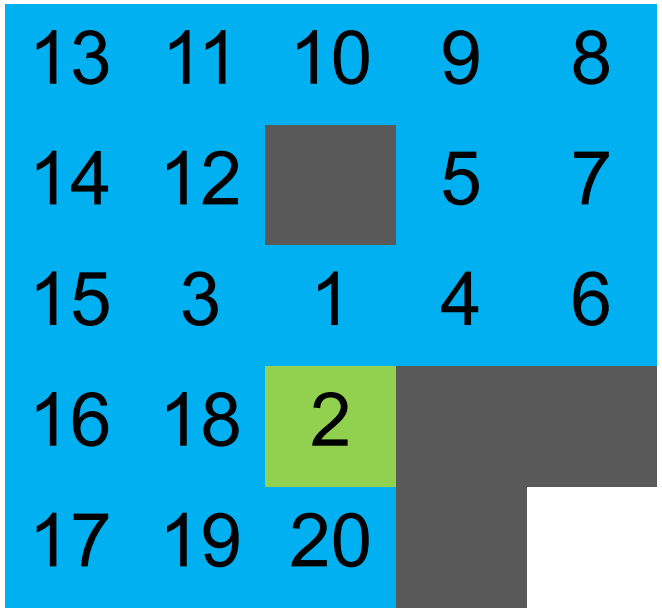
stack : 2

STK | 2



Demo

STK



Stack 版本

```
void dfs(int x, int y){
  std::stack<std::pair<int,int>> STK;
  STK.emplace(x, y);
  while(STK.size()){
    std::tie(x, y) = STK.top();
    STK.pop();
    if(grid[x][y]) continue;
    grid[x][y] = true;
    STK.emplace(x+1, y);
    STK.emplace(x, y+1);
    STK.emplace(x-1, y);
    STK.emplace(x, y-1);
  }
}
```

分類

- 用queue:
  - 廣度優先搜索(Breadth first search,BFS)

- 用stack:
  - 深度優先搜索(Depth first search,DFS)

- 事實上DFS不會像那樣實作

遞迴來實做DFS

- 在進入遞迴的時候，其實函數的資訊都會被記錄在系統提供的stack中
- 因此可以用遞迴實作DFS

```
void dfs(int x, int y) {
  if(grid[x][y]) return;
  grid[x][y] = true;
  dfs(x+1, y);
  dfs(x, y+1);
  dfs(x-1, y);
  dfs(x, y-1);
}
```

+1 -1 很麻煩? 用 for 快速枚舉

```cpp
void bfs(int x, int y) {
  queue<pair<int, int>> Q;
  Q.emplace(x, y);
  int L = 0;
  while (Q.size()) {
    for (int Num = Q.size(); Num--;) {
      tie(x, y) = Q.front();
      Q.pop();
      if (grid[x][y]) continue;
      grid[x][y] = true;
      Level[x][y] = L;
      for (auto [dx, dy] : Dxy)
        Q.emplace(x + dx, y + dy);
    }
    L += 1;
  }
}
```

```cpp
pair<int, int> Dxy[4] =
    {{1, 0}, {0, 1}, {-1, 0}, {0, -1}};
```

```cpp
void dfs(int x, int y) {
  if (grid[x][y]) return;
  grid[x][y] = true;
  for (auto [dx, dy] : Dxy)
    dfs(x + dx, y + dy);
}
```

找出步驟數最少的解

BFS 經典應用

經典題

- https://leetcode.com/problems/water-and-jug-problem/
- 給你兩個容器，容量分別為 jug1Capacity 和 jug2Capacity
- 旁邊有無限供應的水源
- 一開始容器是空的，目標要讓容器的水量總和等於 targetCapacity
- 你只能做以下三件事
  - 將某個容器裝滿水
  - 將某個容器的水倒光
  - 將A容器的水倒到B容器中，直到B容器滿了或是A空了為止

題目問法 :

- 給你 jug1Capacity, jug2Capacity, targetCapacity
- 問法一 : 問你有沒有解(原本題目)
- 問法二 : 問你有解的話最少需要幾個步驟

