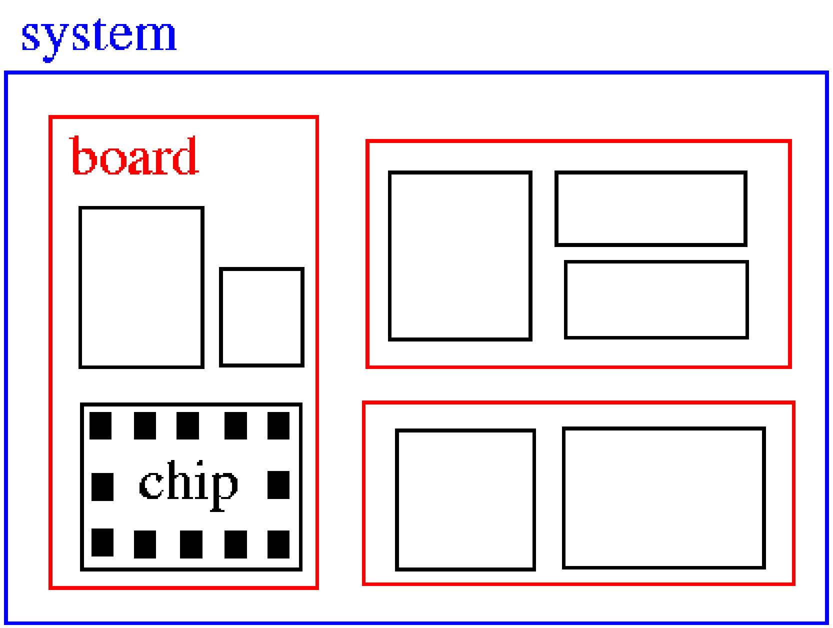
Partitioning

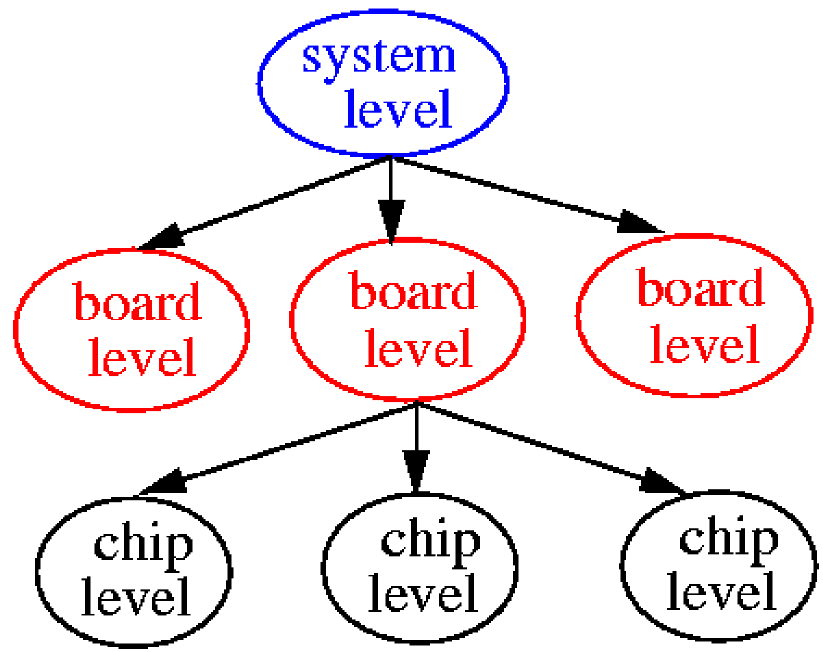
system design

- Decomposition of a complex system into smaller subsystems.
- Each subsystem can be designed independently speeding up the design process.
- Decomposition scheme has to minimize the interconnections among the subsystems.
- Decomposition is carried out hierarchically until each subsystem is of managable size.

Levels of Partitioning

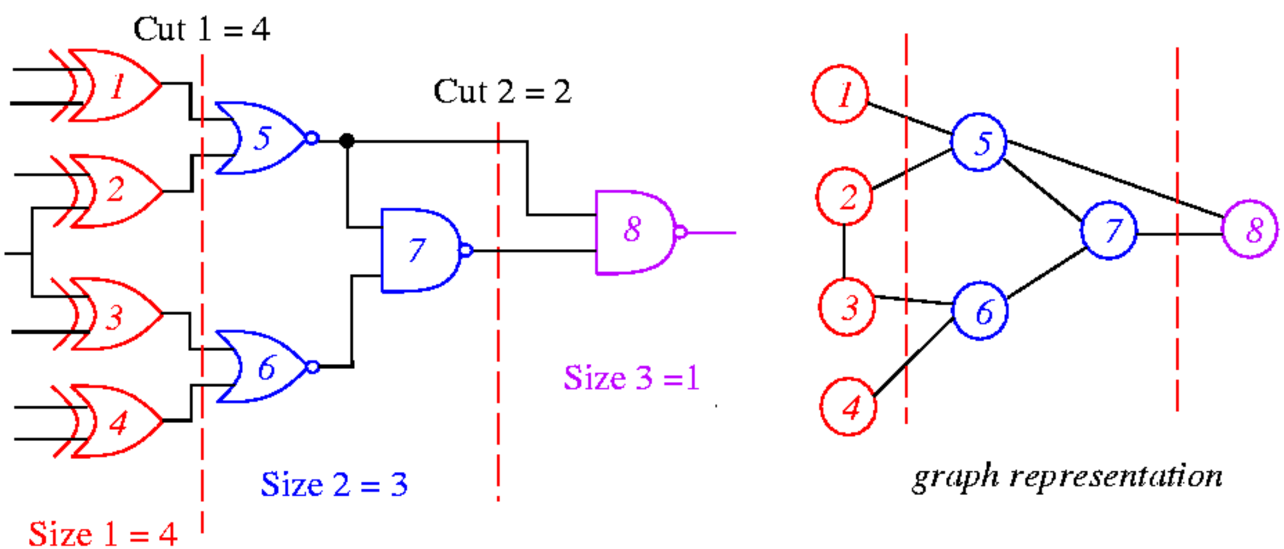
- The levels of partitioning: System, Board, Chip.





Partitioning of a Circuit

- The task of cutting a circuit into smaller parts.
- **Objective:** Partition the circuit into sub-circuits such that every sub-circuit is within a pre-specified size and the # of connections among sub-circuits is minimized.
  – Other possible constraints (e.g., # of pins in a sub-circuit)
  – Other possible objectives (e.g., critical path delay)
- Cutset? Cut size? Size of a sub-circuit?



Problem Definition

- k**-way partitioning:** Given a graph $G(V,E)$, where each vertex $v \in V$ has a **size** $s(v)$ and each edge $e \in E$ has a **weight** $w(e)$, the problem is to divide the set V into k disjoint subsets $V_1, V_2, \ldots, V_k$, such that an objective function is optimized, subject to certain constraints.
- **Bounded size constraint:** The size of the i-th subset is bounded by L_i and U_i $\left(\text{i.e., } L_i \leq \sum_{v \in V_i} s(v) \leq U_i \right)$
- **Min-cut cost between two subsets:** Minimize $\sum_{\forall e=(u,v) \wedge p(u) \neq p(v)} w(e)$, where $p(u)$ is the subset where u is.
- The 2-way, size-constrained partitioning problem is NP-hard, even in its simple form with identical vertex sizes and unit edge weights.

Kernighan-Lin (KL) Algorithm

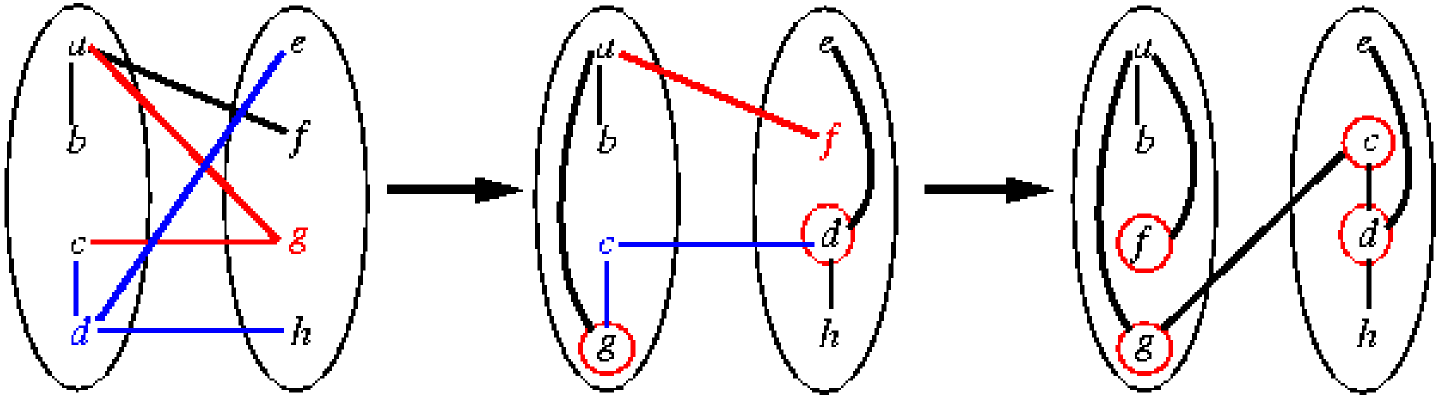
- Kernighan and Lin, “An efficient heuristic procedure for partitioning graphs,” *The Bell System Technical Journal*, vol. 49, no. 2, Feb. 1970.
- An **iterative**, **2-way**, **balanced** partitioning (bi-sectioning) heuristic.
- Restrictions:
  - Assume all vertices are of the same size.
  - Work only for 2-terminal nets.

Key Idea of KL Algorithm

- Start with any initial partitions *A* and *B*.
- A **pass** (exchanging each vertex exactly once) consists of:
  - Exchange a vertex pair which gives the **maximum gain** g_i **(**i.e., largest decrease or smallest increase in cut size), and **lock** them (which thus are prohibited from participating in any further exchanges).
  - This process continues until all vertices are locked.
  - Find the largest partial sum *G* (i.e., find *k* such that $g_1 + \ldots + g_k$ is maximized).
  - Exchange the first *k* pairs.
- Repeat the pass until there is no improvement (i.e., G=0).

KL Algorithm: A Simple Example

- Each edge has a unit weight.



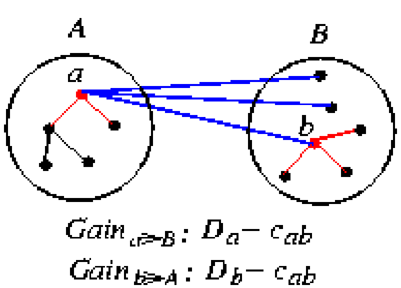
Step #	Vertex pair	Cost reduction	Cut cost
0	-	0	5
1	{d, g}	3	2
2	{c, f}	1	1
3	{b, h}	-2	3
4	{a, e}	-2	5

- Questions: How to compute cost reduction? What pairs to be swapped?

