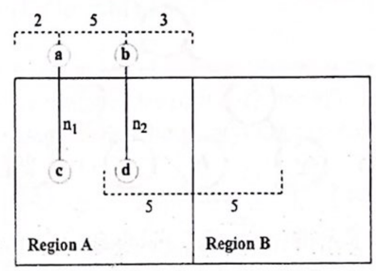
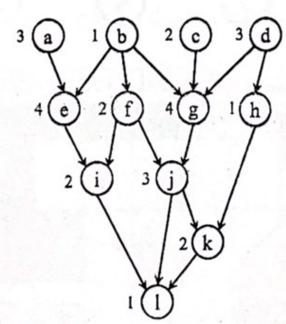
CS 6135 VLSI Physical Design Automation
Final Exam: 10:10 a.m. - 13:10 p.m., December 27, 2022

1. (5 points) Vertex a and vertex b are two vertices in an edge-weighted complete graph, where the weight of each edge is a non-negative integer. Suppose that the vertex set of this graph is partitioned into two subsets, and vertices a and b are not in the same subset. Assume that the respective internal and external costs of vertex a are 2 and 6, and the respective internal and external costs of vertex b are 6 and 3. Can the cut cost be reduced by swapping vertices a and b? Justify your answer.

2. (5 points) Consider the figure below where cells a and b are fixed cells that are placed at the given locations, cells c and d are movable cells that can be moved to the center of either region A or B, net n_1 connects cells a and c, and net n_2 connects cells b and d. The center-to-center distance between regions A and B is 10. Use the exact net-weight model discussed in class to find the cut weights for nets n_1 and n_2 for capturing the wirelength cost precisely.



3. (15 points) Consider the circuit shown below. Assume the area of each gate is 1 unit, the area constraint of each cluster is 3 units, and the interconnection delay between two clusters is 5 units. The gate delay is given next to each gate. Show your work by applying the clustering algorithm discussed in class to the circuit below.



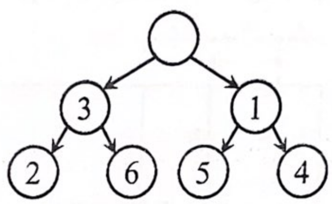
4. (10 points) Given five disjoint sets F, L, R, T, B of rectangular modules, extend the mixed integer linear programming (MILP) method discussed in class to place all modules into a rectangular region of fixed height H such that each module cannot be rotated, each module in F can be placed anywhere, each module in L (R, T, B, respectively) must be placed alone the left (right, top, bottom, respectively) boundary of the region, overlap is disallowed between modules, and the width of the region is minimized. **<u>To answer this question, you only need to give the MILP formulation.</u>**

5. Consider a set of modules in the following table. Assume that each module cannot be rotated.

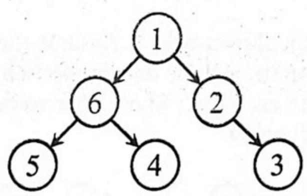
Module	Width	Height
1	2	3
2	3	1
3	2	4
4	4	3
5	2	1
6	1	4

(a) (7 points) Show your work for finding a minimum-area placement for the sequence-pair (125364, 546321).

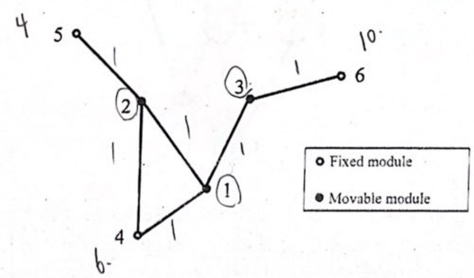
(b) (4 points) Show the placement for the following horizontal O-tree.



(c) (4 points) Show the placement for the following B*-tree.



6. (9 points) Consider a circuit with movable and fixed modules as represented by the graph below. In the graph, each vertex denotes a module, while each edge denotes a two-pin net and is associated with a weight 1.