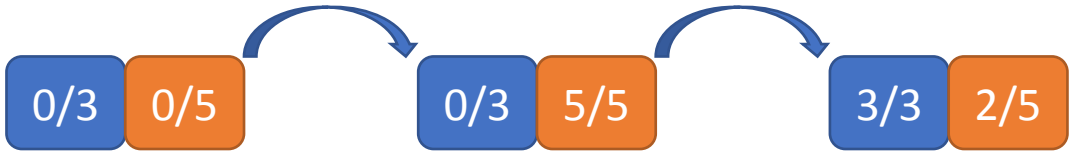
範例測資

- Input: jug1Capacity = 3, jug2Capacity = 5, targetCapacity = 4
- Output: true

模擬一遍 targetCapacity = 4

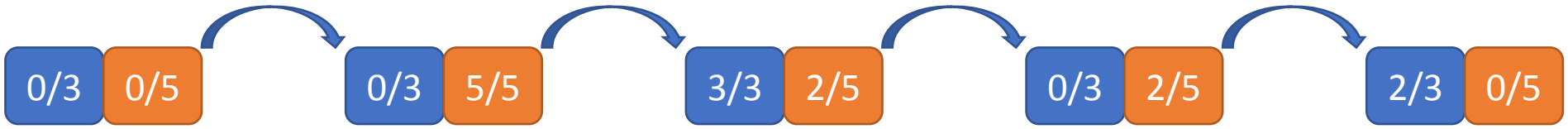
模擬一遍 targetCapacity = 4

模擬一遍 targetCapacity = 4

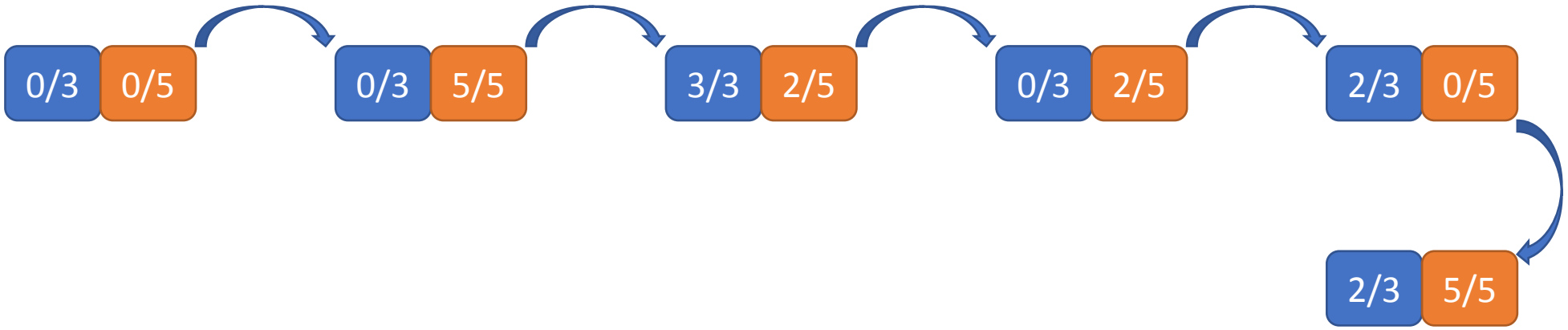


模擬一遍 targetCapacity = 4

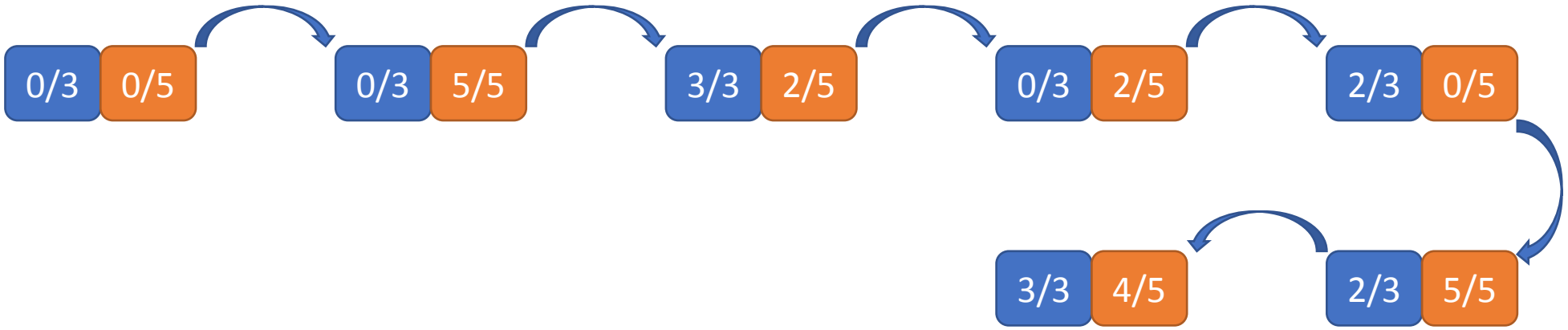
模擬一遍 targetCapacity = 4



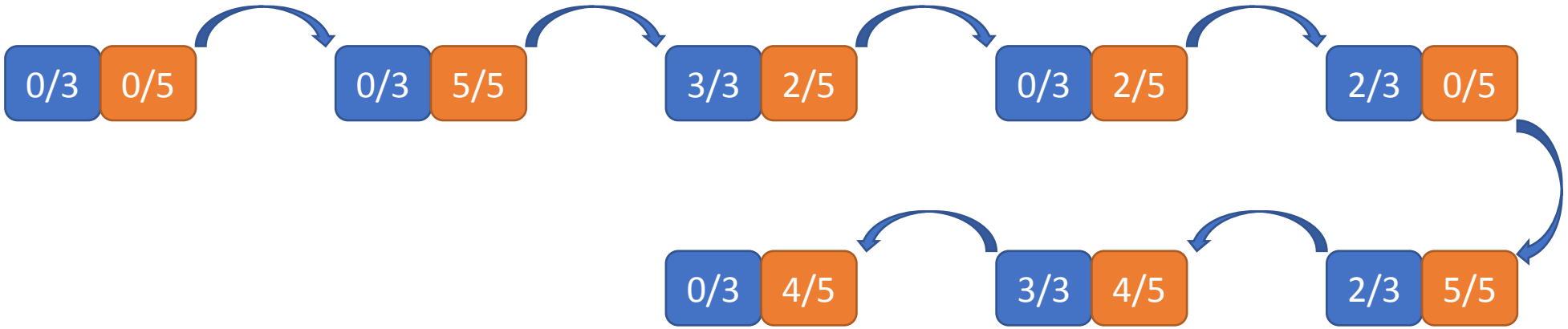
模擬一遍 targetCapacity = 4



模擬一遍 targetCapacity = 4

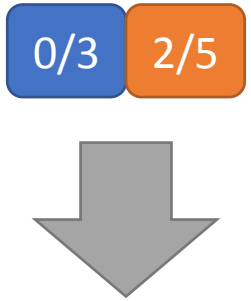


模擬一遍 targetCapacity = 4



想法：每個狀態枚舉所有操作

(jug1, jug2)



(jug1Capacity, jug2)

將jug1倒滿

(jug1, jug2Capacity)

將jug2倒滿

(0, jug2)

將jug1倒光

(jug1, 0)

將jug2倒光

(?, ?)

將jug1倒jug2

(?, ?)

將jug2倒jug1

想法：每個狀態枚舉所有操作

```
jug2New = min(jug1+jug2, jug2Capacity)
jug1New = min(jug1+jug2, jug1Capacity)
```

(jug1, jug2)

(jug1+jug2 - jug2New, jug2New)

將jug1倒jug2

(jug1New, jug1+jug2 - jug1New)

將jug2倒jug1

TLE 的程式碼

```
bool canMeasureWater(int jug1Capacity, int jug2Capacity, int targetCapacity) {
  queue<pair<int, int>> Q;
  set<pair<int, int>> Visited;
  Q.emplace(0, 0);
  while (Q.size()) {
    auto [jug1, jug2] = Q.front(); Q.pop();
    if (Visited.count({jug1, jug2})) continue;
    if (jug1 + jug2 == targetCapacity) return true;
    Visited.emplace(jug1, jug2);
    Q.emplace(jug1Capacity, jug2); // jug1 倒滿
    Q.emplace(jug1, jug2Capacity); // jug2 倒滿
    Q.emplace(0, jug2); // jug1 倒光
    Q.emplace(jug1, 0); // jug2 倒光
    int jug2New = min(jug1 + jug2, jug2Capacity);
    int jug1New = min(jug1 + jug2, jug1Capacity);
    Q.emplace(jug1 + jug2 - jug2New, jug2New); // jug1 倒入 jug2
    Q.emplace(jug1New, jug1 + jug2 - jug1New); // jug2 倒入 jug1
  }
  return false;
}
```