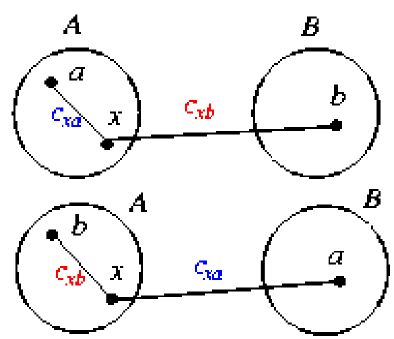
Properties

- Two sets A and B such that $|A|=n=|B|$ and $A\cap B=\emptyset$.
- **External cost** of $a\in A$: $E_a = \sum_{v\in B} c_{av}$.
- **Internal cost** of $a \in A$: $I_a = \sum_{v\in A} c_{av}$.
- D-value of a vertex a: $D_a=E_a-I_a$ (cost reduction for moving a).
- Cost reduction (**gain**) for swapping a and b: $g_{ab}=D_a+D_b-2c_{ab}$.
- If $a\in A$ and $b\in B$ are interchanged, then the new D-values, D', are given by

$$\begin{aligned} D'_x &= D_x + 2c_{xa} - 2c_{xb}, \forall x \in A - \{a\} \\ D'_y &= D_y + 2c_{yb} - 2c_{ya}, \forall y \in B - \{b\}. \end{aligned}$$



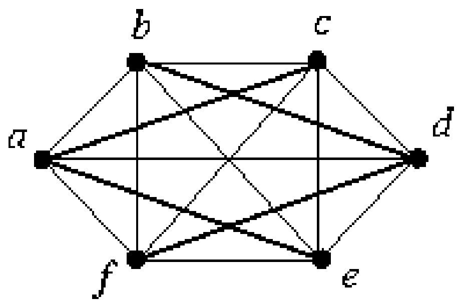
Internal cost vs. External cost



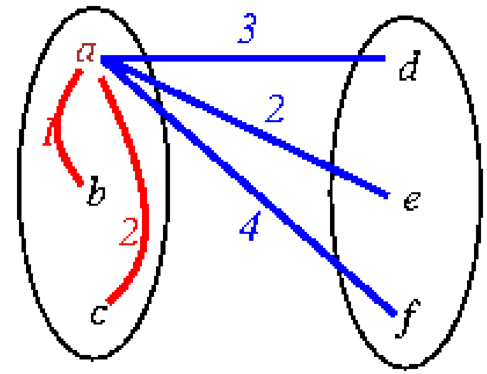
before swap	after swap	$\triangle C$
$-c_{xa}$	$+c_{xa}$	$+2c_{xa}$
$+c_{xb}$	$-c_{xb}$	$-2c_{xb}$

updating D−values

KL Algorithm: A Weighted Example



	a	b	c	d	e	f
a	0	1	2	3	2	4
b	1	0	1	4	2	1
c	2	1	0	3	2	1
d	3	4	3	0	4	3
e	2	2	2	4	0	2
f	4	1	1	3	2	0



costs associated with a

Initial cut cost = (3+2+4)+(4+2+1)+(3+2+1) = 22

- Iteration 1:

I_a=1+2=3; E_a=3+2+4=9; D_a=E_a-I_a=9-3=6

I_b=1+1=2; E_b=4+2+1=7; D_b=E_b-I_b=7-2=5

I_c=2+1=3; E_c=3+2+1=6; D_c=E_c-I_c=6-3=3

I_d=4+3=7; E_d=3+4+3=10; D_d=E_d-I_d=10-7=3

I_e=4+2=6; E_e=2+2+2=6; D_e=E_e-I_e=6-6=0

I_f=3+2=5; E_f=4+1+1=6; D_f=E_f-I_f=6-5=1

Weighted Example (Cont'd)

- Iteration 1:

  I_a=1+2=3; E_a=3+2+4=9; D_a=E_a-I_a=9-3=6

  I_b=1+1=2; E_b=4+2+1=7; D_b=E_b-I_b=7-2=5

  I_c=2+1=3; E_c=3+2+1=6; D_c=E_c-I_c=6-3=3

  I_d=4+3=7; E_d=3+4+3=10; D_d=E_d-I_d=10-7=3

  I_e=4+2=6; E_e=2+2+2=6; D_e=E_e-I_e=6-6=0

  I_f=3+2=5; E_f=4+1+1=6; D_f=E_f-I_f=6-5=1

- $g_{xy}=D_x+D_y-2c_{xy}$.

  $g_{ad} = D_a + D_d - 2c_{ad} = 6 + 3 - 2 * 3 = 3$

  $g_{ae} = 6 + 0 - 2 * 2 = 2$

  $g_{af} = 6 + 1 - 2 * 4 = -1$

  $g_{bd} = 5 + 3 - 2 * 4 = 0$

  $g_{be} = 5 + 0 - 2 * 2 = 1$

  $g_{bf} = 5 + 1 - 2 * 1 = 4$ (maximum)

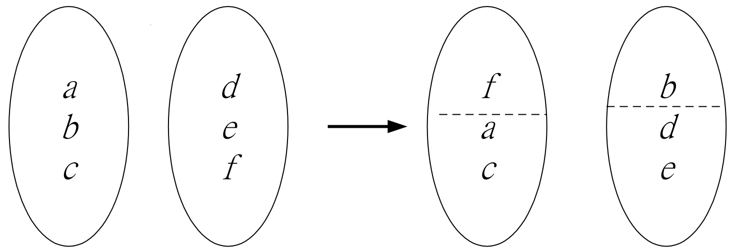
  $g_{cd} = 3 + 3 - 2 * 3 = 0$

  $g_{ce} = 3 + 0 - 2 * 2 = -1$

  $g_{cf} = 3 + 1 - 2 * 1 = 2$

- Swap *b* and *f*! ($\hat{g}_1$ =4)

Weighted Example (Cont'd)



- $D'_x = D_x + 2c_{xp} - 2c_{xq}, \forall x \in A - \{p\}$ (swap p and q, $p \in A$, $q \in B$)

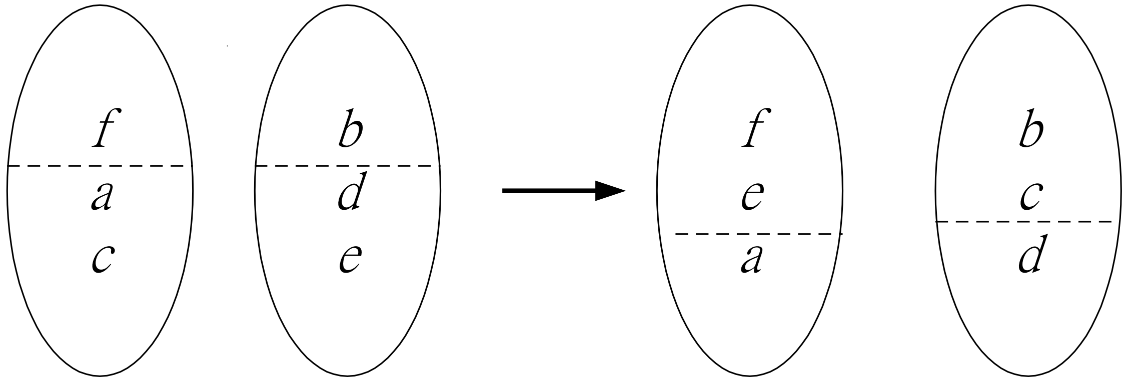
$$D'_a = D_a + 2c_{ab} - 2c_{af} = 6 + 2*1 - 2*4 = 0$$
$$D'_c = D_c + 2c_{cb} - 2c_{cf} = 3 + 2*1 - 2*1 = 3$$
$$D'_d = D_d + 2c_{df} - 2c_{db} = 3 + 2*3 - 2*4 = 1$$
$$D'_e = D_e + 2c_{ef} - 2c_{eb} = 0 + 2*2 - 2*2 = 0$$

- $g_{xy} = D'_x + D'_y - 2c_{xy}$.

$$g_{ad} = D'_a + D'_d - 2c_{ad} = 0 + 1 - 2*3 = -5$$
$$g_{ae} = D'_a + D'_e - 2c_{ae} = 0 + 0 - 2*2 = -4$$
$$g_{cd} = D'_c + D'_d - 2c_{cd} = 3 + 1 - 2*3 = -2$$
$$g_{ce} = D'_c + D'_e - 2c_{ce} = 3 + 0 - 2*2 = -1 \text{ (maximum)}$$

- Swap c and e! ($\hat{g}_2$ = -1)

Weighted Example (Cont'd)



- $D_x^{''} = D_x^{'} + 2c_{xp} - 2c_{xq}, \forall x \in A - \{p\}$

$$D_a^{''} = D_a^{'} + 2c_{ac} - 2c_{ae} = 0 + 2*2 - 2*2 = 0$$

$$D_d^{''} = D_d^{'} + 2c_{de} - 2c_{dc} = 1 + 2*4 - 2*3 = 3$$

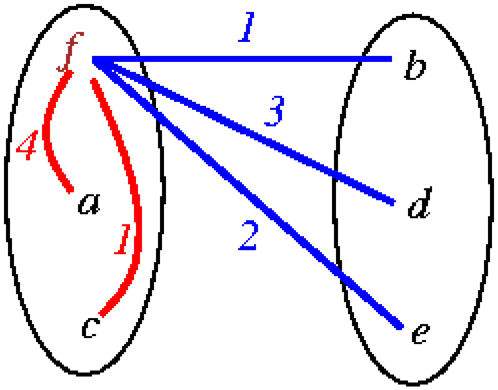
- $g_{xy} = D_x^{''} + D_y^{''} - 2x_{xy}.$

$$g_{ad} = D_a^{''} + D_d^{''} - 2c_{ad} = 0 + 3 - 2*3 = -3 \quad (\hat{g}_3 = -3)$$

- Note that this step is redundant $\left(\sum_{i=1}^{n} \hat{g}_i = 0\right)$
- Summary: $\hat{g}_1 = g_{bf} = 4,\ \hat{g}_2 = g_{ce} = -1,\ \hat{g}_3 = g_{ad} = -3.$
- Largest partial sum max $\sum_{i=1}^{k} \hat{g}_i = 4\ (k = 1) \Rightarrow$ Swap *b* and *f*.

Weighted Example (Cont'd)

	a	b	c	d	e	f
a	0	1	2	3	2	4
b	1	0	1	4	2	1
c	2	1	0	3	2	1
d	3	4	3	0	4	3
e	2	2	2	4	0	2
f	4	1	1	3	2	0



Initial cut cost $= (1+3+2)+(1+3+2)+(1+3+2) = 18\ (22-4)$

- Iteration 2: Repeat what we did at Iteration 1 (Initial cost=22-4=18)

- Summary: $\hat{g}_1 = g_{ce} = -1,\ \hat{g}_2 = g_{ab} = -3,\ \hat{g}_3 = g_{fd} = 4.$

- Largest partial sum= $\max \sum_{i=1}^{k} \hat{g}_i = 0\ (k = 3) \Rightarrow$ Stop!