For the three fixed modules, module 4 is at (6, 2), module 5 is at (4, 5), and module 6 is at (10, 4). Determine the locations of the three movable modules such that the total weighted quadratic wirelength of the circuit is minimized. **<u>To answer this question, you only need to show your work for getting the x-coordinates of the three movable modules.</u>**

7. The log-sum-exponential function of n numbers $z_1, z_2, \ldots, z_n$ is defined as:

$$\mathrm{LSE}_\alpha(z_1, z_2, \ldots, z_n) = \alpha \times \left(\ln\left(\sum_{i=1}^{n} e^{z_i/\alpha}\right)\right).$$

where α is a user-specified parameter. $\mathrm{LSE}_\alpha(z_1, z_2, \ldots, z_n)$ is an approximation of the maximum function $\max(z_1, z_2, \ldots, z_n)$ and α controls the accuracy of the approximation. The error function is defined as:

$$\mathrm{err}_\alpha(z_1, z_2, \ldots, z_n) = \mathrm{LSE}_\alpha(z_1, z_2, \ldots, z_n) - \max(z_1, z_2, \ldots, z_n).$$

Assume that $\max(z_1, z_2, \ldots, z_n) = z_1$ and α is a non-negative real number. Prove that

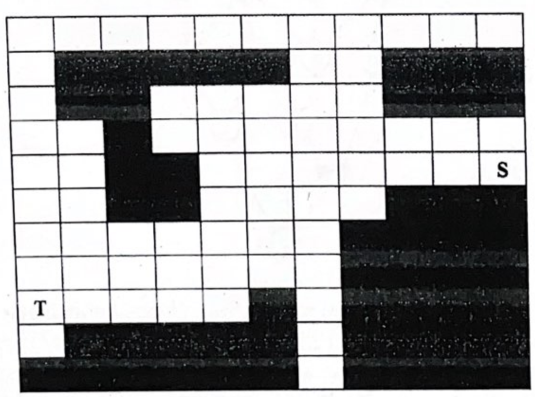
(a) (4 points) $\mathrm{err}_\alpha(z_1, z_2, \ldots, z_n) \geq 0$.

(b) (4 points) $\mathrm{err}_\alpha(z_1, z_2, \ldots, z_n) \leq \alpha \times \ln n$.

8. (8 points) Consider a k-pin net p in a given placement. Suppose the k pins of p are numbered by 1, 2, …, k according to the non-decreasing order of their x-coordinates, and $x_1, x_2, \ldots, x_k$ denote their x-coordinates. Therefore, pin 1 is at the left boundary and pin k is at the right boundary of the net bounding box of p. Let $N = \{(1, k), (1, 2), (2, k), \ldots, (1, k\text{-}1), (k\text{-}1, k)\}$ be the set of 2-pin nets generated from p according to the BoundingBox net model, where each pair of numbers denotes a 2-pin net. Also let $L = \frac{1}{2}\sum_{(i,j)\in N}(w_{(i,j)} \times (x_i - x_j)^2)$, where $w_{(i,j)} = \frac{2}{k-1} \times \frac{1}{|x_i - x_j|}$. Prove that $L = x_k - x_1$.

9. (5 points) Given a set of pins on a plane, it is known that the RMST (Rectilinear Minimum Spanning Tree) wirelength is an upper bound of the RSMT (Rectilinear Steiner Minimum Tree) wirelength. Give a 4-pin net such that its RMST wirelength is 1.5 times its RSMT wirelength.

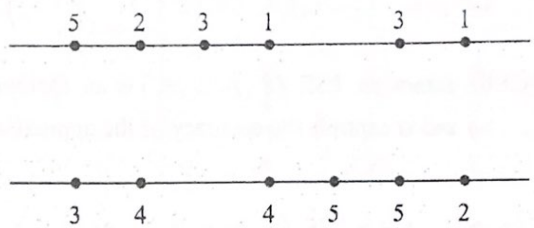
10. Consider the following routing instance with shaded blockages.



(a) (5 points) Show your work for finding a shortest path between S and T using the Lee algorithm.

(b) (7 points) Show your work for finding a least weighted path between S and T using the weighted grid model introduced in class.

11. (8 points) Consider the following two-layer channel routing instance under the reserved HV routing layer model.



Show your work for generating the routing result using the dogleg channel router which considers tracks from the top to the bottom.

國立清華大學試卷

記			分
1	5	2	5
3	10	4	10
5	11	6	4
7	8	8	0
9	5	10	78
11	4	12	
13		14	
15		16	
17		18	
19		20	
總 分	69		

所 系 資工

VLSI
科 目 Physical Design Automation

學 號 110062619

姓 名 楊淨富

日 期 2022. 12. 27

1.

