Spring 2024

BAN 5763 OPTIMIZATION GROUP EXERCISE

SUBMITTED BY:

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Project Objective

The main aim of this project is to create a straightforward discounting model to help a subscription-based business increase its revenue while also keeping customers engaged through thoughtful discounting. Our business groups customers into three categories: high, medium, and low value, with counts of 40,000, 25,000, and 10,000, respectively. The Chief Marketing Officer (CMO) has specified that discounts should not be given to more than 20% of our customer base. Our task is to assess how different levels of discounts can potentially extend the time these customers stay with the business.

By using predictive modeling on the given dataset with the customer lifetime extensions and discount rate for each tier, we plan to figure out how much extending discounts by 1% to 20% with 1% increment might increase the time customers from each tier stay with the company. This analysis will help us devise a model to distribute discounts wisely across the different customer segments. Our model aims to respect the CMO's limit on discounting while trying to maximize revenue. We aim to find a middle ground that keeps customers around for longer without hurting the firm's revenue, aligning with the business's broader aim of increasing revenue and maintaining solid customer relationships over time.

We use a methodical approach involving data analysis, predictive modeling, optimization modeling, and what-if analysis, basing our recommendations on data backed evidence and thorough statistical analysis. Finally, our report lays out what we aim to do, how we plan to do it, the assumptions we're making, the risks involved, and our recommendations, all explained in simple terms that are layman comprehensible.

Our plan acknowledges the importance of smart discounting in keeping a competitive edge. Through careful analysis and the use of advanced modeling techniques, we aim to offer the CMO practical strategies that help keep customers while also driving revenue and growth.
We designed a holistic approach to define the optimal discount offering strategy due to multiple specific business constraints and challenges in opting the best solution. We followed a four-phased approach as shown in ‘Figure 1’ in developing our compelling solutions supported with solid evidence.

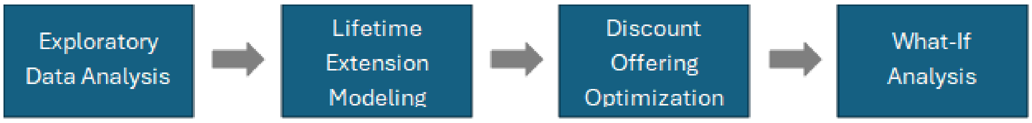


Figure 1: Project Approach

Exploratory Data Analysis

We first did the exploratory data analysis to understand the dynamic nature of discount offerings and customer tiers. This allows us to plan accordingly to devise an accurate modeling in predicting lifetime extensions that can be utilized in discount offering optimization and maximizing the revenue and customer lifetime.

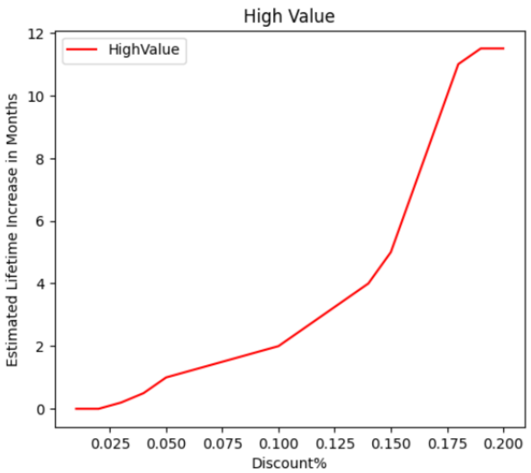


Figure 2: High Value Tier Lifetime Extension vs. Discount Rate

Monthly Revenue: $60

Average Baseline Lifetime: 14 months

The chart illustrates a somewhat non-linear convex curve for high value tier customers, where the increase in customer lifetime months accelerates (with the highest degree at around 16% discount rate) as the discount percentage becomes larger. However, it peaks at around 19% discount rate. Essentially, as discounts grow, customers are likely to stay significantly longer.