Algorithm: Kernighan-Lin(G)
Input: G=(V,E),|V|=2n.
Output: Balanced bi-partition A and B with "small" cut cost.

begin
Bi-partition G into A and B such that $|V_A|=|V_B|$, $V_A \cap V_B = \emptyset$, and $V_A \cup V_B = V$.
repeat
  Compute D_v, $\forall v \in V$.
  for i=1 **to** n **do**
    Find a pair of unlocked vertices $v_{ai} \in V_A$ *and* $v_{bi} \in V_B$ whose exchange makes the largest decrease or smallest increase in cut cost;
    Mark v_{ai} and v_{bi} as locked, store the gain $\hat{g}_i$,and compute the new D_v, for all unlocked $v \in V$;
  Find k, such that $G_k = \sum_{i=1}^{k} \hat{g}_i$ is maximized;
  if $G_k > 0$ **then**
    Move $v_{ai},\ldots,v_{ak}$ from V_A to V_B and $v_{bi},\ldots,v_{bk}$ from V_B to V_A ;
  Unlock v, $\forall v \in V$.
until $G_k \le 0$;
end

Time Complexity

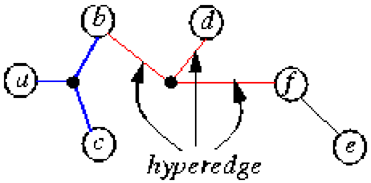
- Line 4: Initial computation of D: $O(n^2)$
- Line 5: The **for-**loop: $O(n)$
- The body of the loop: $O(n^2)$
  - Lines 6-7: Step i takes $O(n-i+1)^2$ time.
- Lines 4-11: Each pass of the repeat loop: $O(n^3)$.
- Suppose the repeat loop terminates after r passes.
- The total running time: $O(rn^3)$.

Extension of KL Algorithm

- *k*-way partitioning
  1. Partition the graph into *k* equal-sized sets. Apply the KL algorithm for each pair of subsets.
  2. Apply the KL algorithm recursively.

Drawbacks of KL Algorithm

- KL handles only unit vertex weights.
  - Vertex weights might represent block sizes, different from blocks to blocks.
  - Reducing a vertex with weight $w(v)$ into a clique with $w(v)$ vertices and edges with a high cost increases the size of the graph substantially.
- KL handles only exact bisections.
  - Need dummy vertices to handle the unbalanced problem.
- KL cannot handle hypergraphs.
  - A hypergraph consists of a set of vertices and a set of hyperedges, where each hyperedge e_i corresponds to a subset N_i of distinct vertices with $|N_i| \geq 2$
- The time complexity of a pass is high, $O(n^3)$.

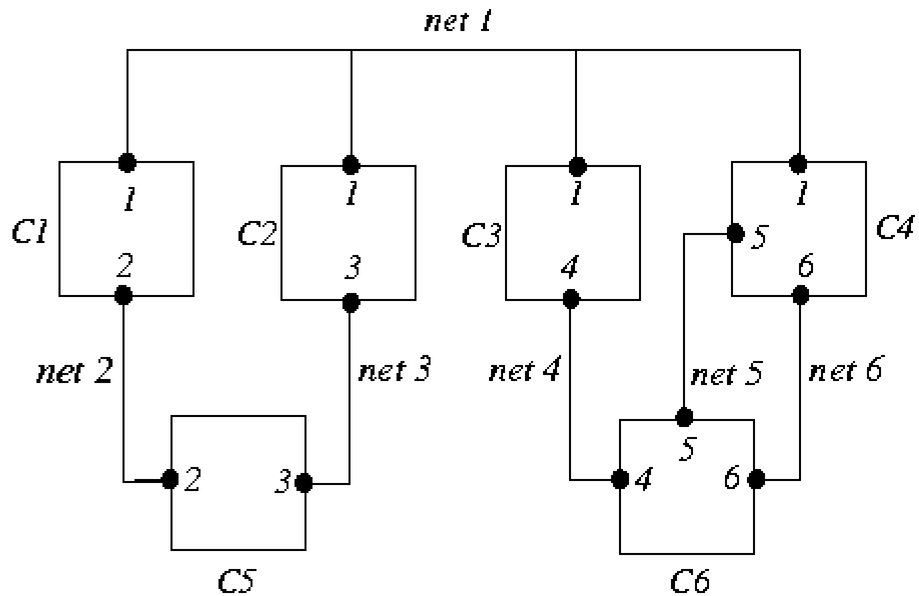


Fiduccia-Mattheyses (FM) Algorithm

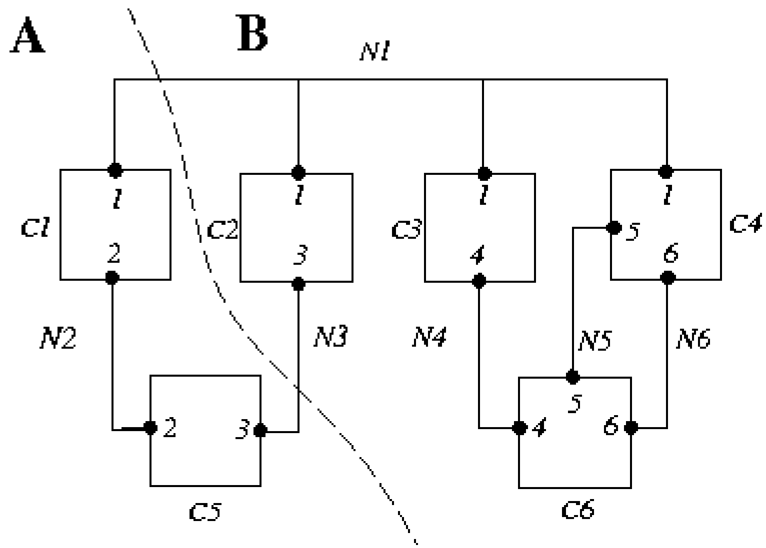
- Fiduccia and Mattheyses. “A linear time heuristic for improving network partitions,” 19th Design Automation Conf.,1982.
- Same as KL:
  - Work in passes.
  - Lock vertices after moved.
  - Actually, only move those vertices up to the maximum partial sum of gain.
- Different from KL:
  - Aim at **reducing net-cut costs**; the concept of cut size is extended to hypergraphs.
  - Only a **single vertex** is moved across the cut in a move.
  - Vertices are weighted.
  - Can handle “unbalanced” partitions; a balance factor is introduced.
  - A special data structure is used to select vertices to be moved across the cut to improve running time.
  - **Time complexity of a pass is** $O(P)$, where P is the total # of pins.

FM: Notation

- $n(i)$: # of cells in Net i; e.g., $n(1)$=4.
- $s(i)$: size of Cell i.
- $p(i)$: # of pins in Cell i; e.g., $p(6)$=3.
- C: total # of cells; e.g., C=6.
- N: total # of nets; e.g., N=6.
- P: total # of pins; $P = p(1)+\ldots+p(C) = n(1)+\ldots+n(N)$.

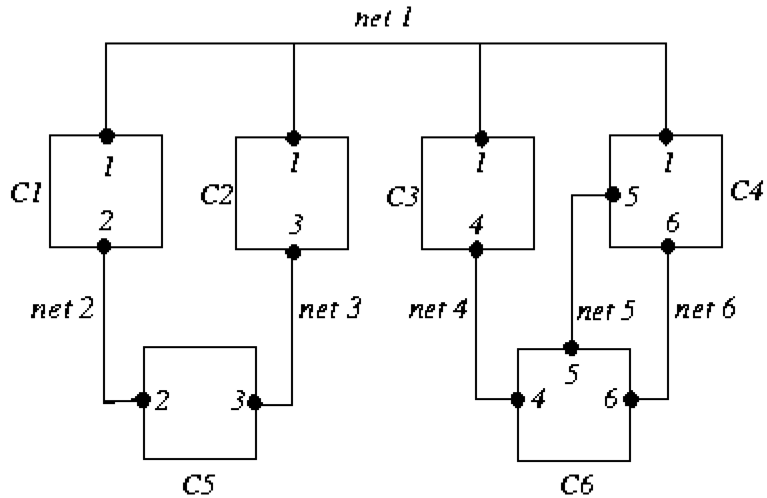


Cut



- **Cutstate** of a net:
  - Net 1 and Net 3 are **cut**.
  - Net 2, Net 4, Net 5, and Net 6 are **uncut.**
- **Cutset**= {Net 1, Net 3}.
- $|A|$=size of $A = s(1)+s(5)$; $|B|=s(2)+s(3)+s(4)+s(6)$.
- Balanced 2-way partitioning: Given a fraction r, $0< r <1$, partition a graph into two sets A and B such that
  - $\frac{|A|}{|A|+|B|} \approx r.$
  - Size of the cutset is minimized.

Input Data Structures



Cell array		Net array	
C1	Nets 1, 2	Net 1	C1, C2, C3, C4
C2	Nets 1, 3	Net 2	C1, C5
C3	Nets 1, 4	Net 3	C2, C5
C4	Nets 1, 5, 6	Net 4	C3, C6
C5	Nets 2, 3	Net 5	C4, C6
C6	Nets 4, 5, 6	Net 6	C4, C6

- Size of the circuit: $P = \sum_{i=1}^{6} n(i) = 14$
- Construction of the two arrays takes $O(P)$ time.

Basic Ideas: Balance and Movement

- Only move a cell at a time, preserving "balance."

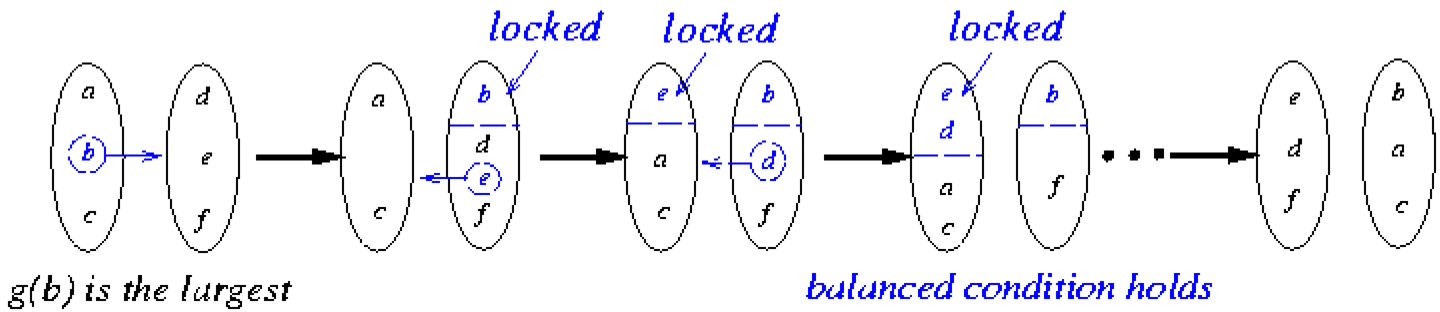
$$\frac{|A|}{|A|+|B|} \approx r$$

$$rW - S_{\max} \leq |A| \leq rW + S_{\max},$$

when $W=|A|+|B|$; $S_{\max}=\max_i s(i)$.