+5

$I_a = 2 \qquad I_b = 6$

$E_a = 6 \qquad E_b = 3$

$D_a = 4 \qquad D_b = -3$

$\because g_{ab} = D_a + D_b - 2C_{ab}$

$= 1 - 2C_{ab} > 0. \quad$ given $C_{ab} \geq 0.$

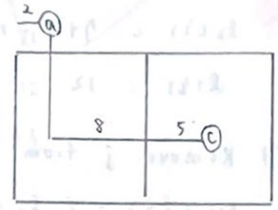
$1 > 2C_{ab}$

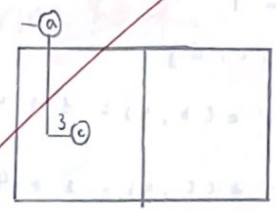
$\therefore$ Cut cost can be reduced. if $\frac{1}{2} > C_{ab} \geq 0.$ #

2. Net Weight Assignment

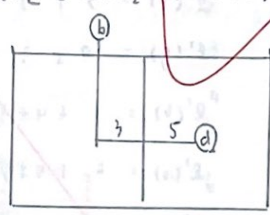
+5

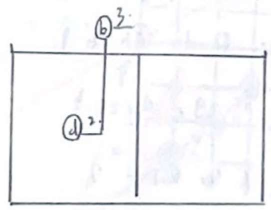
∘ Weight (net_1) = WL (net_1 is cut) − WL (net_1 is not cut) = 13 − 3 = 10 #





∘ Weight (net_2) = WL (net_2 is cut) − WL (net_2 is not cut) = 8 − 2 = 6 #





3.

$l(a) = 3, \quad l(b) = 1, \quad l(c) = 2, \quad l(d) = 3$

Do topological sort to obtain list $T = \{ e, f, g, h, i, j, k, l \}$

① Remove e from T

$N_e / \{e\} = \{a, b\}.$

$l'(a) = l(a) + \Delta(a, e) = 3 + 4 = 7$

$l'(b) = l(b) + \Delta(b, e) = 1 + 4 = 5$

Form list $P = \{a, b\}.$

Labeling :

cluster $(e) = \{ e, a, b \}$

$l_1(e) = \max \{ l'(x) \mid x \in \text{cluster}(e) \cap PI \} = \max (l'(a), l'(b)) = 7$

$l_2(e) = 0.$

$l(e) = 7$

3.

② Remove f from T

+10

$N_f / \{f\} = \{b\}$

$l'(b) = l(b) + \Delta(b, f) = 1 + 2 = 3$.

Form list $P = \{b\}$.

$\text{cluster}(f) = \{f, b\}$.

$l_1(f) = 3$

$l_2(f) = 0$

$l(f) = 3$

③ Remove g from T

$N_g / \{g\} = \{b, c, d\}$.

$l'(b) = l(b) + \Delta(b, g) = 1 + 4 = 5$

$l'(c) = l(c) + \Delta(c, g) = 2 + 4 = \cancel{6}$

$l'(d) = l(d) + \Delta(d, g) = 3 + 4 = \cancel{7}$

Form list $P = \{d, c, b\}$.

$\text{Cluster}(g) = \{g, d, c\}$

$l_1(g) = \max(7, 6) = 7$

$l_2(g) = 5 + 5 = 10$

$l(g) = 10$

④ Remove h from T.

$N_h / \{h\} = \{d\}$.

$l'(d) = l(d) + \Delta(d, h) = 3 + 1 = 4$

Form list $P = \{d\}$.

$\text{cluster}(h) = \{h, d\}$.

$l_1(h) = 4$

$l_2(h) = 0$

$l(h) = 4$

⑤ Remove i from T

$N_i / \{i\} = \{e, f, a, b\}$.

$l'(e) = 7 + 2 = \cancel{9}$

$l'(f) = 3 + 2 = 5$

$l'(a) = 3 + 4 + 2 = \cancel{9}$

$l'(b) = 1 + 4 + 2 = 7$

Form list $P = \{e, a, b, f\}$

$\text{cluster}(i) = \{i, e, \check{a}\}$

$l_1(i) = 9$.