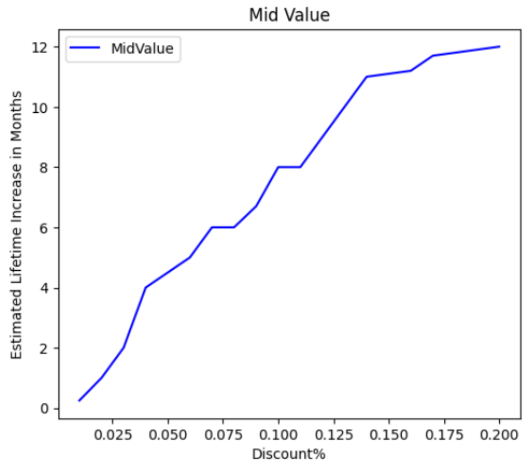


Figure 3: Medium Value Tier Lifetime Extension vs. Discount Rate

Monthly Revenue: $50

Average Baseline Lifetime: 12 months

The chart illustrates a somewhat linear curve for medium value tier customers. As opposed to the high value tier, the customer lifetime months extension increases linearly as the discount percentage becomes larger.

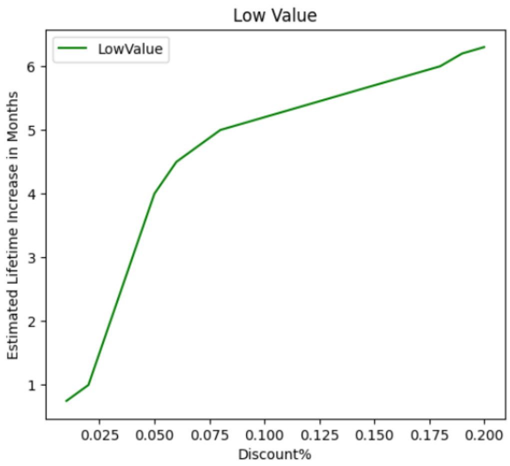


Figure 4: Low Value Tier Lifetime Extension vs. Discount Rate

Monthly Revenue: $40

Average Baseline Lifetime: 6 months

The chart illustrates a non-linear concave curve for low value tier customers. As opposed to the other tiers, the customer lifetime months extension skyrocketed from 2% to around 7% discount rate. Then, the increment rate tapers off after 7%.

Apart from medium value tier customers, based on the observations, lifetime extensions for high value and low value tier customers have to be transformed to be more linear against discount rate, particularly a square-root and square transformation, respectively. This is to ensure that our predictive model can perform and fit better to provide more accurate predictions before moving into optimization model. Note that cube-root transformation was conducted for high value tier customers' lifetime extensions against discount rates, yet the linearity is no better than the square root transformation (Appendix A). To minimize the complexity, we selected square root transformation.