- $g(i)$: gain in moving cell i to the other set,

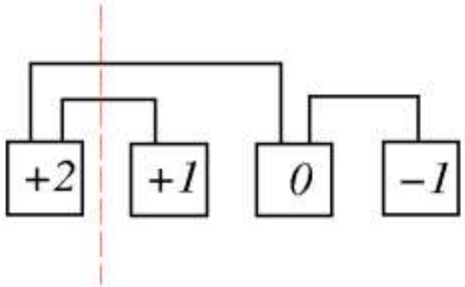
  i.e., size of **old** cutset - size of **new** cutset.

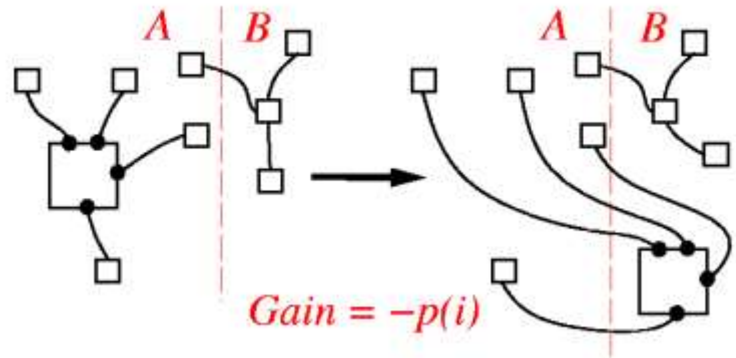


- Suppose $\hat{g}_i$'s: $g(b)$, $g(e)$, $g(d)$, $g(a)$, $g(f)$, $g(c)$ and the largest partial sum is $g(b)+g(e)+g(d)$. Then we should move b,e,d ➔ resulting two sets: $\{a,c,e,d\}$, $\{b,f\}$

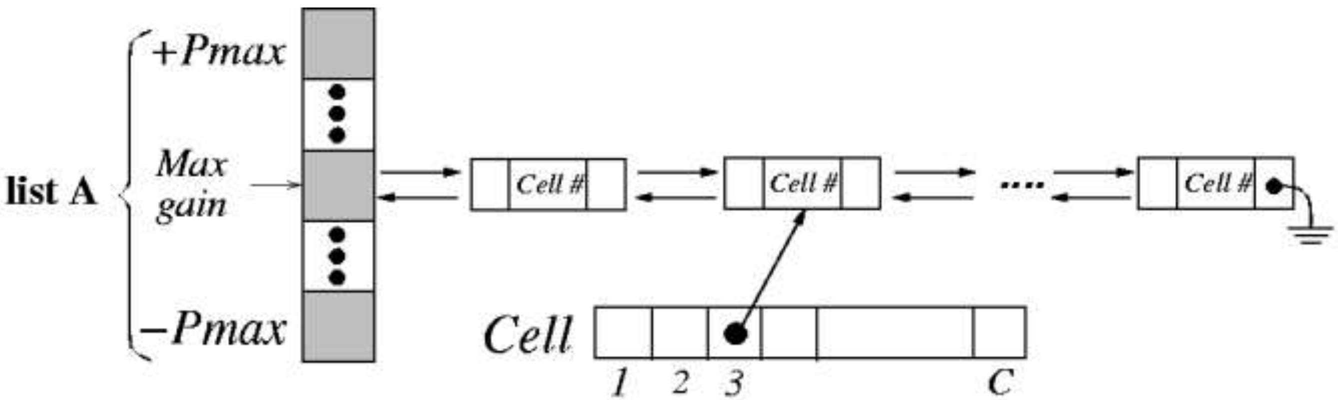
Cell Gains and Data Structure Manipulation

- $-p(i) \leq g(i) \leq p(i)$





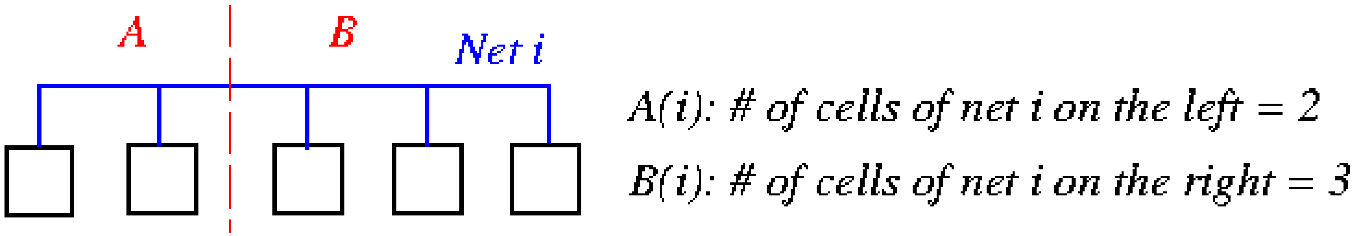
- Two “bucket list” structures, one for set *A* and one for set *B* (P_{max}=$\max_i p(i)$).



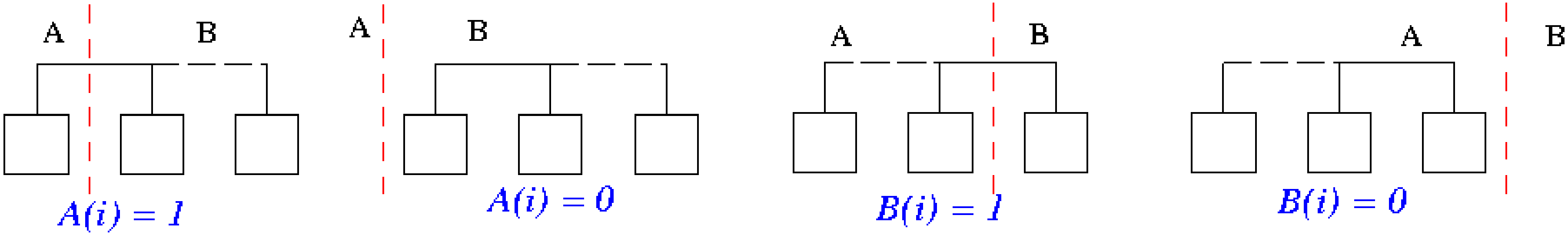
- $O(1)$-time operations: find a cell with Max Gain, remove Cell *i* from the structure, insert Cell *i* into the structure, update $g(i)$ to $g(i) + \triangle$, update the Max Gain pointer.

Net Distribution and Critical Nets

- Distribution of Net i: $(A(i), B(i)) = (2, 3)$.
  - $(A(i), B(i))$ for all i can be computed in $O(P)$ time.



- **Critical Nets**: A net is critical if it has a cell which if moved will change its cutstate.
  - 4 cases: $A(i) = 0$ or 1, $B(i) = 0$ or 1.

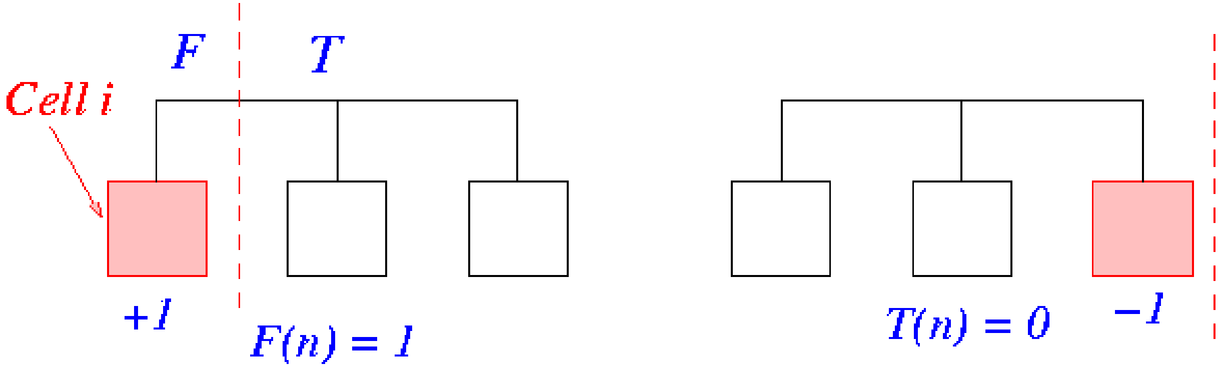


- Gain of a cell depends only on its critical nets.

Computing Cell Gains

- Initialization of all cell gains requires $O(P)$ time:

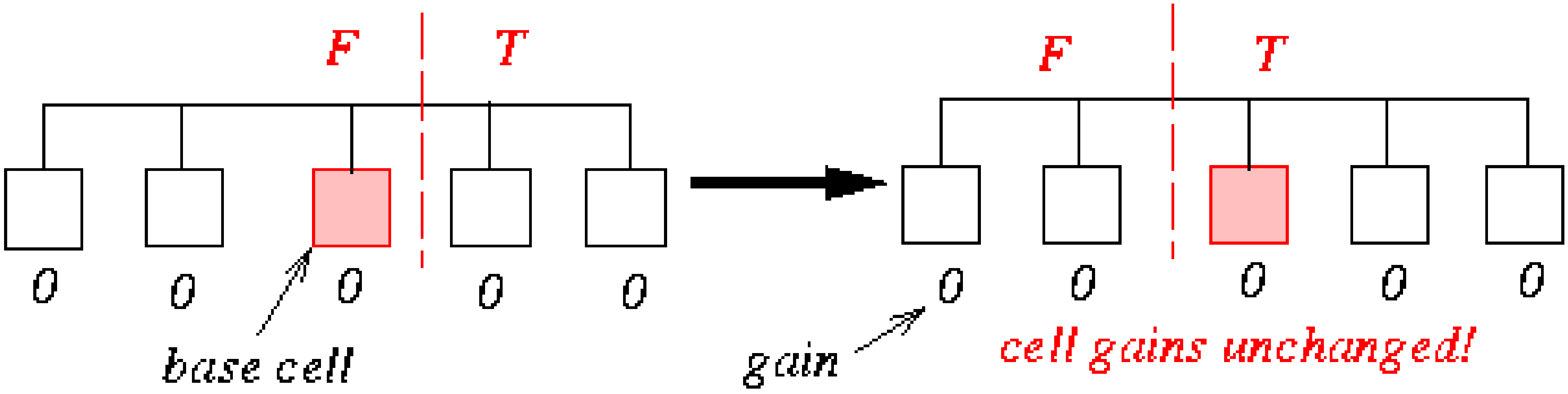
  $g(i) \leftarrow 0$;
  $F \leftarrow$ the "from block" of cell i;
  $T \leftarrow$ the " to block" of cell i;
  for each net n on Cell i **do**
  if $F(n) = 1$ then $g(i) \leftarrow g(i) + 1$;
  if $T(n) = 0$ then $g(i) \leftarrow g(i) - 1$;



- Only need $O(P)$ time to maintain all cell gains in one pass.