$l_2(i) = 7 + 5 = 12$

$l(i) = 12$

⑥ Remove j from T

$N_j / \{j\} = \{f, g, b, c, d\}$

$l'(f) = 3 + 3 = 6$

$l'(g) = 10 + 3 = \cancel{13}$

$l'(b) = 1 + 4 + 3 = 8$

$l'(c) = 2 + 4 + 3 = 9$

$l'(d) = 3 + 4 + 3 = \cancel{10}$

Form list $P = \{g, d, c, b, f\}$.

$\text{cluster}(j) = \{j, g, \check{d}\}$

$l_1(j) = 10$.

$l_2(j) = 9 + 5 = 14$

$l(j) = 14$

⑦ Remove k from T

$N_k / \{k\} = \{j, f, g, h, b, c, d\}$

$l'(j) = 14 + 2 = \cancel{16}$

$l'(f) = 3 + 3 + 2 = 8$

$l'(g) = 10 + 3 + 2 = \cancel{15}$

$l'(h) = 4 + 2 = 6$

$l'(b) = 1 + 4 + 3 + 2 = 10$

$l'(c) = 2 + 4 + 3 + 2 = 11$

$l'(d) = 3 + 4 + 3 + 2 = 12$

Form list $P = \{j, g, d, c, b, f, h\}$

$\text{cluster}(k) = \{k, j, g\}$.

$l_1(k) = 0$.

$l_2(k) = 12 + 5 = 17$

$l(k) = 17$

⑧ Remove l from T

$N_s \cup \{l\} = \{i, j, k, e, f, g, h, a, b, c, d\}$.

$l'(i) = 12 + 1 = 13$

$l'(j) = 14 + 2 + 1 = 17$

$l'(k) = 17 + 1 = 18$

$l'(e) = 7 + 2 + 1 = 10$

$l'(f) = 3 + 3 + 2 + 1 = 9$

$l'(g) = 10 + 3 + 2 + 1 = 16$

$l'(h) = 4 + 2 + 1 = 7$

$l'(a) = 3 + 4 + 2 + 1 = 10$

$l'(b) = 1 + 4 + 3 + 2 + 1 = 11$

$l'(c) = 2 + 4 + 3 + 2 + 1 = 12$

$l'(d) = 3 + 4 + 3 + 2 + 1 = 13$

Form list $P = \{k, j, g, i, d, c, b, a, e, h\}$

cluster $(l) = \{l, k, j\}$.

$l_1(l) = 0$

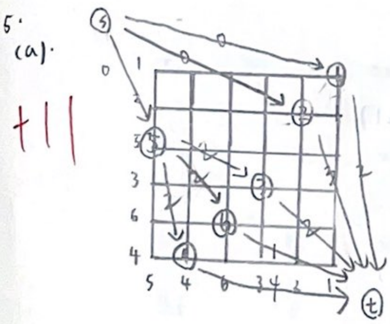
$l_2(l) = 16 + 5 = 21$

$l(l) = 21$

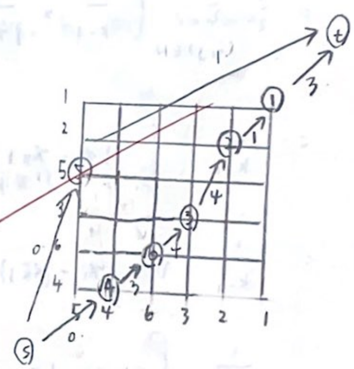
無clustering結果.

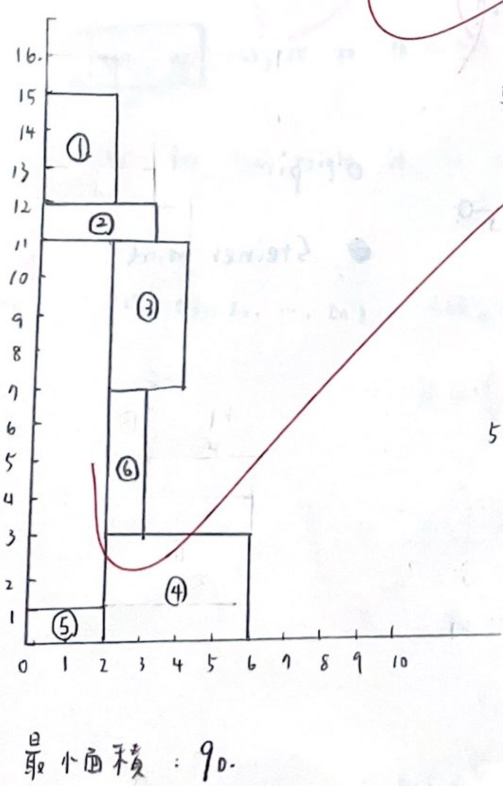
5.
(a).