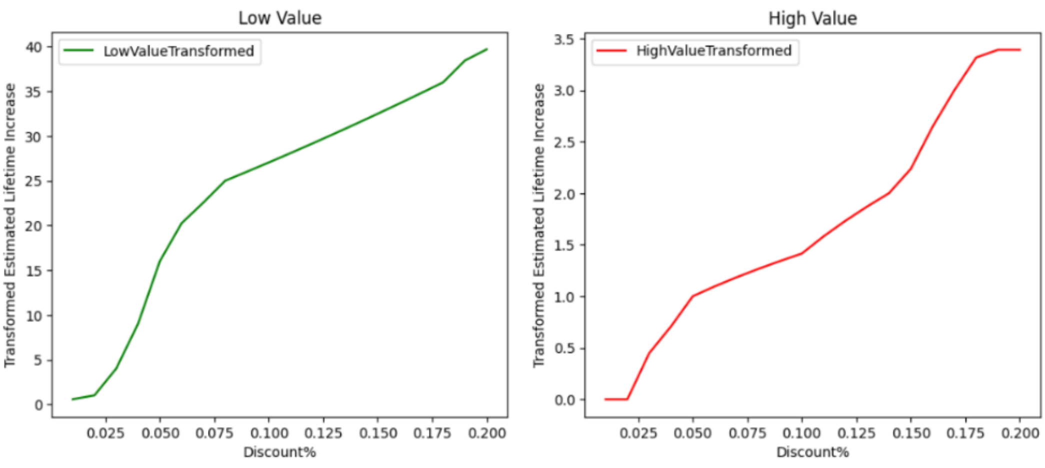


Figure 5: Transformed Lifetime Extension vs. Discount Rate (High & Low Value Tiers)

The transformations show that linearity between lifetime extensions and discount rate for high and low value tiers have improved, thus possessing a better capability to capture the pattern in our predictive modeling.

Methodology

Our end goal is to maximize the revenue from the customers while making sure that the discount provided has the best impact on the lifetime extension of the customers and adhering to certain rules set by the business. As highlighted before, this will be done through a multi-step process which we have elaborated further below.

Predictive Model to Predict Impact of Discount on Lifetime Gain:

To even begin trying to maximize the company's revenue, we need to first establish the impact of the discounts on the lifetimes of different groups of customers. For this, we will be leveraging the historical information on discounts and lifetime that has been provided by the business (explained above in the exploratory data analysis section). Therefore, we start by fitting a linear regression model that can use discounts to predict the number of months that each of the customers in different tiers would extend their membership by. From the above exploratory data analysis, we see that we need to transform the number of months extension for different customer tiers to make them a suitable fit for the linear regression model. We take square of the month's extension for the high value customers and a square root of the months for the low value customers. Three linear regression models (based on the High, Medium, and Low value customers) have been fitted after the transformations mentioned. The details of all the linear regression models can be found in the appendix (Appendix B to Appendix D). We can see that the predictor variable (discount offered) explains 89% or more variation in the predicted/target variable (which is the transformed version of the lifetime extension in months) for each of the customer tiers. This indicates that the linear regression models are a good fit and therefore is a good predictor of the month's extension. We will be leveraging this model in the further steps to establish a relationship between the different values of discounts and the potential lifetime extension.

Optimizing for Revenue:

Now that we have our predictive model, we build our optimization model for maximizing the revenue by providing discounts to the customers but also by following the provided business constraints (no more than 20% of the total customer base should be given a discount). For this, we calculated total revenue as a function of the number of customers in each tier, the baseline lifetime, the additional lifetime due to any discounts provided, and the post discounting monthly average revenue per customer.

We calculate the final total revenue of each of the tiers (considering the lifetime extension based on our predictive model) and sum it up. Our objective function is to maximize this sum. (Figure F in the appendix for reference, where the objective function is defined).

The constraints for this revenue maximization problem are to make sure that not more than 20% of our entire customer base should receive a discount and the total number of people with and without a discount in a customer tier should be equal to original number of customers in that tier (Figure G, in the appendix for reference where the constraints are defined along with the relevant nomenclature).

After solving this optimization problem, our model suggests to not have any discounts for the High Value and Low Value customers. It suggests that there should be a 20% discount for 15,000 of the Middle Value customers which is also 20% of the total customer base of 75,000 (thus respecting the business rules). Based on this, the estimated the total revenue generated would be $57.36 million with this optimization model compared to $51 million without this.

<u>Assumptions Considered:</u>

- If a customer is not provided with any discount, we have assumed that they will get 0 months of lifetime extension and their lifetime would remain the same as the baseline lifetime mentioned in the initial problem statement.
- The final revenue has been calculated assuming that the customers that are getting discounts will be obtaining the benefit of the discounts right from the beginning of their relationship with the company.
- While estimating the lifetime extension and potential revenue gains, we have assumed that we will not be losing any significant chunk of customers due to churn and the original set of customers will be retained.
- We have assumed that providing discounts to one tier of customers will not have any impact on the purchase or subscription behavior of any other group of customers.
- We have also assumed that there would not be a very significant impact of discount leakage where other tiers of customers could obtain the same set of discount benefits through some means and have a significant impact on the final revenue being calculated.