Updating Cell Gains

- To update the gains, we only need to look at those nets, connected to the base cell, which are critical before or after the move.
- **Base cell**: The cell selected for movement from one set to the other.



Algorithm for Updating Cell Gains

Algorithm: Update_Gain

1 **begin** /* move base cell and update neighbors' gains */

2 *F* ← the *Front Block* of the base cell;

3 *T* ← the *To Block* of the base cell;

4 Lock the base cell and complement its block;

5 **for** each net *n* on the base cell **do**

/* check critical nets before the move */

6 **if** *T*(*n*) = 0 then increment gains of all free cells on *n*
elseif *T*(*n*) = 1 **then** decrement gain of the only *T* cell on *n*, if it is free

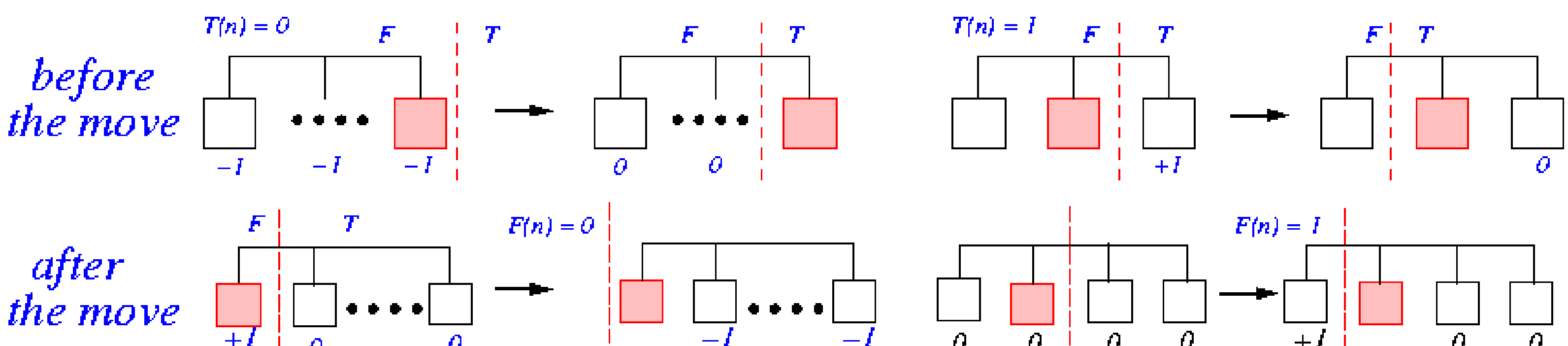
/* change *F*(*n*) and *T*(*n*) to reflect the move */

7 *F*(*n*) ← *F*(*n*) – 1; *T*(*n*) ← *T*(*n*) + 1;

/* check for critical nets after the move */

8 **if** *F*(*n*) = 0 **then** decrement gains of all free cells on *n*
elseif *F*(*n*) = 1 **then** increment gain of the only *F* cell on *n*, if it is free

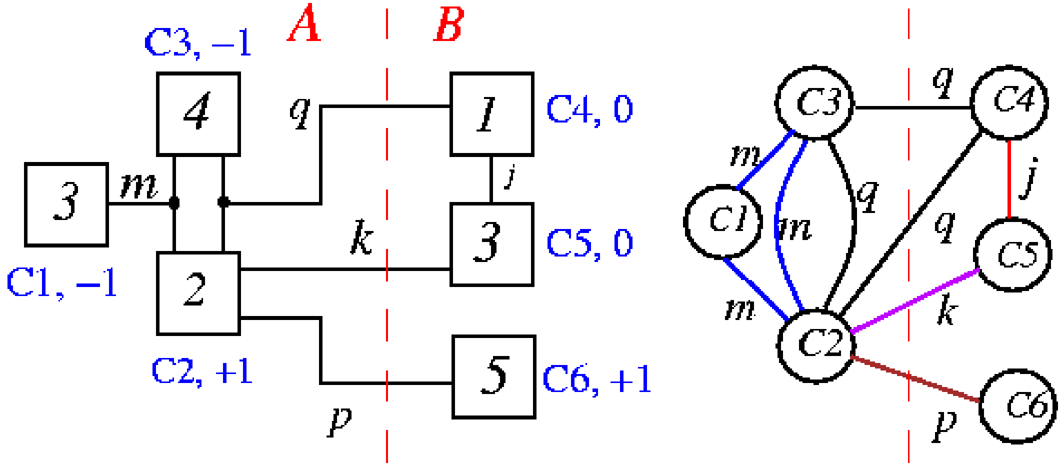
9 **end**



Complexity of Updating Cell Gains

- Once a net has “locked” cells at both sides, the net will remain cut from now on.
- To update the cell gains, it takes $O(n(i))$ work for Net i.
- Total time = $n(1) + n(2) + \ldots + n(N) = O(P)$

FM: An Example

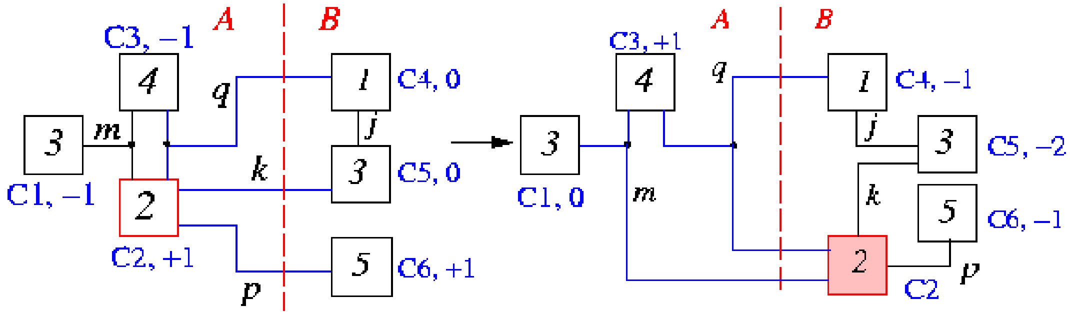


- Computing cell gains: $F(n) = 1 \Rightarrow g(i) + 1$; $T(n) = 0 \Rightarrow g(i) - 1$

Cell	m		q		k		p		j		$g(i)$
	F	T	F	T	F	T	F	T	F	T	
c1	0	−1									−1
c2	0	−1	0	0	+1	0	+1	0			+1
c3	0	−1	0	0							−1
c4			+1	0					0	−1	0
c5					+1	0			0	−1	0
c6							+1	0			+1

- Balanced criterion: $r|V| - S_{max} \le |A| \le r|V| + S_{max}$. Let $r = 0.4 \Rightarrow |A|=9$, $|V|=18$, $S_{max} = 5$, $r|V| = 7.2 \Rightarrow$ Balanced: $2.2 \le 9 \le 12.2$!
- Maximum gain: c_2 and balanced: $2.2 \le 9-2 \le 12.2 \Rightarrow$ Move c_2 from A to B (use size criterion if there is a tie).

FM: An Example (Cont'd)



- Changes in net distribution:

Net	Before move F	Before move T	After move F'	After move T'
k	1	1	0	2
m	3	0	2	1
q	2	1	1	2
p	1	1	0	2

- Updating cell gains on critical nets (run Algorithm Update_Gain):

Cells	Gains due to $T(n)$: k	m	q	p	Gain due to $F(n)$: k	m	q	p	Gain changes: Old	New
c_1		+1							−1	0
c_3		+1					+1		−1	+1
c_4			−1						0	−1
c_5	−1				−1				0	−2
c_6				−1				−1	+1	−1

- Maximum gain: c_3 and balanced! ($2.2 \leq 7\text{-}4 \leq 12.2$)

=> Move c_3 from A to B (use size criterion if there is a tie).

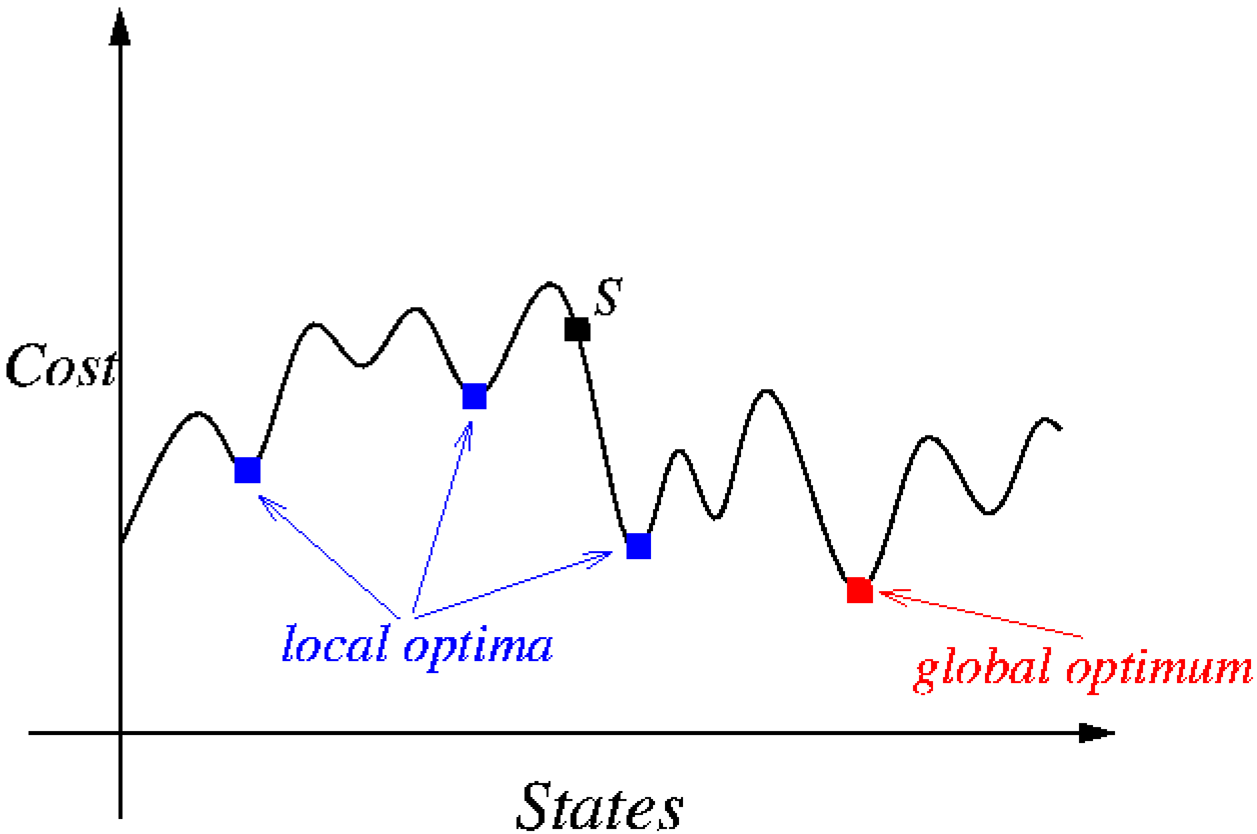
Summary of the Example

Step	Cell	Max gain	$\lvert A\rvert$	Balanced?	Locked cell	A	B
0	-	-	9	-	$\emptyset$	1, 2, 3	4, 5, 6
1	c_2	+1	7	yes	c_2	1, 3	2, 4, 5, 6
2	c_3	+1	3	yes	c_2, c_3	1	2, 3, 4, 5, 6
3	c_1	+1	0	no	-	-	-
$3'$	c_6	-1	8	yes	c_2, c_3, c_6	1, 6	2, 3, 4, 5
4	c_1	+1	5	yes	c_1, c_2, c_3, c_6	6	1, 2, 3, 4, 5
5	c_5	-2	8	yes	c_1, c_2, c_3, c_5, c_6	5, 6	1, 2, 3, 4
6	c_4	0	9	yes	all cells	4, 5, 6	1, 2, 3