+11

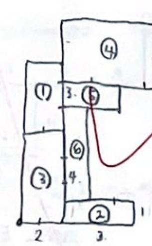


1	0	12
2	0	11
3	2	7
4	2	0
5	0	0
6.	2	3

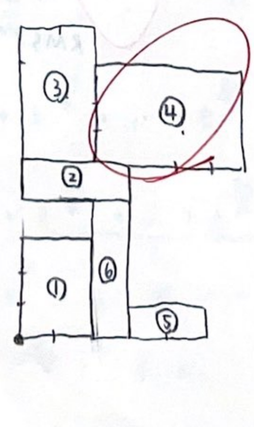


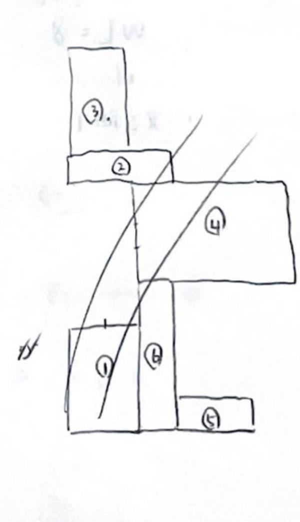


5.(b) O-tree.



5.(c).





最小面積 : 90.

6. +4

$$Q = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 3 & -1 & -1 \\ -1 & 3 & 0 \\ -1 & 0 & 2 \end{bmatrix} \#$$

$$d_x = \begin{pmatrix} -6 \\ -10 \\ -10 \end{pmatrix} \#$$

$$x(cell_1) = -(1 \cdot 6) = -6$$

$$x(cell_2) = -(1 \cdot 4 + 1 \cdot 6) = -10$$

$$x(cell_3) = -(1 \cdot 10) = -10$$

8. +0 +7

$$L^{NBB} = \frac{1}{2} \sum_{(i,j) \in N} \left(w_{(i,j)} \times (x_i - x_j)^2 \right)$$

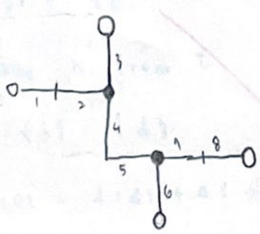
$$= \frac{1}{2} \sum_{(i,j) \in N} \left(\frac{2}{k-1} \times \frac{1}{|x_i - x_j|} \times (x_i - x_j)^2 \right).$$

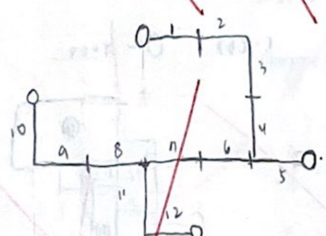
$$= \frac{1}{k-1} \left\{ |x_1 - x_k| + \sum_{2 \le m \le k-1} \left(|x_1 - x_m| + |x_m - x_k| \right) \right\}$$

$$= \frac{1}{k-1} \left\{ |x_1 - x_k| + (k-2) \cdot |x_1 - x_k| \right\}$$

$$= \frac{1}{k-1} \cdot \left\{ (k-1) \cdot |x_1 - x_k| \right\} = |x_1 - x_k|$$

9. +5





o pin

● Steiner point

WL = 8 of RSMT

WL = 12 of RMST

4. Non-overlap constraints

定義： x_i, y_i 为 Module M_i 的左下角座標

h_i, w_i 为 Module M_i 的 height & widen

+10

$$\begin{cases} x_i + w_i \le x_j \\ y_i + h_i \le y_i \\ x_i - w_j \ge x_j \\ y_i - h_j \ge y_j \end{cases} \quad \forall i,j : 1 \le i < j \le n.$$