- Customer behavior is presumed to be predictable and stable, not influenced by external factors not accounted for in the model.

- The costs associated with the product or service are assumed to remain constant regardless of the discounts applied.

- The customer group segmentation is accurate and reflects the true nature of the customer base.

Optimization Model Results

Utilizing the optimization model, we've discovered a potential revenue lift of approximately $6.36 million compared to not offering any discounts without the guidance of the optimization model. To convey the significance of these results, we conducted a what-if analysis to explore alternative scenarios. This analysis presents a comparison between the revenue and average lifetime extension under different conditions, including the absence of discounts, uniform discounts applied to a single customer group, and varied discounts distributed across the high, medium, and low-value customer segments.

From Table-1 below, 'Scenario-1' establishes a baseline, where no discounts are applied, resulting in a total revenue of $51 million, with no subsequent extension of customer lifetime value. On the other hand, the optimal solution, labeled as 'Scenario-9', is prominently highlighted in green, which is the most lucrative option. This scenario not only maximizes revenue amongst all the options considered but also serves as a testament to the effectiveness of the optimization model in enhancing business outcomes. Adhering to the assumption that the firm aims to maximize total revenue, it is logical to focus our discounting efforts on the Mid Value group. These customers bring in $50 monthly revenue, which is just $10 shy of the High Value group ($60) but with a notable difference in the average lifetime extension. At the 20% discount mark, their lifetime with the firm extends by approximately 2.4 months longer than the High Value customers. This insight suggests that Mid Value customers, when given a 20% discount, contribute more to the firm's revenue over time, highlighting the rationale behind the optimization model's result.

Definition	Scenario No.	High Value (40000 customers)		Mid Value (25000 Customers)		Low Value (10000 Customers)		Total Revenue ($)	Avg Lifetime Extension (Months)
		Discount %	No. Of Customers	Discount %	No. Of Customers	Discount %	No. Of Customers		
No Discounts (Baseline)	Scenario-1	0%	All	0%	All	0%	All	$51,000,000	0
	Scenario-2	5%	20% of Total Customers	-	-	-	-	$51,461,135	0.5
	Scenario-3	-	-	5%	20% of Total Customers	-	-	$53,441,598	4.1
	Scenario-4	-	-	-	-	5%	20% of Total Customers (Capped at 10,000)	$52,276,530	3.7
	Scenario-5	10%	20% of Total Customers	-	-	-	-	$51,795,982	2.5
Full Discount Numbers Allocated To Just 1 Group	Scenario-6	-	-	10%	20% of Total Customers	-	-	$54,986,594	7.2
	Scenario-7	-	-	-	-	10%	20% of Total Customers (Capped at 10,000)	$52,497,090	4.8
	Scenario-8	20%	20% of Total Customers	-	-	-	-	$56,550,369	11.2
	Scenario-9	**-**	**-**	**20%**	**20% of Total Customers**	**-**	**-**	**$57,360,857**	**13.6**
	Scenario-10	-	-	-	-	20%	20% of Total Customers (Capped at 10,000)	$52,614,392	6.5
Mixed Discounts Allocated To More than 1 Group	Scenario-11	20%	13.33% of Total Customers	20%	0.67% of Total Customers	-	-	$56,820,532	12
	Scenario-12	20%	0.67% of Total Customers	20%	13.33% of Total Customers	-	-	$57,090,694	12.8

Table 1: Scenario Based What if Analysis on Total Revenue

<u>Advantages:</u>

- The primary advantage of employing the optimization model is its efficiency in navigating through thousands of potential scenarios. Manually analyzing each option to determine the one that maximizes revenue could take months of continuous effort. In contrast, the optimization model delivers a solution in under 10 seconds.