- g_1=1, g_2=1, g_3=-1, g_4=1, g_5=-2, g_6=0 => Maximum partial sum G_k = +2, k = 2 or 4.
- Since k = 4 results in a better balanced => Move c_1, c_2, c_3, c_6 => A={6}, B={1,2,3,4,5}.
- **Repeat the whole process until new** $G_k \leq 0$.

Simulated Annealing Revisited

- Kirkpatrick, Gelatt, and Vecchi, “Optimization by simulated annealing,” *Science*, May 1983.



Simulated Annealing Basics

- Non-zero probability for “up-hill” moves.
- Probability depends on
  1. magnitude of the “up-hill” movement
  2. total search time

$$Prob(S \rightarrow S') = \begin{cases} 1 & \text{if } \Delta C \leq 0 \quad /*\ ``down-hill'' \ moves */ \\ e^{-\frac{\Delta C}{T}} & \text{if } \Delta C > 0 \quad /*\ ``up-hill'' \ moves */ \end{cases}$$

- $\triangle C = cost(S') - cost(S)$
- T: Control parameter (temperature)
- Annealing schedule: $T = T_0, T_1, T_2, \ldots$, where $T_i = r^i T_0$, $r < 1$.

Generic Simulated Annealing Algorithm

Algorithm: Simulated_Annealing

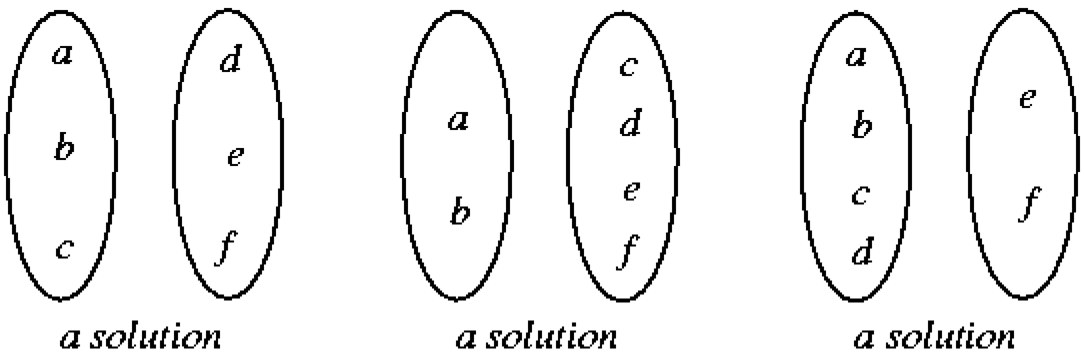
begin
Get an initial solution S;
Get an initial temperature $T > 0$;
while not yet "frozen" **do**
    for $1 \leq i \leq P$ **do**
        Pick a random neighbor S' of S;
        $\triangle \leftarrow cost(S') - cost(S)$;
        /* down hill move */
        if $\triangle \leq 0$ **then** $S \leftarrow S'$
        /* uphill move */
        if $\triangle > 0$ **then** $S \leftarrow S'$ with probability $e^{-\frac{\triangle}{T}}$;
    $T \leftarrow rT$; /* reduce temperature */
return S
end

Basic Ingredients for Simulated Annealing

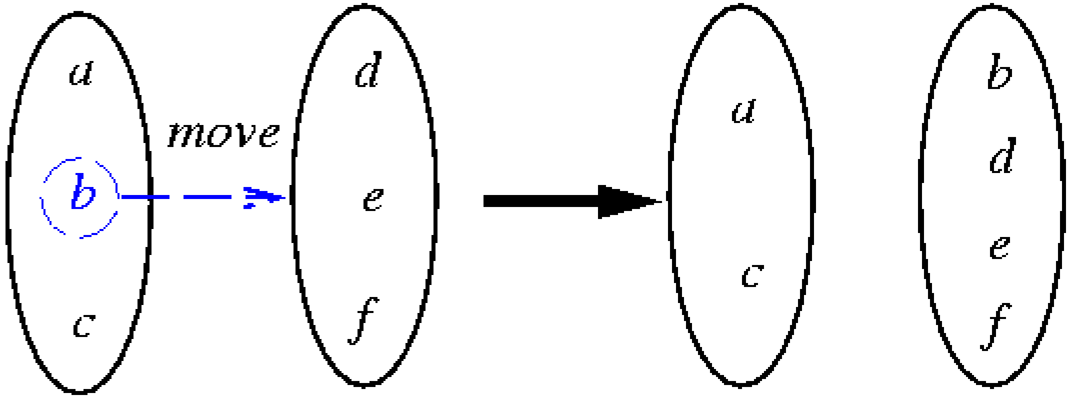
- Solution space
- Neighborhood structure
- Cost function
- Annealing schedule

Partitioning by Simulated Annealing

- Kirkpatrick, Gelatt, and Vecchi, "Optimization by simulated annealing," *Science*, May 1983.
- **Solution space**: set of all partitioning solutions



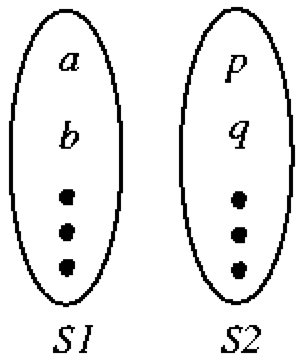
- **Neighborhood structure:**



Randomly move one cell to the other side

Partitioning by Simulated Annealing (cont'd)

- **Cost function**: $f = C + \lambda B$
  - C: the solution cost as used before.
  - B: a measure of how balance the solution is
  - λ: a constant



$$B = (|S1| - |S2|)^2$$

- **Annealing schedule:**
  - $T_n = r^n T_0$, $r = 0.9$.
  - At each temperature, either
    1. There are 10 accepted moves/cell on the average, or
    2. # of attempts $\geq$100 * total # of cells.
  - The system is “frozen” if very low acceptances at 3 consecutive temperatures.

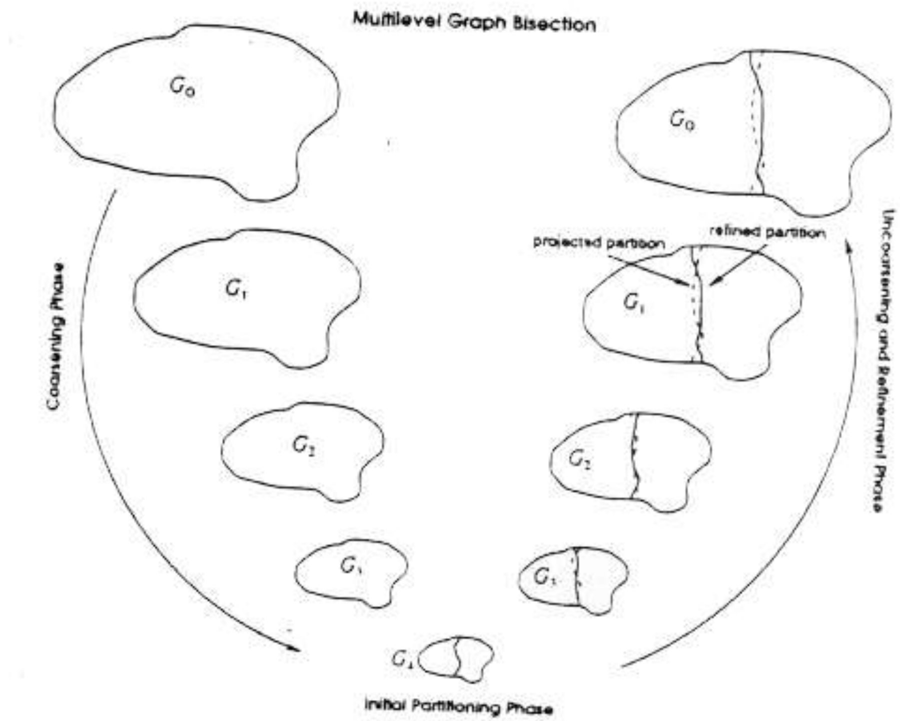
Multilevel Partitioning

- **Three phases** (for bipartitioning)

  - **Coarsening:** construct a sequence of smaller (coarser) graphs.

  - **Initial partitioning:** construct a bipartitioning solution for the coarsest graph.