$$\sum_{Module_i \in \{F,L,R,T,B\}} w_i = W \qquad \sum_{Module_i \in \{F,L,R,T,B\}} h_i = H.$$

$$n = |F| + |L| + |R| + |T| + |B|$$

non-overlap $\Rightarrow$

$$\begin{cases} x_i + w_i \le x_j + W(p_{ij} + q_{ij}) & ③ \\ y_i + h_i \le y_j + H(1 + p_{ij} - q_{ij}) & ④ \\ x_i - w_j \ge x_j - W(1 - p_{ij} + q_{ij}) & ⑤ \\ y_i - h_j \ge y_j - H(2 - p_{ij} - q_{ij}) & ⑥ \end{cases}$$

$x_i + w_i \le w \quad 1 \le i \le n$ ①

$y_i + h_i \le H \quad 1 \le i \le n.$ ②

$x_i \ge 0,\ y_i \ge 0,\ 1 \le i \le n$ ⑦

$p_{ij}, q_{ij} \in \{0, 1\}.$ ⑧

w_i, h_i are known.

$x_k = 0. \quad \forall$ Module $k \in \{L\}$. i.e., $k = 1 \sim |L|$ ⑨

$x_m = W - \text{width}(\text{module}_m) \quad \forall$ module $m \in \{R\}$. ⑩

$y_n = H - \text{height}(\text{module}_n) \quad \forall$ module $n \in \{T\}$. ⑪

$y_\ell = 0. \quad \forall$ module $\ell \in \{B\}$. ⑫

$\boxed{\min\ w}$ subject to ① ~ ⑫

↑

by fix the height H so as to minimize the width w.

+8

7.(a) $err_\alpha(z_1, z_2, \cdots, z_n) = LSE_\alpha(z_1, z_2, \cdots, z_n) - \max(z_1, z_2, \cdots, z_n)$

$$= \alpha \times \left(\ln \left(\sum_{i=1}^{n} e^{\frac{z_i}{\alpha}} \right) \right) - z_1$$

$$= \alpha \times \left(\ln \left(e^{\frac{z_1}{\alpha}} + e^{\frac{z_2}{\alpha}} + \cdots + e^{\frac{z_n}{\alpha}} \right) \right) - z_1$$

$$= \ln \left(\underline{e^{\frac{z_1}{\alpha}} + e^{\frac{z_2}{\alpha}} + \cdots + e^{\frac{z_n}{\alpha}}} \right)^{\alpha} - z_1 \quad ①$$

$\because \ln \left(e^{\frac{z_1}{\alpha}} \right)^{\alpha} = z_1$ 且 $\alpha \ge 0$. 且 $\alpha \in R$

$\therefore \ln \left(e^{\frac{z_1}{\alpha}} + \sum_{i=2}^{n} e^{\frac{z_i}{\alpha}} \right)^{\alpha} \ge z_1$

7(b) 在背面

$\therefore$ ① ≥ 0. 即 $err_\alpha(z_1, z_2, \cdots z_n) \ge 0$.

7-(b).

$err_\alpha(z_1, z_2, \cdots, z_n)$

$$= \alpha \times \left(\ln\left(\sum_{i=1}^{n} e^{\frac{z_i}{\alpha}}\right)\right) - \max(z_1, z_2, \cdots, z_n)$$

$$= \alpha \times \left(\ln\left(e^{\frac{z_1}{\alpha}} + e^{\frac{z_2}{\alpha}} + \cdots + e^{\frac{z_n}{\alpha}}\right)\right) - z_1$$

$$\leq \alpha \times \left(\ln\left(e^{\frac{z_1}{\alpha}} + e^{\frac{z_1}{\alpha}} + \cdots + e^{\frac{z_1}{\alpha}}\right)\right) - z_1$$

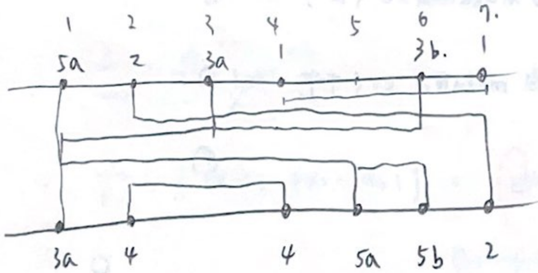
$$\leq \alpha\left(\ln e^{\frac{z_1}{\alpha}} + \ln n\right) - z_1$$

$$\leq \alpha\left(\ln e^{\frac{z_1}{\alpha}} + \ln n\right) - z_1$$

$$\leq \alpha \cdot \ln n$$

11.