- Utilizing regression models that can explain more than 89% of lifetime extension in tandem with the optimization model, creates a robust and adaptable framework. This system is designed for ease of use, allowing the business the flexibility to experiment with discounts exceeding 20%, which can then be seamlessly integrated into the optimization model for analysis.

- Should the business opt to ensure a minimum level of participation from each customer group (High, Medium, Low Value), this can be accommodated by adding a constraint within the optimization model. Such flexibility allows for strategic inclusion across segments.

- Lastly, it's vital to leverage optimization models to inform business decisions rather than relying solely on intuition. These models provide a comprehensive array of scenarios, enriching the decision-making process. When these insights are combined with in-depth business knowledge, they pave the way for well-rounded strategies that are likely to be in the best interest of the company.

Risks:

- A considerable risk arises when the discount strategy is overly concentrated on a single customer group. Although this brings in the most revenue, it can potentially lead to dissatisfaction among the remaining customer groups, possibly increasing the rate of early customer attrition.

- The regression model's training is confined to the data, exploring the relationship between the 1%-20% discount range with the Customer Lifetime Extension. While extrapolating to the 25%-30% discount range could prove beneficial, proceeding beyond this threshold may not yield reliable results as customer behavior in response to higher discounts could deviate significantly due to discount rate saturation. Additionally, the effectiveness of discounts is inherently limited if the product does not align with the customer's needs, rendering discounts ineffective as a means of persuasion.

- Finally, there's a risk associated with habituating customers to high discount rates, which can erode profit margins over time. Dependence on discounts might set a new perceived value for the product, leading to customer attrition if those discounts are abruptly removed rather than gradually. This price sensitivity makes it crucial to manage discount strategies carefully to avoid establishing unsustainable pricing expectations.

Recommendations

With the firm's aspirations for enhanced revenue generation at the forefront, these recommendations highlight a strategic approach, outlining objectives into immediate, medium-term, and long-term plans. Initially, the focus will be on capitalizing on revenue-enhancing opportunities for swift gains, subsequently transitioning to emphasize customer satisfaction, thereby ensuring sustained growth and customer loyalty in the long run.