  - **Uncoarsening & refinement:** the bipartitioning solution is successively projected to the next-level finer graph, and at each level an iterative refinement algorithm (such as KL or FM) is used to further improve the solution.



hMETIS

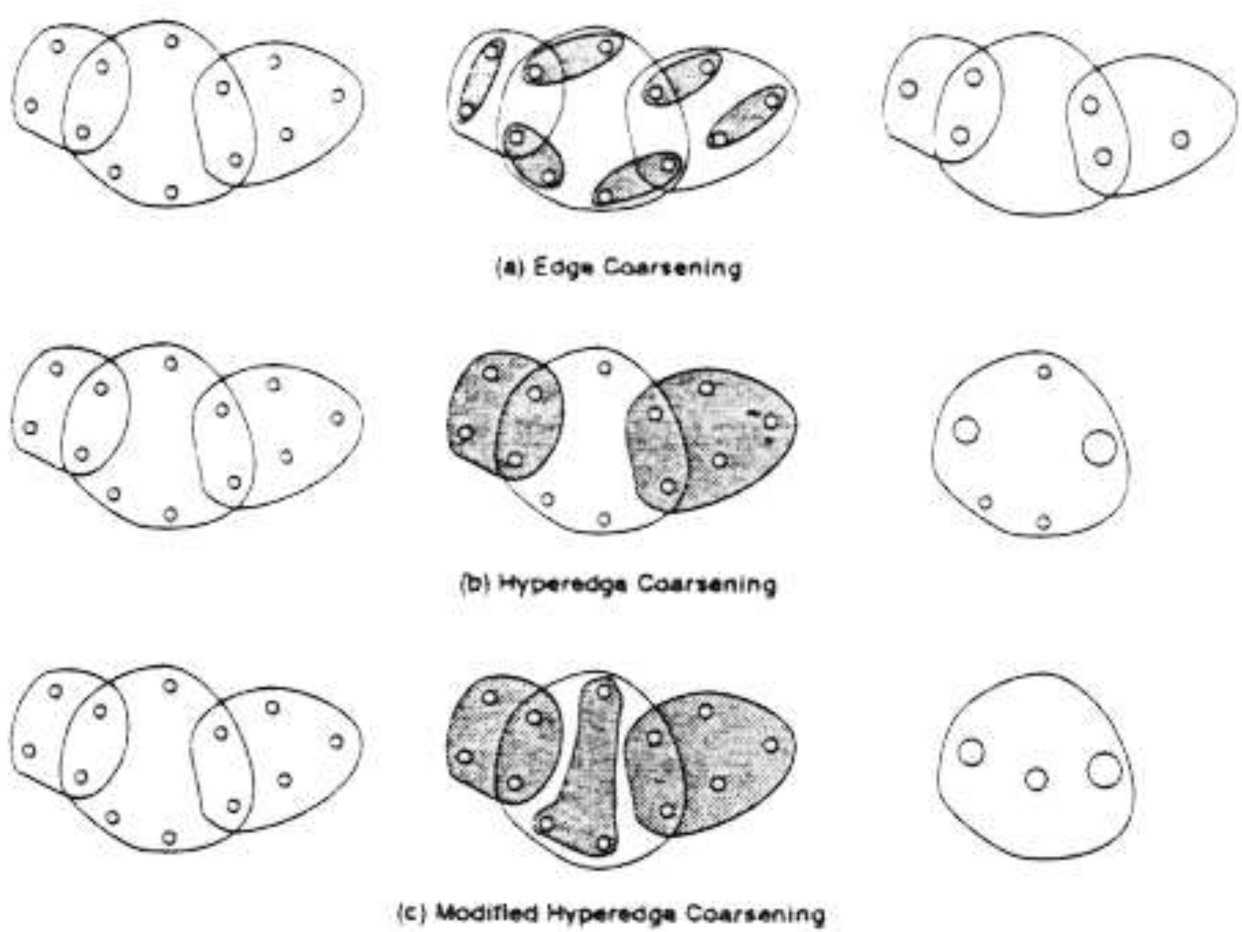
- Kayrpis, Aggarwal, Kumar and Shekhar, "Multilevel hypergraph partitioning: application in VLSI domain," DAC, 1997.
- Three coarsening algorithms:

  - **Edge coarsening:** A maximal matching of the vertices.

  - **Hyperedge coarsening**: a set of hyperedges is selected, and the vertices belonging to a selected hyperedge are merged into a cluster. (Preference: hyperedges with large weights and hyperedges of small size.)

  - **Modified hyperedge coarsening:** hyperedge coarsening + merging the remaining vertices of each hyperedge into a cluster.

hMETIS (Cont'd)



(a) Edge Coarsening

(b) Hyperedge Coarsening

(c) Modified Hyperedge Coarsening

hMETIS (Cont'd)

- Two uncoarsening & refinement algorithms:

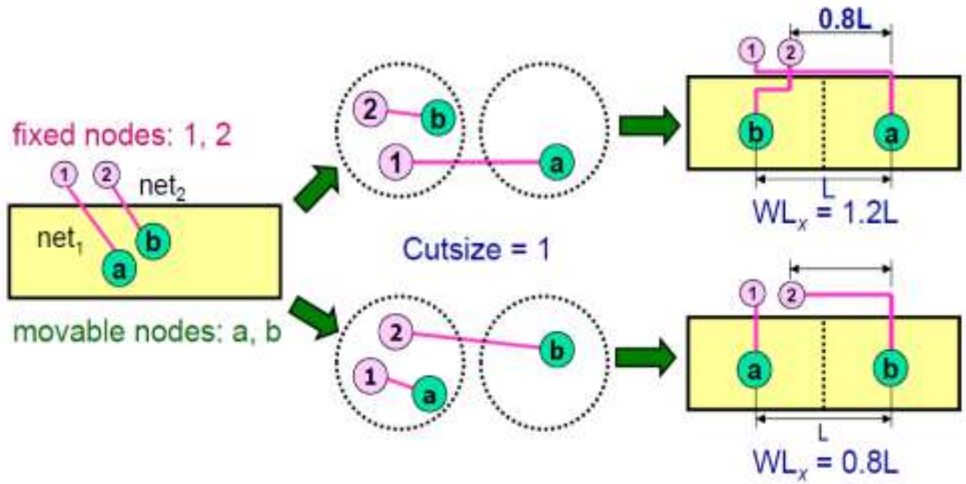
  - FM algorithm with modifications:

    * Restrict the maximum number of passes to 2.
    * Stop each pass when no improvement is made from the first *k* moves.

  - Hyperedge refinement: move groups of vertices between subsets so that an entire hyperedge is removed from the cut set.

Partitioning for Wirelength Minimization

- Chen, Chang, Lin, "IMF: Interconnection-driven floorplanning for large-scale building-module designs," ICCAD-05
- Minimizing cut size is *not* equivalent to minimizing wirelength (WL)

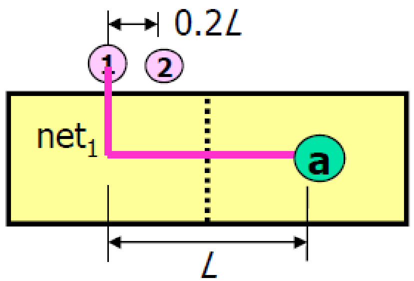


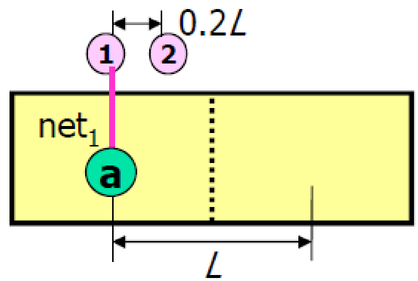
- Problem: **hyperedge weight is a constant value!**
  - Shall map the min-cut cost to wirelength (WL) change
  - Shall assign the hyperedge weight as the value of wirelength contribution if the hyperedge is cut

Net Weight Assignment

- net_1 connects a movable node *a* and a fixed node 1.

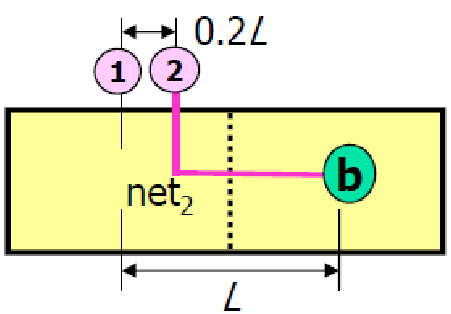
$$\text{Weight}(net_1) = \text{WL}(net_1 \text{ is cut}) - \text{WL}(net_1 \text{ is not cut})$$
$$= L - 0L = L$$

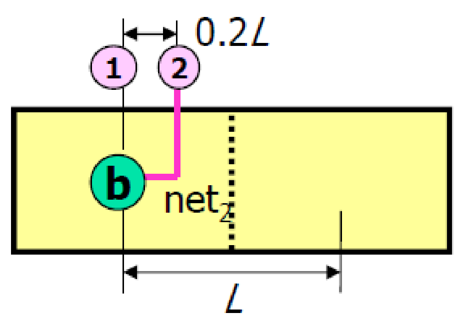




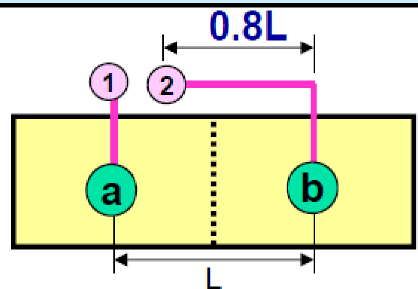
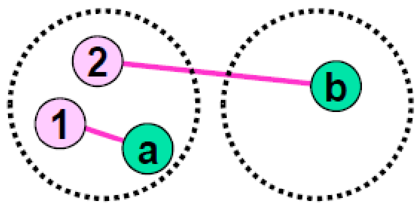
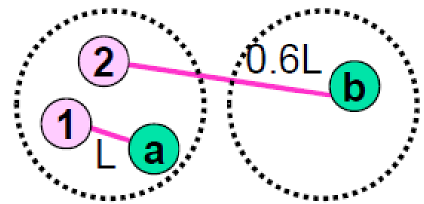
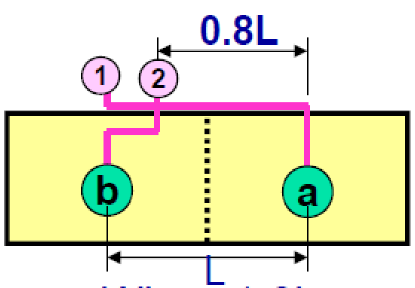
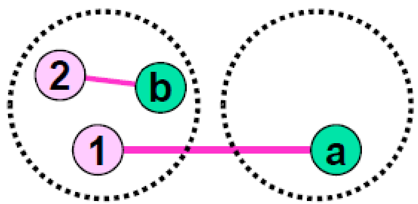
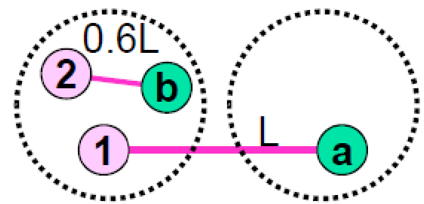
- net_2 connects a movable node *b* and a fixed node 2.

$$\text{Weight}(net_2) = \text{WL}(net_2 \text{ is cut}) - \text{WL}(net_2 \text{ is not cut})$$
$$= 0.8L - 0.2L = 0.6L$$





Examples

Physical Partitions	Traditional Terminal Propagation	Exact Net-Weight Modeling
 WL_x = 0.8L	 Cutsize = 1	 Cut weight = 0.6L
 WL_x = 1.2L	 Cutsize = 1	 Cut weight = 1.0L

Cut weight is proportitional to the wirelength (WL)

WL = Cut weight + 0.2L

(0.2L is the WL lower bound: placing a & b in the left side)

Relationship Between WL and Cut Weight

- Theorem: $WL_i = w_{1,i} + n_{cut,i}$
  - $n_{cut,i}$: cut weight for net i
  - $w_{1,j}$: the wirelength lower bound for net i