+4



~~1: [4,7]~~

~~2: [2,7]~~

~~3a: [1,3]~~

3b: [3,6]

~~4: [2,4]~~

~~5a: [1,5]~~

~~5b: [5,6]~~

~~3a: [1,3]~~

~~5a: [1,5]~~

~~4: [2,4]~~

→ ~~2: [2,7]~~

3b: [3,6]

~~1: [4,7]~~

5b: [5,6]

Track 1: 5a.

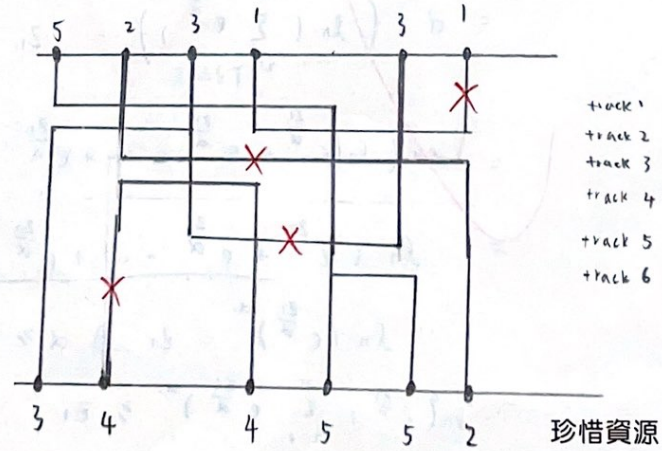
Track 2: 3a, 1.

Track 3: 2.

Track 4: 4.

Track 5: 3b.

Track 6: 5b.



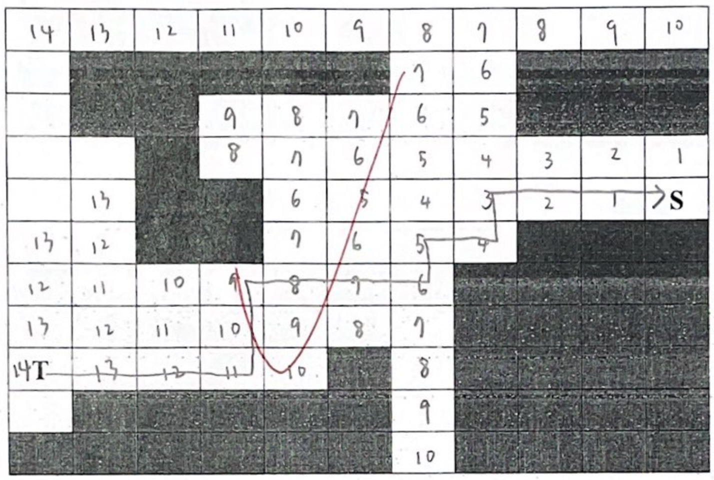
CS6135 VLSI Physical Design Automation

Final Exam: 10:10 a.m. – 13:10 p.m., December 27, 2022

Answer Sheet

Student ID: 110062619 Name: 楊淨富

10. (a) Lee algorithm +5



(b) Weighted grid model +2 traceback +1

Weight

3	2	2	2	2	2	3	3	2	2	2
2						2	2			
2			1	2	2	3	2			
3	1		1	3	3	3	3	2	2	1
3	2			2	3	3	3	2	1	**S**
3	2			2	3	3	1			
3	3	2	2	3	3	2				
2	3	3	3	3	2	2				
T	1	2	2	1		1				
1						1				
						1				

Route.

31	28	26	24	22	20	18	16	18	20	22
						15	13			
			19	18	16	14	11			
	30		18	17	15	12	9	5	3	1
32	29			14	12	9	6	3	1	**S**
30	27			15	13	10	7			
28	25	22	20	18	15	12				
30	28	25	22	19	16	14				
T	25	24	22	20		15				
						16				
						17				