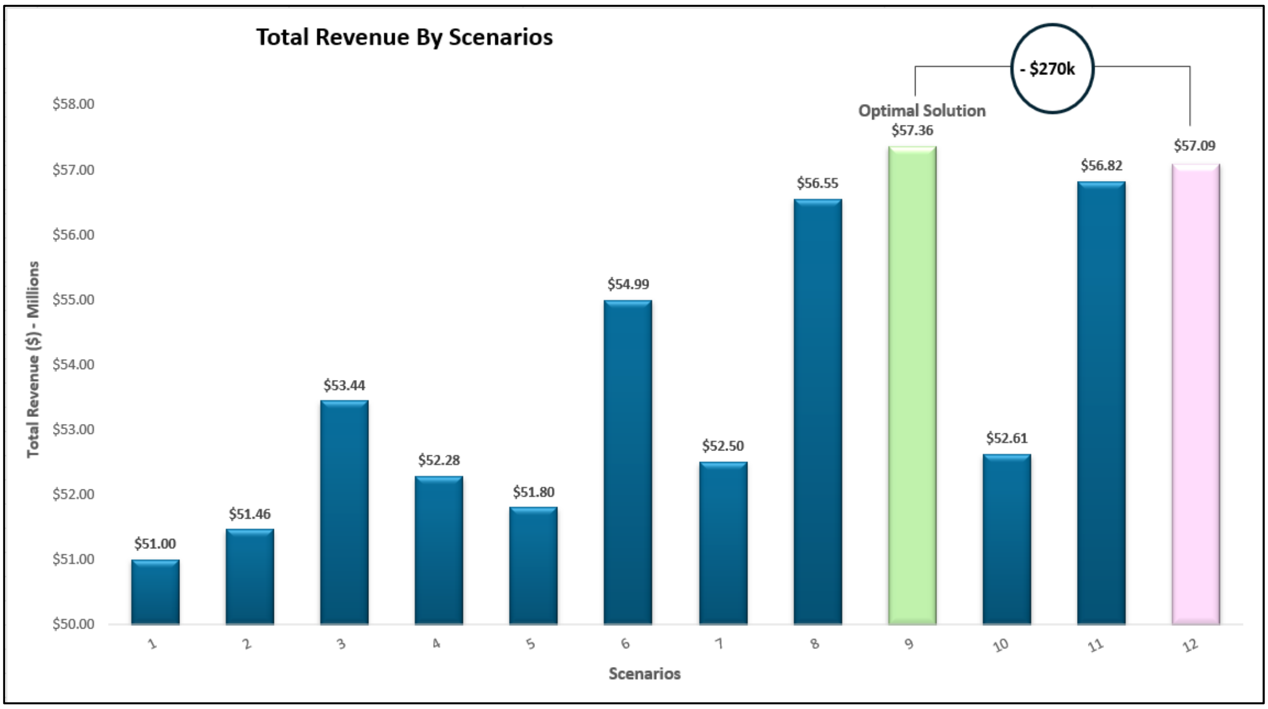


Figure 6: Revenue Across Scenarios

‘Figure 6’ above is an extension of ‘Table-1’ allowing us to visualize the effectiveness of different scenarios and how much revenue can be generated from each of them. The bar highlighted in green is the optimal solution allocating 20% discount fully to the Mid Value group while the one in purple involves a mixed discount allocation, which will be covered in the recommendation below.

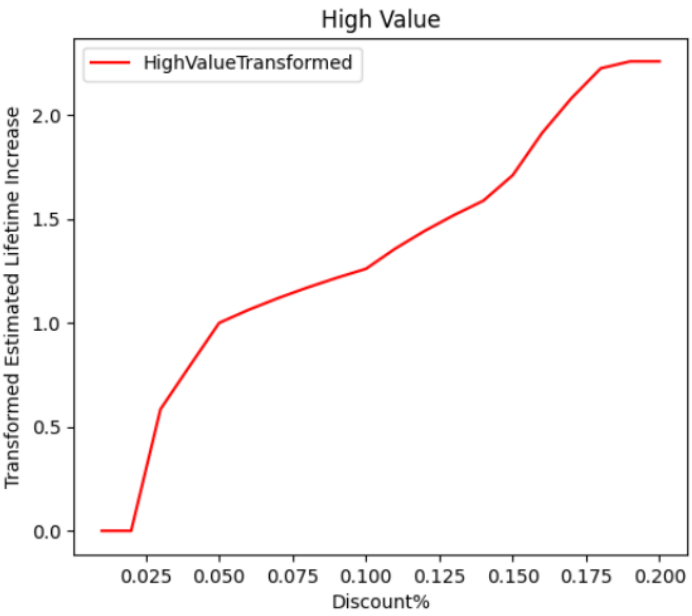
For the firm's short-term objective of maximizing revenue, our recommendation aligns with the optimization model's strategy, specifically targeting the Mid Value customer group with a 20% discount for 15,000 customers i.e. 20% of the entire customer base. This strategic allocation is projected to extend each customer's lifetime by an additional 13.6 months, generating an estimated $6.36 million in surplus revenue over the no-discount baseline. However, this strategy should be approached with caution due to potential dissatisfaction among other customer groups.

Considering a slow transition to other customer groups in the medium term still maintains focus on revenue while gradually incorporating a broader customer engagement perspective, which could prove beneficial for the firm. Figure-3 offers insights into this strategy, notably in Scenario-12 (highlighted in purple), where distributing 20% discounts among both the High Value (5,000 customers) and Mid Value (10,000 customers) groups yields a total revenue of $57.09 million, marginally $270,162 less than the optimal scenario. Further, from Scenario-12 in Table-1, we can see that this diversified focus would extend the overall customer lifetime by an average of 12.8 months across both groups, rather than just investing all eggs into one basket. A strategy like this is a small step towards investing in customer satisfaction allowing for a smooth transition to the long-term goals.