- Then, we have

Constant

$$min(\sum WL_i) = min(\sum(w_{1,i} + n_{cut,i})) = \sum w_{1,i} + min(\sum n_{cut,i})$$

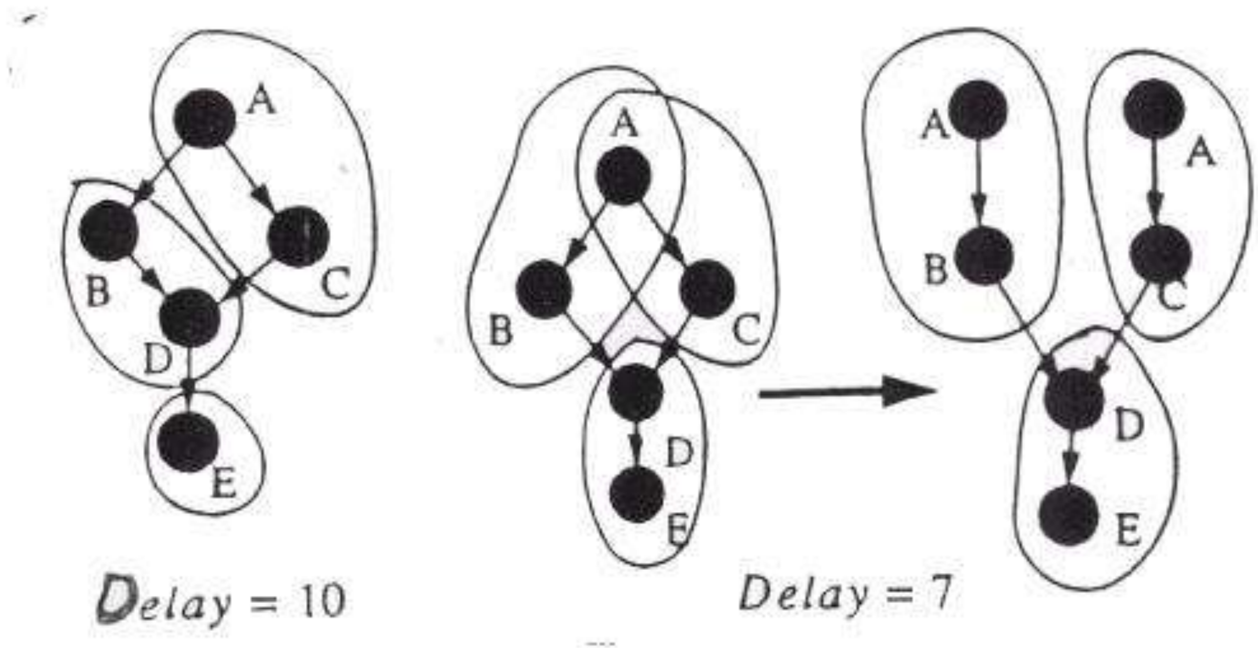
Finding the minimum wirelength is equivalent to finding the cut weight

Clustering for Delay Minimization

- Allow gate duplication.
- Gate duplication may help reduce delay.

D=3; M=2; $\delta(v)$=1, $w(v)$=1, for each v.

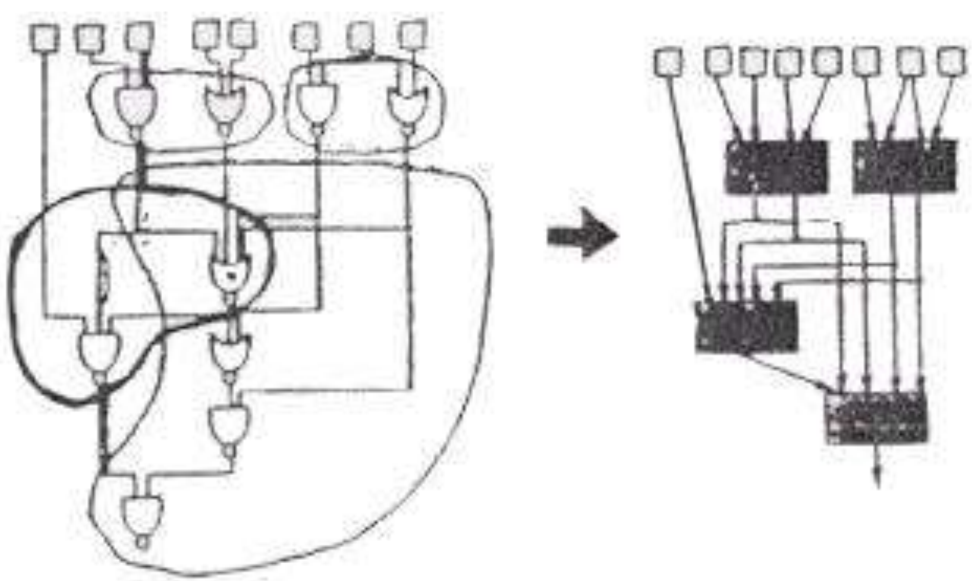
Without gate duplication With gate duplication



Unit Delay Model

- No gate delay.
- No interconnection delay within a cluster.
- Delay of 1 unit for an interconnection between 2 clusters.
- An optimal algorithm for area constraint only (Lawler, Levitt and Turner, IEEE TC, 1966).
- An optimal algorithm for pin constraint only (Cong and Ding, ICCAD, 1992).

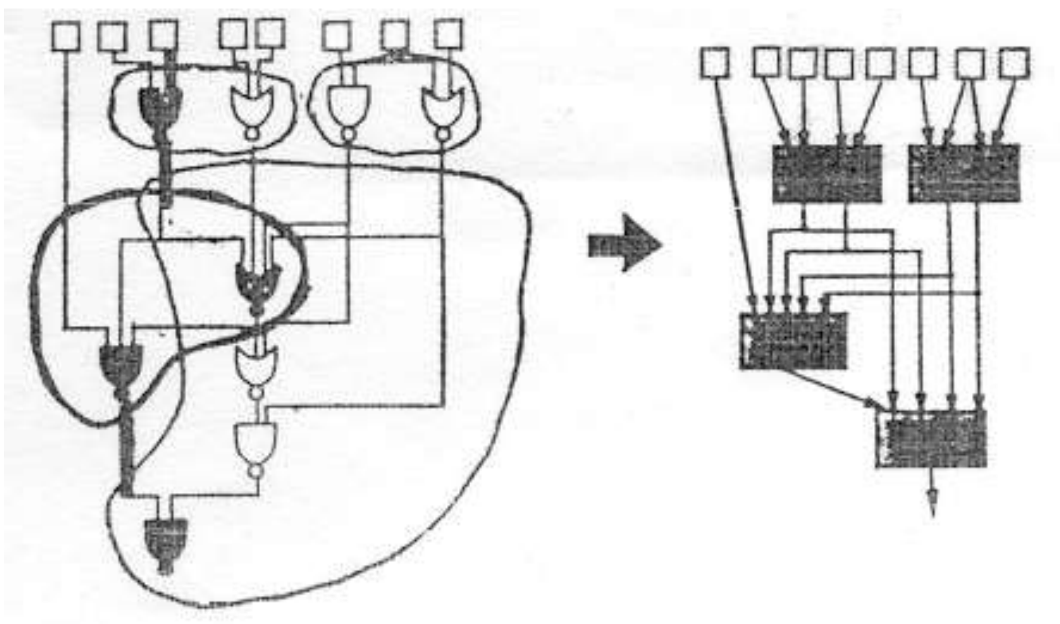
Circuit delay = 3.



General Delay Model

- Each gate has a delay.
- No interconnection delay within a cluster.
- Delay of *D* units for an interconnection between 2 clusters.
- A heuristic algorithm for area constraint only (Murgai, Brayton and Sangivanni-Vincentelli, ICCAD, 1991).

$D = 2$, $\delta(v) = 1$, circuit delay = 6+4 = 10.



General Delay Model (Cont'd)

- Rajaraman and Wong, "Optimal clustering for delay minimization," DAC, 1993.
- Optimal algorithm: $O(n^2 \log n + nm)$, where n is # of gates, m is # of interconnections.
- Definitions:

  - M: the area constraint on a cluster.

  - $W(C)$: the total area of the gates in cluster C.

  - N: a given combinational circuit.

  - N_v: v and all its *predecessors* in N.

  - $\delta(v)$: the delay of v.

  - $\Delta(u,v)$: maximum delay along any path from the output of u to the output of v, ignoring delays on interconnections.

  - $w(v)$: the area of v.

  - $l(v)$: the delay at v in an optimal clustering of N_v.

    For each *primary input* v, $l(v)=\delta(v)$.

  - $l'(u)=l(u)+\Delta(u,v)$, for each u in N_v-$\{v\}$.

General Delay Model (Cont'd)

- Algorithm: labeling phase + clustering phase.
- **Labeling phase:** compute *l*(*v*) for each *v* in a topological order.

  \- *P*: the set of nodes in N_v-{*v*} sorted in non-increasing order in the value of *l'*.

Algorithm *Labeling*(v);
begin
$done \leftarrow false$;
$cluster(v) \leftarrow \{v\}$;
while (*not done*)
    Remove the first node u in P;
    if $(W(cluster(v)) + w(u)) \leq M)$
        $cluster(v) \leftarrow cluster(v) \cup \{u\}$;
        if P is empty
            $done \leftarrow true$;
        endif
    else
        $done \leftarrow true$;
    endif
endwhile
$l_1(v) \leftarrow max\{l'(x) \mid x \in cluster(v) \cap \mathcal{PI}\}$;
$l_2(v) \leftarrow \cancel{l'(u) + D}$; $max\{l'(u)+D \mid u \in N_v - cluster(v)\}$
$l(v) \leftarrow max\{l_1(v)\ , l_2(v)\}$;
end

General Delay Model (Cont'd)

- **Clustering phase:** generate the clusters based on the information obtained in the labeling phase.
- Overall algorithm:

begin
Compute the maximum delay matrix Δ. $\Delta(i,j)$ is the maximum delay along any path from the output of i to the output of j;
for each $PI\ i$, **do** $l(i) \leftarrow \delta(i)$;
Sort the non-PI nodes of N in topological order to obtain list T;
while T is non-empty
  Remove the first node v from T;
  Compute N_v;
  for each node $u \in N_v \setminus \{v\}$ do
    $l'(u) \leftarrow l(u) + \Delta(u,v)$;
  Sort the nodes in $N_v \setminus \{v\}$ in order of decreasing value of l' to form list P;
  Call $Labeling(v)$;
endwhile
$L \leftarrow \mathcal{PO}$;
$S \leftarrow \phi$;
while L is not empty
  Remove a node v from L; $N - cluster(v)$
  $S \leftarrow S \cup \{cluster(v)\}$;
  for all nodes x in N, such that x is adjacent to y, for some $y \in cluster(v)$, $L \leftarrow L \cup \{x\}$;
endwhile
end

General Delay Model (Cont'd)

- An example: M=3, D=3