In the long term, we recommend the development of a nuanced customer segmentation framework, leveraging demographics, product preferences, and monthly revenue to tailor discounts effectively. According to an article published by Cornell University, written by Douglas Stayman (2013) [1], by segmenting the mass market into well-defined groups of customers, a business can focus resources on creating the best possible value proposition for each segment to deliver benefits that meet their specific needs. They may spend a little more to do this, but if the benefits delivered to customers exceed the cost to provide them, profits will still be maximized. Further, including the nuances of this robust customer segmentation into our current optimization framework with minor tweaks can help the business understand how to maximize its revenue while also focusing on customer satisfaction, retention, and loyalty. Hence, this holistic approach caters more towards a realistic goal which could potentially bring in more revenue in the long run and boost product market fit.

Appendix

Appendix A – Cube-Root Transformation for Lifetime Extension vs. Discount Rate for High-Value Tier Customer



Appendix B – High Value Regression Model

```
                            OLS Regression Results
==============================================================================
Dep. Variable:     HighValueTransformed   R-squared:                       0.966
Model:                              OLS   Adj. R-squared:                  0.964
Method:                   Least Squares   F-statistic:                     517.1
Date:                  Mon, 11 Mar 2024   Prob (F-statistic):           1.04e-14
Time:                          18:30:10   Log-Likelihood:                 4.9638
No. Observations:                    20   AIC:                            -5.928
Df Residuals:                        18   BIC:                            -3.936
Df Model:                             1
Covariance Type:              nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.1616      0.092     -1.748      0.098      -0.356       0.033
Discount%     17.5477      0.772     22.739      0.000      15.926      19.169
==============================================================================
Omnibus:                        4.179   Durbin-Watson:                   0.455
Prob(Omnibus):                  0.124   Jarque-Bera (JB):                1.423
Skew:                           0.030   Prob(JB):                        0.491
Kurtosis:                       1.695   Cond. No.                         17.5
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

------------------------------------------------------------------------------------------
Intercept: -0.16158478540620969, Beta: 17.54773093450779
```

Appendix C – Mid Value Regression Model

```
                            OLS Regression Results
==============================================================================
Dep. Variable:     MidValueTransformed   R-squared:                       0.957
Model:                             OLS   Adj. R-squared:                  0.955
Method:                  Least Squares   F-statistic:                     399.9
Date:                 Mon, 11 Mar 2024   Prob (F-statistic):           9.65e-14
Time:                         18:30:11   Log-Likelihood:                -23.366
No. Observations:                   20   AIC:                             50.73
Df Residuals:                       18   BIC:                             52.72
Df Model:                            1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.8774      0.381      2.302      0.033       0.077       1.678
Discount%     63.6203      3.181     19.998      0.000      56.937      70.304
==============================================================================
Omnibus:                        2.038   Durbin-Watson:                   0.436
Prob(Omnibus):                  0.361   Jarque-Bera (JB):                1.677
Skew:                          -0.593   Prob(JB):                        0.432
Kurtosis:                       2.221   Cond. No.                         17.5
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

--------------------------------------------------------------------------------------------
Intercept: 0.8773684210526331, Beta: 63.62030075187968
```

Appendix D – Low Value Regression Model

```
                            OLS Regression Results
==============================================================================
Dep. Variable:     LowValueTransformed   R-squared:                       0.902
Model:                             OLS   Adj. R-squared:                  0.896
Method:                  Least Squares   F-statistic:                     165.4
Date:                 Mon, 11 Mar 2024   Prob (F-statistic):           1.64e-10
Time:                         18:30:12   Log-Likelihood:                -54.628
No. Observations:                   20   AIC:                             113.3
Df Residuals:                       18   BIC:                             115.2
Df Model:                            1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          3.7575      1.819      2.065      0.054      -0.064       7.579
Discount%    195.3357     15.186     12.863      0.000     163.430     227.241
==============================================================================
Omnibus:                        2.651   Durbin-Watson:                   0.188
Prob(Omnibus):                  0.266   Jarque-Bera (JB):                1.180
Skew:                          -0.085   Prob(JB):                        0.554
Kurtosis:                       1.822   Cond. No.                         17.5
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

--------------------------------------------------------------------------------------------
Intercept: 3.757500000000006, Beta: 195.33571428571423
```

Appendix E – Binary Decision Variable To Activate Discount Given or Not

- **Decision Variables**:
  - Number of customers to give discounts to in each Tier and for each Discount Depth % (0% to 20%)

    Let $C_{tier,disc}$ denote the number of customers in tier $tier \in T$ and discount depth $disc \in D$

  - Binary decision variable that indicates whether a discount was given or not

    $$discApplied_{tier,disc} = \begin{cases} 1 & \text{if disc} > 0 \\ 0 & \text{if disc} = 0 \end{cases} \quad \forall tier \in T \text{ and } \forall disc \in D$$

Appendix F – Objective Function

- **Objective:**
  - Maximize the total revenue obtained by increasing the lifetime through the discounts offered

$$\text{HighTierRevFunction} = \sum_{disc\in D} R_{tier} \times (1 - disc) \times C_{tier,disc} \times (L_{tier} + ((Intercept_{tier} + (disc \times BetaDisc_{tier}))^2) \times discApplied_{tier,disc}) \text{ where, } tier \in High$$

$$\text{MidTierRevFunction} = \sum_{disc\in D} R_{tier} \times (1 - disc) \times C_{tier,disc} \times (L_{tier} + (Intercept_{tier} + (disc \times BetaDisc_{tier})) \times discApplied_{tier,disc}) \text{ where, } tier \in Mid$$

$$\text{LowTierRevFunction} = \sum_{disc\in D} R_{tier} \times (1 - disc) \times C_{tier,disc} \times (L_{tier} + ((Intercept_{tier} + (disc \times BetaDisc_{tier}))^{0.5}) \times discApplied_{tier,disc}) \text{ where, } tier \in Low$$

$$\text{Maximize, } (HighTierRevFunction + MidTierRevFunction + LowTierRevFunction)$$

Appendix G - Constraints

- **Constraints:**
  - No more than 20% of total customer base should be given a discount

$$\sum_{\text{tier}\in T} \sum_{\text{disc}\in D} C_{tier,disc} \times discApplied_{tier,disc} \leq (\sum_{\text{tier}\in T} TotC_{tier}) \times 0.20$$

  - Total number of discounted and non-discounted customers in each tier should add up to the total number of customers present in each tier

$$\sum_{\text{disc}\in D} C_{tier,disc} = TotC_{tier} \quad \forall \text{tier} \in T$$

Where,

- $TotC$ denotes the number total number of customers in each tier
- T denotes the three tiers available i.e. $High, Mid, Low$
- D denotes the depth of discount that can be offered to a customer from 0% to 20% at increments of 1% each. Example: $(0.0, 0.01, 0.02, ...0.20)$
- R denotes the average monthly revenue per customer for each tier
- L denotes the average baseline lifetime for each tier
- $Intercept$ is the value of the intercept of the discount vs lifetime increase estimation model for each tier
- $BetaDisc$ is the value of the beta coefficient of discount in the discount vs lifetime increase estimation model for each tier

Reference

[1] "Segment Customers to Increase Leads, Sales, and Satisfaction," *eCornell*. https://ecornell.cornell.edu/news-center/press-room/segment-customers-to-increase-leads-sales-and-satisfaction/